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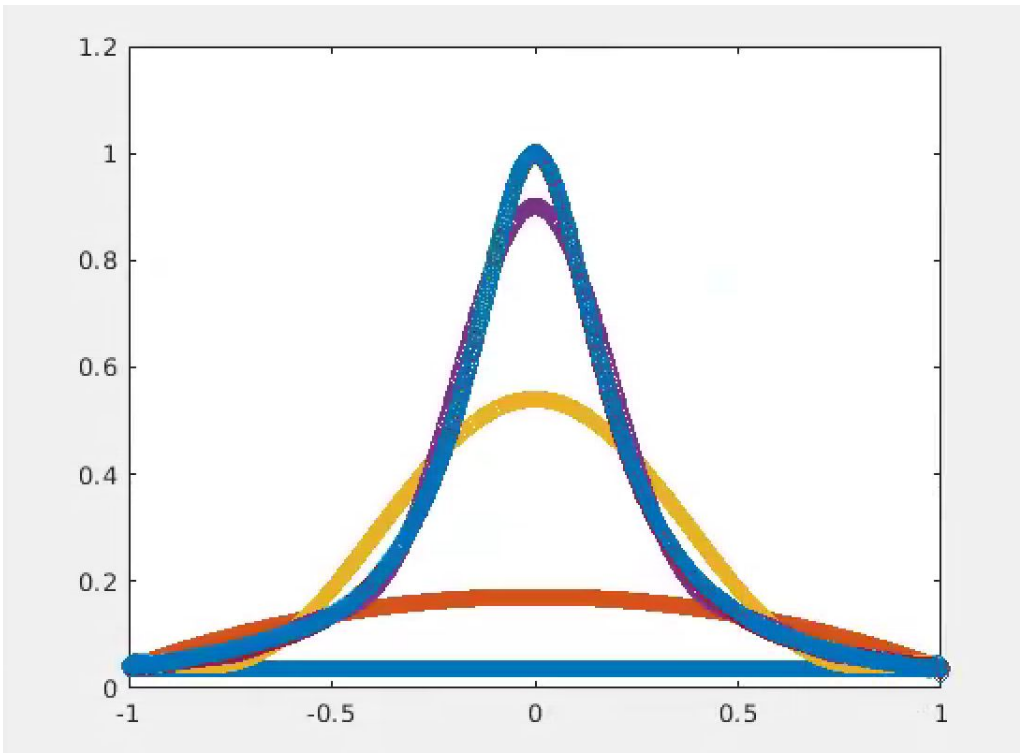
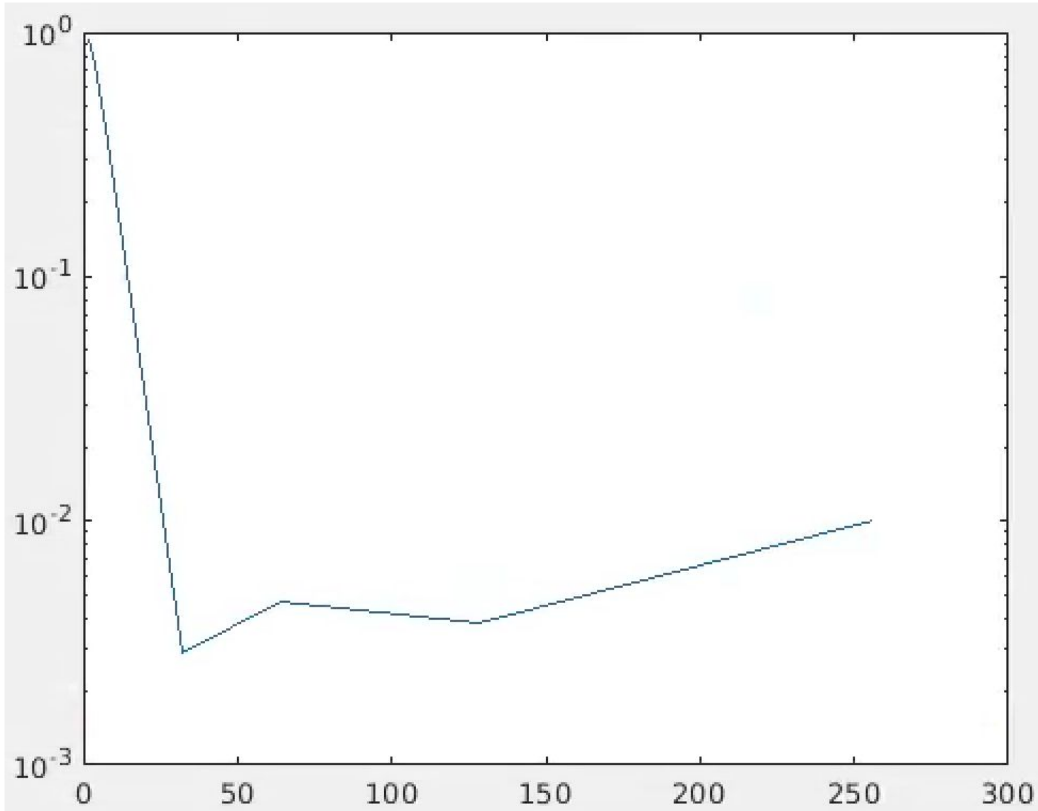
Assignment 1
Jan 24, 2019
CS 3200

1. You have been given code for generating polynomial interpolants by inverting the Vandermonde matrix, all using equispaced points on [−1, 1]. You will be interpolating the Runge function f(x) = 1 / (1 + 25(x^2))
   a. Write code to build the Vandermonde matrix on Chebyshev extrema



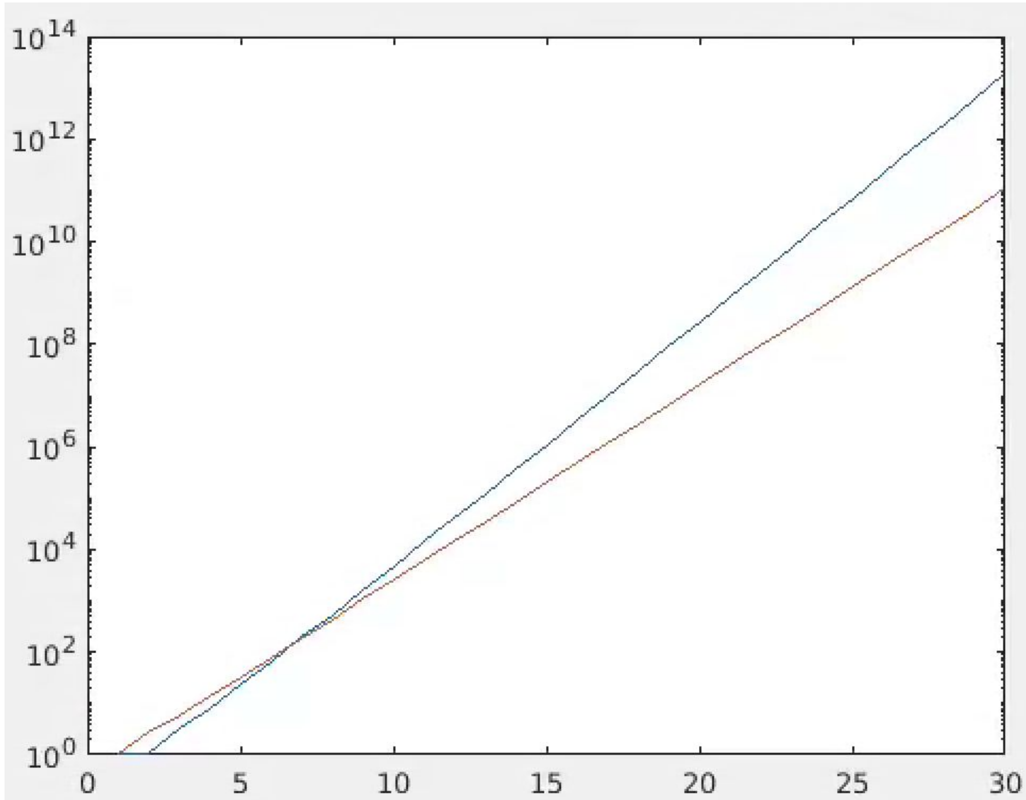


The magnitude as N + 1 derivatives grows rapidly

$$\prod_{k=0}^{N}(x - x_k)$$

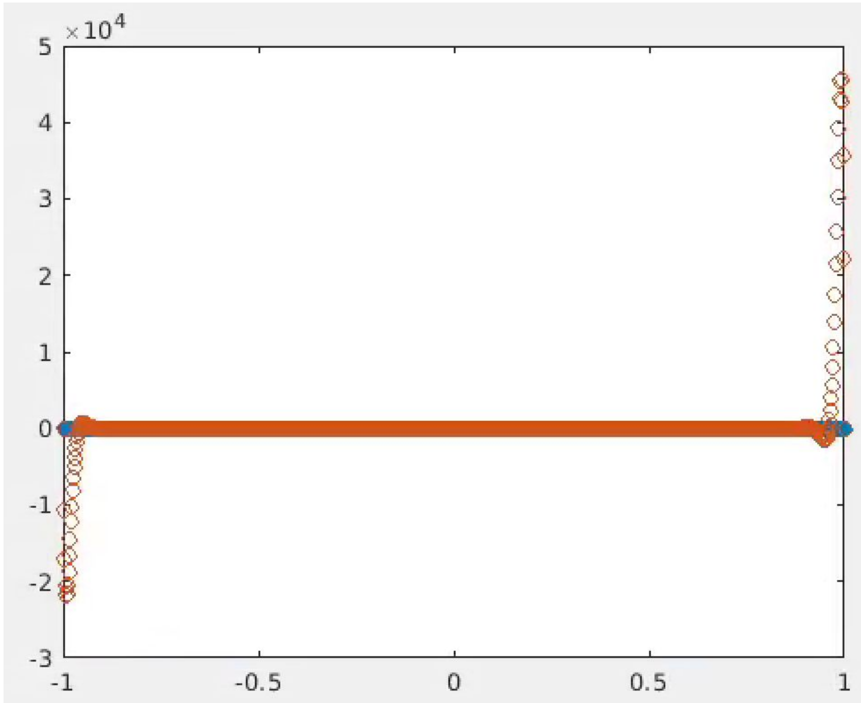
The full error term grows rapidly as N increase, causing Runge Phenomenon. Using Chebyshev extrema, the points are no longer evenly spaced, but now cluster more towards the end of the interval. The Chebyshev extrema helps prevent the runge phenomenon and makes it so the error decreases as N increase, which is the results we are looking for.

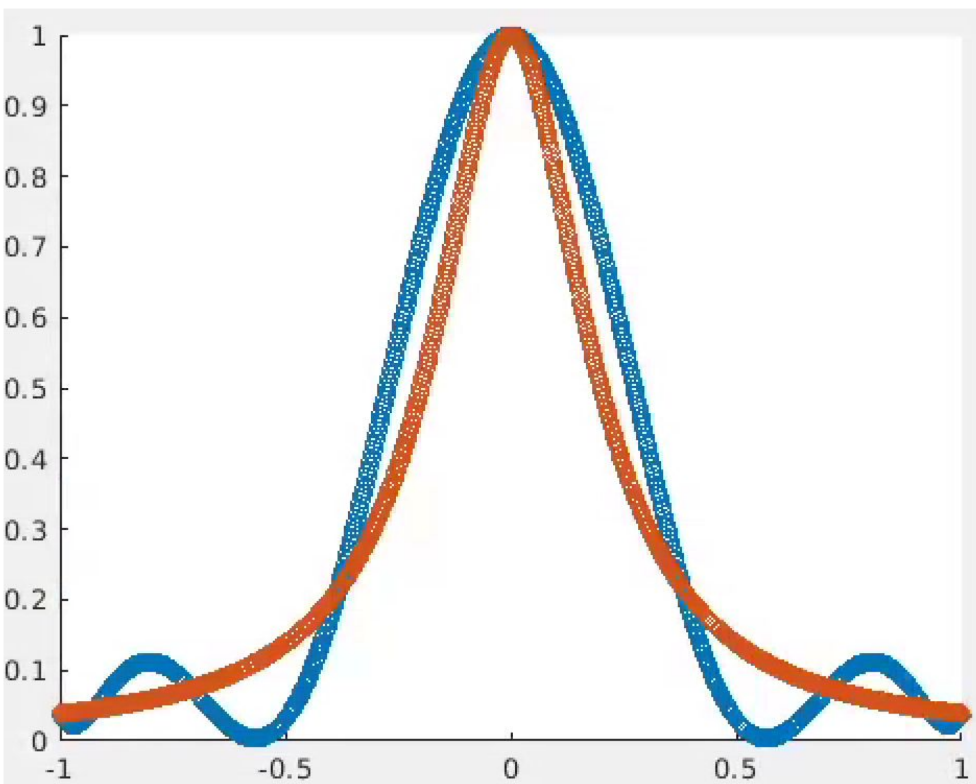
b. Generate a plot of condition number of the Vandermonde matrix vs number of nodes for both types of points (equispaced and Chebyshev)



The condition number is a way to quantify how close a matrix is to singular. As you can see that as N increases, the number gets big fast, which is the type of result we were expecting. This means that the Vandermonde answer will not be correct to one digit of precision. You also see the Chebyshev does better than the Vandermonde which is also expected.

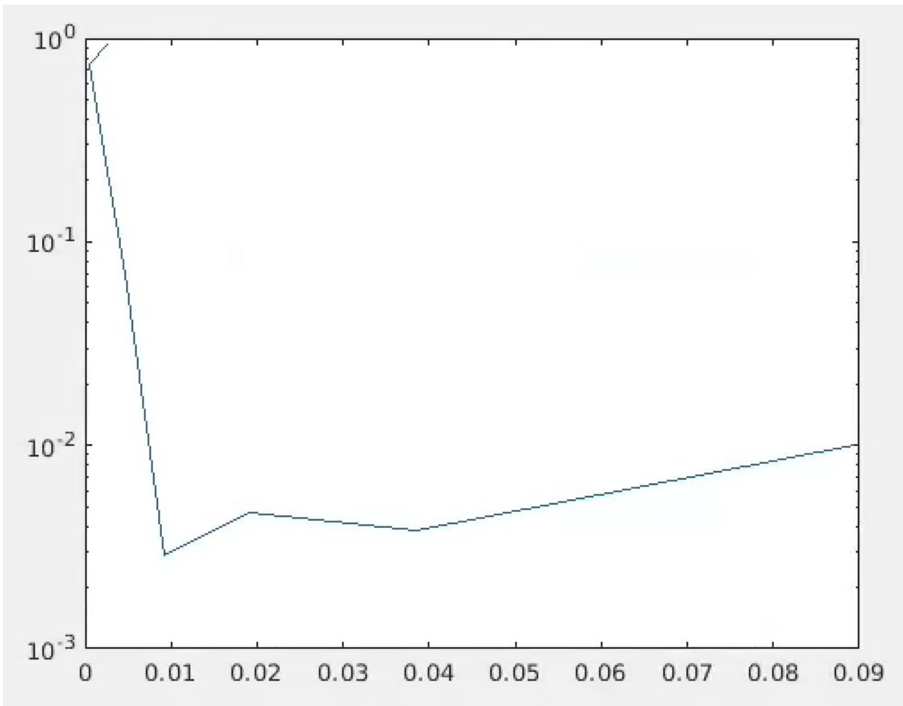
c. Generate a plot of the polynomial interpolants for N = 9 and N = 50 for both types of points





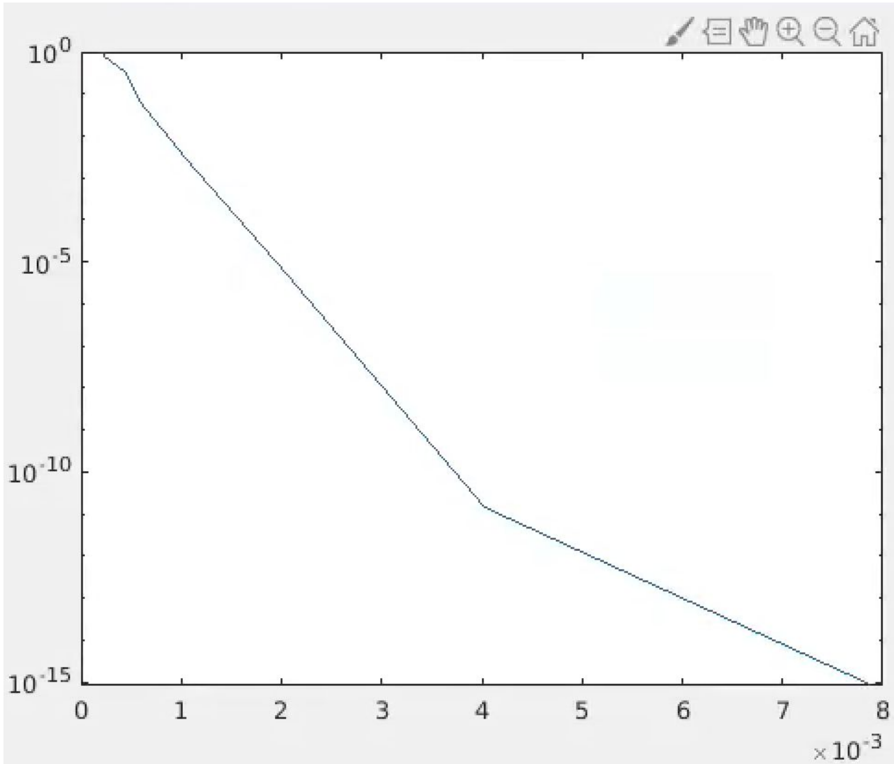
As you can see for N = 9 both Vandermonde and Chebyshev seem to get more accurate, but For N = 50 you can see that Vandermonde get out of control in accuracy due to Runge Phenomenon, but Chebyshev start becoming more accurate as expected

d. Build a polynomial interpolant at Chebyshev extrema (instead of equispaced points) using the Vandermonde matrix. Evaluate this interpolant at 10,000 equispaced nodes. Produce a plot of total time for interpolation at increasing numbers of nodes and evaluation at 10,000 equispaced nodes (measured using Matlab's functions) vs the accuracy (measured as the l2 relative error in the polynomial interpolant)



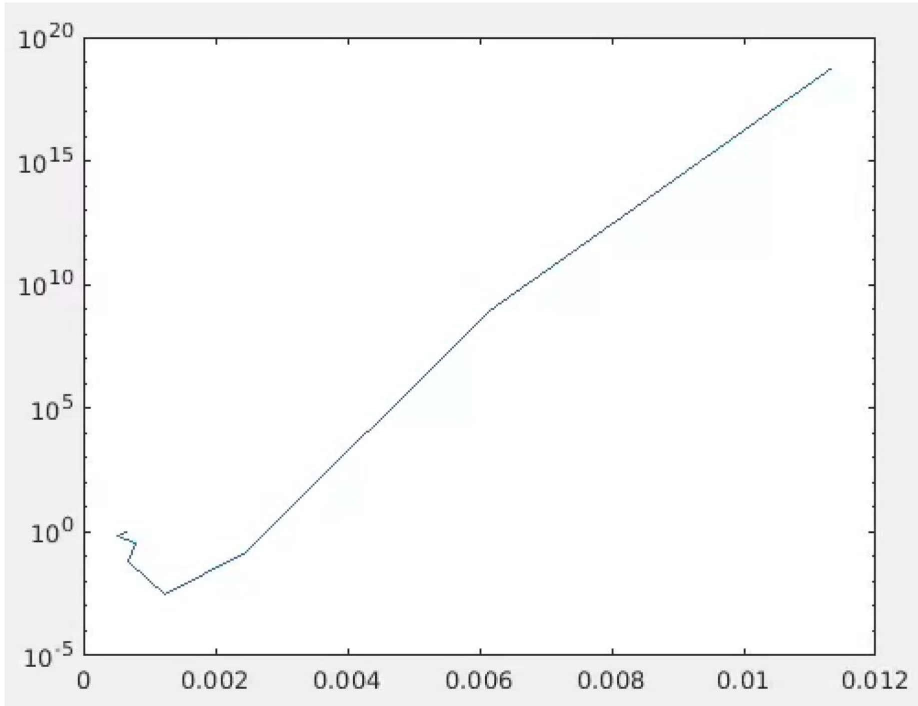
semilogy(Time,Error), As the time goes up then the error goes down, then slowly starts going back up again.

e. You have also been provided code to perform barycentric Lagrange interpolation at Chebyshev extrema. Produce a plot of total time vs accuracy (as above) at 10,000 equispaced nodes. How does it compare to the Vandermonde approach



semilogy(Time,Error), as the times goes up then the calculation goes down to almost nothing.

f. Matlab has a built-in function called polyfit to do polynomial interpolation. How does polyfit compare in terms of accuracy and computational cost to the barycentric approach? How does it compare to the Vandermonde approach? Produce plots similar to those above to justify your answers



semilogy(Time,Error), as time goes up the error goes down, the error goes down then starts to go up again. Similar to the one in problem D but goes up at a faster rate.