Métodos Numéricos - Tarea # 7

Sea $\mathbb{K} = \begin{bmatrix} k+k' & -k' \\ -k' & k+k' \end{bmatrix}$, calcular sus eigenvalores y eigenvectores.

$$\det(\mathbb{K} - \lambda I) = \det \begin{bmatrix} k+k'-\lambda & -k' \\ -k' & k+k'-\lambda \end{bmatrix} = (k+k'-\lambda)^2 - (k')^2 = 0$$

$$k^2 + kk' - k\lambda + kk' + k'^2 - k'\lambda - k\lambda - k'\lambda + \lambda^2 - k'^2 = 0 \rightarrow \lambda^2 - \lambda(2k+2k') + 2kk' + k^2 = 0$$

$a = 1 \quad b = -(2k+2k') \quad c = 2kk' + k^2$

$$\lambda_{1,2} = \frac{(2k+2k') \pm \sqrt{[-(2k+2k')]^2 - 4(1)(2kk'+k^2)}}{2(1)}$$

$$\lambda_{1,2} = \frac{2k+2k' \pm \sqrt{4k^2+8kk'+4k'^2-8kk'-4k^2}}{2(1)} = \frac{2k+2k' \pm \sqrt{4k'^2}}{2} \rightarrow \begin{matrix} \lambda_1 = k+2k' \\ \lambda_2 = k \end{matrix}$$

Eigenvalores

$$\begin{bmatrix} -k' & -k' \\ -k' & -k' \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{matrix} -k'x - k'y = 0 \\ -k'x = k'y \\ x = -y \end{matrix} \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ y \end{bmatrix} = y \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} k' & -k' \\ -k' & k' \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{matrix} k'x - k'y = 0 \\ k'x = k'y \\ x = y \end{matrix} \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ x \end{bmatrix} = x \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Eigenvectores

Comprobación $\mathbb{K}\vec{v} = \lambda \vec{v}$

- Para $\lambda = k + 2k$

$$\underbrace{\begin{bmatrix} k+k' & -k' \\ -k' & k+k' \end{bmatrix}}_{\mathbb{K}} \underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{\vec{v}_1} = \begin{bmatrix} k - k' - k' \\ k' + k + k' \end{bmatrix} = \begin{bmatrix} -k-2k' \\ k+2k' \end{bmatrix} \qquad \underbrace{k+2k'}_{\lambda_1} \underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{\vec{v}_1} = \begin{bmatrix} -k-2k' \\ k+2k' \end{bmatrix}$$

- Para $\lambda = k$

$$\underbrace{\begin{bmatrix} k+k' & -k' \\ -k' & k+k' \end{bmatrix}}_{\mathbb{K}} \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\vec{v}_2} = \begin{bmatrix} k+k'-k' \\ -k'+k+k' \end{bmatrix} = \begin{bmatrix} k \\ k \end{bmatrix} \qquad \underbrace{k}_{\lambda_2} \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\vec{v}_2} = \begin{bmatrix} k \\ k \end{bmatrix}$$