Tarea 7.

Catherine Etienne Chuzeville Rosas

Calcular los eigenvalores y eigenvectores de

$$-\begin{pmatrix} k+k' & -k' \\ -k' & k+k' \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Sea $A = \begin{pmatrix} -k-k' & k' \\ k' & -k-k' \end{pmatrix}$ y $\lambda I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$

$$A - \lambda I = \begin{pmatrix} -k-k' & k' \\ k' & -k-k' \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -k-k'-\lambda & k' \\ k' & -k-k'-\lambda \end{pmatrix}$$

luego $\Rightarrow \det(A-\lambda) = \begin{vmatrix} -k-k'-\lambda & k' \\ k' & -k-k'-\lambda \end{vmatrix} = (-k-k'-\lambda)(-k-k'-\lambda) - (k')(k')$

$$\det(A-\lambda) = k^2 + kk' + k\lambda + kk' + k'^2 + k'\lambda + k\lambda + k'\lambda + \lambda^2 - k'^2$$

$$\det(A-\lambda) = \lambda^2 + (2k+2k')\lambda + (k^2+2kk')$$

$$\lambda_{1,2} = \frac{-(2k+2k') \pm \sqrt{(2k+2k')^2 - 4(k^2+2kk')}}{2}$$

$$\lambda_{1,2} = \frac{-(2k+2k') \pm \sqrt{\cancel{4k^2} + \cancel{8kk'} + 4k'^2 - \cancel{4k^2} - \cancel{8kk'}}}{2}$$

$$\lambda_{1,2} = \frac{-(2k+2k') \pm \sqrt{4k'^2}}{2} = \frac{-(2k+2k') \pm 2k'}{2}$$

$$\lambda_1 = \frac{-2k - \cancel{2k'} + \cancel{2k'}}{2} \quad \Rightarrow \quad \lambda_2 = -\frac{2k - 2k' - 2k'}{2}$$

$$\lambda_1 = -\frac{2k}{2} \qquad \lambda_2 = \frac{-2k-4k'}{2} = \frac{2(-k-2k')}{2}$$

$$\boxed{\lambda_1 = -k} \qquad \boxed{\lambda_2 = -k-2k'}$$

eigenvalor

eigenvalor

Ahora, se calcula $A - \lambda I$.

$\Rightarrow$ Para el primer eigenvalor

$$A - (-KI) = A + KI$$

$$\begin{pmatrix} -K-K' & K' \\ K' & -K-K' \end{pmatrix} + \begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix} = \begin{pmatrix} -K' & K' \\ K' & -K' \end{pmatrix}$$

$\Rightarrow$ luego $\begin{pmatrix} -K' & K' \\ K' & -K' \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$-K'x_1 + K'x_2 = 0$$
$$K'x_1 - K'x_2 = 0$$

$\Rightarrow$ De aquí que $x_1 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_2 \end{pmatrix} = x_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

y el eigenvector correspondiente a λ_1 es

$$\vec{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\Rightarrow$ Para el segundo eigenvalor

$$A - (-K - 2K')I = A + (K + 2K')I$$

$$\begin{pmatrix} -K-K' & K' \\ K' & K-K' \end{pmatrix} + \begin{pmatrix} K+2K' & 0 \\ 0 & K+2K' \end{pmatrix} = \begin{pmatrix} K' & K' \\ K' & K' \end{pmatrix}$$

luego $\begin{pmatrix} K' & K' \\ K' & K' \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$K'x_1 + K'x_2 = 0$$
$$K'x_1 = -K'x_2$$
$$x_1 = -x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

y el eigenvector correspondiente a λ_2 es

$$\vec{V}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

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