Tarea: Métodos de Matrices con osciladores acoplados

Calcular eigenvalores & eigenvectores de

$$-\begin{pmatrix} K+K' & -K' \\ -K' & K+K' \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$\mathbb{K}$ $\vec{x}$

$$\lambda I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$\begin{pmatrix} -(K+K') & +K' \\ +K' & -(K+K') \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\det(\mathbb{K} - \lambda I) = 0$$

$$\det(\mathbb{K} - \lambda I) = \det\begin{vmatrix} -K-K'-\lambda & +K' \\ K' & -K-K'-\lambda \end{vmatrix}$$

$$= (-K-K'-\lambda)(-K-K'-\lambda) - (K')^2 = 0$$

$$= (-K-K'-\lambda)^2 - (K')^2$$

$$= (-K-K'-\lambda+K')(-K-K'-\lambda-K')$$

$$(-K-\lambda)(-K-2K'-\lambda) = 0$$

$$\lambda_1 = -K \qquad \lambda_2 = -K-2K'$$

Ahora los eigenvectores:

$$\begin{bmatrix} -K' & +K' \\ K' & -K' \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-K'x + K'y = 0$$
$$K'x - K'y = 0$$
$$\therefore \quad x = y$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ y \end{bmatrix} = y\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} K' & K' \\ K' & K' \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$K'x + K'y = 0$$
$$K'x + K'y = 0$$
$$x = -y$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ y \end{bmatrix} = y\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \& \quad \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$