# **Instrumental Variables**

UNDERSTANDING IV



### Roadmap

Where do (Good) Instruments Come From?

True Lotteries

**Natural Experiments** 

Panel Data

**2SLS Mechanics** 

Just-Identified IV

Overidentification

Weak and Many Instruments

Weak IVs

Many IVs

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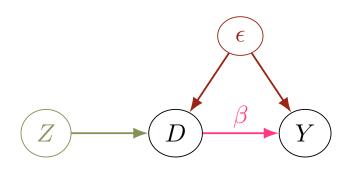
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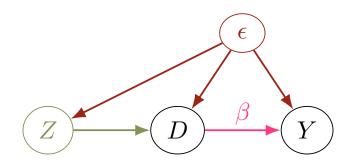
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 More modern IV texts take care to distinguish between these two conceptually distinct requirements...

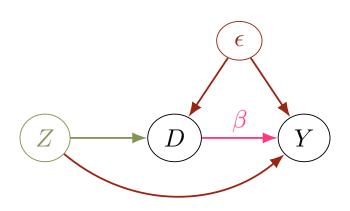
## A Valid Instrument



# A Violation of As-Good-As-Random Assignment



# A Violation of Exclusion



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- Relevance holds when  $Z_i$  has some effect on  $D_i$
- "Gold standard" IV: a randomized offer to participate in a program, with  $\mathcal{D}_i$  recording program participation
  - Exclusion restriction likely to hold for any  $Y_i$ , by construction
  - Relevance almost guaranteed (provided people want the program!)

Charter School Lotteries

Abdulkadiroglu et al. (2016) are interested in whether going to a "charter" middle school increases standardized test scores

- Charter students tend to score better, but we worry about selection
- History of doubting educational inputs, since Coleman (1966)

#### Charter School Lotteries

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We study a particular charter (UP Academy), which is "takeover"

• Two offer IVs: "immediate" (on lottery night) and from a waitlist

## Lottery IV Estimates of UP Test Score Effects

TABLE 8—LOTTERY IV ESTIMATES OF UP EFFECTS

			OLS (2)	2SLS			
		Comparison group mean (1)		First stage			
				Immediate offer (3)	Waitlist offer (4)	Enrollment effect (5)	
Panel A. All grades (Sixth through eighth)	Math (N = 2,202)	0.059	0.301 (0.022)	0.760 (0.063)	0.562 (0.067)	0.270 (0.056)	
	ELA $(N = 2,205)$	0.103	0.148 (0.020)	0.759 (0.063)	0.562 (0.067)	0.118 (0.051)	

#### 2. Natural Experiments

Without appealing to literal randomization, we may credibly argue  $Z_i$  is as-good-as-randomly assigned conditional on some  $\mathbf{W}_i$ 

- Such "natural experiments" rely on a selection-on-observables argument (for  $Z_i$ , instead  $D_i$ )
- ullet Still worry about exclusion:  $Z_i$  cannot affect  $Y_i$  except through  $D_i$

Quarter-of-Birth

Angrist and Krueger (1991) famously estimate labor market returns to schooling with a creative IV: student quarter-of-birth

- Compulsory schooling requirements prevent students from dropping before the day they turn 16 (used to be more binding)
- Fixed school start dates mean students who drop out at 16 get more or less schooling depending on their birth date

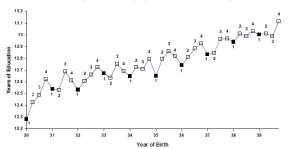
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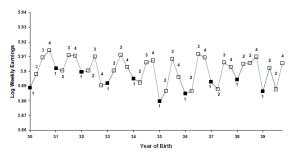
- Compulsory schooling requirements prevent students from dropping before the day they turn 16 (used to be more binding)
- Fixed school start dates mean students who drop out at 16 get more or less schooling depending on their birth date
- Quarter-of-birth seems quasi-randomly assigned is it excludable?
   See Buckles and Hungerman (2013)...

#### The Quarter-of-Birth Natural Experiment: Visualized

A. Average Education by Quarter of Birth (first stage)



B. Average Weekly Wage by Quarter of Birth (reduced form)



# Quarter-of-Birth IV Estimates of Returns to Schooling

Table 4.1.1: 2SLS estimates of the economic returns to schooling

	OLS				2SLS	
	(1)	(2)	(3)	(4)	(5)	(6)
Years of education	0.075 (0.0004)	0.072 (0.0004)	0.103 (0.024)	0.112 (0.021)	0.106 (0.026)	0.108 (0.019)
Covariates:						
9 year of birth dummies 50 state of birth dummies		<b>√</b>			<b>√</b>	<b>√</b>
In struments:			dummy for QOB=1	dummy for QOB=1 or QOB=2	dummy for QOB=1	full set of QOB dummies

3. Panel Data

We might also combine IV + difference-in-differences identification

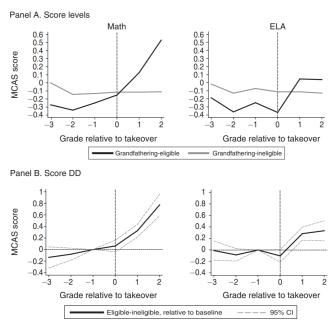
- ullet E.g. instrument with  $Z_i imes Post_t$ , controlling for  $Z_i$  and  $Post_t$  FEs
- This requires two parallel trends assumptions, for the RF and FS
- Still need to worry about the exclusion restriction, as always

Charter School Takeovers

In Abdulkadiroglu et al. (2016), we complement the lottery analysis of takeover charters with an instrumented diff-in-diff analysis

- Students enrolled in the "legacy" public school were eligible for being "grandfathered" into UP, without having to apply to the charter
- We compare their trends in test scores & enrollment to a matched comparison group of observably-similar students at other schools

# Grandfathering IV: Visualized



# Grandfathering IV Estimates of UP Test Score Effects

TABLE 7—GRANDFATHERING IV ESTIMATES OF UP EFFECTS

				2SLS	
		Comparison group mean (1)	OLS (2)	First stage (3)	Enrollment effect (4)
Panel A. All grades					
(Seventh through eighth)	Math $(N = 1,543)$	-0.233	0.400 $(0.032)$	1.051 (0.040)	0.321 (0.039)
	ELA $(N = 1,539)$	-0.214	0.296 (0.035)	1.040 (0.041)	0.394 (0.044)

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$$Y_i = \beta D_i + \mathbf{W}_i' \boldsymbol{\gamma} + \varepsilon_i$$
 (second stage) 
$$D_i = \pi Z_i + \mathbf{W}_i' \boldsymbol{\mu} + \eta_i$$
 (first stage)

where  $W_i$  includes a constant.

The reduced form is:

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Same identification logic as before:

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• Can use Frisch-Waugh-Lovell to "partial out"  $\mathbf{W}_i$  from  $Y_i$ ,  $X_i$ ,  $D_i$ , and so get back to an IV regression without controls

Sometimes we have more than one instrument  $Z_{i\ell}$ , for  $\ell=1,\ldots,L$ .

This leads to an "overidentified" IV regression:

$$Y_i = \beta D_i + \mathbf{W}_i' \boldsymbol{\gamma} + \varepsilon_i$$
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where 
$$\mathbf{Z}_i = [Z_{i1}, \dots, Z_{iL}]'$$
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Overidentification can yield tests of IV validity

• Intuitively, 2SLS checks whether all the  $Z_{i\ell}$  yields the same IV estimate, which is sensible in a constant-effects model...

## Putting the "2S" in "2SLS"

You'll notice I haven't actually defined 2SLS beyond the simple case

- Before we had  $\beta^{IV}=\frac{Cov(Z_i,Y_i)}{Cov(Z_i,D_i)}$  leading to  $\widehat{\beta}^{IV}=\frac{\widehat{Cov}(Z_i,Y_i)}{\widehat{Cov}(Z_i,D_i)}$
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The proof of this follows from some (simple) linear algebra

• Intuitively, regressing  $Y_i$  on  $\widehat{\pi}^{OLS}Z_i$  gives a scaled RF:

$$\widehat{\beta}^{IV} = \frac{\widehat{\rho}^{OLS}}{\widehat{\pi}^{OLS}}$$

## Avoid Manual 2SLS!

Although easy, you should never do such "manual 2SLS" yourself!

- Your point estimates will be right, but your SEs won't be!
- Also might forget to include some controls in the second stage, etc

Just let Stata/R do everything for you...

## 2SLS Done Right

#### IV (2SLS) estimation

Estimates efficient for homoskedasticity only Statistics robust to heteroskedasticity

Number of obs = F( 2. 5.16 0.0083 Total (centered) SS Centered R2 = -2,5922 576796958.9 Total (uncentered) SS 3183192639 Uncentered R2 = 0.3491 Residual SS 2071965250 Root MSE 5480

clear all
sysuse auto
ivreg2 price (mpg=rep78) weight, r

price	Coef.	Robust Std. Err.	z	P> z	[95% Conf.	Interval]
mpg	1404.283	1499.569	0.94	0.349	-1534.819	4343.384
weight	10.38214	8.57869	1.21	0.226	-6.431778	27.19607
_cons	-55229.89	57542.19	-0.96	0.337	-168010.5	57550.73

	_coms	-5.	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	37342	.15	. 50	0.55	, -10	0010.5	3,330.73
Under	identif	ication	test	(Kleiber	gen-Paap	rk LM	1 stat	istic):		1.200
							C	hi-sq(1)	P-val =	0.2734
Weak	identif	ication	test	(Cragg-D	onald Wal	d F s	tatis	tic):		1.459
				(Kleiber	gen-Paap	rk Wa	ald F	statisti	c):	1.083
Stock	-Yogo w	eak ID	test	critical	values: 1	0% ma	ximal	IV size		16.38
					1	5% ma	ximal	IV size		8.96
					2	0% ma	ximal	IV size		6.66
					2	5% ma	ximal	IV size		5.53

Source: Stock-Yogo (2005). Reproduced by permission. NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Instrumented: mpg
Included instruments: weight
Excluded instruments: rep78

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#### Weak Instruments

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• Specifically the first stage  $\ \ {\mbox{F-statistic}}$  , which tests  $\pi^{OLS}=0$ 

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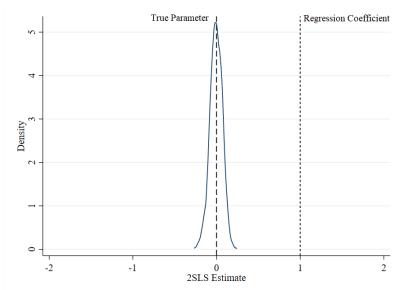
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Much made of this over the years, but Angrist and Kolesár (2022) argue recently that we shouldn't worry too much

- The SE increase tends to be large enough to "cover up" the bias
- Just-id. 2SLS is "approximately median-unbiased"

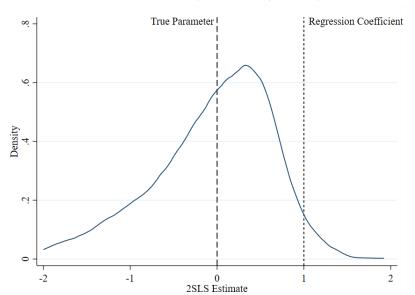
# Weak Instruments: Visualized

Monte Carlo: 
$$Y_i=arepsilon_i$$
,  $D_i=\Pi Z_i+\eta_i$ :  $\Pi=Var(arepsilon_i)=Var(\eta_i)=1$ 



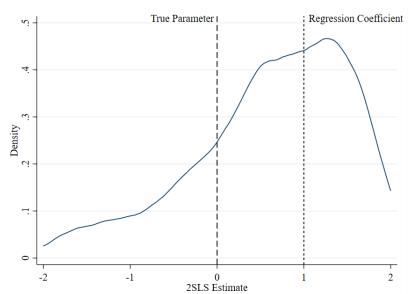
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Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_i + \eta_i$ :  $\Pi = 0.1$  (Weaker)



# Weak Instruments: Visualized

Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_i + \eta_i$ :  $\Pi = 0.01$  (Very Weak)



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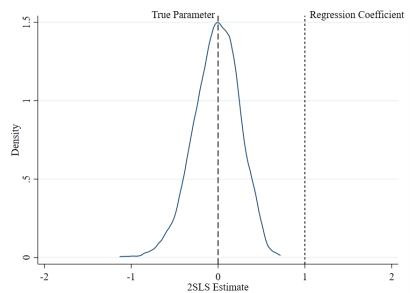
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As we'll see, this bias is especially relevant in judge IV designs

- Potentially many judge assignment indicators as the instrument
- Leave-out corrections (e.g. Angrist et al. 1999) have been adapted to this setting in recent years (e.g. Kolesár 2013)

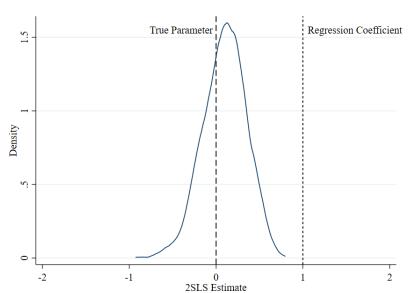
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Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_{i1} + \eta_i$ : IV with one  $Z_{i1}$ 



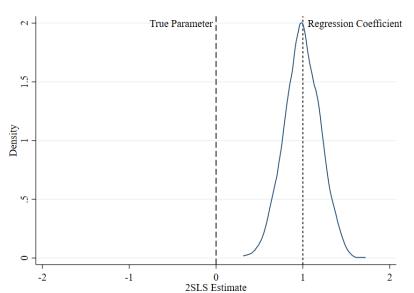
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Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_{i1} + \eta_i$ : IV with 100  $Z_{ij}$ 



## What to Do?

#### Check your F's after every IV regression

- State of the art: Montiel Olea and Pflueger '15; weakivtest in Stata
- Staiger-Stock rule-of-thumb (F > 10) still seems widely held
- See Lee et al. (2020) and Keane and Neal (2022) for some discussions of additional subtleties

#### If your F is small, some things to consider:

- Is there a different instrument that's stronger?
- Is there a better functional form for the instrument you have?
- Do interactions with covariates help? (note: beware many-weak!)
- Does changing the covariate set help? (note: beware invalidity!)