Instrumental Variables

Introduction

MIXTAPE SESSIONS

Roadmap

- Introductions
  - Who Am I?
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  - What is This Course?
- Regression Review
  - Models vs. Estimands vs. Estimators
  - Regression Identification and Endogeneity
- Intro to IV
  - Instrument Validity and Relevance
  - The 2SLS Estimator

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  - → IV-based analyses of discrimination and bias
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- Two 75-minute coding labs, applying what we've learned
  - → I will be live-coding in Stata, but R code will also be available
  - → Goal: demonstrate both methods & how I think about applying them

Schedule

9:00-10:00am	Lecture 1: Regression Review; Regression Endogeneity; Introduction to IV
10:00-10:10am	*Break*
10:10-11:10am	Lecture 2: Understanding Instrument Validity; 2SLS Mechanics; Applications
11:10-11:15am	*Break*
11:15am-12:30pm	Coding Lab 1: Angrist and Krueger (1991)
12:30-1:30pm	*Lunch*
1:30-2:30pm	Lecture 3: Heterogeneous Treatment Effects; Characterizing Compliers; MTEs
2:30-2:40pm	*Break*
2:40pm-3:40pm	Lecture 4: Judge Leniency Designs; Shift-Share IV; New IV Frontiers
3:40-3:45pm	*Break*
3:45-5:00pm	Coding Lab 2: Stevenson (2018)
5:00-5:15pm	Closing Remarks

Roadmap

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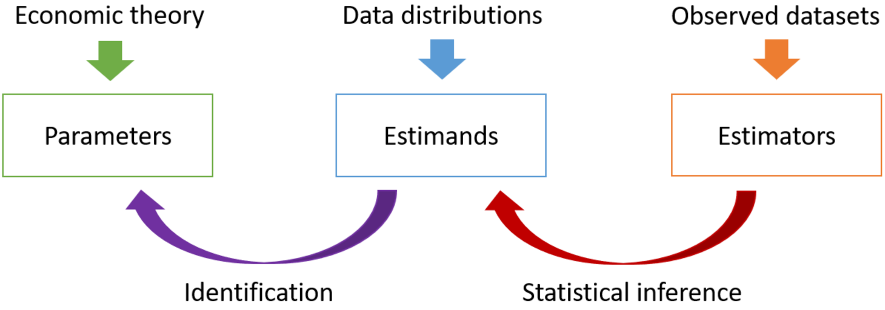
  - → Make assumptions to link parameters & estimands ("identification")
- *Estimators* are functions of the observed data itself (the "sample")
  - → E.g. a difference in sample means or ratio of OLS coefficients
  - → Since data are random, so are estimators. Each has a distribution
  - → Use knowledge of estimator distributions to make learn about estimands ("inference") and—hopefully—identified parameters

Identification vs. Estimation



This course will mostly focus on identification, but we'll cover some IV estimation / inference issues as well

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- The OLS *estimator* $\widehat{\beta}^{OLS}$ consistently estimates the regression *estimand* β^{OLS} under relatively weak conditions (e.g. *i.i.d.* data)
  - $\rightarrow$ Stata tells us $\widehat{\beta}^{OLS}$ and what we can infer about β^{OLS} from it
  - $\rightarrow$ It *doesn't* directly tell us about the relationship between β^{OLS} and β

Population Regression

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- Key point: we can always define $\boldsymbol{\beta}^{OLS}$ for any Y_i and $\mathbf{X}_i$ (assuming no perfect collinearity); this is what Stata estimates
  - → Specifically it computes $\widehat{\boldsymbol{\beta}}^{OLS} = (\frac{1}{N}\sum_i \mathbf{X}_i\mathbf{X}_i')^{-1}(\frac{1}{N}\sum_i \mathbf{X}_i Y_i)$ and uses large-sample asymptotics (LLN/CLT) to get a standard error

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- But what if this *estimand* is not what we want?
  - $\rightarrow$ What if $\boldsymbol{\beta}^{OLS}$ fails to coincide with our economic parameter of interest (e.g. returns to mixtape workshops)?

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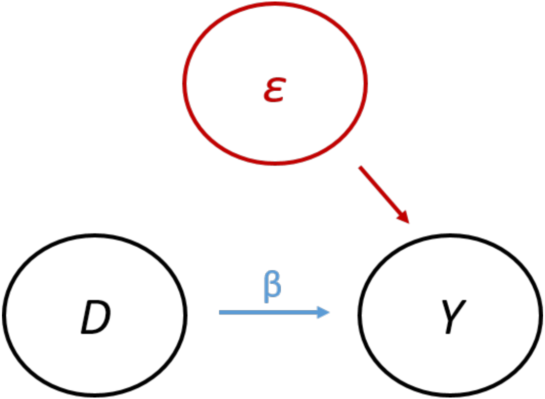
- Adding more controls (e.g. demographics) may or may not help
  - → Projecting ε_i on X_i, we get $Y_i = \alpha + \beta D_i + \gamma X_i + \tilde{\varepsilon}_i$, $Cov(X_i, \tilde{\varepsilon}_i) = 0$
  - → Whether or not $Cov(D_i, \tilde{\varepsilon}_i) = 0$ depends on whether X_i sufficiently accounts for the confounding relationship $Cov(D_i, \varepsilon_i) \neq 0$

Regression "Exogeneity"



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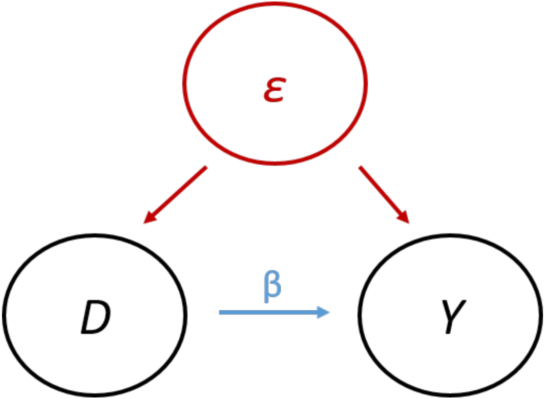

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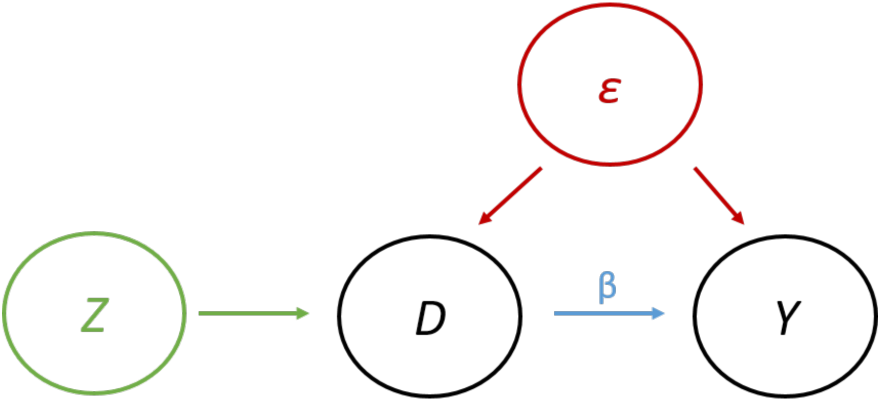
- Indeed, this leads us to IV estimands (and estimators)

The IV Solution



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Instrument Validity and Relevance

- Causal/structural model $Y_i = \alpha + \beta D_i + \varepsilon_i$ and a candidate IV Z_i
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- We then have identification:

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  - $\rightarrow$ Compare to the OLS estimand: $\beta^{OLS} = \frac{Cov(D_i, Y_i)}{Var(D_i)}$

“Reduced Form” and “First Stage”

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- Sometimes we refer to the IV estimand as the “second stage”:

$$Y_i = \alpha^{IV} + \beta^{IV} D_i + U_i$$

where now $Cov(Z_i, U_i) = 0$. Thus “IV=RF/FS” ($\beta^{IV} = \rho^{OLS}/\pi^{OLS}$)

The 2SLS Estimator

- As with OLS, we estimate IV by sample analog:

$$\widehat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)} = \frac{\widehat{\rho}^{OLS}}{\widehat{\pi}^{OLS}}$$

where $\widehat{Cov}(X_i, W_i) = \frac{1}{N}\sum_i X_i W_i - \left(\frac{1}{N}\sum_i X_i\right)\left(\frac{1}{N}\sum_i W_i\right)$,
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- We will soon consider extensions of all of this, with controls / multiple instruments / etc

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- Here $\beta^{IV} = \frac{Cov(Z_i,Y_i)/Var(Z_i)}{Cov(Z_i,D_i)/Var(Z_i)} = \frac{E[Y_i|Z_i=1]-E[Y_i|Z_i=0]}{E[D_i|Z_i=1]-E[D_i|Z_i=0]}$ has a special name, because Z_i is binary: the *Wald estimand*
  - → First stage $E[D_i \mid Z_i = 1] - E[D_i \mid Z_i = 0]$: effect of eligibility on the *probability* of military service (b/c D_i is binary)
  - → Reduced form $E[Y_i \mid Z_i = 1] - E[Y_i \mid Z_i = 0]$: effect of eligibility on adult earnings (measured in 1971, 1981...)

Angrist (1990): The “Draft Lottery Paper”

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- IV interprets the latter causal effect in terms of the former

Draft Lottery Reduced Form, First Stage, and IV

IV Estimates of the Effects of Military Service on the Earnings of White Men born in 1950

	Earnings		Veteran Status		Wald Estimate of Veteran Effect
Earnings year	Mean	Eligibility Effect	Mean	Eligibility Effect	
	(1)	(2)	(3)	(4)	(5)
1981	16,461	-435.8 (210.5)	.267	.159 (.040)	-2,741 (1,324)
1971	3,338	-325.9 (46.6)			-2050 (293)
1969	2,299	-2.0 (34.5)			

Note: Adapted from Table 5 in Angrist and Krueger (1999) and author tabulations. Standard errors are shown in parentheses. Earnings data are from Social Security administrative records. Figures are in nominal dollars. Veteran status data are from the Survey of Program Participation. There are about 13,500 individuals in the sample.

Draft Lottery Reduced Form and First Stage Visualized

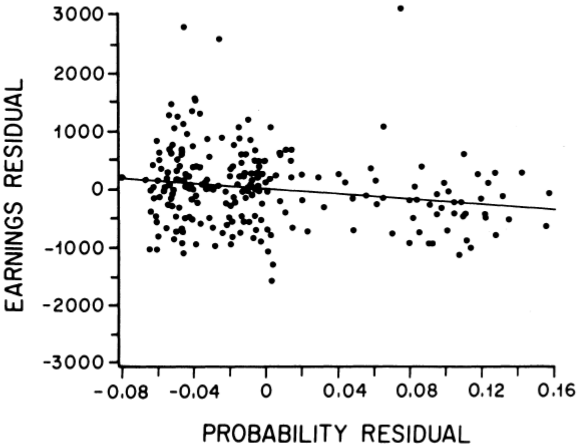

FIGURE 3. EARNINGS AND THE PROBABILITY OF VETERAN STATUS BY LOTTERY NUMBER

Notes: The figure plots mean W-2 compensation in 1981–4 against probabilities of veteran status by cohort and groups of five consecutive lottery numbers for white men born 1950–3. Plotted points consist of the average residuals (over four years of earnings) from regressions on period and cohort effects. The slope of the least-squares regression line drawn through the points is −2,384, with a standard error of 778, and is an estimate of α in the equation

$$\bar{y}_{ctj} = \beta_c + \delta_t + \hat{p}_{cj}\alpha + \bar{u}_{ctj}.$$