Instrumental Variables

INTRODUCTION



Roadmap

Introductions

Who Am I?

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What is This Course?

Regression Review

Models vs. Estimands vs. Estimators

Regression Identification and Endogeneity

Intro to IV

Instrument Validity and Relevance

The 2SLS Estimator

Groos Family Assistant Professor of Economics, Brown University

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- Lottery- and non-lottery IVs in studies of educational quality (Angrist et al. 2016, 2017, 2021, 2022; Abdulkadiroğlu et al. 2016)
- Quasi-experimental evaluations of healthcare quality (Hull 2020; Abaluck et al. 2021, 2022)
- IV-based analyses of discrimination and bias (Arnold et al. 2020, 2021, 2022; Hull 2021; Bohren et al. 2022)
- Shift-share instruments and related designs
 (Borusyak et al. 2022; Borusyak and Hull 2021, 2022; Goldsmith-Pinkham et al. 2022)

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A constant student of IV (and econometrics more generally)

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- I will be live-coding in Stata, but R code will also be available
- Goal: demonstrate both methods & how I think about applying them

Schedule

5:00-5:15pm

Closing Remarks

Lecture 1: Regression Review; Regression Endogeneity; Introduction to IV
Break
Lecture 2: Understanding Instrument Validity; 2SLS Mechanics; Applications
Break
Coding Lab 1: Angrist and Krueger (1991)
Lunch
Lecture 3: Heterogeneous Treatment Effects; Characterizing Compliers; MTEs
Break
Lecture 4: Judge Leniency Designs; Shift-Share IV; New IV Frontiers
Break
Coding Lab 2: Stevenson (2018)

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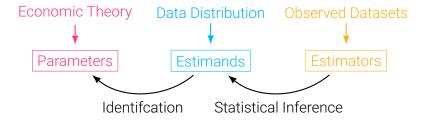
The 2SLS Estimator

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 - → Make assumptions to link parameters & estimands ("identification")
- Estimators are functions of the observed data itself (the "sample")
 - → E.g. a difference in sample means or ratio of OLS coefficients
 - $\,\rightarrow\,$ Since data are random, so are estimators. Each has a distribution
 - → Use knowledge of estimator distributions to make learn about estimands ("inference") and—hopefully—identified parameters

The IV Solution



This course will mostly focus on identification, but we'll cover some IV estimation / inference issues as well

Let's Get Concrete

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- Parameter: returns to taking this class β , measured in some outcome Y_i (e.g. lifetime top-5 pubs / earnings / twitter followers)
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We see a sample of Y_i , D_i , and some other covariates W_{1i}, \ldots, W_{Ki}

• We fire up Stata and reg y d w1-wk, r. How do we interpret the output?

The OLS estimator $\hat{\beta}^{OLS}$ consistently estimates the regression estimand β^{OLS} under relatively weak conditions (e.g. *i.i.d.* data)

- Stata tells us $\hat{\beta}^{OLS}$ and what we can infer about β^{OLS} from it
- ullet It doesn't directly tell us about the relationship between eta^{OLS} and eta

The population regression of Y_i on $\mathbf{X}_i = [1, D_i, W_{1i}, \dots, W_{Ki}]'$ is given by $Y_i = \mathbf{X}_i' \beta^{OLS} + U_i$ where $E[\mathbf{X}_i U_i] = 0$

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Key point: we can always define β^{OLS} for any Y_i and \mathbf{X}_i (assuming no perfect collinearity); this is what Stata estimates

• Specifically it computes $\widehat{\beta}^{OLS} = (\frac{1}{N} \sum_i \mathbf{X}_i \mathbf{X}_i')^{-1} (\frac{1}{N} \sum_i \mathbf{X}_i Y_i)$ and uses large-sample asymptotics (LLN/CLT) to get a standard error



But what if this estimand is not what we want?

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• What if β^{OLS} fails to coincide with our economic parameter of interest (e.g. returns to mixtape workshops)?

The model parameter in $Y_i=\alpha+\beta D_i+\varepsilon_i$ need not coincide with the regression coefficient in $Y_i=\alpha^{OLS}+\beta^{OLS}D_i+U_i$

• I.e. we may not have $Cov(D_i, \varepsilon_i) = 0$ (always have $Cov(D_i, U_i) = 0$)

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Selection bias (a.k.a. omitted variables bias): students with higher latent earnings potential ε_i are more likely to take this class D_i

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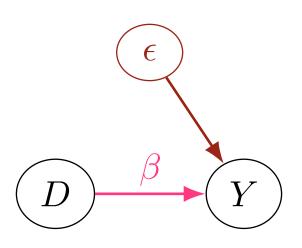
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Can I just Control My Way Out of This?

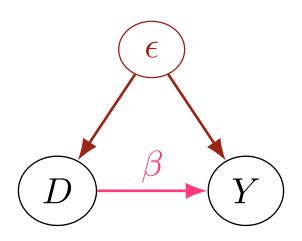
Adding more controls (e.g. demographics) may or may not help

- Projecting ε_i on X_i , we get $Y_i = \alpha + \beta D_i + \gamma X_i + \tilde{\varepsilon}_i$, $Cov(X_i, \tilde{\varepsilon}_i) = 0$
- Whether or not $Cov(D_i, \tilde{\varepsilon}_i) = 0$ depends on whether X_i sufficiently accounts for the confounding relationship $Cov(D_i, \varepsilon_i) \neq 0$

Regression "Exogeneity"



Regression "Endogeneity"



...But Sometimes, You Get What you Need

Imagine this course was "oversubscribed," and admission was determined by lottery

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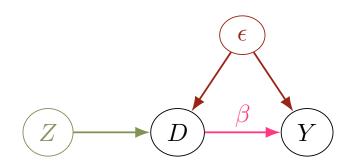
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Indeed, this leads us to IV estimands (and estimators)

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We then have identification:

$$\begin{split} Cov(Z_i,Y_i) &= Cov(Z_i,\alpha + \beta D_i + \varepsilon_i) = \beta Cov(Z_i,D_i) + Cov(Z_i,\varepsilon_i) \\ &= \beta Cov(Z_i,D_i) \text{, Implying } \beta = \frac{Cov(Z_i,Y_i)}{Cov(Z_i,D_i)} \end{split}$$

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• Compare to the OLS estimand: $\beta^{OLS} = \frac{Cov(D_i, Y_i)}{Var(D_i)}$

"Reduced Form" and "First Stage"

Note we can write

$$\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} = \frac{Cov(Z_i, Y_i) / Var(Z_i)}{Cov(Z_i, D_i) / Var(Z_i)} = \frac{\rho^{OLS}}{\pi^{OLS}}$$

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$$Y_i = \kappa^{OLS} + \rho^{OLS} Z_i + V_i$$
 "reduced form"
$$D_i = \mu^{OLS} + \pi^{OLS} Z_i + W_i$$
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Sometimes we refer to the IV estimand as the "second stage":

$$Y_i = \alpha^{IV} + \beta^{IV} D_i + U_i$$

where now $Cov(Z_i, U_i) = 0$. Thus "IV=RF/FS" $(\beta^{IV} = \rho^{OLS}/\pi^{OLS})$

The 2SLS Estimator

As with OLS, we estimate IV by sample analog:

$$\widehat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)} = \frac{\widehat{\rho}^{OLS}}{\widehat{\pi}^{OLS}}$$

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We will soon consider extensions of all of this, with controls / multiple instruments / etc

Angrist (1990): The "Draft Lottery Paper"

Angrist famously used Vietnam-era draft eligibility as an instrument to estimate the earnings effects of military service

• Let Z_i be an indicator for draft eligibility, D_i be an indicator for military service, and Y_i measure later-life earnings

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- First stage $E[D_i \mid Z_i = 1] E[D_i \mid Z_i = 0]$: effect of eligibility on the probability of military service (b/c D_i is binary)
- Reduced form $E[Y_i \mid Z_i = 1] E[Y_i \mid Z_i = 0]$: effect of eligibility on adult earnings (measured in 1971, 1981...)

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IV interprets the latter causal effect in terms of the former

IV Estimates of the Effects of Military Service on the Earnings of White Men born in 1950

Earnings year	Earnings		Veteran Status		Wald Estimate of
	Mean (1)	Eligibility Effect (2)	Mean (3)	Eligibility Effect (4)	Veteran Effect
1971	3,338	-325.9 (46.6)			-2050 (293)
1969	2,299	-2.0 (34.5)			

Note: Adapted from Table 5 in Angrist and Krueger (1999) and author tabulations. Standard errors are shown in parentheses. Earnings data are from Social Security administrative records. Figures are in nominal dollars. Veteran status data are from the Survey of Program Participation. There are about 13,500 individuals in the sample.

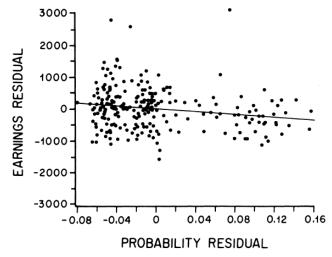


FIGURE 3. EARNINGS AND THE PROBABILITY OF VETERAN STATUS BY
LOTTERY NUMBER

Notes: The figure plots mean W-2 compensation in 1981–4 against probabilities of veteran status by cohort and groups of five consecutive lottery numbers for white men born 1950–3. Plotted points consist of the average residuals (over four years of earnings) from regressions on period and cohort effects. The slope of the least-squares regression line drawn through the points is -2,384, with a standard error of 778, and is an estimate of α in the equation

$$\bar{y}_{cti} = \beta_c + \delta_t + \hat{p}_{ci}\alpha + \bar{u}_{cti}.$$