

# Instrumental Variables

*UNDERSTANDING IV*

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# Roadmap

## Where do (Good) Instruments Come From?

- True Lotteries

- Natural Experiments

- Panel Data

## 2SLS Mechanics

- Just-Identified IV

- Overidentification

## Weak and Many Instruments

- Weak IV

- Many IVs

# Subtleties of the Validity Condition

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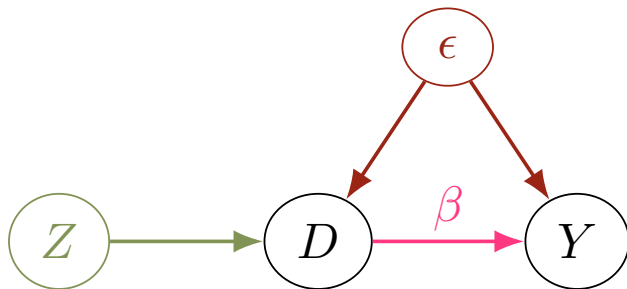
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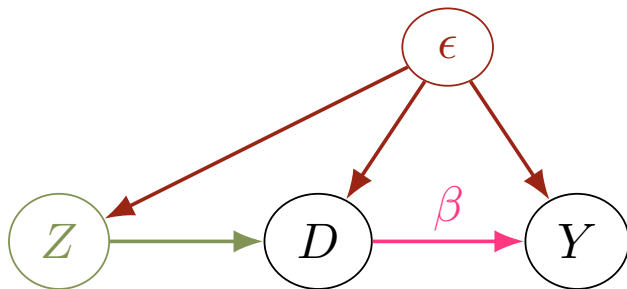
- More modern IV texts take care to distinguish between these two conceptually distinct requirements...

## A Valid Instrument

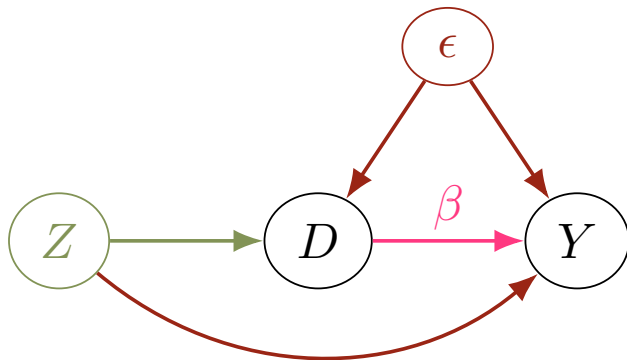




# A Violation of As-Good-As-Random Assignment



## A Violation of Exclusion



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“Gold standard” IV: a randomized offer to participate in a program, with  $X_i$  recording program participation

- Exclusion restriction likely to hold for any  $Y_i$ , by construction
- Relevance almost guaranteed (provided people want the program!)

# Example

## *Charter School Lotteries*

Abdulkadiroglu et al. (2016) are interested in whether going to a “charter” middle school increases standardized test scores

- Charter students tend to score better, but we worry about selection
- History of doubting educational inputs, since Coleman (1966)

# Example

## Charter School Lotteries

We leverage an institutional feature of charters: *admission lotteries*

- When more kids want to enroll than there are seats, admission offers  $Z_i \in \{0, 1\}$  are effectively drawn from a hat
- Offers plausibly only affect later test scores  $Y_i$  by changing charter enrollment  $D_i \in \{0, 1\}$ , so are plausibly valid instruments
- We need to control for lottery fixed effects (“risk sets”) to make  $Z_i$  as-good-as-randomly assigned – more on this soon



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We study a particular charter (UP Academy), which is “takeover”

- Two offer IVs: “immediate” (on lottery night) and from a waitlist

# Lottery IV Estimates of UP Test Score Effects

TABLE 8—LOTTERY IV ESTIMATES OF UP EFFECTS

		2SLS				
		First stage			Enrollment effect (5)	
		Comparison group mean (1)	OLS (2)	Immediate offer (3)		Waitlist offer (4)
<i>Panel A. All grades</i>						
(Sixth through eighth)	Math (N = 2,202)	0.059	0.301 (0.022)	0.760 (0.063)	0.562 (0.067)	0.270 (0.056)
	ELA (N = 2,205)	0.103	0.148 (0.020)	0.759 (0.063)	0.562 (0.067)	0.118 (0.051)

# Where do IVs Come From?

## 2. Natural Experiments

Without appealing to literal randomization, we may credibly argue  $Z_i$  is as-good-as-randomly assigned conditional on some  $\mathbf{W}_i$

- Such “natural experiments” rely on a selection-on-observables argument (for  $Z_i$ , instead  $D_i$ )
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# Example

## *Quarter-of-Birth*

Angrist and Krueger (1991) famously estimate labor market returns to schooling with a creative IV: student quarter-of-birth

- Compulsory schooling requirements prevent students from dropping before the day they turn 16 (used to be more binding)
- Fixed school start dates mean students who drop out at 16 get more or less schooling depending on their birth date

# Example

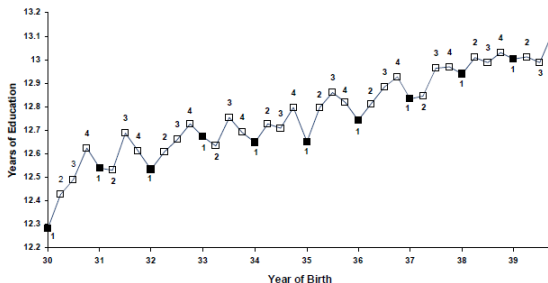
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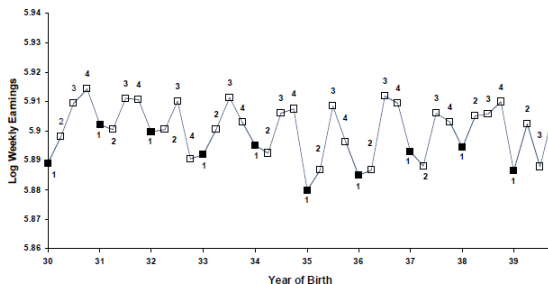
- Compulsory schooling requirements prevent students from dropping before the day they turn 16 (used to be more binding)
- Fixed school start dates mean students who drop out at 16 get more or less schooling depending on their birth date
- Quarter-of-birth seems quasi-randomly assigned — is it excludable?  
See Buckles and Hungerman (2013)...

# The Quarter-of-Birth Natural Experiment: Visualized

A. Average Education by Quarter of Birth (first stage)



B. Average Weekly Wage by Quarter of Birth (reduced form)



# Quarter-of-Birth IV Estimates of Returns to Schooling

Table 4.1.1: 2SLS estimates of the economic returns to schooling

	OLS		2SLS			
	(1)	(2)	(3)	(4)	(5)	(6)
Years of education	0.075 (0.0004)	0.072 (0.0004)	0.103 (0.024)	0.112 (0.021)	0.106 (0.026)	0.108 (0.019)
<i>Covariates:</i>						
9 year of birth dummies		✓			✓	✓
50 state of birth dummies		✓			✓	✓
<i>Instruments:</i>						
			dummy for QOB=1	dummy for QOB=1 or QOB=2	dummy for QOB=1	full set of QOB dummies



# Where do IVs Come From?

## 3. Panel Data

We might also combine IV + difference-in-differences identification

- E.g. instrument with  $Z_i \times Post_t$ , controlling for  $Z_i$  and  $Post_t$  FEs
- This requires two parallel trends assumptions, for the RF and FS
- Still need to worry about the exclusion restriction, as always

# Example

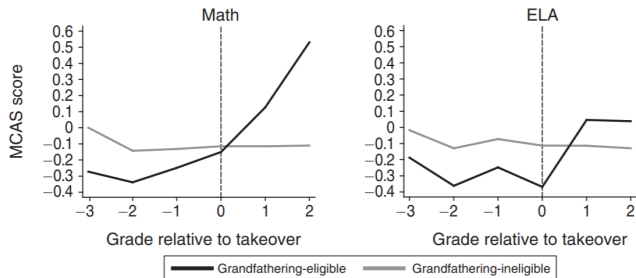
## *Charter School Takeovers*

Abdulkadiroglu et al. (2016) complement their lottery analysis of takeover charters with an instrumented diff-in-diff analysis

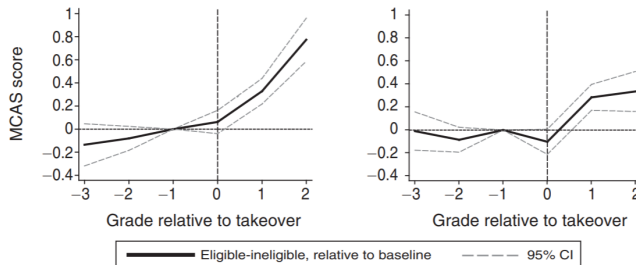
- Students enrolled in the “legacy” public school were eligible for being “grandfathered” into UP, without having to apply to the charter
- We compare their trends in test scores & enrollment to a matched comparison group of observably-similar students at other schools

# Grandfathering IV: Visualized

Panel A. Score levels



Panel B. Score DD



# Grandfathering IV Estimates of UP Test Score Effects

TABLE 7—GRANDFATHERING IV ESTIMATES OF UP EFFECTS

		Comparison group mean (1)	OLS (2)	2SLS	
				First stage (3)	Enrollment effect (4)
<i>Panel A. All grades</i>					
(Seventh through eighth)	Math (N = 1,543)	−0.233	0.400 (0.032)	1.051 (0.040)	0.321 (0.039)
	ELA (N = 1,539)	−0.214	0.296 (0.035)	1.040 (0.041)	0.394 (0.044)

# Roadmap

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Natural Experiments

Panel Data

2SLS Mechanics

Just-Identified IV

Overidentification

Weak and Many Instruments

Weak IV

Many IVs

## Just-Identified IV

The Stata `ivregress`/`ivreg2` commands (or `fixes::feols` in R) allows for controls and multiple treatments / instruments

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$$Y_i = \beta D_i + \mathbf{W}_i' \boldsymbol{\gamma} + \varepsilon_i \quad (\text{second stage})$$

$$X_i = \pi Z_i + \mathbf{W}_i' \boldsymbol{\mu} + \eta_i \quad (\text{first stage})$$

where  $\mathbf{W}_i$  includes a constant.

## Just-Identified IV

The reduced form is:

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- Can use Frisch-Waugh-Lovell to “partial out”  $\mathbf{W}_i$  from  $Y_i$ ,  $X_i$ ,  $D_i$ , and so get back to an IV regression without controls

# Overidentification

Sometimes we have more than one instrument  $Z_{i\ell}$ , for  $\ell = 1, \dots, L$ .

This leads to an “overidentified” IV regression:

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where  $\mathbf{Z}_i = [Z_{i1}, \dots, Z_{iL}]'$ . Reduced form:  $Y_i = \mathbf{Z}_i' \boldsymbol{\rho} + \mathbf{W}_i' \boldsymbol{\kappa} + \nu_i$

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Overidentification can yield tests of IV validity

- Intuitively, 2SLS checks whether all the  $Z_{i\ell}$  yields the same IV estimate, which is sensible in a constant-effects model...



## Putting the “2S” in “2SLS”

You'll notice I haven't actually defined 2SLS beyond the simple case

- Before we had  $\beta^{IV} = \frac{\text{Cov}(Z_i, Y_i)}{\text{Cov}(Z_i, D_i)}$  leading to  $\hat{\beta}^{IV} = \frac{\widehat{\text{Cov}}(Z_i, Y_i)}{\widehat{\text{Cov}}(Z_i, D_i)}$
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- First regress  $D_i$  on all instruments  $Z_{i\ell}$  and controls  $W_{ik}$
- Then regress  $Y_i$  on the “fitted values”  $\hat{D}_i$  and controls  $W_{ik}$

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The proof of this follows from some (simple) linear algebra

- Intuitively, regressing  $Y_i$  on  $\hat{\pi}^{OLS} Z_i$  gives a scaled RF:

$$\hat{\beta}^{IV} = \frac{\hat{\rho}^{OLS}}{\hat{\pi}^{OLS}}$$

# Avoid Manual 2SLS!

Although easy, you should never do such “manual 2SLS” yourself!

- Your point estimates will be right, but your SEs won't be!
- Also might forget to include some controls in the second stage, etc

Just let Stata/R do everything for you...

# 2SLS Done Right

## IV (2SLS) estimation

Estimates efficient for homoskedasticity only  
Statistics robust to heteroskedasticity

Total (centered) SS	=	576796958.9	Number of obs =	69
Total (uncentered) SS	=	3183192639	F( 2, 66) =	5.16
Residual SS	=	2071965250	Prob > F =	0.0083
			Centered R2 =	-2.5922
			Uncentered R2 =	0.3491
			Root MSE =	5480

```
clear all
sysuse auto
```

```
ivreg2 price (mpg=rep78) weight, r
```

price	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
mpg	1404.283	1499.569	0.94	0.349	-1534.819	4343.384
weight	10.38214	8.57869	1.21	0.226	-6.431778	27.19607
_cons	-55229.89	57542.19	-0.96	0.337	-168010.5	57550.73

Underidentification test (Kleibergen-Paap rk LM statistic): 1.200  
Chi-sq(1) P-val = 0.2734

Weak identification test (Cragg-Donald Wald F statistic): 1.459  
(Kleibergen-Paap rk Wald F statistic): 1.083  
Stock-Yogo weak ID test critical values: 10% maximal IV size 16.38  
15% maximal IV size 8.96  
20% maximal IV size 6.66  
25% maximal IV size 5.53

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 0.000  
(equation exactly identified)

Instrumented: mpg  
Included instruments: weight  
Excluded instruments: rep78

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- Typically use the rule-of-thumb of  $F < 10$  (Staiger and Stock 1997)
- In this case the second-stage SEs will be large and the 2SLS estimate will tend to be biased towards the corresponding OLS



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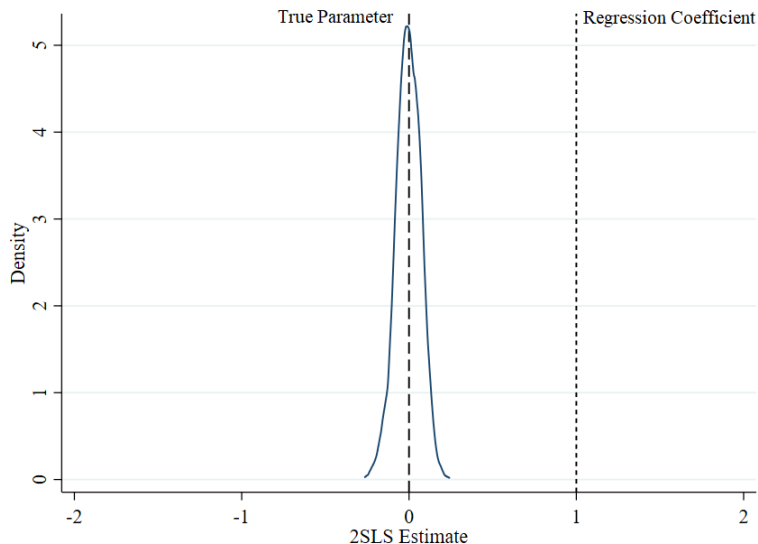
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Much made of this over the years, but Angrist and Kolesár (2022) argue recently that we shouldn’t worry too much

- The SE increase tends to be large enough to “cover up” the bias
- Just-id. 2SLS is “approximately median-unbiased”

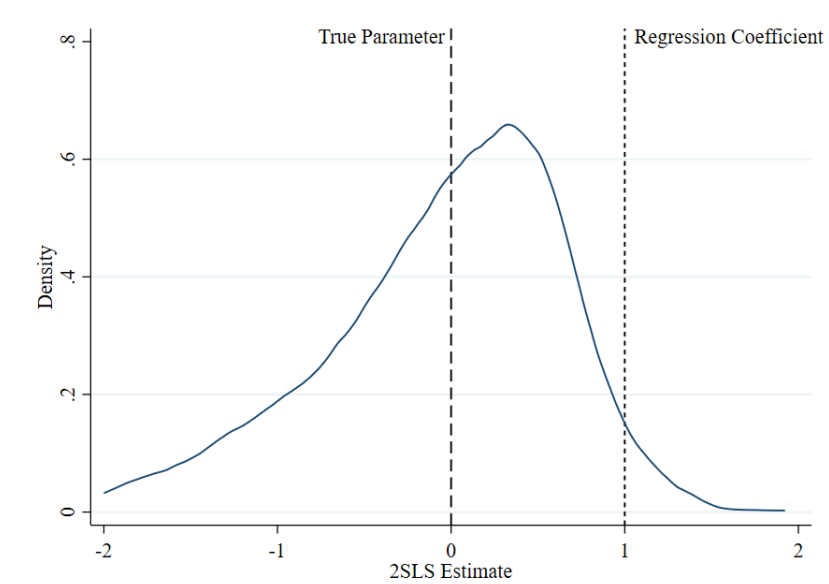
# Weak Instruments: Visualized

Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_i + \eta_i$ :  $\Pi = \text{Var}(\varepsilon_i) = \text{Var}(\eta_i) = 1$



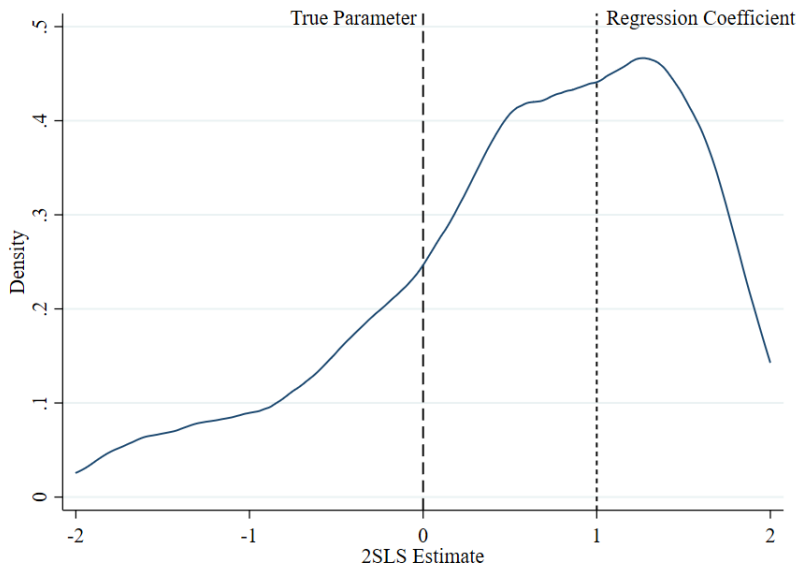
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Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_i + \eta_i$ :  $\Pi = 0.01$  (Very Weak)



# Many IVs

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- Also tends to manifest in low first-stage  $F$ 's, so also good to check

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Many-IV bias is also towards OLS, but unlike before SEs go *down*

- Intuitively, a more flexible FS tends to fit  $D_i$  better  $\rightarrow$  more power
- But we can have *overfitting* with lots of  $Z_i \rightarrow$  essentially recreate  $D_i$

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- Also tends to manifest in low first-stage  $F$ 's, so also good to check

Many-IV bias is also towards OLS, but unlike before SEs go *down*

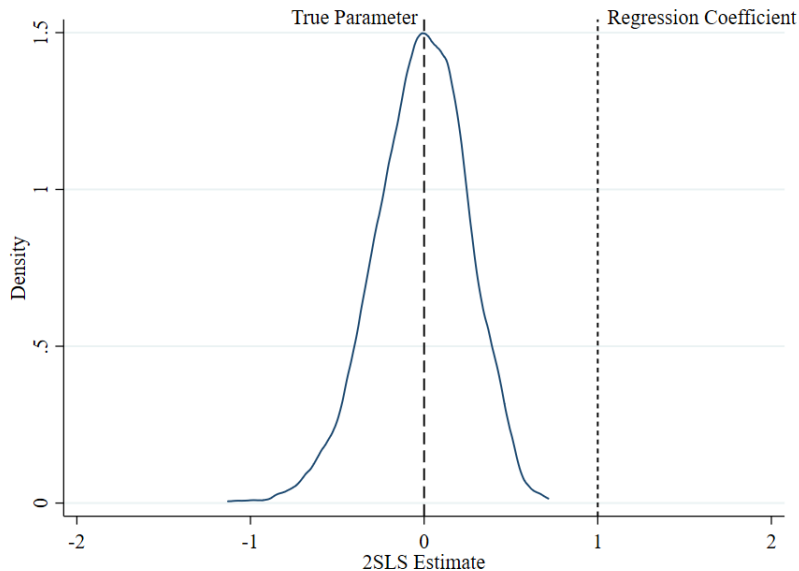
- Intuitively, a more flexible FS tends to fit  $D_i$  better  $\rightarrow$  more power
- But we can have *overfitting* with lots of  $Z_i \rightarrow$  essentially recreate  $D_i$

As we'll see, this bias is especially relevant in judge IV designs

- Potentially many judge assignment indicators as the instrument
- Leave-out corrections (e.g. Angrist et al. 1999) have been adapted to this setting in recent years (e.g. Kolesár 2013)

# Weak and Many Instruments VIII

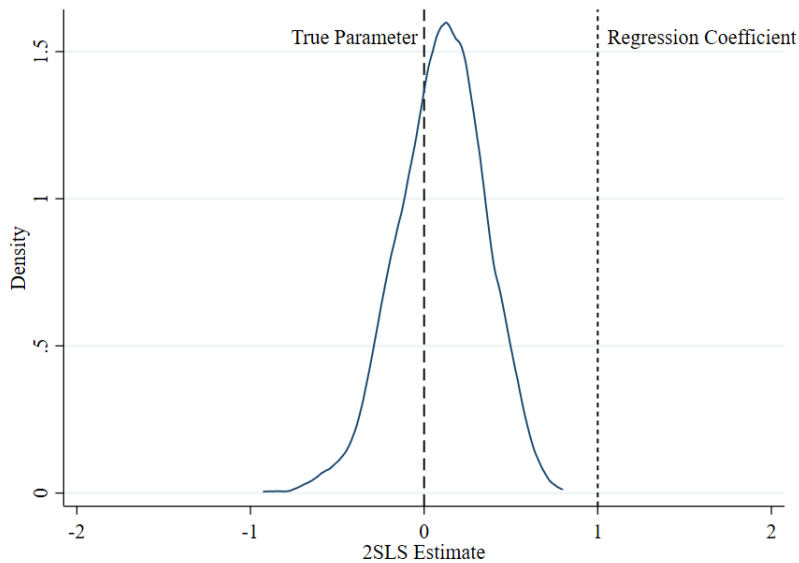
Monte Carlo:  $Y_i = \varepsilon_i$ ,  $X_i = \Pi Z_{i1} + \eta_i$ : IV with one  $Z_{i1}$





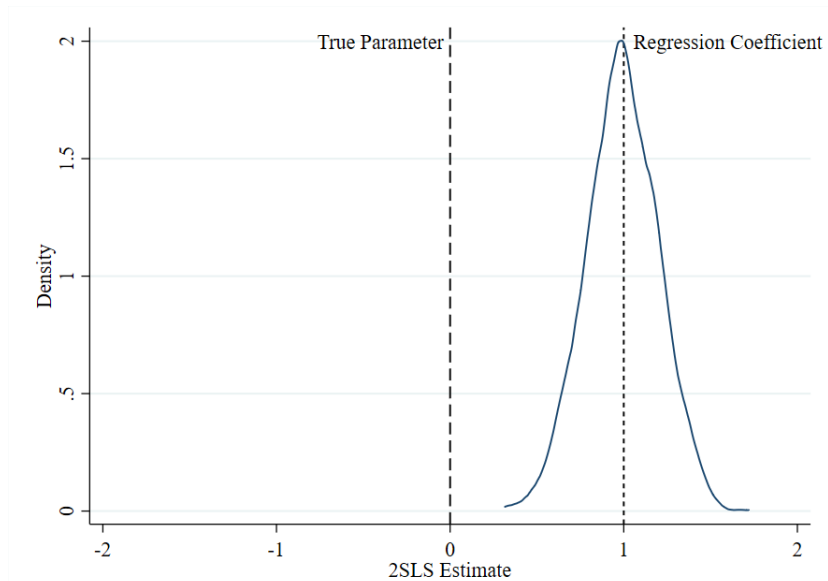
# Weak and Many Instruments IX

Monte Carlo:  $Y_i = \varepsilon_i$ ,  $X_i = \Pi Z_{i1} + \eta_i$ : IV with ten  $Z_{ij}$



# Weak and Many Instruments X

Monte Carlo:  $Y_i = \varepsilon_i$ ,  $X_i = \Pi Z_{i1} + \eta_i$ : IV with 100  $Z_{ij}$



# What to Do?

Check your  $F$ 's after every IV regression

- Staiger-Stock rule-of-thumb ( $F > 10$ ) still seems widely held
- See Lee et al. (2020) for a recent alternative approach

If your  $F$  is small, some things to consider:

- Is there a different instrument that's stronger?
- Is there a better functional form for the instrument you have?
- Do interactions with covariates help? (note: beware many-weak!)
- Does changing the covariate set help? (note: beware invalidity!)