Instrumental Variables

Introduction

MIXTAPE SESSIONS

Roadmap

Who Am I?

Groos Family Assistant Professor of Economics, Brown University

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- Quasi-experimental evaluations of healthcare quality

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A constant student of IV (and econometrics more generally)

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Two 40-minute coding labs, applying what we've learned

- 20 min: you seeing how far you can get on your own, or with your classmate's help (use Discord rooms!)
- 20 min: me live-coding solutions in Stata (we will also post R code)

Schedule

Wednesday 4/27	6:00-7:00pm	Lecture 1: Regression Review; Regression Endogeneity; Introduction to IV
	7:00-7:10pm	*Break*
	7:10-8:10pm	Lecture 2: Understanding Instrument Validity; 2SLS Mechanics; Applications
	8:10-8:20pm	*Break*
	8:20-9:00pm	Coding Lab 1: Angrist and Krueger (1991)
Thursday 4/28	6:00-7:00pm	Lecture 3: Heterogeneous Treatment Effects; Characterizing Compliers; MTEs
	7:00-7:10pm	*Break*
	7:10-8:10pm	Lecture 4: Judge Leniency Designs; Shift-Share IV; New IV Frontiers
	8:10-8:20pm	*Break*
	8:20-9:00pm	Coding Lab 2: Stevenson (2018)
	9:00-9:15pm	Closing Remarks

Roadmap

Models vs. Estimands vs. Estimators

Three distinct objects (though not always clearly distinguished)

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- Parameters come from models of how observed data are generated
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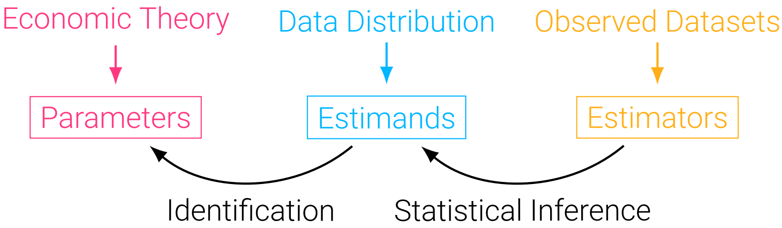
- Estimators are functions of the observed data itself (the "sample")
  - → E.g. a difference in sample means or ratio of OLS coefficients
  - → Since data are random, so are estimators. Each has a distribution
  - → Use knowledge of estimator distributions to learn about estimands ("inference") and—hopefully—identified parameters

The Lay of the Land



This course will mostly focus on identification, but we'll cover some IV estimation / inference issues as well

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- Parameter: returns to taking this class β, measured in some outcome Y_i (e.g. lifetime top-5 pubs / earnings / twitter followers)
- Simple causal/structural model: $Y_i = \alpha + \beta D_i + \varepsilon_i$, where $D_i \in \{0, 1\}$ indicates taking this class

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We see a sample of Y_i, D_i, and some other covariates $W_{1i}, \ldots, W_{Ki}$

- We fire up Stata and type `reg y d w*, r`
- How do we interpret the output?

Population Regression

The OLS *estimator* $\widehat{\beta}^{OLS}$ consistently estimates the regression *estimand* β^{OLS} under relatively weak conditions (e.g. *i.i.d.* data)

- Stata tells us $\widehat{\beta}^{OLS}$ and what we can infer about β^{OLS} from it
- It *doesn't* directly tell us about the relationship between β^{OLS} and β

Population Regression

The population regression of Y_i on $\mathbf{X}_i = [1, D_i, W_{1i}, \ldots, W_{Ki}]'$ is given by $Y_i = \mathbf{X}_i'\beta^{OLS} + U_i$ where $E[\mathbf{X}_i U_i] = 0$

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Key point: we can always define β^{OLS} for any Y_i and $\mathbf{X}_i$ (assuming no perfect collinearity); this is what Stata estimates

- Specifically it computes $\widehat{\beta}^{OLS} = (\frac{1}{N}\sum_i \mathbf{X}_i\mathbf{X}_i')^{-1}(\frac{1}{N}\sum_i \mathbf{X}_i Y_i)$ and uses large-sample asymptotics (LLN/CLT) to get a standard error

You Can’t Always Get What you Want...

But what if this *estimand* is not what we want?

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- What if β^{OLS} fails to coincide with our economic parameter of interest (e.g. returns to mixtape workshops)?

You Can't Always Get What you Want...

The model parameter in $Y_i = \alpha + \beta D_i + \varepsilon_i$ need not coincide with the regression coefficient in $Y_i = \alpha^{OLS} + \beta^{OLS} D_i + U_i$

- I.e. we may not have $Cov(D_i, \varepsilon_i) = 0$ (always have $Cov(D_i, U_i) = 0$)

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Selection bias (a.k.a. omitted variables bias): students with higher latent earnings potential ε_i are more likely to take this class D_i

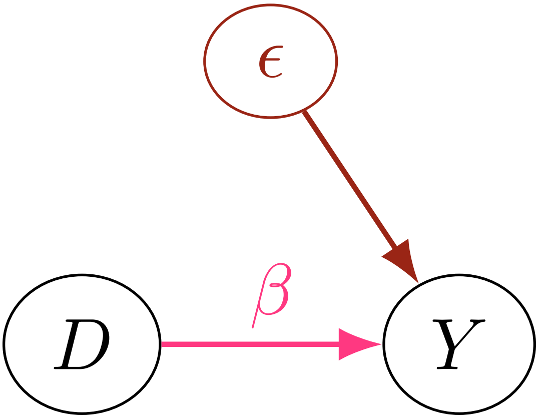
- $Cov(D_i, \varepsilon_i) > 0$ means $\beta^{OLS} > \beta$: overstate the returns-to-mixtape

Can I just Control My Way Out of This?

Adding more controls (e.g. demographics) may or may not help

- Projecting ε_i on X_i, we get $Y_i = \alpha + \beta D_i + \gamma X_i + \tilde{\varepsilon}_i, Cov(X_i, \tilde{\varepsilon}_i) = 0$
- Whether or not $Cov(D_i, \tilde{\varepsilon}_i) = 0$ depends on whether X_i sufficiently accounts for the confounding relationship $Cov(D_i, \varepsilon_i) \neq 0$

Regression “Exogeneity”



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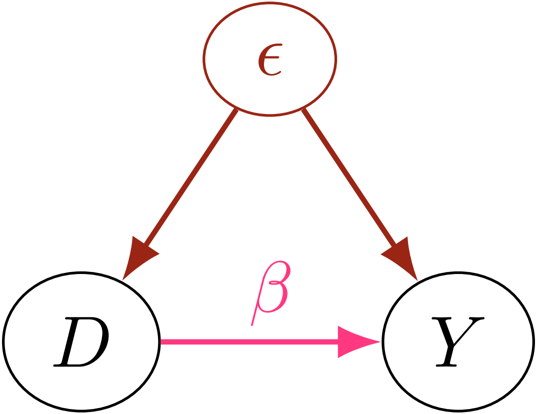

...But Sometimes, You Get What you Need

Imagine this course was "oversubscribed," and admission was determined by lottery

- Among those interested in taking the course, a random sample denoted by $Z_i = 1$ was given access
- The rest, with $Z_i = 0$ not initially given access (maybe got in later)

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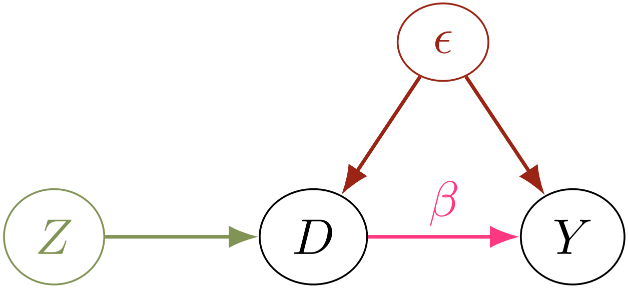
Indeed, this leads us to IV estimands (and estimators)

The IV Solution



Roadmap

Introductions

- Who Am I?
- What is This Course?

Regression Review

- Models vs. Estimands vs. Estimators
- Regression Identification and Endogeneity

Introduction to IV

- Instrument Validity and Relevance
- The 2SLS Estimator

Instrument Validity and Relevance

Causal/structural model $Y_i = \alpha + \beta D_i + \varepsilon_i$ and a candidate IV Z_i

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We then have identification:

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We then have identification:

$$Cov(Z_i, Y_i) = Cov(Z_i, \alpha + \beta D_i + \varepsilon_i) = \beta Cov(Z_i, D_i) + Cov(Z_i, \varepsilon_i)$$

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$$= \beta Cov(Z_i, D_i) \implies \beta = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$$

The IV Estimand

The (simple) IV *estimand* is:

$$\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$$

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- Compare to the OLS estimand: $\beta^{OLS} = \frac{Cov(D_i, Y_i)}{Var(D_i)}$

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Note that we can write:

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where ρ^{OLS} and π^{OLS} are two OLS estimands:

$$Y_i = \kappa^{OLS} + \rho^{OLS} Z_i + V_i \quad \text{“reduced form”}$$

$$D_i = \mu^{OLS} + \pi^{OLS} Z_i + W_i \quad \text{“first stage”}$$

IV estimand as the "Second Stage"

Sometimes we refer to the IV estimand as the "second stage":

$$Y_i = \alpha^{IV} + \beta^{IV} D_i + U_i$$

where now $Cov(Z_i, U_i) = 0$. Thus "IV=RF/FS" ($\beta^{IV} = \rho^{OLS} / \pi^{OLS}$)

The 2SLS Estimator

As with OLS, we estimate IV by sample analog:

$$\widehat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)} = \frac{\widehat{\rho}^{OLS}}{\widehat{\pi}^{OLS}}$$

where $\widehat{Cov}(X_i, W_i) = \frac{1}{N}\sum_i X_i W_i - \left(\frac{1}{N}\sum_i X_i\right)\left(\frac{1}{N}\sum_i W_i\right)$,
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We will soon consider extensions of all of this, with controls / multiple instruments / etc

Angrist (1990): The “Draft Lottery Paper”

Angrist famously used Vietnam-era draft eligibility as an instrument to estimate the earnings effects of military service

- Let $Z_i \in \{0, 1\}$ be an indicator for draft eligibility, $D_i \in \{0, 1\}$ be an indicator for military service, and Y_i measure later-life earnings

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- First stage $E[D_i \mid Z_i = 1] - E[D_i \mid Z_i = 0]$: effect of eligibility on the *probability* of military service (b/c D_i is binary)
- Reduced form $E[Y_i \mid Z_i = 1] - E[Y_i \mid Z_i = 0]$: effect of eligibility on adult earnings (measured in 1971, 1981...)

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IV interprets the latter causal effect in terms of the former

IV Estimates of the Effects of Military Service on the Earnings of White Men born in 1950

Earnings year	Earnings: Mean	Earnings: Eligibility Effect	Veteran Status: Mean	Veteran Status: Eligibility Effect	Wald Estimate of Veteran Effect
	(1)	(2)	(3)	(4)	(5)
1981	16,461	-435.8 (210.5)	.267	.159 (.040)	-2,741 (1,324)
1971	3,338	-325.9 (46.6)			-2050 (293)
1969	2,299	-2.0 (34.5)			

Note: Adapted from Table 5 in Angrist and Krueger (1999) and author tabulations. Standard errors are shown in parentheses. Earnings data are from Social Security administrative records. Figures are in nominal dollars. Veteran status data are from the Survey of Program Participation. There are about 13,500 individuals in the sample.

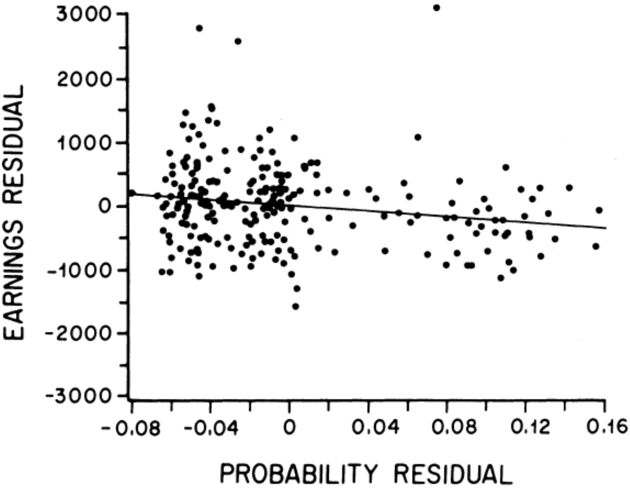

FIGURE 3. EARNINGS AND THE PROBABILITY OF VETERAN STATUS BY LOTTERY NUMBER

Notes: The figure plots mean W-2 compensation in 1981–4 against probabilities of veteran status by cohort and groups of five consecutive lottery numbers for white men born 1950–3. Plotted points consist of the average residuals (over four years of earnings) from regressions on period and cohort effects. The slope of the least-squares regression line drawn through the points is −2,384, with a standard error of 778, and is an estimate of α in the equation

$$\bar{y}_{ctj} = \beta_c + \delta_t + \hat{p}_{cj}\alpha + \bar{u}_{ctj}.$$