

# Instrumental Variables

*INTRODUCTION*

---



# Roadmap

## Introductions

- Who Am I?

- Who Am I?

- What is This Course?

## Regression Review

- Models vs. Estimands vs. Estimators

- Regression Identification and Endogeneity

## Intro to IV

- Instrument Validity and Relevance

- The 2SLS Estimator

# Who Am I?

Groos Family Assistant Professor of Economics, Brown University

# Who Am I?

Groos Family Assistant Professor of Economics, Brown University

A big fan of instrumental variable (IV) methods

# Who Am I?

Groos Family Assistant Professor of Economics, Brown University

A big fan of instrumental variable (IV) methods

- Lottery- and non-lottery IVs in studies of educational quality

(Angrist et al. 2016, 2017, 2021, 2022; Abdulkadiroğlu et al. 2016)

- Quasi-experimental evaluations of healthcare quality

(Hull 2020; Abaluck et al. 2021, 2022)

- IV-based analyses of discrimination and bias

(Arnold et al. 2020, 2021, 2022; Hull 2021; Bohren et al. 2022)

- Shift-share instruments and related designs

(Borusyak et al. 2022; Borusyak and Hull 2021, 2022; Goldsmith-Pinkham et al. 2022)

# Who Am I?

Groos Family Assistant Professor of Economics, Brown University

A big fan of instrumental variable (IV) methods

- Lottery- and non-lottery IVs in studies of educational quality

(Angrist et al. 2016, 2017, 2021, 2022; Abdulkadiroğlu et al. 2016)

- Quasi-experimental evaluations of healthcare quality

(Hull 2020; Abaluck et al. 2021, 2022)

- IV-based analyses of discrimination and bias

(Arnold et al. 2020, 2021, 2022; Hull 2021; Bohren et al. 2022)

- Shift-share instruments and related designs

(Borusyak et al. 2022; Borusyak and Hull 2021, 2022; Goldsmith-Pinkham et al. 2022)

A constant student of IV (and econometrics more generally)

# What is This Course?

A one-day intensive on IV, with focus on recent practical advances

# What is This Course?

A one-day intensive on IV, with focus on recent practical advances

- Far from comprehensive; stay tuned for more “mixtape tracks” that take deeper dives on particular topics (judge IV, etc)
- Emphasis on *practical*: IV is meant to be used, not just studied!



# What is This Course?

A one-day intensive on IV, with focus on recent practical advances

- Far from comprehensive; stay tuned for more “mixtape tracks” that take deeper dives on particular topics (judge IV, etc)
- Emphasis on *practical*: IV is meant to be used, not just studied!

Four one-hour lectures: from IV basics to recent topics

# What is This Course?

A one-day intensive on IV, with focus on recent practical advances

- Far from comprehensive; stay tuned for more “mixtape tracks” that take deeper dives on particular topics (judge IV, etc)
- Emphasis on *practical*: IV is meant to be used, not just studied!

Four one-hour lectures: from IV basics to recent topics

- Please ask questions in the Discord chat!
- I will try to stick to the schedule but may improvise slightly

# What is This Course?

A one-day intensive on IV, with focus on recent practical advances

- Far from comprehensive; stay tuned for more “mixtape tracks” that take deeper dives on particular topics (judge IV, etc)
- Emphasis on *practical*: IV is meant to be used, not just studied!

Four one-hour lectures: from IV basics to recent topics

- Please ask questions in the Discord chat!
- I will try to stick to the schedule but may improvise slightly

Two 75-minute coding labs, applying what we've learned

# What is This Course?

A one-day intensive on IV, with focus on recent practical advances

- Far from comprehensive; stay tuned for more “mixtape tracks” that take deeper dives on particular topics (judge IV, etc)
- Emphasis on *practical*: IV is meant to be used, not just studied!

Four one-hour lectures: from IV basics to recent topics

- Please ask questions in the Discord chat!
- I will try to stick to the schedule but may improvise slightly

Two 75-minute coding labs, applying what we've learned

- I will be live-coding in Stata, but R code will also be available
- Goal: demonstrate both methods & how I think about applying them

# Schedule

9:00-10:00am	Lecture 1: Regression Review; Regression Endogeneity; Introduction to IV
10:00-10:10am	<i>Break</i>
10:10-11:10am	Lecture 2: Understanding Instrument Validity; 2SLS Mechanics; Applications
11:10-11:15am	<i>Break</i>
11:15am-12:30pm	Coding Lab 1: Angrist and Krueger (1991)
12:30-1:30pm	<i>Lunch</i>
1:30-2:30pm	Lecture 3: Heterogeneous Treatment Effects; Characterizing Compliers; MTEs
2:30-2:40pm	<i>Break</i>
2:40pm-3:40pm	Lecture 4: Judge Leniency Designs; Shift-Share IV; New IV Frontiers
3:40-3:45pm	<i>Break</i>
3:45-5:00pm	Coding Lab 2: Stevenson (2018)
5:00-5:15pm	Closing Remarks

# Roadmap

## Introductions

- Who Am I?

- Who Am I?

- What is This Course?

## Regression Review

- Models vs. Estimands vs. Estimators

- Regression Identification and Endogeneity

## Intro to IV

- Instrument Validity and Relevance

- The 2SLS Estimator

# Models vs. Estimands vs. Estimators

Three distinct objects (though not always clearly distinguished)

# Models vs. Estimands vs. Estimators

Three distinct objects (though not always clearly distinguished)

- **Parameters** come from models of how observed data are generated
  - E.g. a structural supply/demand model or potential outcomes
  - They set the target for an empirical analysis: what we want to know



# Models vs. Estimands vs. Estimators

Three distinct objects (though not always clearly distinguished)

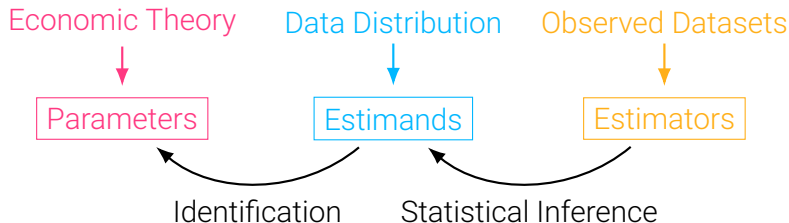
- **Parameters** come from models of how observed data are generated
  - E.g. a structural supply/demand model or potential outcomes
  - They set the target for an empirical analysis: what we want to know
- **Estimands** are functions of the population distribution of observable data
  - E.g. a difference in means or ratio of population regression coef's
  - Make assumptions to link parameters & estimands ("identification")

# Models vs. Estimands vs. Estimators

Three distinct objects (though not always clearly distinguished)

- **Parameters** come from models of how observed data are generated
  - E.g. a structural supply/demand model or potential outcomes
  - They set the target for an empirical analysis: what we want to know
- **Estimands** are functions of the population distribution of observable data
  - E.g. a difference in means or ratio of population regression coef's
  - Make assumptions to link parameters & estimands ("identification")
- **Estimators** are functions of the observed data itself (the "sample")
  - E.g. a difference in sample means or ratio of OLS coefficients
  - Since data are random, so are estimators. Each has a distribution
  - Use knowledge of estimator distributions to make learn about estimands ("inference") and—hopefully—identified parameters

# The IV Solution



This course will mostly focus on identification, but we'll cover some IV estimation / inference issues as well

# Let's Get Concrete

Human capital theory (e.g. Becker, 1957) tells us that taking one-day IV intensives are likely to boost later-life productivity

# Let's Get Concrete

Human capital theory (e.g. Becker, 1957) tells us that taking one-day IV intensives are likely to boost later-life productivity

- Parameter: returns to taking this class  $\beta$ , measured in some outcome  $Y_i$  (e.g. lifetime top-5 pubs / earnings / twitter followers)
- Simple causal/structural model:  $Y_i = \alpha + \beta D_i + \varepsilon_i$ , where  $D_i \in \{0, 1\}$  indicates taking this class

# Let's Get Concrete

Human capital theory (e.g. Becker, 1957) tells us that taking one-day IV intensives are likely to boost later-life productivity

- Parameter: returns to taking this class  $\beta$ , measured in some outcome  $Y_i$  (e.g. lifetime top-5 pubs / earnings / twitter followers)
- Simple causal/structural model:  $Y_i = \alpha + \beta D_i + \varepsilon_i$ , where  $D_i \in \{0, 1\}$  indicates taking this class

We see a sample of  $Y_i$ ,  $D_i$ , and some other covariates  $W_{1i}, \dots, W_{Ki}$

- We fire up Stata and type `reg y d w1-wk, r`. How do we interpret the output?

# Population Regression

The OLS estimator  $\hat{\beta}^{OLS}$  consistently estimates the regression estimand  $\beta^{OLS}$  under relatively weak conditions (e.g. *i.i.d.* data)

- Stata tells us  $\hat{\beta}^{OLS}$  and what we can infer about  $\beta^{OLS}$  from it
- It *doesn't* directly tell us about the relationship between  $\beta^{OLS}$  and  $\beta$

# Population Regression

The population regression of  $Y_i$  on  $\mathbf{X}_i = [1, D_i, W_{1i}, \dots, W_{Ki}]'$  is given by  $Y_i = \mathbf{X}_i' \beta^{OLS} + U_i$  where  $E[\mathbf{X}_i U_i] = 0$



# Population Regression

The population regression of  $Y_i$  on  $\mathbf{X}_i = [1, D_i, W_{1i}, \dots, W_{Ki}]'$  is given by  $Y_i = \mathbf{X}_i' \beta^{OLS} + U_i$  where  $E[\mathbf{X}_i U_i] = 0$

- Equivalently,  $\beta^{OLS} = E[\mathbf{X}_i \mathbf{X}_i']^{-1} E[\mathbf{X}_i Y_i]$  and  $U_i = Y_i - \mathbf{X}_i' \beta^{OLS}$
- $\beta^{OLS}$  contains regression coefficients;  $U_i$  is the regression residual

# Population Regression

The population regression of  $Y_i$  on  $\mathbf{X}_i = [1, D_i, W_{1i}, \dots, W_{Ki}]'$  is given by  $Y_i = \mathbf{X}_i' \beta^{OLS} + U_i$  where  $E[\mathbf{X}_i U_i] = 0$

- Equivalently,  $\beta^{OLS} = E[\mathbf{X}_i \mathbf{X}_i']^{-1} E[\mathbf{X}_i Y_i]$  and  $U_i = Y_i - \mathbf{X}_i' \beta^{OLS}$
- $\beta^{OLS}$  contains regression coefficients;  $U_i$  is the regression residual

Key point: we can always define  $\beta^{OLS}$  for any  $Y_i$  and  $\mathbf{X}_i$  (assuming no perfect collinearity); this is what Stata estimates

- Specifically it computes  $\hat{\beta}^{OLS} = (\frac{1}{N} \sum_i \mathbf{X}_i \mathbf{X}_i')^{-1} (\frac{1}{N} \sum_i \mathbf{X}_i Y_i)$  and uses large-sample asymptotics (LLN/CLT) to get a standard error

You Can't Always Get What you Want...

But what if this *estimand* is not what we want?

# You Can't Always Get What you Want...

But what if this *estimand* is not what we want?

- What if  $\beta^{OLS}$  fails to coincide with our economic parameter of interest (e.g. returns to mixtape workshops)?

# You Can't Always Get What you Want...

The model parameter in  $Y_i = \alpha + \beta D_i + \varepsilon_i$  need not coincide with the regression coefficient in  $Y_i = \alpha^{OLS} + \beta^{OLS} D_i + U_i$

- I.e. we may not have  $Cov(D_i, \varepsilon_i) = 0$  (always have  $Cov(D_i, U_i) = 0$ )

# You Can't Always Get What you Want...

The model parameter in  $Y_i = \alpha + \beta D_i + \varepsilon_i$  need not coincide with the regression coefficient in  $Y_i = \alpha^{OLS} + \beta^{OLS} D_i + U_i$

- I.e. we may not have  $Cov(D_i, \varepsilon_i) = 0$  (always have  $Cov(D_i, U_i) = 0$ )

Selection bias (a.k.a. omitted variables bias): students with higher latent earnings potential  $\varepsilon_i$  are more likely to take this class  $D_i$

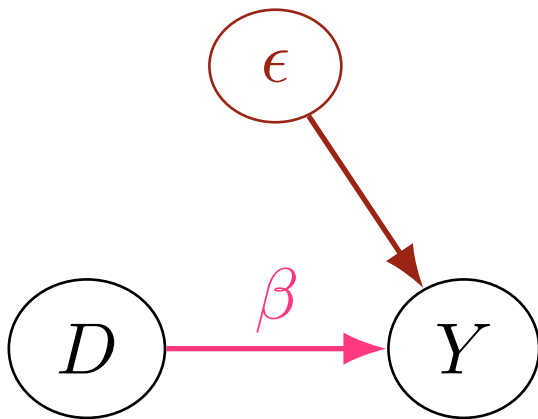
- $Cov(D_i, \varepsilon_i) > 0$  means  $\beta^{OLS} > \beta$ : overstate the returns-to-mixtape

# Can I just Control My Way Out of This?

Adding more controls (e.g. demographics) may or may not help

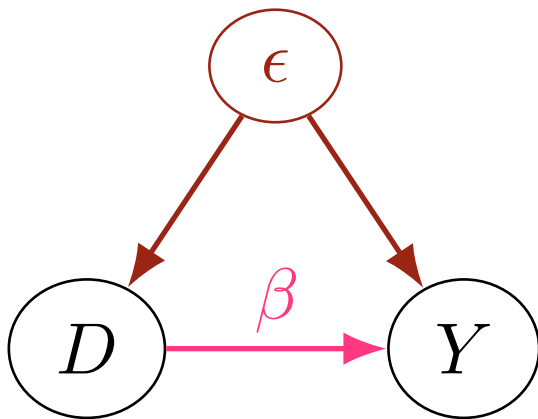
- Projecting  $\varepsilon_i$  on  $X_i$ , we get  $Y_i = \alpha + \beta D_i + \gamma X_i + \tilde{\varepsilon}_i$ ,  $Cov(X_i, \tilde{\varepsilon}_i) = 0$
- Whether or not  $Cov(D_i, \tilde{\varepsilon}_i) = 0$  depends on whether  $X_i$  sufficiently accounts for the confounding relationship  $Cov(D_i, \varepsilon_i) \neq 0$

## Regression “Exogeneity”





## Regression “Endogeneity”



## ...But Sometimes, You Get What you Need

Imagine this course was “oversubscribed,” and admission was determined by lottery

- Among those interested in taking the course, a random sample denoted by  $Z_i = 1$  was given access
- The rest, with  $Z_i = 0$  not initially given access (maybe got in later)

## ...But Sometimes, You Get What you Need

Imagine this course was “oversubscribed,” and admission was determined by lottery

- Among those interested in taking the course, a random sample denoted by  $Z_i = 1$  was given access
- The rest, with  $Z_i = 0$  not initially given access (maybe got in later)

Intuitively, this external shock  $Z_i$  should be helpful for identifying  $\beta$

- Affects  $D_i$ , so relevant to the “treatment” of interest
- Randomly assigned, so unconfounded by selection (unlike  $D_i$ )

## ...But Sometimes, You Get What you Need

Imagine this course was “oversubscribed,” and admission was determined by lottery

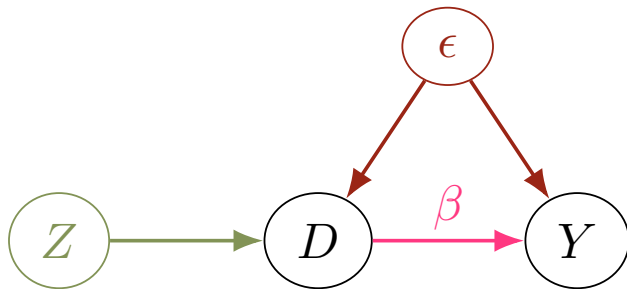
- Among those interested in taking the course, a random sample denoted by  $Z_i = 1$  was given access
- The rest, with  $Z_i = 0$  not initially given access (maybe got in later)

Intuitively, this external shock  $Z_i$  should be helpful for identifying  $\beta$

- Affects  $D_i$ , so relevant to the “treatment” of interest
- Randomly assigned, so unconfounded by selection (unlike  $D_i$ )

Indeed, this leads us to IV estimands (and estimators)

# The IV Solution



# Roadmap

## Introductions

- Who Am I?

- Who Am I?

- What is This Course?

## Regression Review

- Models vs. Estimands vs. Estimators

- Regression Identification and Endogeneity

## Intro to IV

- Instrument Validity and Relevance

- The 2SLS Estimator

# Instrument Validity and Relevance

Causal/structural model  $Y_i = \alpha + \beta D_i + \varepsilon_i$  and a candidate IV  $Z_i$

- Single  $D_i$  and  $Z_i$  and no further controls, for now

# Instrument Validity and Relevance

Causal/structural model  $Y_i = \alpha + \beta D_i + \varepsilon_i$  and a candidate IV  $Z_i$

- Single  $D_i$  and  $Z_i$  and no further controls, for now

Two key assumptions:

- Relevance:  $Z_i$  and  $D_i$  are correlated:  $Cov(Z_i, D_i) \neq 0$
- Validity:  $Z_i$  and  $\varepsilon_i$  are *uncorrelated*:  $Cov(Z_i, \varepsilon_i) = 0$



# Instrument Validity and Relevance

Causal/structural model  $Y_i = \alpha + \beta D_i + \varepsilon_i$  and a candidate IV  $Z_i$

- Single  $D_i$  and  $Z_i$  and no further controls, for now

Two key assumptions:

- Relevance:  $Z_i$  and  $D_i$  are correlated:  $Cov(Z_i, D_i) \neq 0$
- Validity:  $Z_i$  and  $\varepsilon_i$  are *uncorrelated*:  $Cov(Z_i, \varepsilon_i) = 0$

We then have identification:

$$\begin{aligned} Cov(Z_i, Y_i) &= Cov(Z_i, \alpha + \beta D_i + \varepsilon_i) = \beta Cov(Z_i, D_i) + Cov(Z_i, \varepsilon_i) \\ &= \beta Cov(Z_i, D_i), \text{ implying } \beta = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} \end{aligned}$$

# Instrument Validity and Relevance

Causal/structural model  $Y_i = \alpha + \beta D_i + \varepsilon_i$  and a candidate IV  $Z_i$

- Single  $D_i$  and  $Z_i$  and no further controls, for now

Two key assumptions:

- Relevance:  $Z_i$  and  $D_i$  are correlated:  $Cov(Z_i, D_i) \neq 0$
- Validity:  $Z_i$  and  $\varepsilon_i$  are *uncorrelated*:  $Cov(Z_i, \varepsilon_i) = 0$

We then have identification:

$$\begin{aligned} Cov(Z_i, Y_i) &= Cov(Z_i, \alpha + \beta D_i + \varepsilon_i) = \beta Cov(Z_i, D_i) + Cov(Z_i, \varepsilon_i) \\ &= \beta Cov(Z_i, D_i), \text{ implying } \beta = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} \end{aligned}$$

$$\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} \text{ is the (simple) IV estimand}$$

# Instrument Validity and Relevance

Causal/structural model  $Y_i = \alpha + \beta D_i + \varepsilon_i$  and a candidate IV  $Z_i$

- Single  $D_i$  and  $Z_i$  and no further controls, for now

Two key assumptions:

- Relevance:  $Z_i$  and  $D_i$  are correlated:  $Cov(Z_i, D_i) \neq 0$
- Validity:  $Z_i$  and  $\varepsilon_i$  are *uncorrelated*:  $Cov(Z_i, \varepsilon_i) = 0$

We then have identification:

$$\begin{aligned} Cov(Z_i, Y_i) &= Cov(Z_i, \alpha + \beta D_i + \varepsilon_i) = \beta Cov(Z_i, D_i) + Cov(Z_i, \varepsilon_i) \\ &= \beta Cov(Z_i, D_i), \text{ Implying } \beta = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} \end{aligned}$$

$\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$  is the (simple) IV estimand

- Compare to the OLS estimand:  $\beta^{OLS} = \frac{Cov(D_i, Y_i)}{Var(D_i)}$

## “Reduced Form” and “First Stage”

Note we can write

$$\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} = \frac{Cov(Z_i, Y_i)/Var(Z_i)}{Cov(Z_i, D_i)/Var(Z_i)} = \frac{\rho^{OLS}}{\pi^{OLS}}$$

where  $\rho^{OLS}$  and  $\pi^{OLS}$  are two OLS estimands:

## “Reduced Form” and “First Stage”

Note we can write

$$\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} = \frac{Cov(Z_i, Y_i)/Var(Z_i)}{Cov(Z_i, D_i)/Var(Z_i)} = \frac{\rho^{OLS}}{\pi^{OLS}}$$

where  $\rho^{OLS}$  and  $\pi^{OLS}$  are two OLS estimands:

$$Y_i = \kappa^{OLS} + \rho^{OLS} Z_i + V_i \quad \text{“reduced form”}$$

$$D_i = \mu^{OLS} + \pi^{OLS} Z_i + W_i \quad \text{“first stage”}$$

## “Reduced Form” and “First Stage”

Note we can write

$$\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} = \frac{Cov(Z_i, Y_i)/Var(Z_i)}{Cov(Z_i, D_i)/Var(Z_i)} = \frac{\rho^{OLS}}{\pi^{OLS}}$$

where  $\rho^{OLS}$  and  $\pi^{OLS}$  are two OLS estimands:

$$Y_i = \kappa^{OLS} + \rho^{OLS} Z_i + V_i \quad \text{“reduced form”}$$

$$D_i = \mu^{OLS} + \pi^{OLS} Z_i + W_i \quad \text{“first stage”}$$

Sometimes we refer to the IV estimand as the “second stage”:

$$Y_i = \alpha^{IV} + \beta^{IV} D_i + U_i$$

where now  $Cov(Z_i, U_i) = 0$ . Thus “IV=RF/FS” ( $\beta^{IV} = \rho^{OLS}/\pi^{OLS}$ )

# The 2SLS Estimator

As with OLS, we estimate IV by sample analog:

$$\hat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)} = \frac{\hat{\rho}^{OLS}}{\hat{\pi}^{OLS}}$$

where  $\widehat{Cov}(X_i, W_i) = \frac{1}{N} \sum_i X_i W_i - (\frac{1}{N} \sum_i X_i) (\frac{1}{N} \sum_i W_i)$ ,  
 $\hat{\rho}^{OLS} = \widehat{Cov}(Z_i, Y_i) / \widehat{Var}(Z_i)$ , and  $\hat{\pi}^{OLS} = \widehat{Cov}(Z_i, D_i) / \widehat{Var}(Z_i)$

# The 2SLS Estimator

As with OLS, we estimate IV by sample analog:

$$\hat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)} = \frac{\hat{\rho}^{OLS}}{\hat{\pi}^{OLS}}$$

where  $\widehat{Cov}(X_i, W_i) = \frac{1}{N} \sum_i X_i W_i - (\frac{1}{N} \sum_i X_i) (\frac{1}{N} \sum_i W_i)$ ,  
 $\hat{\rho}^{OLS} = \widehat{Cov}(Z_i, Y_i) / \widehat{Var}(Z_i)$ , and  $\hat{\pi}^{OLS} = \widehat{Cov}(Z_i, D_i) / \widehat{Var}(Z_i)$

- This is what Stata does when you type `ivreg2 y (d=z), r`
- Standard errors come from the usual large-sample asymptotics



# The 2SLS Estimator

As with OLS, we estimate IV by sample analog:

$$\hat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)} = \frac{\hat{\rho}^{OLS}}{\hat{\pi}^{OLS}}$$

where  $\widehat{Cov}(X_i, W_i) = \frac{1}{N} \sum_i X_i W_i - (\frac{1}{N} \sum_i X_i) (\frac{1}{N} \sum_i W_i)$ ,  
 $\hat{\rho}^{OLS} = \widehat{Cov}(Z_i, Y_i) / \widehat{Var}(Z_i)$ , and  $\hat{\pi}^{OLS} = \widehat{Cov}(Z_i, D_i) / \widehat{Var}(Z_i)$

- This is what Stata does when you type `ivreg2 y (d=z), r`
- Standard errors come from the usual large-sample asymptotics

We will soon consider extensions of all of this, with controls / multiple instruments / etc

## Angrist (1990): The “Draft Lottery Paper”

Angrist famously used Vietnam-era draft eligibility as an instrument to estimate the earnings effects of military service

- Let  $Z_i$  be an indicator for draft eligibility,  $D_i$  be an indicator for military service, and  $Y_i$  measure later-life earnings

# Angrist (1990): The “Draft Lottery Paper”

Angrist famously used Vietnam-era draft eligibility as an instrument to estimate the earnings effects of military service

- Let  $Z_i$  be an indicator for draft eligibility,  $D_i$  be an indicator for military service, and  $Y_i$  measure later-life earnings

Here  $\beta^{IV} = \frac{Cov(Z_i, Y_i)/Var(Z_i)}{Cov(Z_i, D_i)/Var(Z_i)} = \frac{E[Y_i|Z_i=1] - E[Y_i|Z_i=0]}{E[D_i|Z_i=1] - E[D_i|Z_i=0]}$  has a special name, because  $Z_i$  is binary: the **Wald estimand**

- First stage  $E[D_i | Z_i = 1] - E[D_i | Z_i = 0]$ : effect of eligibility on the *probability* of military service (b/c  $D_i$  is binary)
- Reduced form  $E[Y_i | Z_i = 1] - E[Y_i | Z_i = 0]$ : effect of eligibility on adult earnings (measured in 1971, 1981...)

# Angrist (1990): The “Draft Lottery Paper”

Angrist famously used Vietnam-era draft eligibility as an instrument to estimate the earnings effects of military service

- Let  $Z_i$  be an indicator for draft eligibility,  $D_i$  be an indicator for military service, and  $Y_i$  measure later-life earnings

Here  $\beta^{IV} = \frac{Cov(Z_i, Y_i)/Var(Z_i)}{Cov(Z_i, D_i)/Var(Z_i)} = \frac{E[Y_i|Z_i=1] - E[Y_i|Z_i=0]}{E[D_i|Z_i=1] - E[D_i|Z_i=0]}$  has a special name, because  $Z_i$  is binary: the **Wald estimand**

- First stage  $E[D_i | Z_i = 1] - E[D_i | Z_i = 0]$ : effect of eligibility on the *probability* of military service (b/c  $D_i$  is binary)
- Reduced form  $E[Y_i | Z_i = 1] - E[Y_i | Z_i = 0]$ : effect of eligibility on adult earnings (measured in 1971, 1981...)

IV interprets the latter causal effect in terms of the former

#### IV Estimates of the Effects of Military Service on the Earnings of White Men born in 1950

Earnings year	Earnings		Veteran Status		Wald Estimate of Veteran Effect
	Mean	Eligibility Effect	Mean	Eligibility Effect	
	(1)	(2)	(3)	(4)	(5)
1981	16,461	-435.8 (210.5)	.267	.159 (.040)	-2,741 (1,324)
1971	3,338	-325.9 (46.6)			-2050 (293)
1969	2,299	-2.0 (34.5)			

Note: Adapted from Table 5 in Angrist and Krueger (1999) and author tabulations. Standard errors are shown in parentheses. Earnings data are from Social Security administrative records. Figures are in nominal dollars. Veteran status data are from the Survey of Program Participation. There are about 13,500 individuals in the sample.

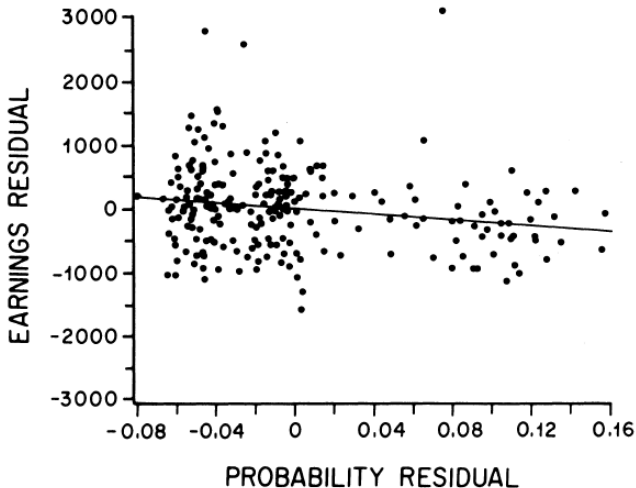


FIGURE 3. EARNINGS AND THE PROBABILITY OF VETERAN STATUS BY LOTTERY NUMBER

*Notes:* The figure plots mean W-2 compensation in 1981–4 against probabilities of veteran status by cohort and groups of five consecutive lottery numbers for white men born 1950–3. Plotted points consist of the average residuals (over four years of earnings) from regressions on period and cohort effects. The slope of the least-squares regression line drawn through the points is  $-2,384$ , with a standard error of  $778$ , and is an estimate of  $\alpha$  in the equation

$$\bar{y}_{ctj} = \beta_c + \delta_t + \hat{p}_{cj}\alpha + \bar{u}_{ctj}.$$