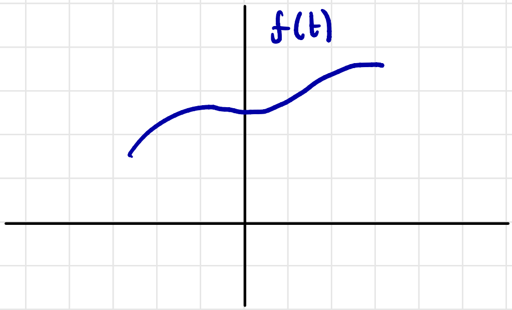
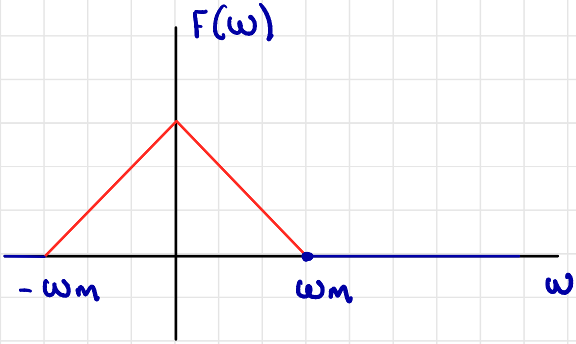
Teorema del Muestreo Uniforme en el tiempo.

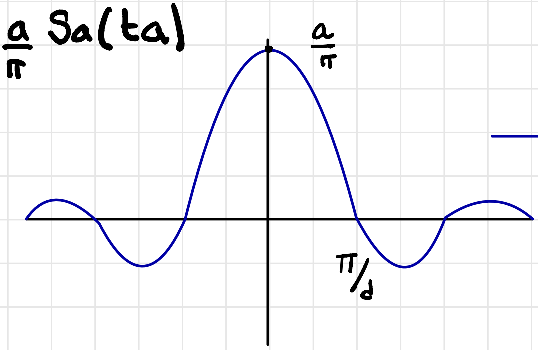
Este teorema afirma que si una señal $f(t)$ limitada en banda, es decir no contiene componentes frecuenciales superiores a f_M ciclos por segundo, $f(t)$ puede ser reconstruida a partir de sus muestras, si $f(t)$ fue muestreada a más de dos veces f_M

Otra forma de expresar el Concepto de Ancho de Banda Limitado es que $\mathcal{F}\{f(t)\} = F(\omega) = 0$ si $|\omega| > \omega_m$

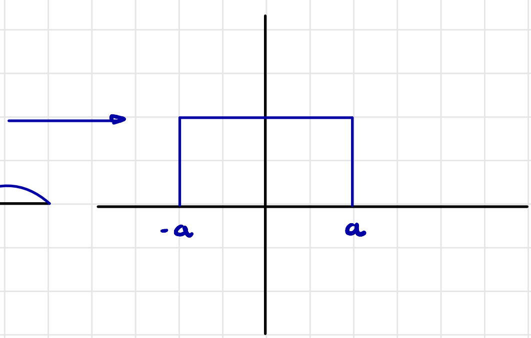
$\omega_M = 2\pi \cdot f_M$;



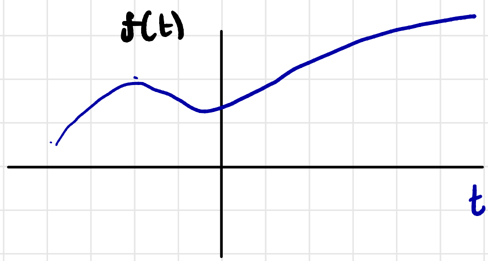


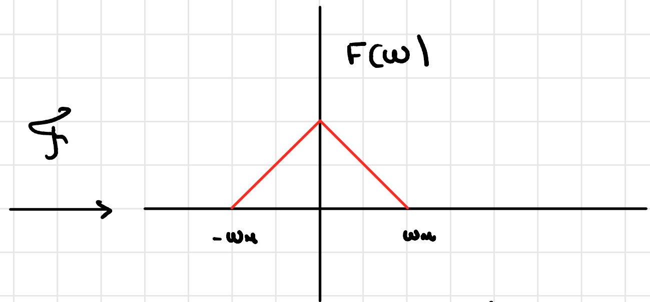


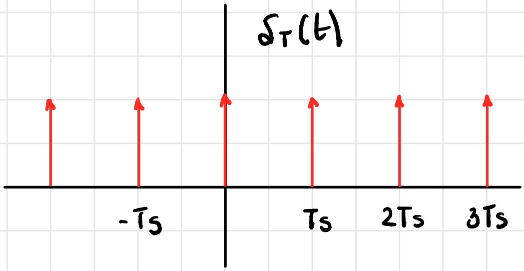
Señal Limitada en Banda

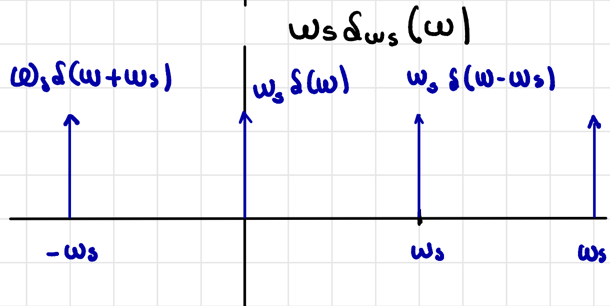


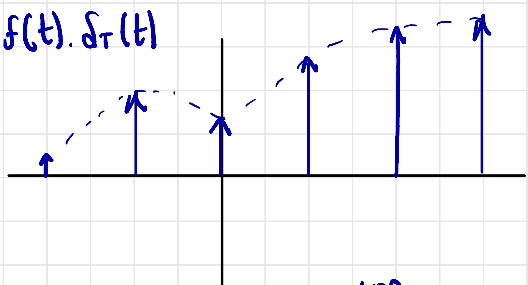
Caso 1: $\omega_s = 2\omega_M$











$$\delta_T(t).f(t) = f(t).\sum_{n=-\infty}^{+\infty} \delta(t-nT_s) \xrightarrow{\mathcal{F}} \omega_s.\delta_{\omega_s}(\omega) * F(\omega) . \frac{1}{2\pi}$$

$$\delta_T(t).f(t) = \sum_{n=-\infty}^{+\infty} f(t).\delta(t-nT_s) \xrightarrow{\mathcal{F}} \frac{1}{2\pi} . F(\omega) * \omega_s . \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_s)$$

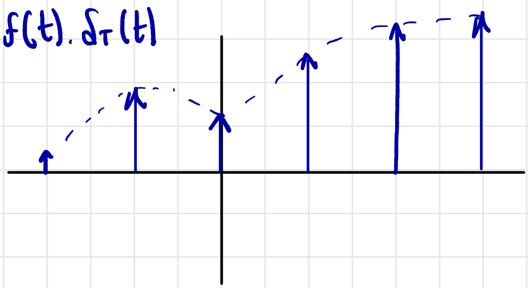
$$\boxed{\delta_T(t).f(t) = \sum_{n=-\infty}^{+\infty} f(nT_s).\delta(t-nT_s)} \xrightarrow{\mathcal{F}} \frac{\omega_s}{2\pi} . F(\omega) * \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_s)$$

$$\xrightarrow{\mathcal{F}} \frac{1}{T_s} . \sum_{n=-\infty}^{+\infty} F(\omega) * \delta(\omega - n\omega_s)$$

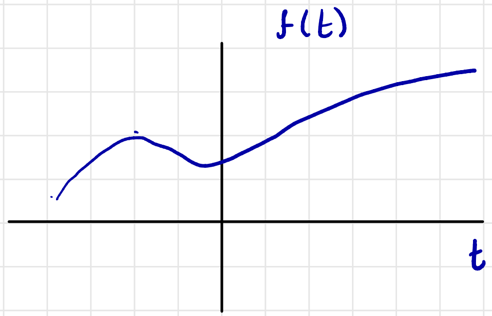
$$\xrightarrow{\mathcal{F}} \boxed{\frac{1}{T_s} \sum_{n=-\infty}^{+\infty} F(\omega - n\omega_s)}$$

$$\delta_T(t) \cdot f(t) \longrightarrow \frac{1}{T_s} \sum F(\omega - \omega_s)$$

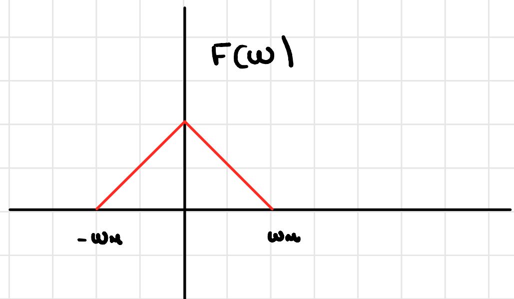
$$\delta_T(t) \cdot f(t) \xrightarrow{\mathcal{F}} \frac{1}{T_s} F_{\omega_s}(\omega)$$

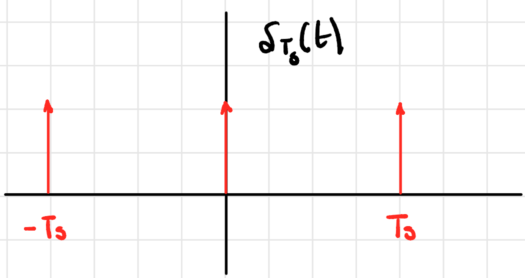


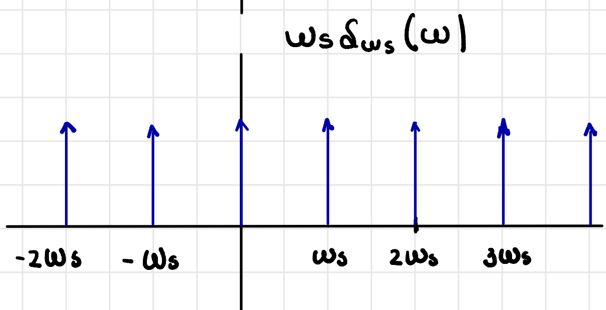
Caso 2 $\omega_s < 2\omega_M$

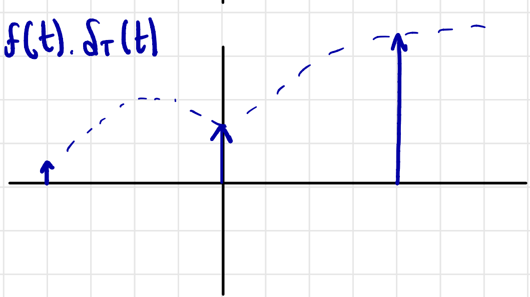


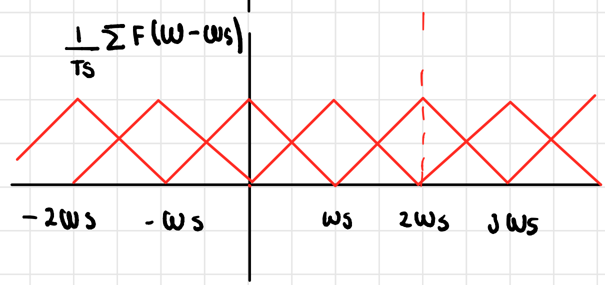
$\xrightarrow{\mathcal{F}}$



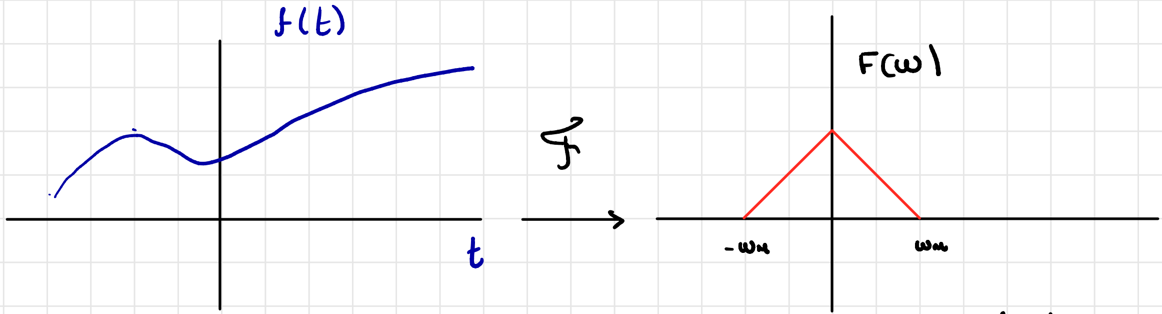


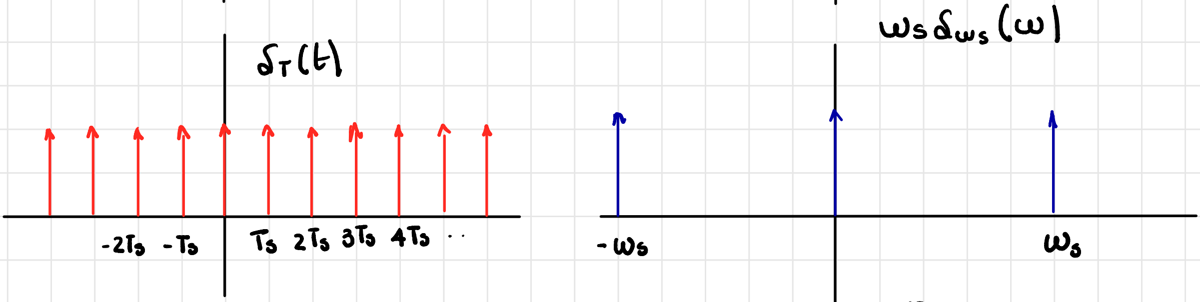


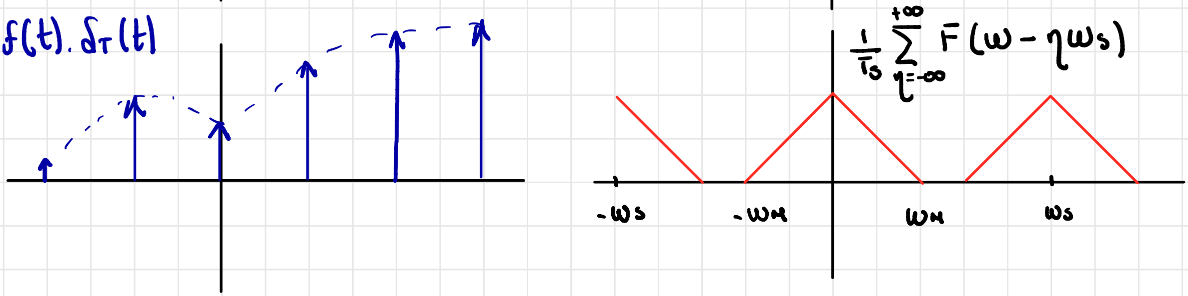




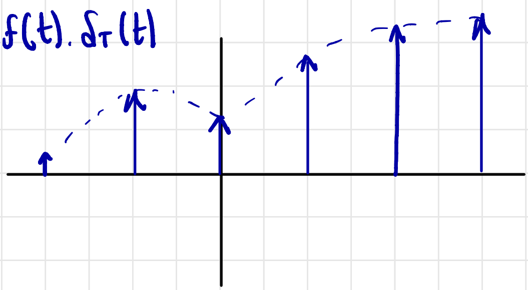
Caso $\omega_s > 2\omega_m$

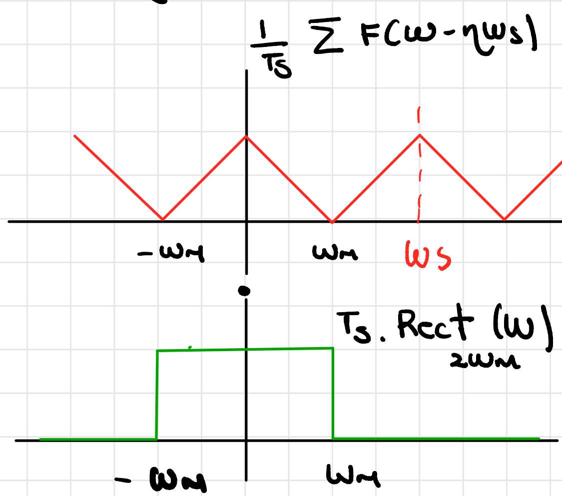




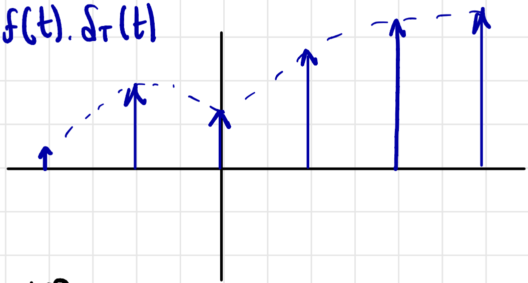


Recuperación de la señal. CASO 1 → CASO 3

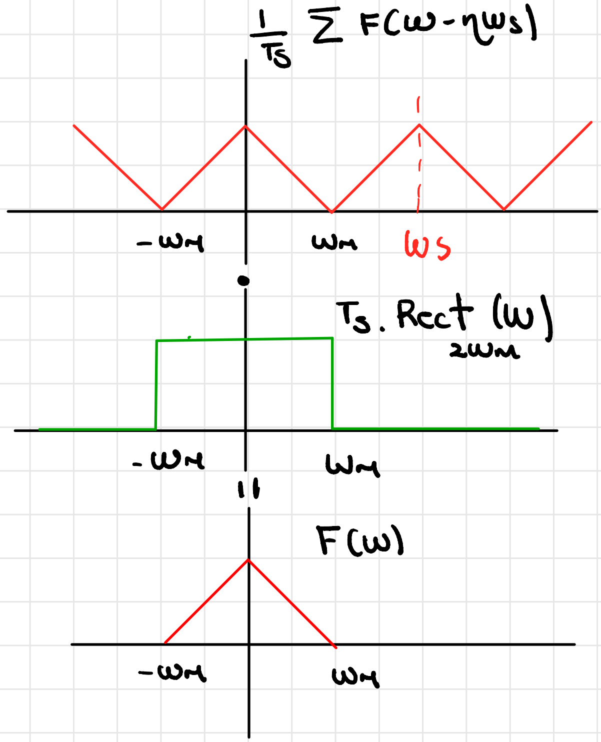




$f(t).\delta_T(t)$



$$\sum_{n=-\infty}^{+\infty} f(nT_s)\,\delta(t-nT_s)$$



$$\frac{1}{T_s}\cdot\sum_{n=-\infty}^{+\infty} F(\omega-n\omega_s).\; T_s \text{Rect}_{2\omega_M}(\omega) = F(\omega)$$

$$f(t).\delta_T(t) \;*\; \boxed{} = f(t)$$

$$\text{Rect}_d(t) \longrightarrow d\,Sa(\omega\, d/2)$$

$$2\omega_M Sa(t\omega_M) \longrightarrow 2\pi\, \text{Rect}_{2\omega_M}(t)$$

$$\frac{\omega_M}{\pi} Sa(t\omega_M) \longrightarrow \text{Rect}_{2\omega_M}(\omega)$$

$$f(t)\delta_T(t) * \frac{\omega_M}{\pi} . T_s . Sa(t\omega_M) = f(t) \qquad T_s = \frac{1}{2f_m}$$

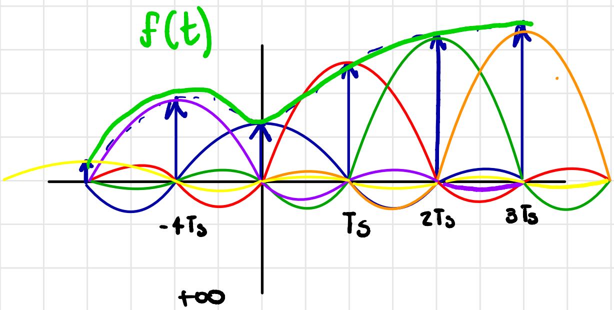
$$f(t).\delta_T(t) * \frac{2\pi . f_M .}{2\pi . f_m} . Sa(t\omega_M)$$

$$f(t).\delta_T(t) * Sa(t\omega_M) = f(t)$$

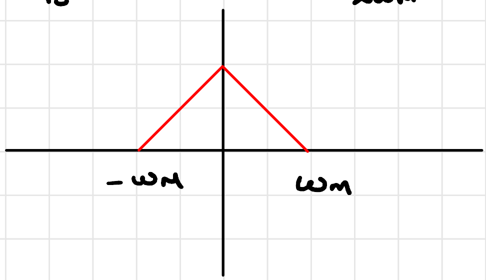
$$f(t) = \delta_T(t) f(t) * Sa(t\omega_m) =$$

$$f(t) = \sum_{n=-\infty}^{+\infty} f(nT_s) \cdot \delta(t - nT_s) * Sa(t\omega_m)$$

$$f(t) = \sum_{n=-\infty}^{+\infty} f(nT_s)\, Sa\{(t - nT_s) . \omega_m\}$$



$$\frac{1}{T_s} F(\omega - \omega_m) . T_s Rect_{2\omega_M}(\omega) = F(\omega)$$



$$f(t) = \sum_{n=-\infty}^{+\infty} f(nT_s) Sa\{(t - nT_s)\omega_m\}$$

Recordar que $T_s = \frac{1}{2f_M}$

$$f(t) = \sum_{n=-\infty}^{+\infty} f(nT_s) Sa\{(t - \frac{n}{2f_m})\omega_m\}$$