Forma trigonométrica de Fourier 1ª FTF

Base trigonométrica $\eta \in \mathbb{Z}^+$

$$\boxed{\{1, \cos w_0 t, \cos 2w_0 t, \cos 3w_0 t, \dots \cos \eta w_0 t, \; \operatorname{sen} w_0 t, \operatorname{sen} 2w_0 t, \operatorname{sen} 3w_0 t, \dots \operatorname{sen} \eta w_0 t\}}$$

Ortogonal en un T

Representación

$$x(t) = a_0 + \sum_{\eta=1}^{+\infty} a_\eta \cos \eta w_0 t + \sum_{\eta=1}^{+\infty} b_\eta \operatorname{sen} \eta w_0 t$$

$$a_0 = \frac{1}{T}\int_T x(t) \qquad a_\eta = \frac{2}{T}\int_T x(t).\cos \eta w_0 t$$

$$b_\eta = \frac{2}{T}\int_T x(t).\operatorname{Sen} \eta w_0 t$$

Forma Compacta. 2da FTF

Base $\{\cos(\eta w_0 t + \theta_\eta)\}$ $\eta \in \mathbb{Z}^+$

Ortogonal en un T

Representación

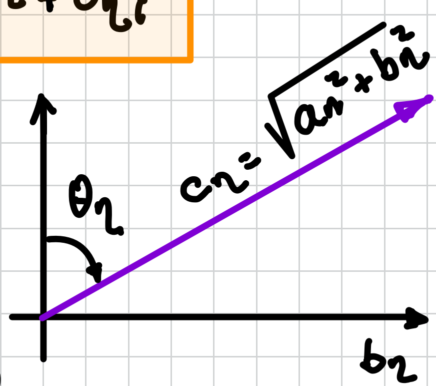
$$\boxed{x(t) = C_0 + \sum_{\eta=1}^{+\infty} C_\eta . \cos(\eta w_0 t + \theta_\eta)}$$

$C_0 = a_0$

$C_\eta = \sqrt{a_\eta^2 + b_\eta^2}$

$\theta_\eta = -\arctan(b_n/a_n)$ $a_n > 0$

$\theta_\eta = \pi - \arctan(b_n/a_n)$ $a_n < 0$



Forma compleja de Fourier 3ª FTF

Base Compleja de Fourier

$$\left\{ \ldots e^{-jn\omega_0 t}, \ldots F_{-3} e^{-j3\omega_0 t}, F_{-2} e^{-j2\omega_0 t}, F_{-1} e^{-j\omega_0 t}, 1, \ldots e^{jn\omega_0 t} \ldots F_3 e^{j3\omega_0 t}, F_2 e^{j2\omega_0 t}, F_1 e^{j\omega_0 t} \right\}$$

$n \in \mathbb{Z}$
Ortogonal en un T

Representación

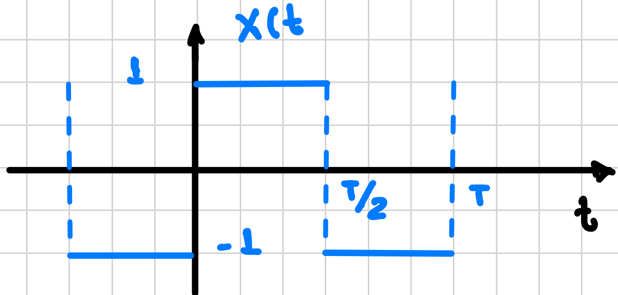
$$x(t) = \ldots F_{-2} e^{-j2\omega_0 t} + F_{-1} e^{-j\omega_0 t} + F_0 + F_1 e^{j\omega_0 t} + F_2 e^{-j2\omega_0 t} + \ldots$$

$$\boxed{x(t) = \sum_{n=-\infty}^{+\infty} F_n \cdot e^{jn\omega_0 t}}$$

Donde $F_n = \frac{1}{T} \int_T x(t) \cdot e^{-jn\omega_0 t} \, dt$

Ejercicio dada la señal

$$x(t) = \begin{cases} 1 & 0 < t < T/2 \\ -1 & T/2 < t < T \\ x(t) = x(t+T) & \end{cases}$$



Graficar los espectros de $x(t)$ utilizando la

- Representación trigonométrica de Fourier o 1ª FTF
- Representación compacta de Fourier o 2ª FTF
- Representación Compleja de Fourier o 3ª FTF

Representación trigonométrica de Fourier

$$\boxed{x(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{+\infty} b_n \operatorname{sen} n\omega_0 t}$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = 0$$

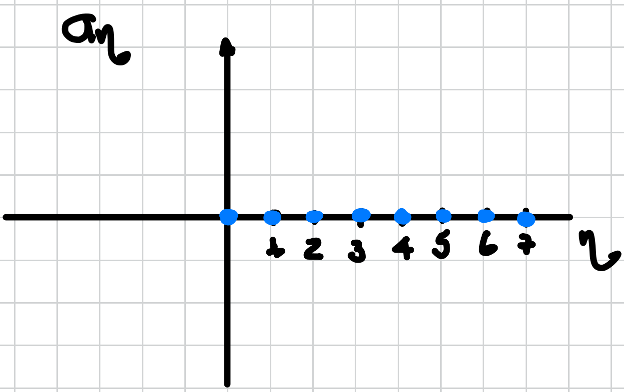
$$a_n = \frac{2}{T}\int_T x(t)\cdot\cos n\omega_0 t\, dt = 0$$

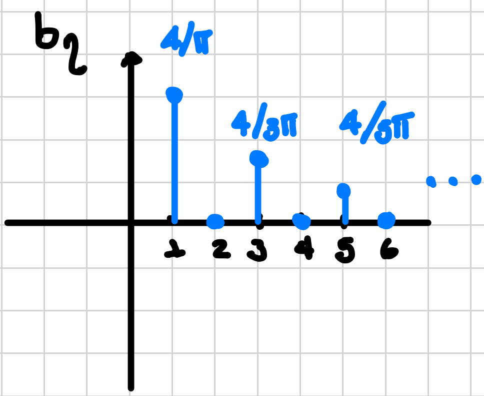
$$b_n = \frac{2}{T}\int_T x(t)\cdot \operatorname{sen} n\omega_0 t\,dt = \begin{cases} b_{2n}=0 \\ b_{2n-1} = \dfrac{4}{(2n-1)\pi}\end{cases}$$

$$x(t) = \sum_{n=1}^{+\infty} \frac{4}{\pi(2n-1)}\cdot \operatorname{sen}[(2n-1)\omega_0 t]$$

$$\boxed{x(t) = \frac{4}{\pi}\operatorname{sen}\omega_0 t + \frac{4}{3\pi}\operatorname{sen}3\omega_0 t + \frac{4}{5\pi}\operatorname{sen}5\omega_0 t + \cdots}$$

Dado lo anterior su gráficos espectrales son





Representación Compacta de Fourier.

$$\boxed{x(t) = C_0 + \sum_{n=1}^{+\infty} C_n \cdot \cos(n\omega_0 t + \theta_n)}$$

Por lo tanto

$$C_0 = \sqrt{a_0^2}$$

$$C_n = \sqrt{a_n^2 + b_n^2} = \sqrt{b_n^2}$$

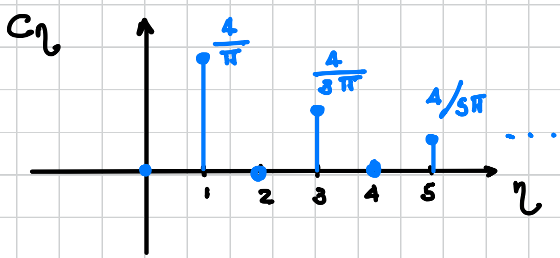
$$C_{2n} = \sqrt{a_{2n}^2 + b_{2n}^2} = 0$$ dado que $a_{2n} = 0$
$b_{2n} = 0$

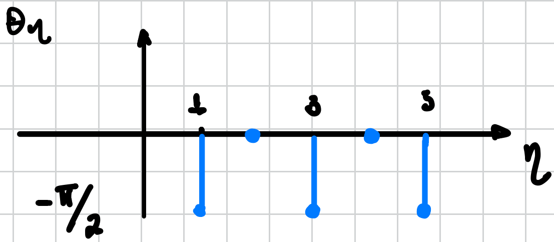
$$C_{2n-1} = \sqrt{\left(\frac{4}{(2n-1)\pi}\right)^2} = \left|\frac{4}{(2n-1)\pi}\right|$$

$$\theta_{2n-1} = -\arctan\left(b_{2n-1}/a_{2n-1}\right) = -\pi/2$$

$$x(t) = 4/\pi \cos(\omega_0 t - \pi/2) + 4/3\pi \cos(3\omega_0 t - \pi/2) + \frac{4}{5\pi}\cos(5\omega_0 t - \pi/2) + \dots$$

En general $\boxed{x(t) = \sum_{n=1}^{+\infty} \frac{4}{(2n-1)\pi} \cdot \cos((2n-1)\omega_0 t - \pi/2)}$





Representación Compleja de Fourier.

$$\boxed{x(t) = \sum_{n=-\infty}^{+\infty} F_n \cdot e^{jn\omega_0 t}}$$

Por lo tanto $\boxed{F_n = \frac{a_n - jb_n}{2}}$ $\boxed{F_{-n} = \frac{a_n + jb_n}{2}}$

$$F_0 = a_0 = 0$$

$$F_{2n} = \frac{a_{2n} - jb_{2n}}{2} = 0$$

$$F_{2n-1} = \frac{a_{2n-1} - jb_{2n-1}}{2} = \frac{-jb_{2n-1}}{2}$$

$$F_{2n-1} = \frac{-j}{2}\frac{4}{(2n-1)\pi} = -j\frac{2}{(2n-1)\pi}$$

$F_{2n-1} = F_n$ para n: impar

$$|F_{2n-1}| = \sqrt{\left(\frac{2}{(2n-1)\pi}\right)^2} = \frac{2}{(2n-1)\pi}$$

$$\theta_{2n-1} = -\pi/2$$

$$F_{-(2n-1)} = \frac{a_{2n-1} + jb_{2n-1}}{2} = \frac{jb_{2n-1}}{2} = \frac{j}{2}\frac{4}{(2n-1)\pi}$$

$$|F_{-(2n-1)}| = \frac{2}{(2n-1)\pi}$$

$$\theta_{-(2n-1)} = \pi/2$$

$F_{-(2n-1)} = F_{-n}$ para n impar

$$x(t) = \frac{2}{5\pi} \cdot e^{j\pi/2} \cdot e^{-j5\omega_0 t} + \frac{2}{3\pi} \cdot e^{j\pi/2} \cdot e^{-j3\omega_0 t} + \frac{2}{\pi} e^{j\pi/2} e^{-j\omega_0 t}$$

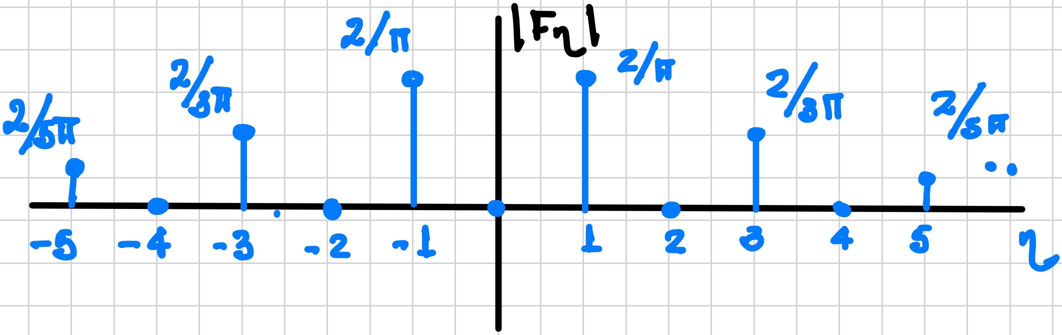
$$+ \frac{2}{5\pi} \cdot e^{-j\pi/2} e^{j5\omega_0 t} + \frac{2}{3\pi} e^{-j\pi/2} e^{j3\omega_0 t} + \frac{2}{\pi} e^{-j\pi/2} \cdot e^{j\omega_0 t}$$

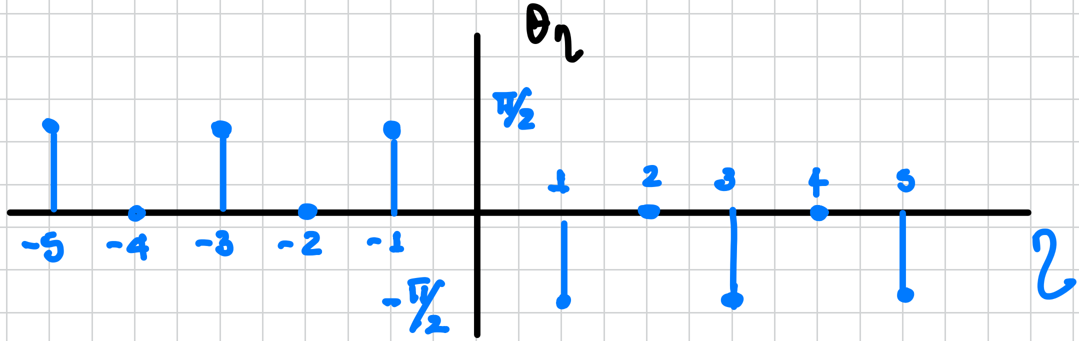
En general $x(t)$ tendrá 2 Partes

La primera: $$\sum_{n=1}^{+\infty} F_{-n} \cdot e^{-jn\omega_0 t} = \sum_{n=1}^{+\infty} \frac{2 \cdot e^{j\pi/2}}{(2n-1)\pi} \cdot e^{-j(2n-1)\omega_0 t}$$

y la segunda $$\sum_{n=1}^{+\infty} F_n \cdot e^{jn\omega_0 t} = \sum_{n=1}^{+\infty} \frac{2 \cdot e^{-j\pi/2}}{(2n-1)\pi} \cdot e^{j(2n-1)\omega_0 t}$$

$$x(t) = \sum_{n=1}^{+\infty} \frac{2 \cdot e^{j\pi/2}}{(2n-1)\pi} \cdot e^{-j(2n-1)\omega_0 t} + \sum_{n=1}^{+\infty} \frac{2 \cdot e^{-j\pi/2}}{(2n-1)\pi} \cdot e^{j(2n-1)\omega_0 t}$$





$$x(t) = \sum_{n=1}^{+\infty} \frac{2}{(2n-1)\pi} e^{j\pi/2} \cdot e^{-j(2n-1)w_0 t} + \sum_{n=1}^{+\infty} \frac{2}{2n-1} e^{-j\frac{\pi}{2}} \cdot e^{j(2n-1)w_0 t}$$

$$x(t) = \sum_{n=1}^{+\infty} \frac{2}{(2n-1)\pi} \cdot e^{-j[(2n-1)w_0 t - \pi/2]} + \sum_{n=1}^{+\infty} \frac{2}{(2n-1)\pi} \cdot e^{j[(2n-1)w_0 t - \pi/2]}$$

$$x(t) = \sum_{n=1}^{+\infty} \frac{2 \cdot 2}{(2n-1)\pi} \cdot \left(\frac{e^{j((2n-1)w_0 t - \pi/2)} + e^{-j((2n-1)w_0 t - \pi/2)}}{2} \right)$$

$$\boxed{x(t) = \sum_{n=1}^{+\infty} \frac{4}{(2n-1)\pi} \cdot \cos\left((2n-1)w_0 t - \pi/2\right)}$$

Ejercicio:

Dada la señal $x(t)$, graficar los espectros en función de la potencia del espectro para las representaciones trigonométrica de Fourier, Compacta de Fourier y Compleja de Fourier. Verificar la señal representada.

$$\boxed{x(t) = \begin{cases} 1/T \cdot t & 0 < t < T \\ x(t+T) = x(t) & \end{cases}}$$