Análisis de Señales

La serie discreta de Fourier.

$$x(t) \longrightarrow F_\eta \qquad \eta = 0, \pm 1, \pm 2 \ldots$$

$$x(t) = \sum_{\eta=-\infty}^{+\infty} F_\eta \cdot e^{j\eta\omega_0 t} \longrightarrow x(n) = \sum \quad ?$$

$$e^{j\omega t} \longrightarrow e^{j2\pi \cdot F \cdot t}$$

$$e^{j2\pi \cdot F \cdot nT_s}$$

$$e^{j2\pi \cdot \frac{F}{F_s} \cdot n}$$

$$e^{j2\pi f \cdot n}$$

$$e^{j2\pi \frac{K}{N} \cdot n}$$

$$\boxed{x(n) = \sum_{K=0}^{N-1} F_K \, e^{j2\pi \cdot \frac{K}{N} \cdot n}}$$

El rango de Frecuencias de las señales continuas va desde $-\infty$ a $+\infty$

El rango de frecuencias de las señales discretas en el tiempo se limita al intervalo $-\pi$ a π

Una señal discreta en el tiempo de periodo Fundamental N puede constar de componentes separadas $\frac{2\pi}{N}$ ó $f = \frac{1}{N}$

En consecuencia, la representación en series de Fourier de una señal periódica discreta en el tiempo contendrá como máximo N componentes de frecuencia

$x(n)$ se describe totalmente a partir de N Armónicas

Porque los espectros son periódicos de Periodo N.

Mundo Análogo	Mundo Discreto
$x(t) = \sum_{\eta=-\infty}^{+\infty} F_\eta \cdot e^{j\eta\omega_0 t}$	$\boxed{x(n) = \sum_{K=0}^{N-1} F_K \cdot e^{j2\pi \frac{K}{N} \cdot \eta}}$

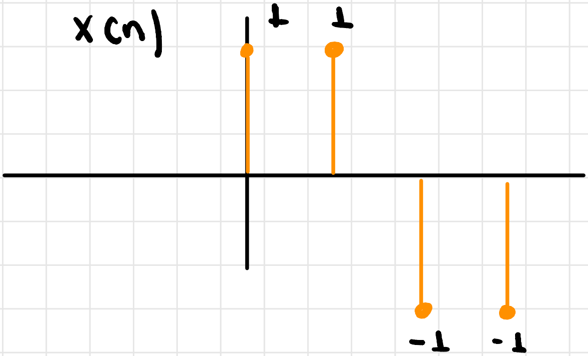
$$F_n = \frac{1}{T}\int_T x(t)\cdot e^{-jn\omega_0 t}\,dt$$

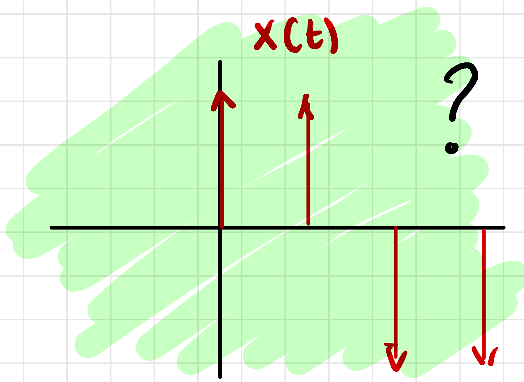
$$\boxed{F_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\cdot e^{-j2\pi\frac{k}{N}\cdot n}}$$

Ejemplo.

Dada la función $x(n) = \{\underset{\uparrow}{1}, 1, -1, -1\}$ $N=4$

Hallar su representación





No confundir con

$$F_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\cdot e^{-j2\pi\frac{k}{N}\cdot n} \qquad n = 0 \,..\, N-1$$

$$F_0 = \frac{1}{4}\cdot\{x(0)\cdot e^{-j2\pi\frac{0}{4}\cdot 0} + x(1)\,e^{-j2\pi\cdot\frac{0}{4}\cdot 1} + x(2)\cdot e^{-j2\pi\cdot\frac{0}{4}\cdot 2} + x(3)\cdot e^{-j2\pi\cdot\frac{0}{4}\cdot 3}\}$$

$$F_0 = \frac{1}{4}\{1\cdot e^0 + 1\cdot e^0 + (-1)e^0 + (-1)e^0\}$$

$$F_0 = 0 \quad \checkmark$$

$$F_1 = \frac{1}{4} \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi \frac{1}{4} \cdot n} \qquad n = 0 \ldots N-1.$$

$$F_1 = \frac{1}{4} \{ x(0) \cdot e^{-j2\pi \frac{1}{4} \cdot 0} + x(1) \cdot e^{-j2\pi \cdot \frac{1}{4} \cdot 1} +$$

$$x(2) \cdot e^{-j2\pi \frac{1}{4} \cdot 2} + x(3)\, e^{-j2\pi \cdot \frac{1}{4} \cdot 3} \}$$

$$F_1 = \frac{1}{4} \{ 1\, e^{0} + 1\, e^{-j\pi/2} + (-1) e^{-j\pi} + (-1)\, e^{-j3\pi/2} \}$$

$$F_1 = \frac{1}{4} \{ 1 + 1 e^{-j\pi/2} - 1 e^{-j\pi} - 1 e^{-j3\pi/2} \}$$

$$e^{-j\pi/2} = \cos \pi/2 - j \operatorname{sen} \pi/2 = -j$$

$$e^{-j\pi} = \cos \pi - j \operatorname{sen} \pi = -1$$

$$e^{-j3\pi/2} = \cos 3\pi/2 - j \operatorname{sen} 3\pi/2 = j$$

$$F_1 = \frac{1}{4} (1 - j + 1 - j) = \frac{1}{4} (2 - 2j) = \frac{1}{2} (1 - j)$$

$$F_1 = |F_1|\, e^{j\theta_1}$$

$$|F_1| = \sqrt{1 + 1} \times \frac{1}{2} = \frac{\sqrt{2}}{2}$$

$$\theta_1 = \arctan(-1/1) = -\pi/4$$

$$F_1 = \frac{\sqrt{2}}{2} \cdot e^{-j\pi/4}$$

$$F_2 = \frac{1}{4} \sum_{n=0}^{N-1} x(n) . e^{-j2\pi \frac{2}{4} . n} \qquad n = 0 \ldots N-1$$

$$F_2 = \frac{1}{4} \{ x(0) . e^{-j2\pi . \frac{2}{4} . 0} + x(1) . e^{-j2\pi . \frac{2}{4} . 1} +$$

$$x(2) . e^{-j2\pi \frac{2}{4} . 2} + x(3) . e^{-j2\pi . \frac{2}{4} . 3} \}$$

$$F_2 = \frac{1}{4} \{ 1 . e^{-j0} + 1 \, e^{-j\pi} - 1 \, e^{-j2\pi} - 1 \, e^{-j3\pi} \}$$

$$e^{-j\pi} = \cos\pi - j \operatorname{sen}\pi = -1$$

$$e^{-j2\pi} = \cos 2\pi - j \operatorname{sen} 2\pi = 1$$

$$e^{-j3\pi} = \cos 3\pi - j \operatorname{sen} 3\pi = -1$$

$$F_2 = \frac{1}{4} \{ 1 + 1 . (-1) - 1 . 1 - 1 . (-1) \}$$

$$F_2 = \frac{1}{4} \{ 1 - 1 - 1 + 1 \} = 0$$

$$\boxed{F_2 = 0 .}$$

$$F_3 = \frac{1}{4} \sum_{n=0}^{N-1} x(n) . e^{j2\pi \frac{3}{4} . n}$$

$$F_3 = \frac{1}{4} \{ x(0) . e^{-j2\pi \frac{3}{4} . 0} + x(1) e^{-j2\pi . \frac{3}{4} . 1} + x(2) e^{-j2\pi \frac{3}{4} 2}$$

$$F_3 = \frac{1}{4}\left\{ x(0)\cdot e^{-j2\pi\frac{3}{4}\cdot 0} + x(1)\, e^{-j2\pi\cdot\frac{3}{4}\cdot 1} + x(2)\, e^{-j2\pi\frac{3}{4}2} + x(3)\cdot e^{-j2\pi\frac{3}{4}\cdot 3} \right\}$$

$$F_3 = \frac{1}{4}\left\{ 1\, e^{j0} + 1\, e^{-j3\pi/2} - 1\, e^{-j3\pi} - 1\, e^{-j9\pi/2} \right\}$$

$$e^{-j3\pi/2} = \cos 3\pi/2 - j \operatorname{sen} 3\pi/2 = j$$

$$e^{-j3\pi} = \cos 3\pi - j \operatorname{sen} 3\pi = -1$$

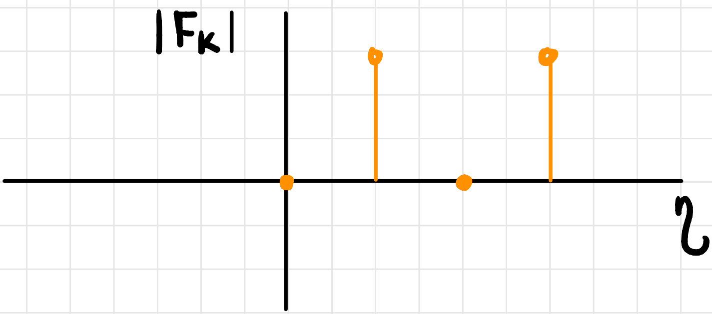
$$e^{-j\frac{9\pi}{2}} = \cos\frac{9\pi}{2} - j \operatorname{sen}\frac{9\pi}{2} = -j$$

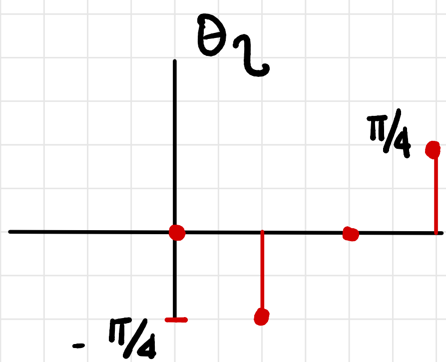
$$F_3 = \frac{1}{4}\left\{ 1 + j - 1(-1) - 1(-j) \right\}$$

$$F_3 = \frac{1}{4}\left\{ 2 + 2j \right\} \qquad F_3 = \frac{1}{2}(1+j)$$

$$F_3 = \frac{\sqrt{2}}{2}\, e^{j\pi/4}$$

$$F_K = \left\{ 0,\ \frac{\sqrt{2}}{2} e^{-j\pi/4},\ 0,\ \frac{\sqrt{2}}{2} e^{j\pi/4} \right\}$$





Para expresar x(n).

$$x(n) = \sum_{K=0}^{N-1} F_K \, e^{j2\pi \cdot \frac{K}{N} \cdot n}$$

$$x(n) = \sum_{K=0}^{3} F_K \cdot e^{j2\pi \frac{K}{4} \cdot n} \qquad K = 0 .. N-1$$

$$x(0) = \left\{ F_0 \cdot e^{j2\pi \cdot \frac{0}{4} \cdot 0} + F_1 e^{j2\pi \frac{1}{4} \cdot 0} + F_2 e^{j2\pi \frac{2}{4} \cdot 0} \right.$$

$$\left. + F_3 \cdot e^{j2\pi \frac{3}{4} \cdot 0} \right\}$$

$$x(0) = \left\{ 0 \cdot e^0 + \frac{\sqrt{2}}{2} e^{-j\pi/4} \cdot e^0 + 0 + \frac{\sqrt{2}}{2} e^{j\pi/4} \right\}$$

$$x(0) = \frac{\sqrt{2}}{2} \left(e^{-j\pi/4} + e^{j\pi/4} \right)$$

$$x(0) = \frac{\sqrt{2}}{2} \times 2 \cos \pi/4$$

$$x(0) = \frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}} = 1.$$

$$x(1) = \sum_{K=0}^{N-1} F_K \cdot e^{j2\pi \frac{K}{4} \cdot 1}$$

$$x(1) = \left\{ F_0 \cdot e^{j2\pi \cdot \frac{0}{4} \cdot 1} + F_1 e^{j2\pi \frac{1}{4} \cdot 1} + 0 \cdot e^{j2\pi \frac{2}{4} \cdot 1} + F_3 e^{j2\pi \frac{3}{4} \cdot 1} \right\}$$

$$x(1) = \left\{ \frac{\sqrt{2}}{2} \cdot e^{-j\pi/4} \cdot e^{j\pi/2} + \frac{\sqrt{2}}{2} e^{j\pi/4} \cdot e^{j3\pi/2} \right\}$$

$$x(1) = \left\{ \frac{\sqrt{2}}{2} \cdot e^{-j\pi/4} \cdot e^{j\pi/2} + \frac{\sqrt{2}}{2} e^{j\pi/4} \cdot e^{j3\pi/2} \right\}$$

$$x(1) = \left\{ \frac{\sqrt{2}}{2} e^{j\pi/4} + \frac{\sqrt{2}}{2} e^{j7\pi/4} \right\}$$

$$x(1) = \frac{\sqrt{2}}{2} \left\{ 2 . \cos \pi/4 \right\} = 1 .$$

$$x(2) = \sum_{k=0}^{N-1} F_k e^{j2\pi \frac{k}{4} . 2}$$

$$x(2) = \left\{ \frac{\sqrt{2}}{2} e^{-\pi/4} \cdot e^{j2\pi . \frac{1}{4} . 2} + \frac{\sqrt{2}}{2} e^{j\pi/4} \cdot e^{j2\pi \frac{3}{4} . 2} \right\}$$

$$x(2) = \left\{ \frac{\sqrt{2}}{2} e^{-\pi/4} \cdot e^{j\pi} + \frac{\sqrt{2}}{2} e^{j\pi/4} \cdot e^{j3\pi} \right\}$$

$$x(2) = \left\{ \frac{\sqrt{2}}{2} e^{j3\pi/4} + \frac{\sqrt{2}}{2} e^{j13\pi/4} \right\}$$

$$x(2) = \frac{\sqrt{2}}{2} \left\{ e^{j3\pi/4} + e^{j13\pi/4} \right\}$$

$$e^{j3\pi/4} = -\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}}$$

$$e^{j13\pi/4} = -\frac{1}{\sqrt{2}} - j \frac{1}{\sqrt{2}}$$

$$\overline{e^{j3\pi/4} + e^{j13\pi/4} = -\frac{2}{\sqrt{2}}} \Rightarrow x(2) = \frac{\sqrt{2}}{2} \cdot -\frac{2}{\sqrt{2}} = -1$$

$$X(3) = \sum_{k=0}^{N-1} F_k \cdot e^{j2\pi \frac{k}{4} \cdot 3}$$

$$X(3) = \left\{ \frac{\sqrt{2}}{2} e^{-j\pi/4} \cdot e^{j2\pi \frac{1}{4} \cdot 3} + \frac{\sqrt{2}}{2} e^{j\pi/4} \cdot e^{j2\pi \frac{3}{4} \cdot 3} \right\}$$

$$X(3) = \frac{\sqrt{2}}{2} \left\{ e^{-j\pi/4} \cdot e^{j3\pi/2} + e^{j\pi/4} \cdot e^{j9\pi/2} \right\}$$

$$X(3) = \frac{\sqrt{2}}{2} \left\{ e^{+j5\pi/4} + e^{j19\pi/4} \right\}$$

$$X(3) = \frac{\sqrt{2}}{2} \left\{ -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \right\}$$

$$X(3) = \frac{\sqrt{2}}{2} \cdot -\frac{2}{\sqrt{2}}$$

$$X(3) = -1.$$

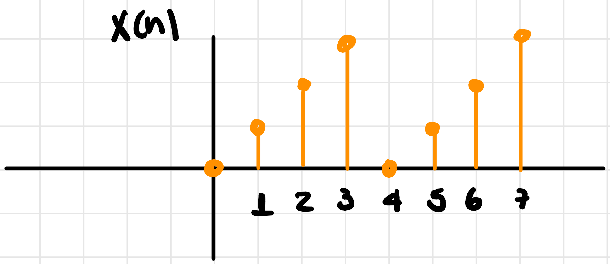
$$\boxed{x(n) = \{1, 1, -1, -1\}}$$

Ejercicio.

Desarrollar un programa en Colab, debidamente Comentado.

Donde dado una señal discreta x(n), se calcule los Componentes frecuenciales; Valide y Grafique Los Resultados.

Calcule Manualmente, los componentes armonicos de la señal $x(n)$.



$x(n) = \{0, 1, 2, 3\}$ $\quad N = 4.$

Verifique nuevamente la reconstrucción de $x(n)$.

Teorema de Parseval.

$$P_{x(n)} = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

$$P_{señal} = \sum_{k=0}^{N-1} |F_k|^2 \cdot \cancel{P_n}^{\,1}$$

$$P_{señal} = \frac{1}{4}\left((1)^2 + (1)^2 + (-1)^2 + (-1)^2 \right) = \frac{4}{4} = 1$$

$$P_{señal} = \left(0 + \left(\frac{\sqrt{2}}{2}\right)^2 + 0 + \left(\sqrt{2}/2\right)^2 \right)$$

$$P_{señal} = \frac{2}{4} + \frac{2}{4} = 1 \checkmark$$

Finalmente, estudiar el siguiente código, en especial la función fft.

https://colab.research.google.com/drive/1J2buim7mr0yzNK4vs649inezz7pAI7Qd#scrollTo=VLAJrZ6mNGRm