

Phenomenology of Majorana zero modes in full-shell hybrid nanowires

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Outline

- ① Engineering topologically protected edge states**
- ② Signals in the LDOS: CdGM analogs**
- ③ Opening the topological minigap**
- ④ Conclusions**

The Kitaev chain

- Chain of N spin-less fermions (p -wave superconductivity):

$$H = -\mu \sum_{j=1}^N \left(c_j^\dagger c_j - \frac{1}{2} \right) + \sum_{j=1}^{N-1} \left[-t (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \Delta (c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger) \right]$$



R. Aguado 2017, *Rivista del Nuovo Cimento*.
E. Prada *et al.* 2020, *Nature Reviews Physics*.
A. Y. Kitaev 2001, *Physics-Uspekhi*.

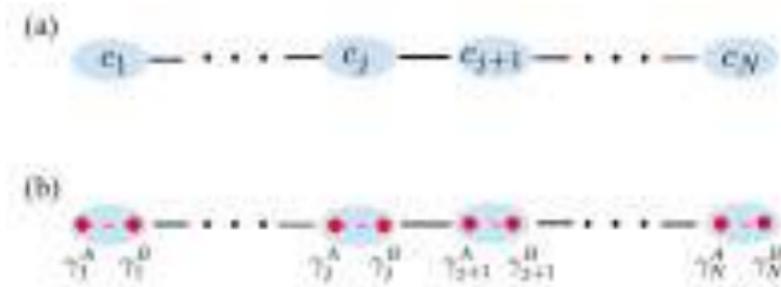
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- ▶ Majorana representation:

$$c_j = \frac{1}{2} (\gamma_j^A + i\gamma_j^B), \quad c_j^\dagger = \frac{1}{2} (\gamma_j^A - i\gamma_j^B)$$



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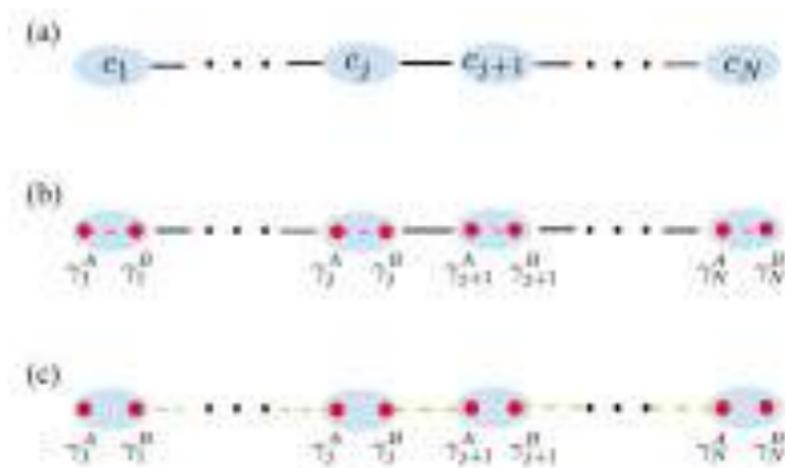
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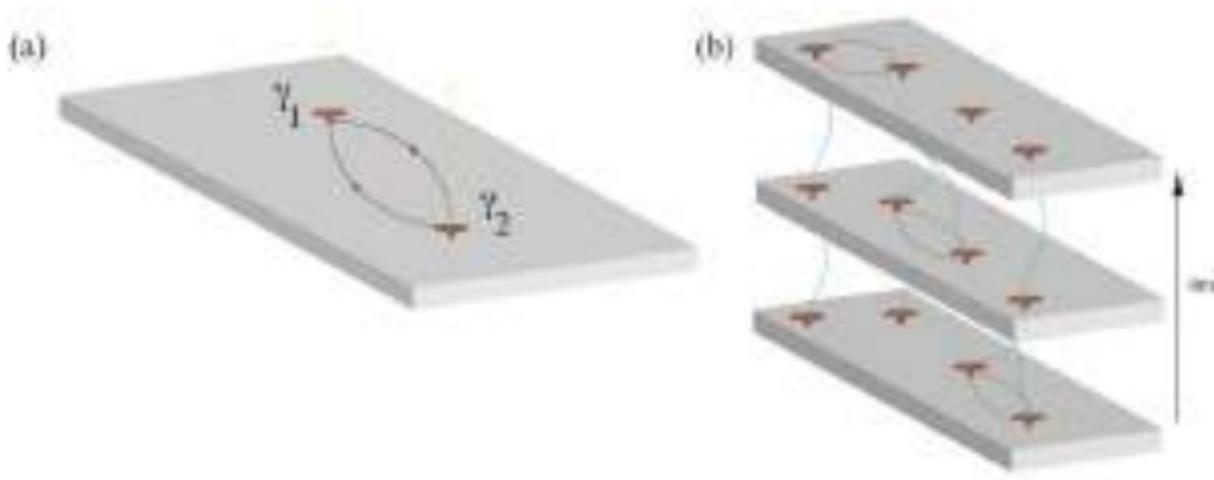
$$c_j = \frac{1}{2} (\gamma_j^A + i\gamma_j^B), \quad c_j^\dagger = \frac{1}{2} (\gamma_j^A - i\gamma_j^B)$$

- Hamiltonian in terms of Majorana operators:

$$H = -\frac{i\mu}{2} \sum_{j=1}^N \gamma_j^A \gamma_j^B + \frac{i}{2} \sum_{j=1}^{N-1} [(\Delta + t)\gamma_j^B \gamma_{j+1}^A + (\Delta - t)\gamma_j^A \gamma_{j+1}^B]$$



Majoranas for qubits



- ▶ MZM are non-Abelian anyons.
- ▶ Gap closing/reopening \Rightarrow topological protection.

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We need a p -wave superconductor!

- ▶ The superconducting pairing term in the Kitaev chain is spinless:
$$\Delta \left(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger \right).$$

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- ▶ p -wave is very rare in nature. We need to engineer it.

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- ▶ Fu and Kane: s -wave pairing behaves as p -wave when projected onto the basis of helical electrons.

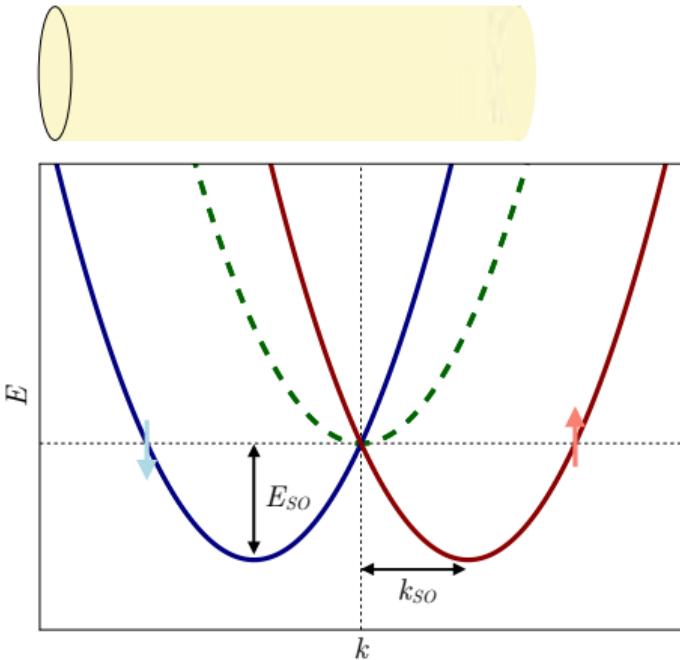
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- ▶ Lutchyn and Oreg: proximitize semiconductors with strong spin-orbit coupling.

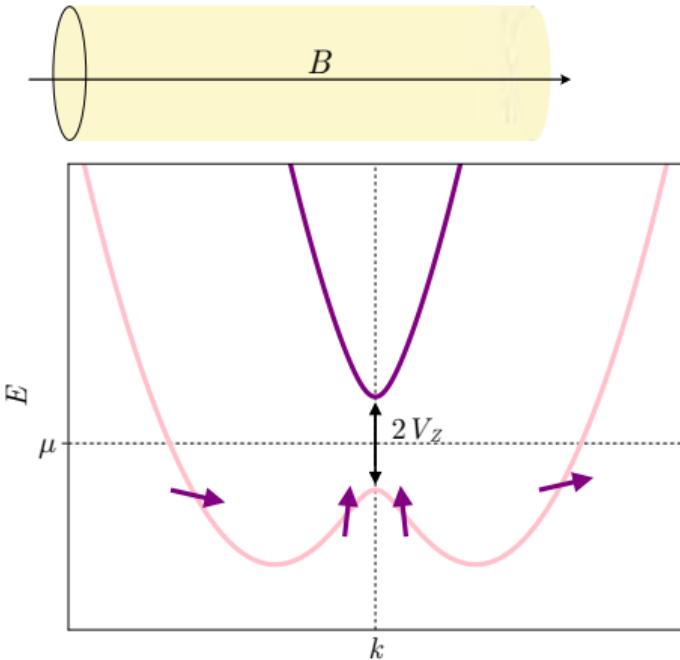
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Rashba, Zeeman and helical bands

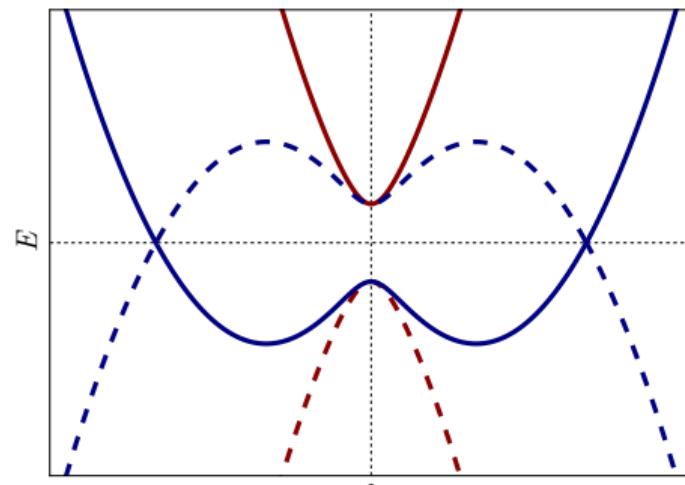
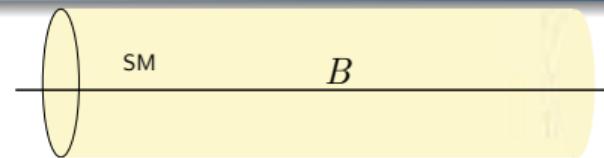
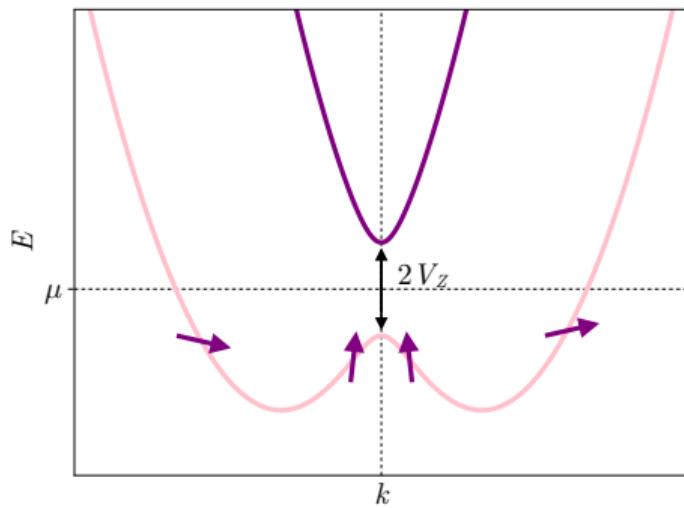
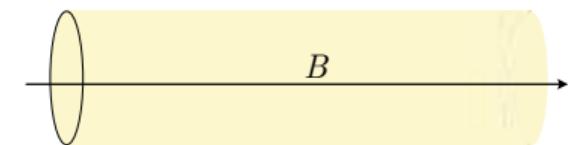


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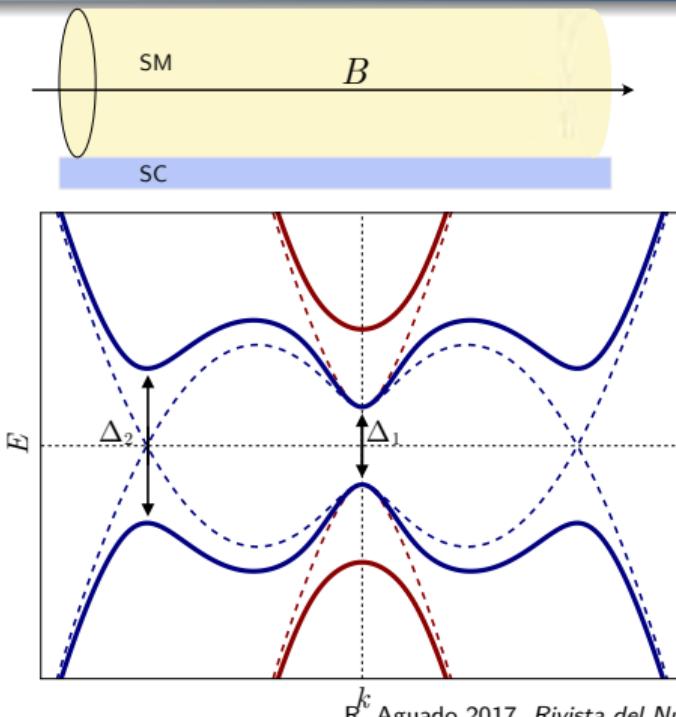
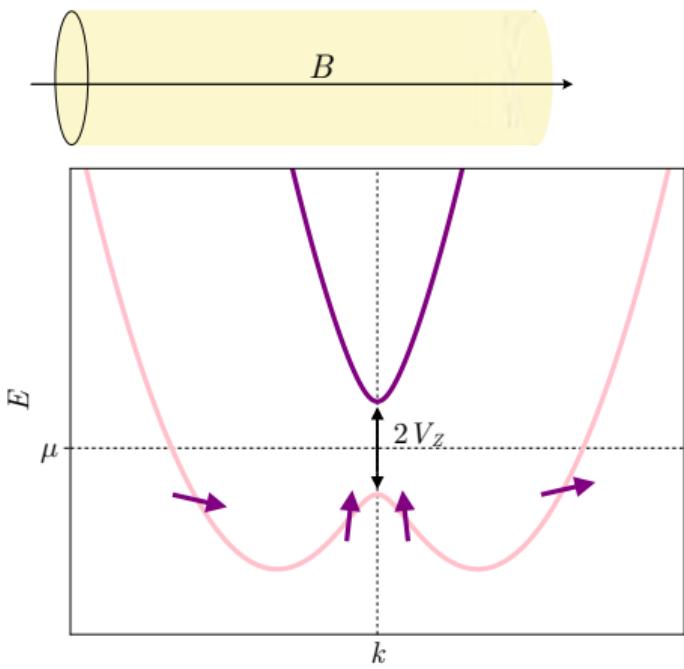


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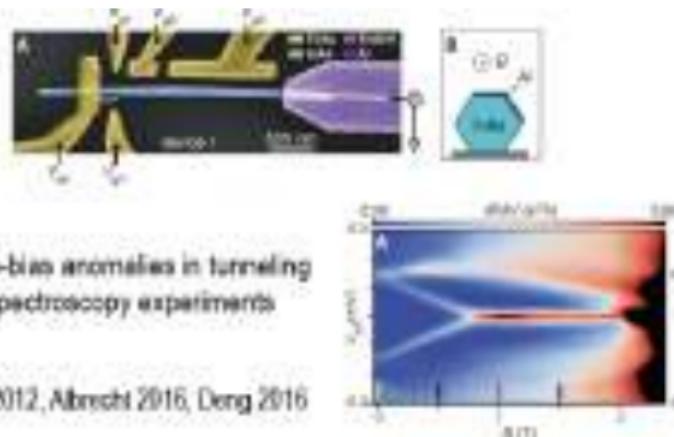
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Searching for Majoranas

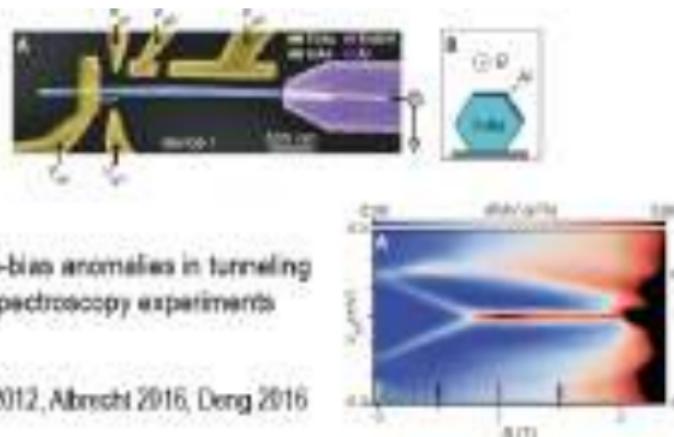
- ▶ Strong experimental interest.



Claims: V. Mourik *et al.* 2012, *Science*. S. M. Albrecht *et al.* 2016, *Nature*. M. T. Deng *et al.* 2016, *Science*. "Unclaims": E. J. H. Lee *et al.* 2012, *Phys. Rev. Lett.* M. Valentini, F. Peñaranda, *et al.* 2021, *Science*. M. Valentini, M. Borovkov, *et al.* 2022, *Nature*.

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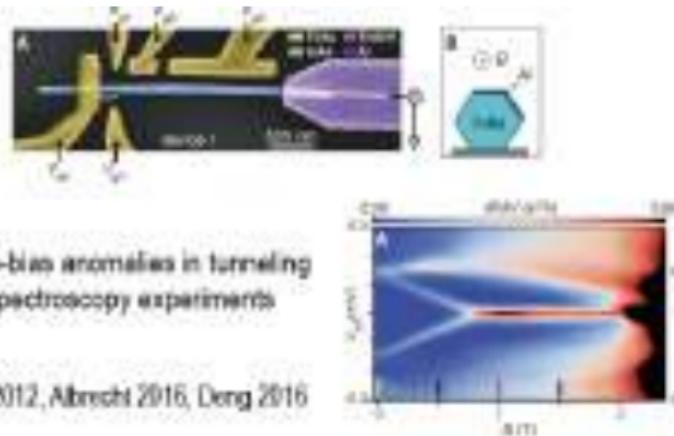
- ▶ Strong experimental interest.
- ▶ Zero-bias anomalies detected with non-topological explanations.



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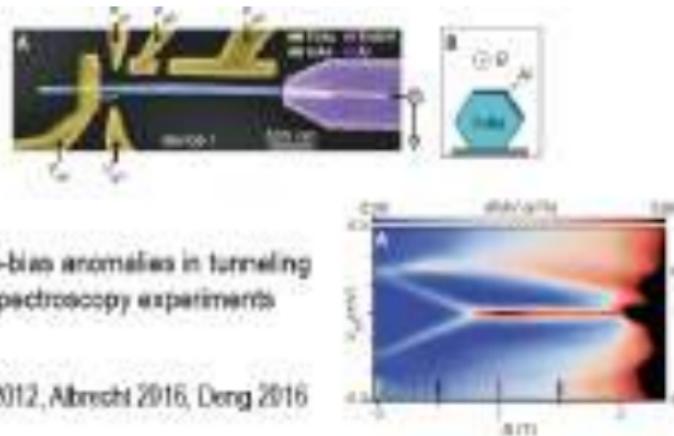
- ▶ Drawbacks:



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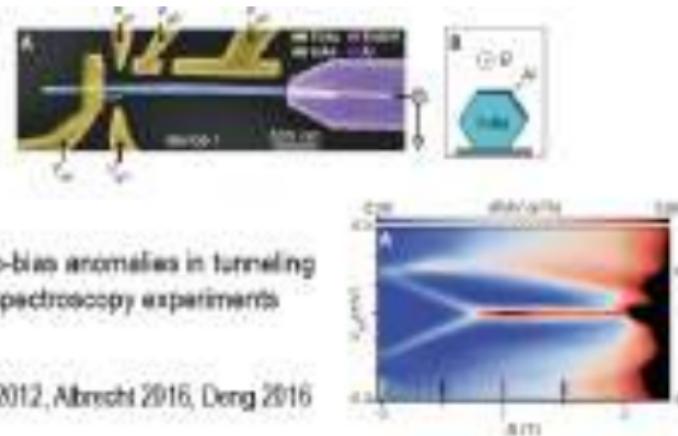
- ▶ Drawbacks:
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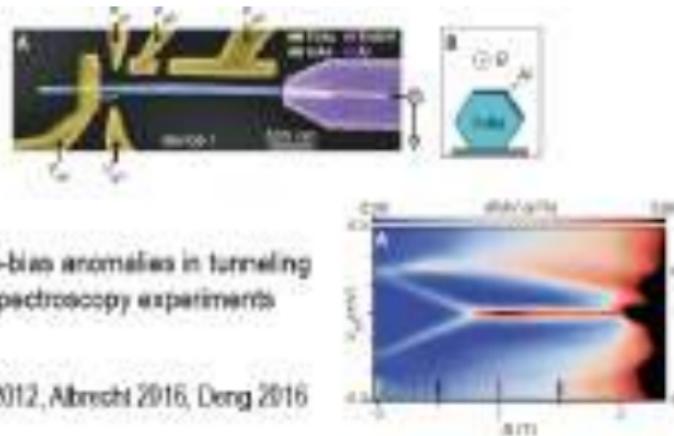
- ▶ Drawbacks:
 - ▶ Multimode effects.
 - ▶ Electrostatic environment.



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Searching for Majoranas

- ▶ Drawbacks:
 - ▶ Multimode effects.
 - ▶ Electrostatic environment.
 - ▶ Renormalized parameters.



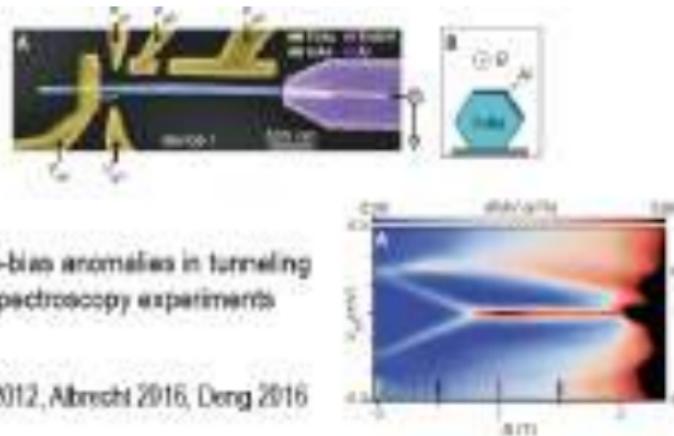
Zero-bias anomalies in tunneling spectroscopy experiments

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 - ▶ High magnetic fields.



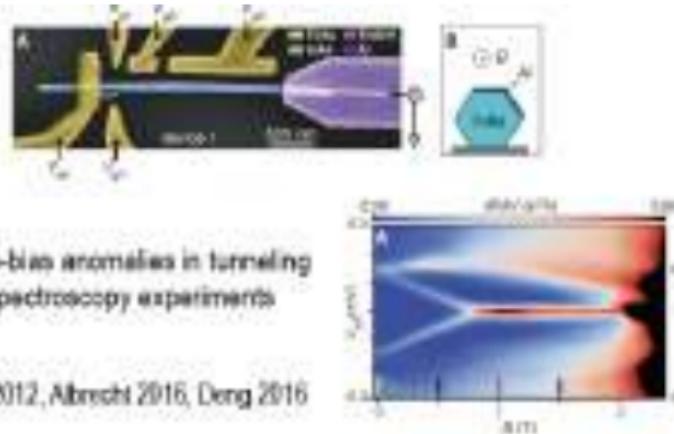
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 - ▶ Orbital effects.



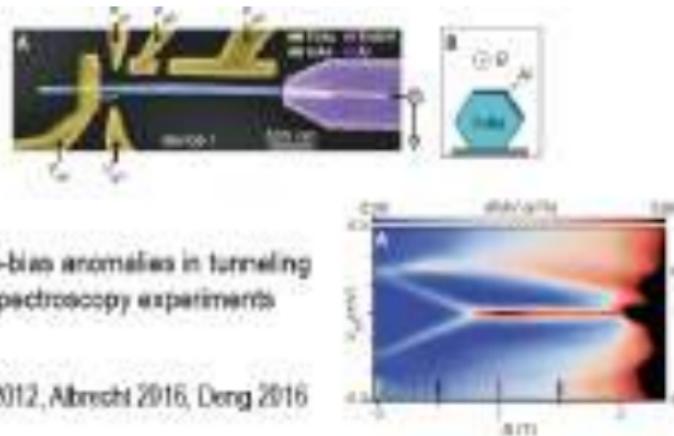
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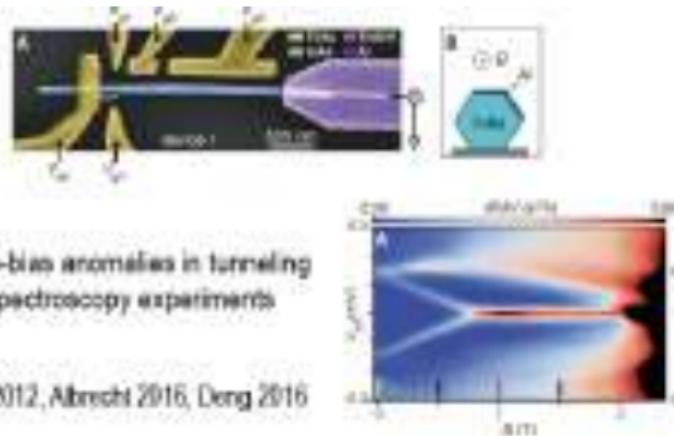
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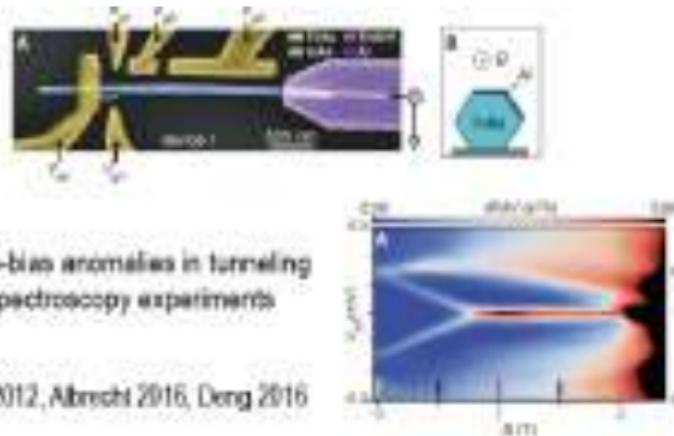
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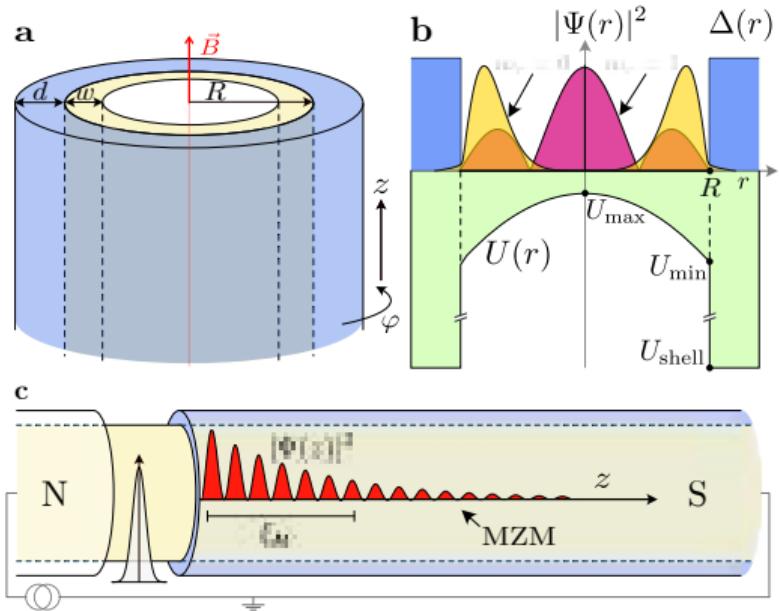
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 - ▶ QD physics.



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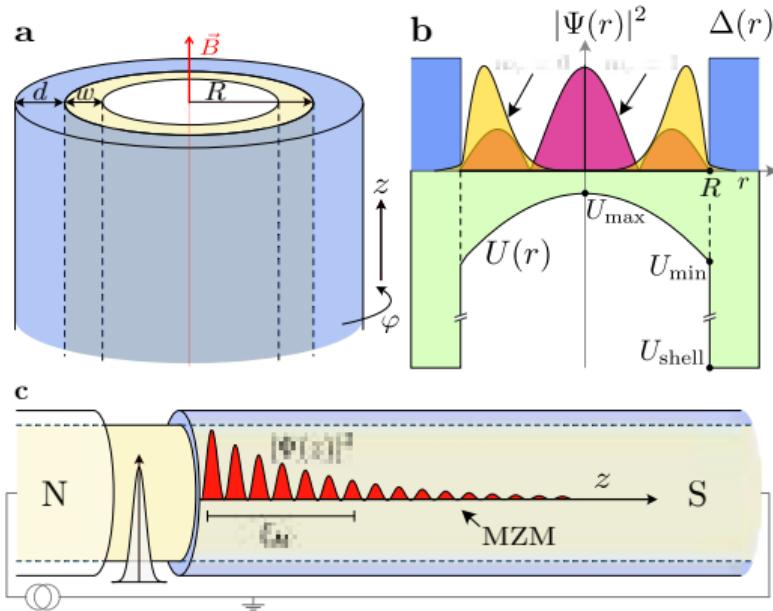
The full-shell nanowire



► Key points:

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P. San-Jose et al. 2023, *Phys. Rev. B*.
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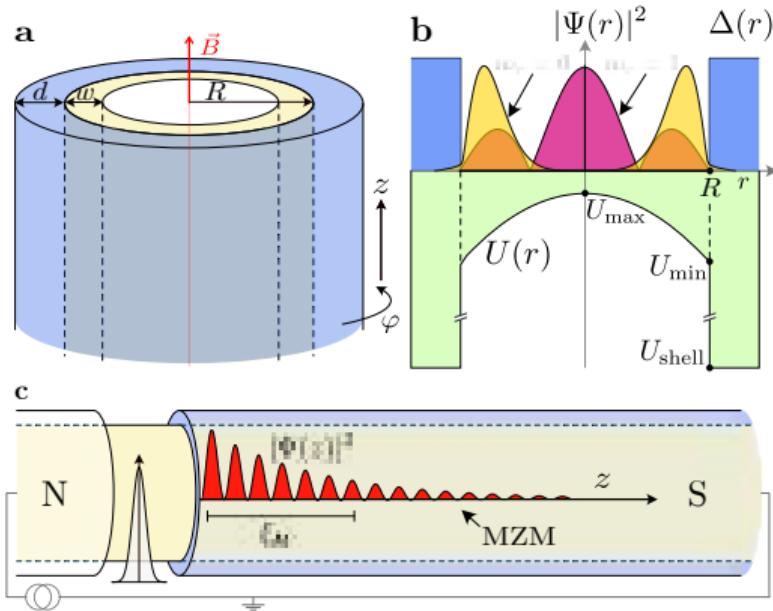
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- ▶ Key points:
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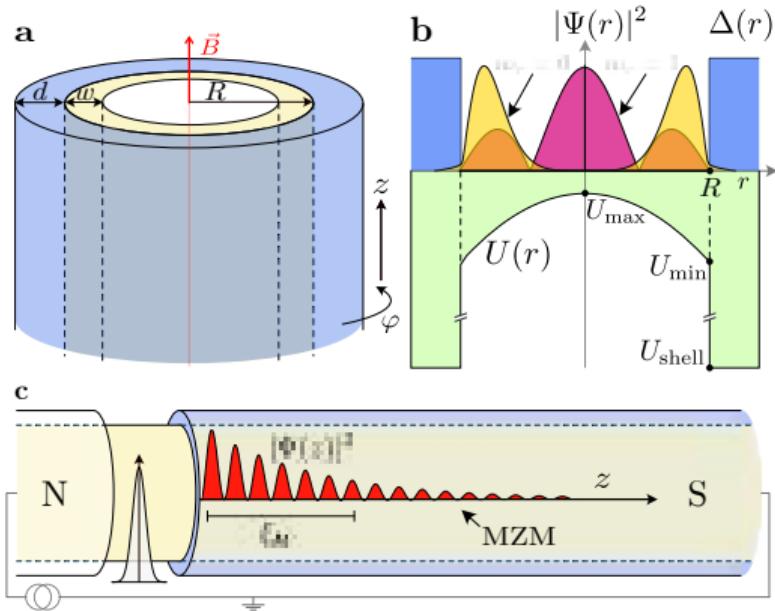
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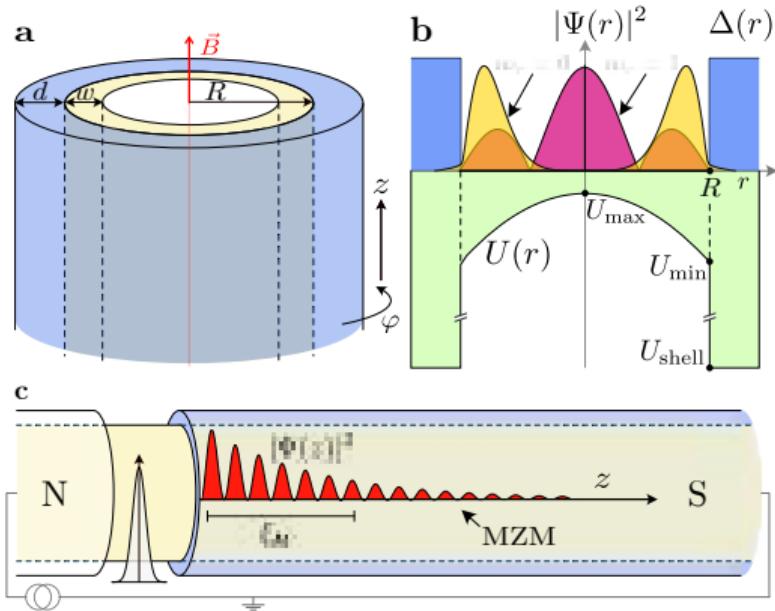
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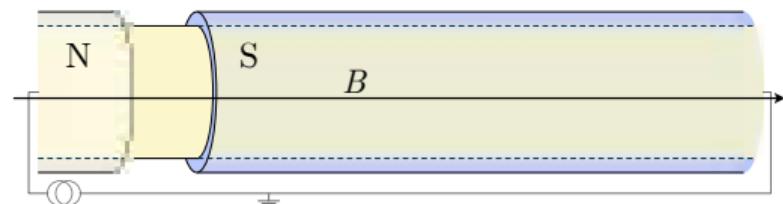
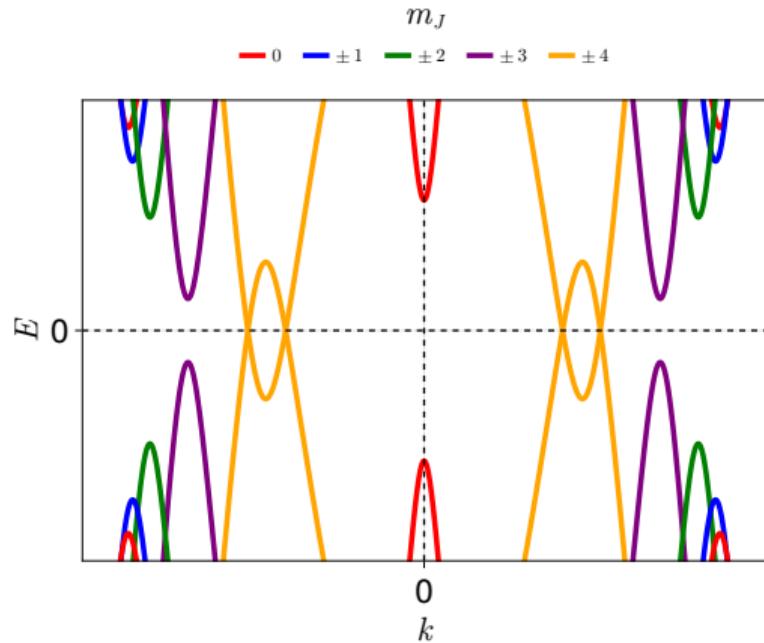
The full-shell nanowire



- ▶ Key points:
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 - ▶ Topological transition driven by orbital effect **not Zeeman**
 - ▶ Needs lower magnetic fields.
 - ▶ Only one angular mode can be topological.

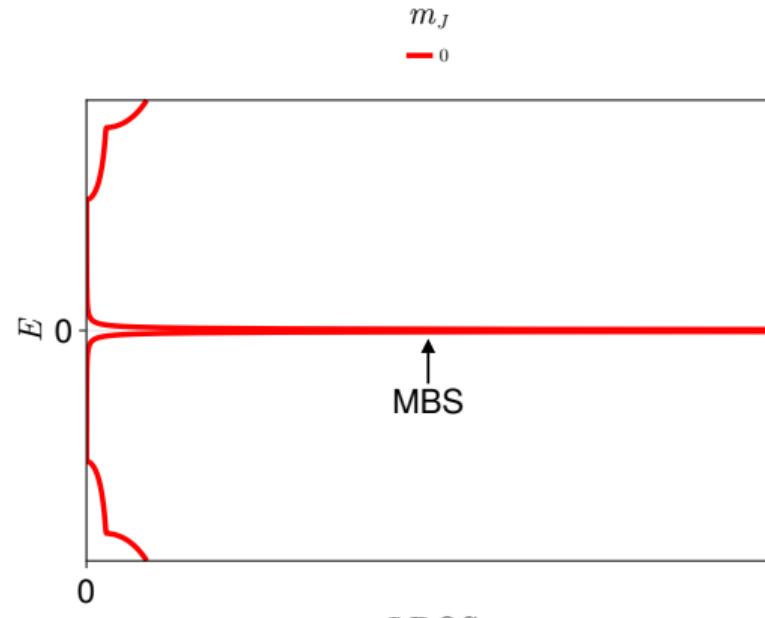
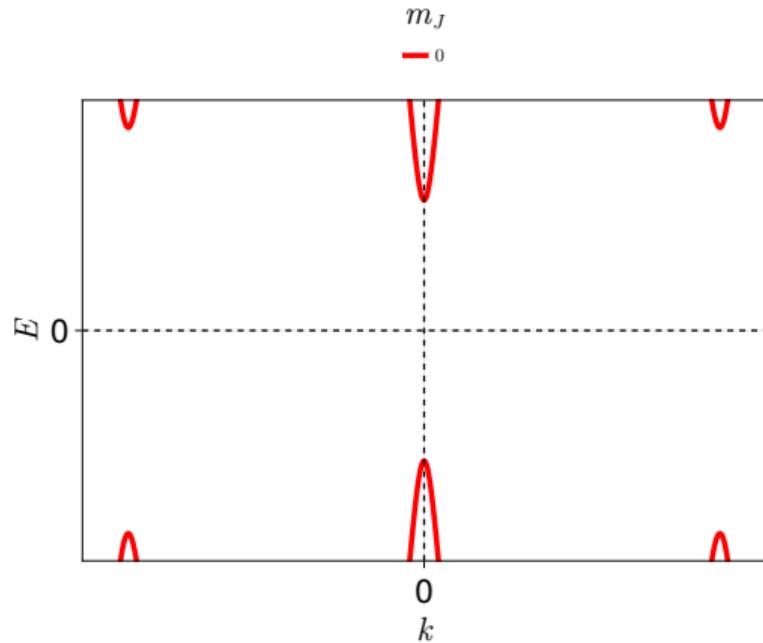
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The CdGM analog states



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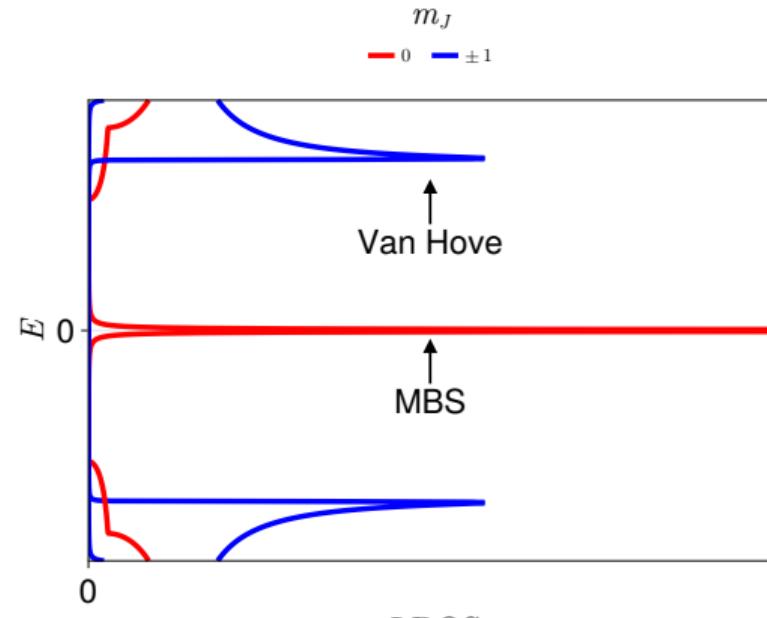
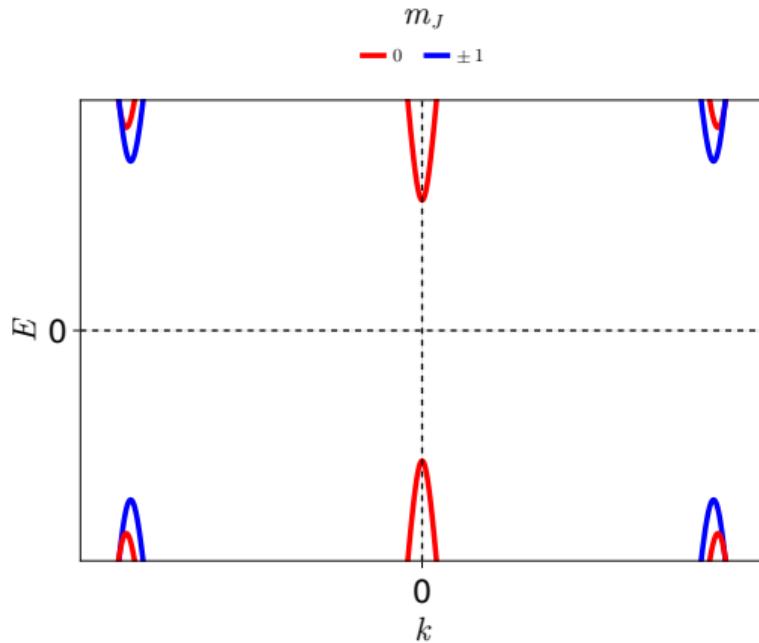
The CdGM analog states



LDOS

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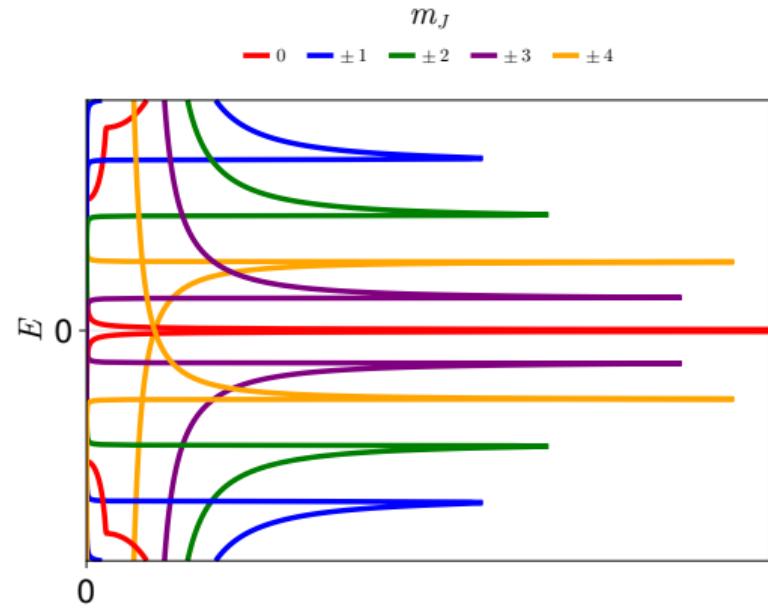
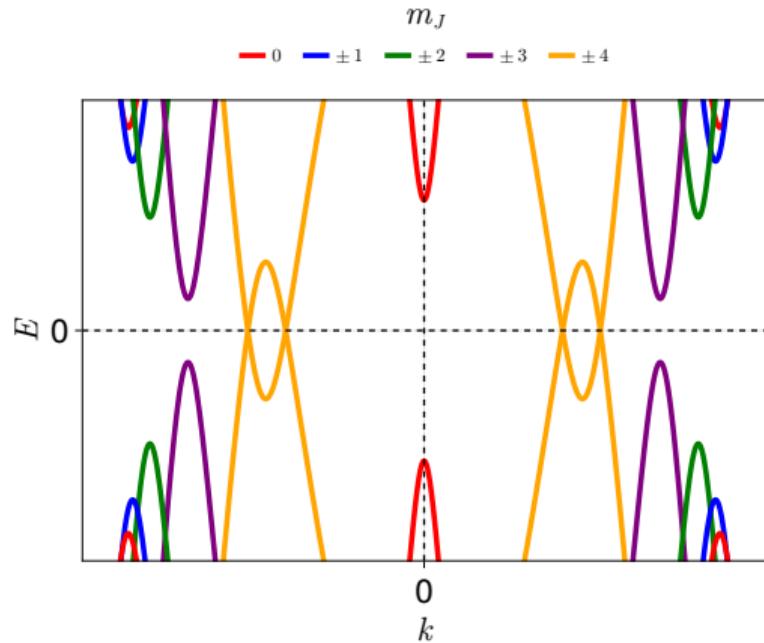
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LDOS

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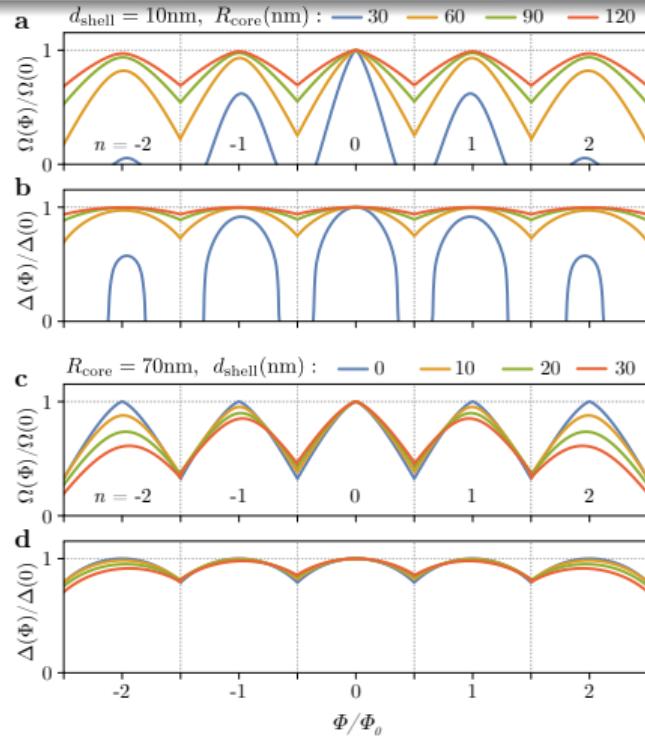
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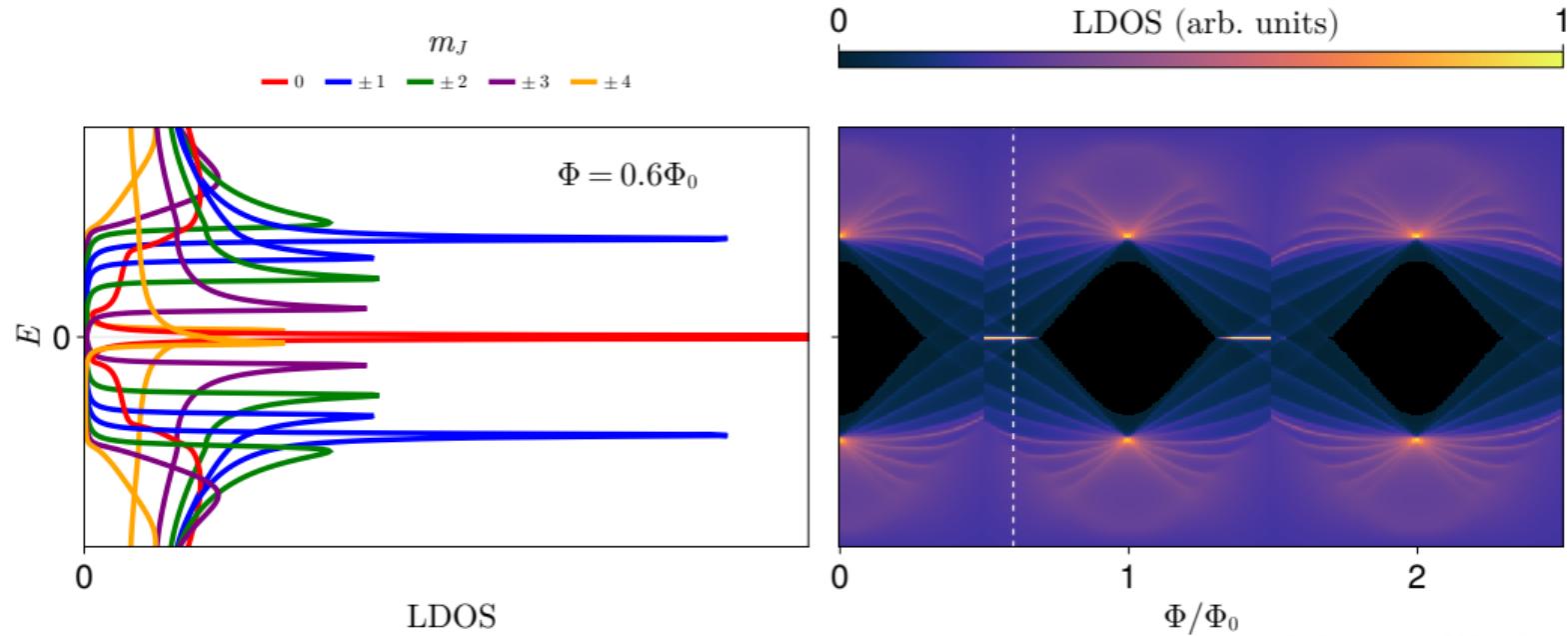
Digression: the Little-Parks effect



- ▶ Cylinder \Leftrightarrow vortex.
- ▶ Too thin for full Meissner.
- ▶ Quantized winding of the order parameter: $\Delta = |\Delta| e^{in\varphi}$.
- ▶ $n \in \mathbb{Z}$ and jumps every flux quantum Φ_0 .
- ▶ Quasi-quantization of flux \Rightarrow pairing presents LP lobes.
- ▶ Depends on R , SC thickness d and ξ_d , the SC coherence length.

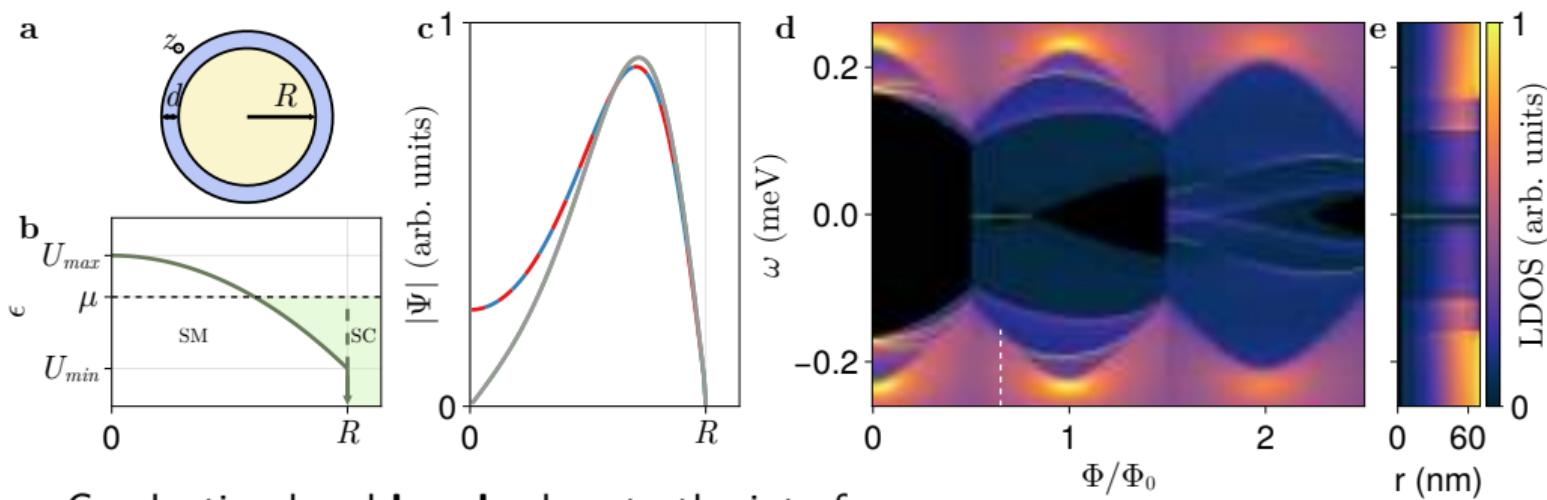
W. A. Little and R. D. Parks 1962, *Phys. Rev. Lett.*
 R. D. Parks and W. A. Little 1964, *Phys. Rev.*

LDOS vs. flux



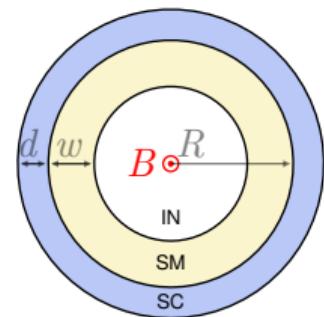
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Pushing the WF to the interface

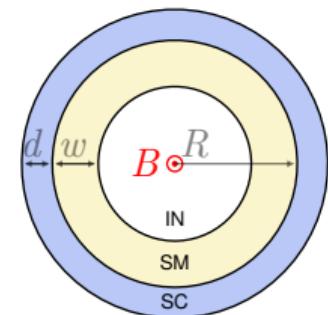
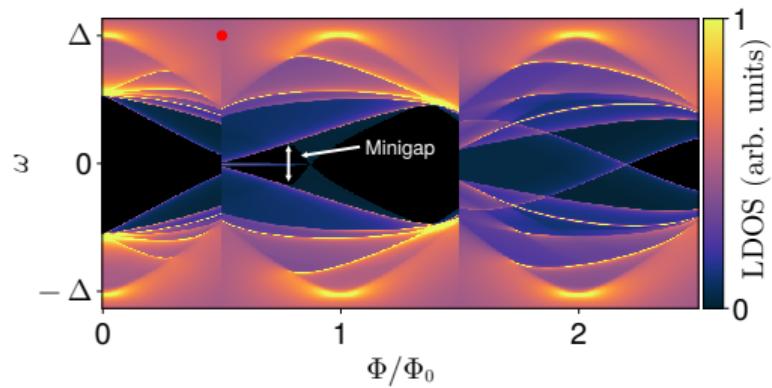


- ▶ Conduction band **bends** close to the interface.

Protected islands in the tubular-core

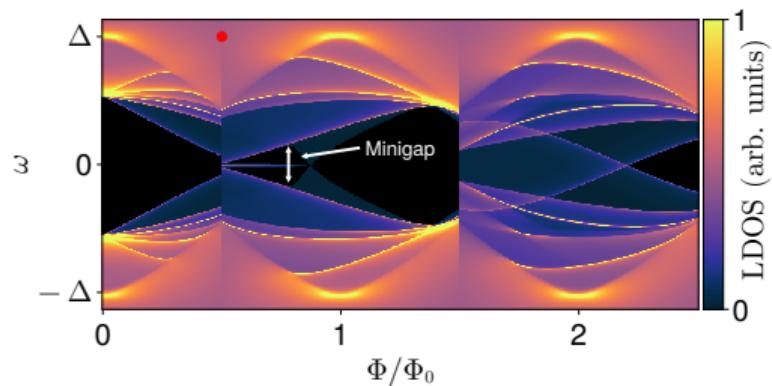


Protected islands in the tubular-core

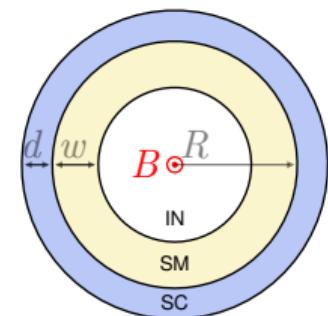


C. Payá *et al.* 2024, *Phys. Rev. B*.

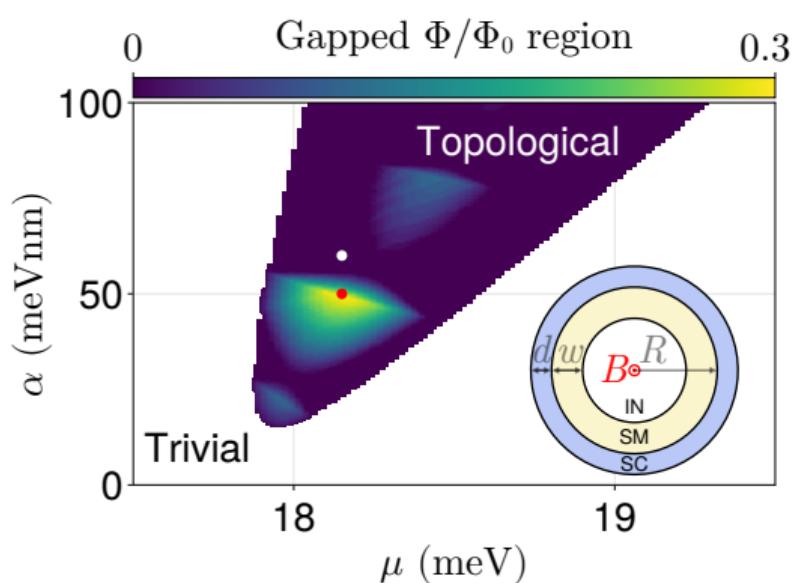
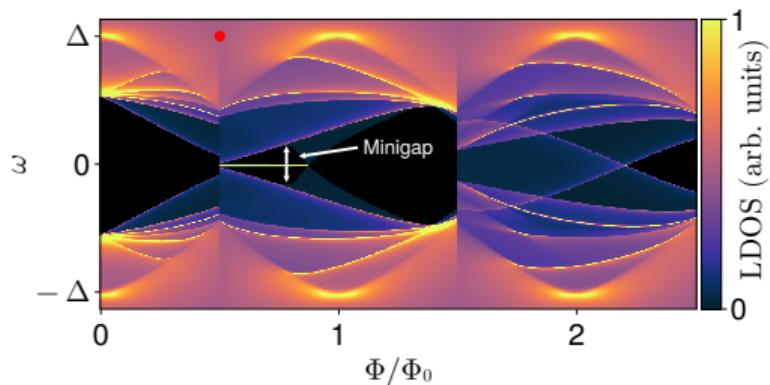
Protected islands in the tubular-core



- We need to push the charge to the interface.



Protected islands in the tubular-core



- ▶ We need to push the charge to the interface.
- ▶ Topologically protected islands appear-

Summary

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 1. Majorana zero modes coexist with CdGM analog states.
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Take home message

Majorana physics of full-shell nanowires is very rich. For pristine configurations, the tubular-core model is the optimal candidate in comparison to the solid-core geometry.

People involved

Project Leader



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Theory:

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Ongoing experiments:

Jesper Nygård (Niels Bohr Institute)



Phenomenology of Majorana zero modes in full-shell hybrid nanowires

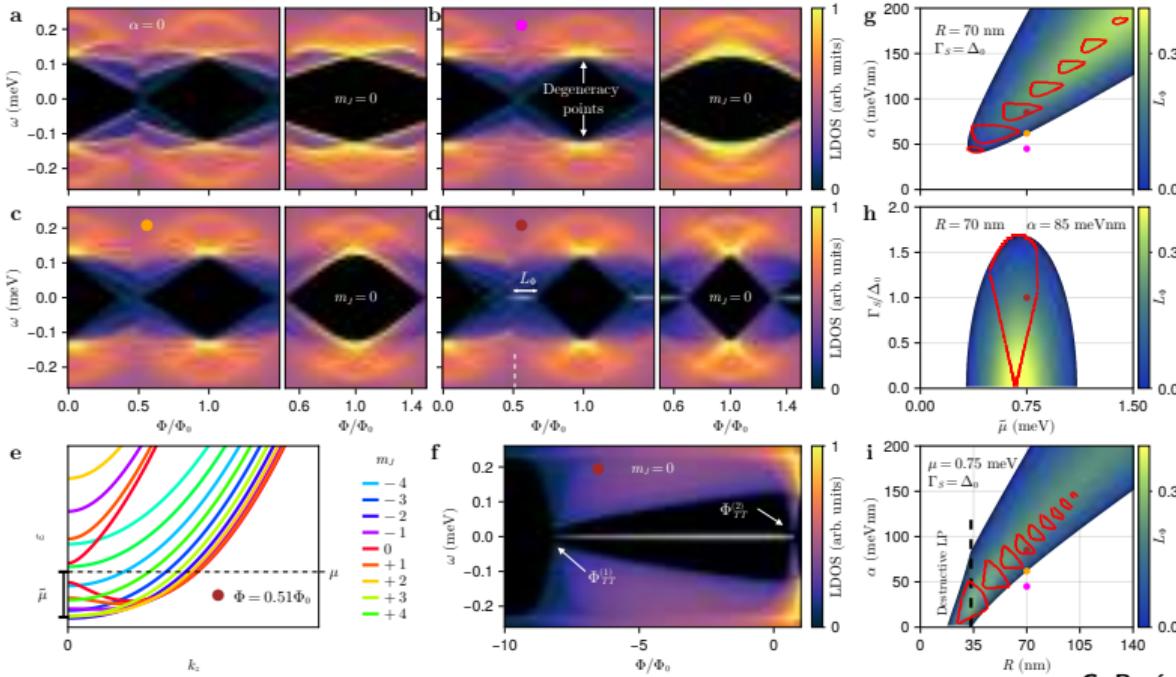
Carlos Payá

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July 9, 2024

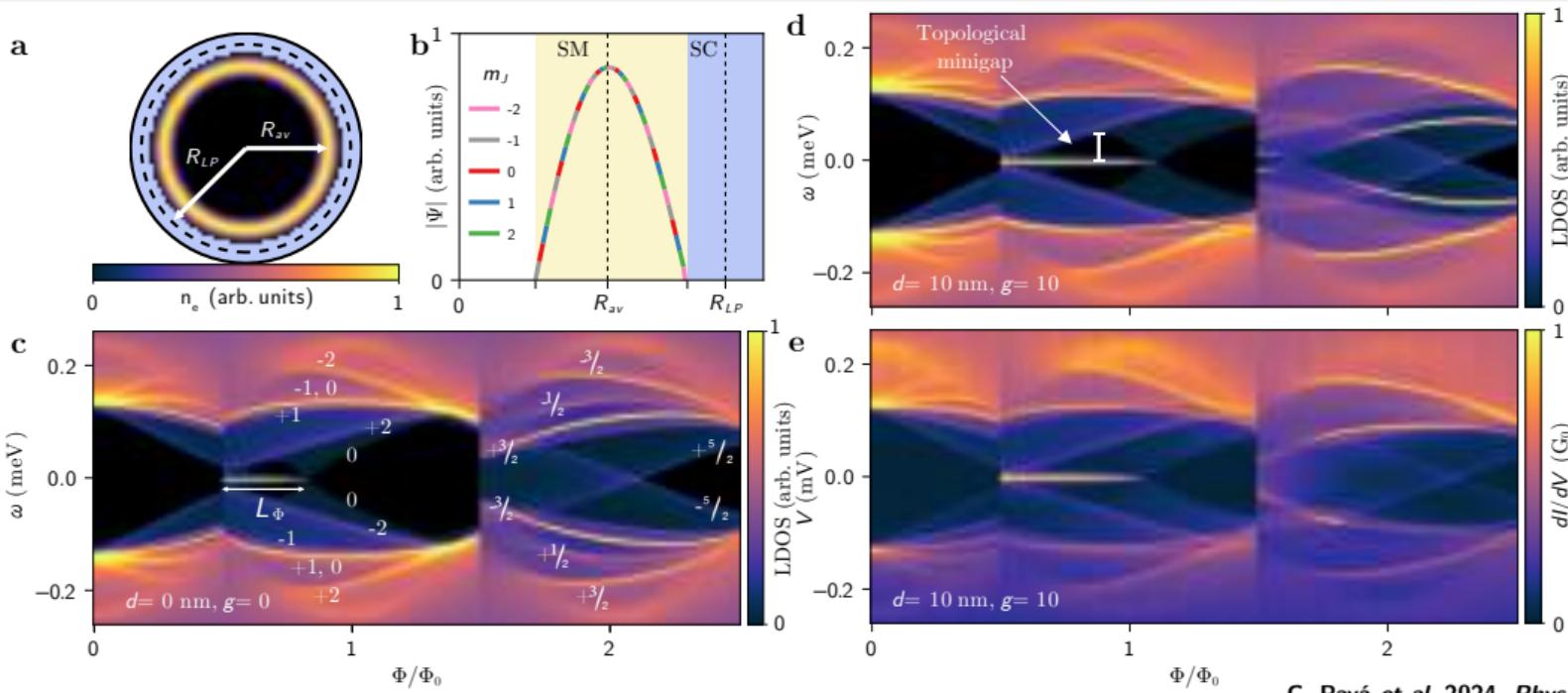


Hollow-core results



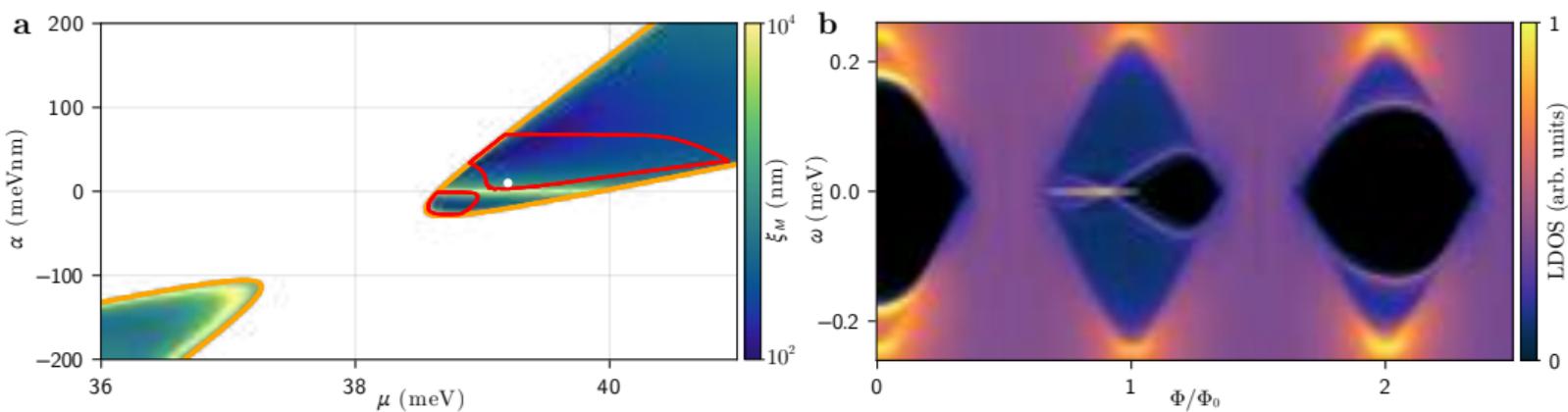
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Modified hollow-core results

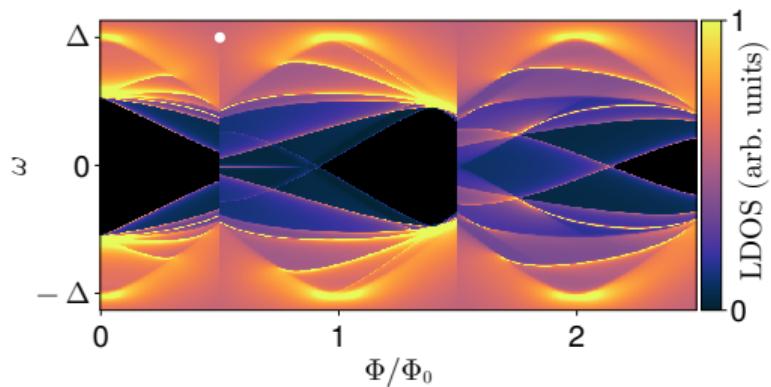


C. Payá et al. 2024, Phys. Rev. B.

Destructive Little-Parks

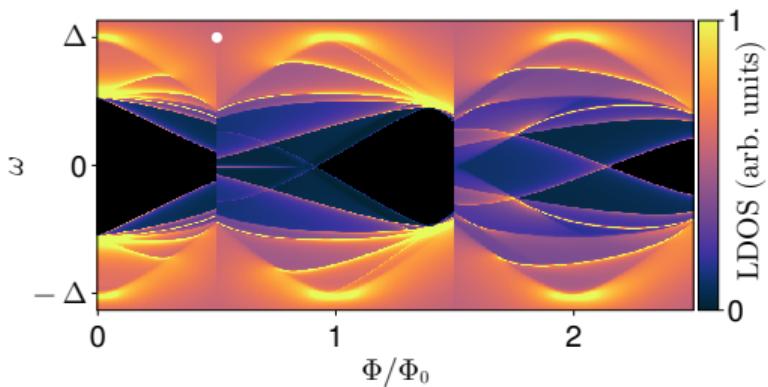


The tubular-core model



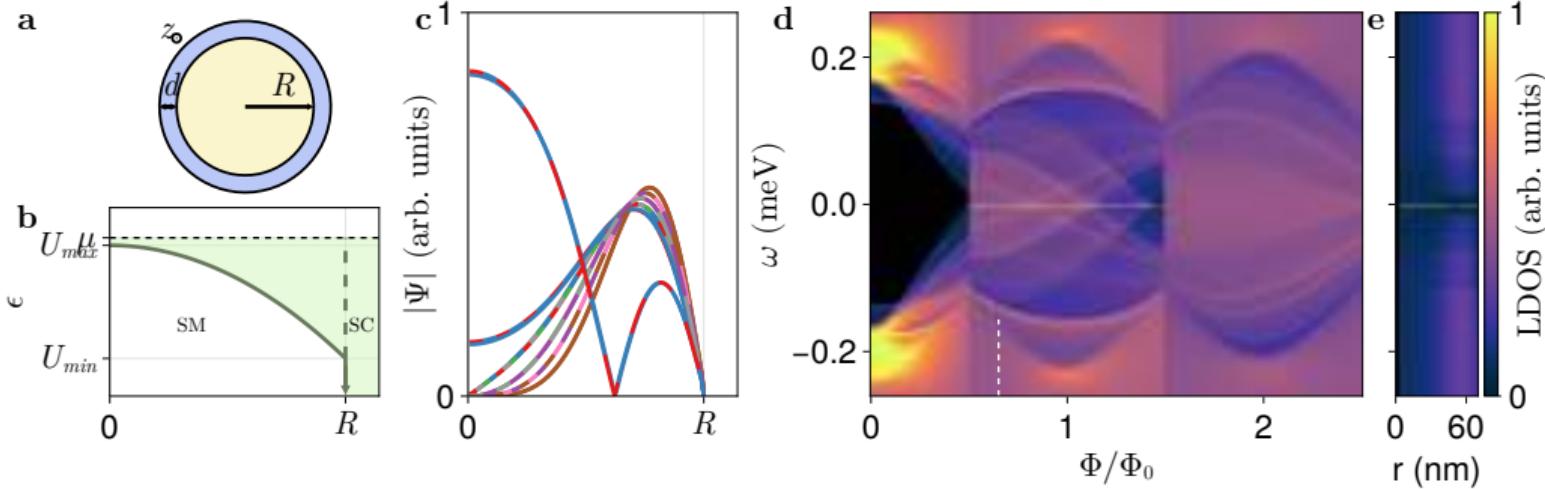
- ▶ Adding a width to the semiconductor.

The tubular-core model

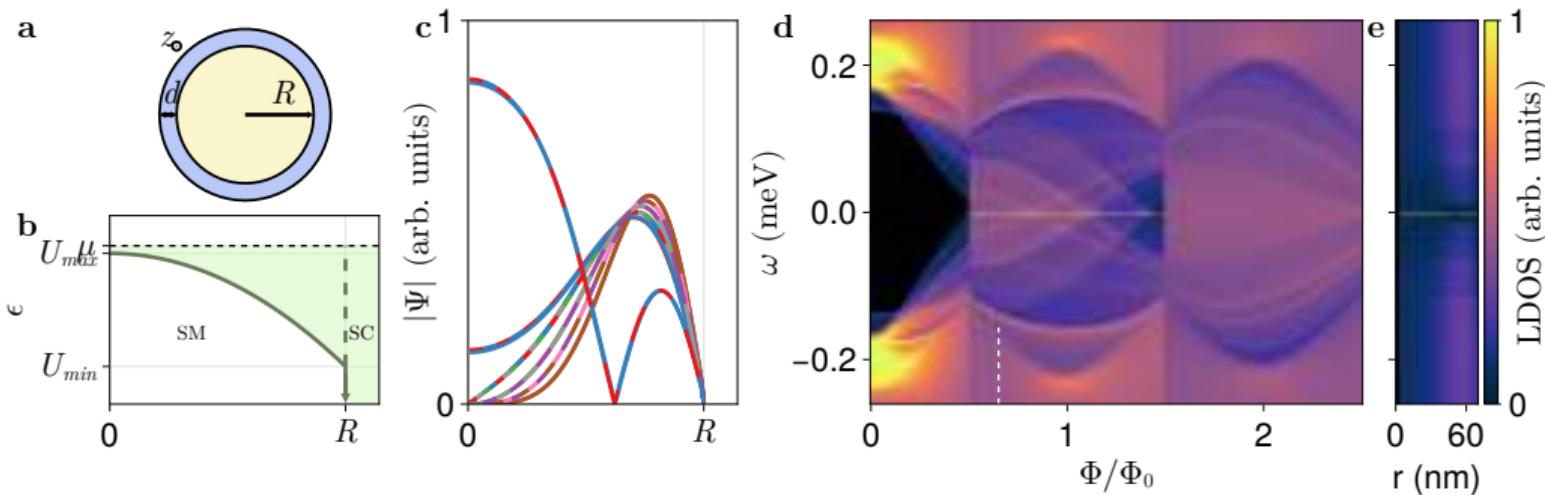


- ▶ Adding a width to the semiconductor.
- ▶ Most common scenario: CdGMs fill the MZM minigap.
- ▶ No topological protection

Second radial mode: protection lost

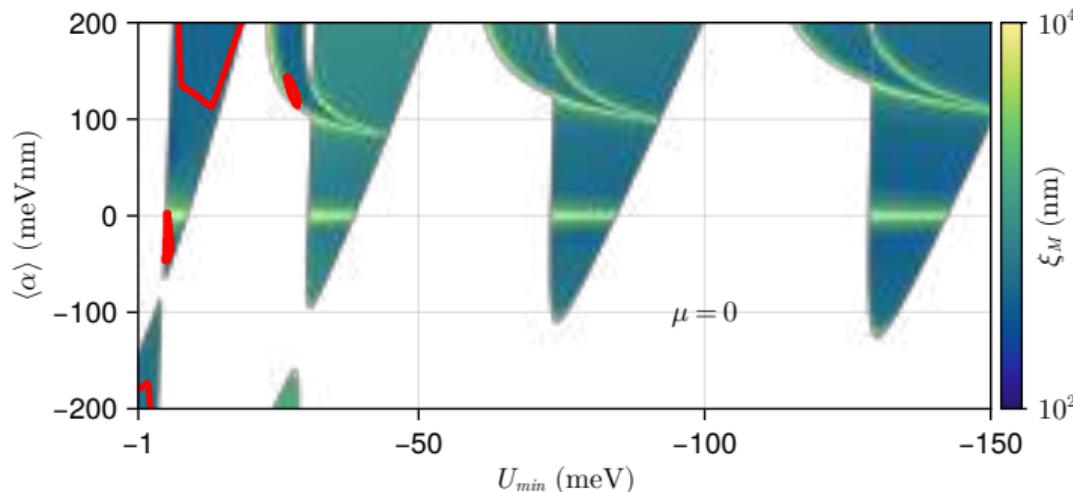


Second radial mode: protection lost



- When the second radial mode is occupied, the ZEP expands over the full lobe, but CdGMs cover it.

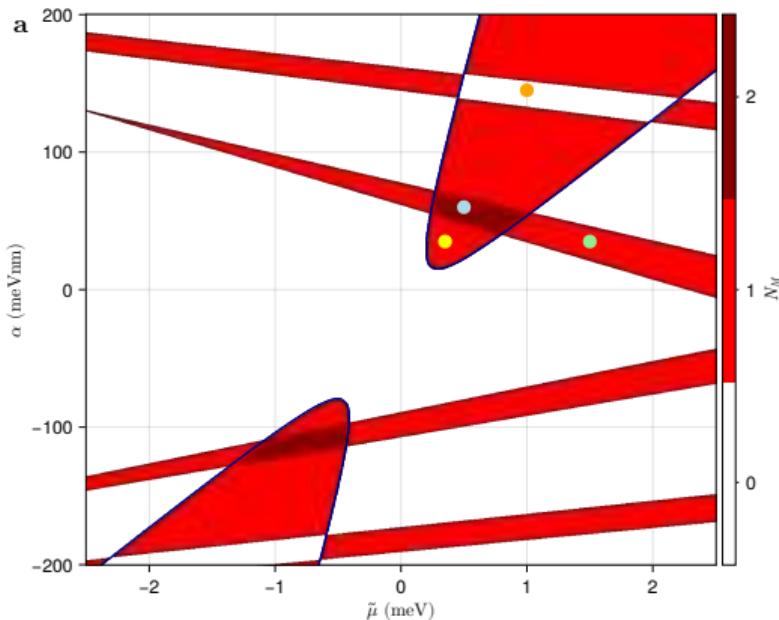
Band-bending: not enough islands



- ▶ Notice axis are mean α and U_{min} , the minimum of the dome-profile.
- ▶ One wedge per radial mode. No islands outside the first radial mode.

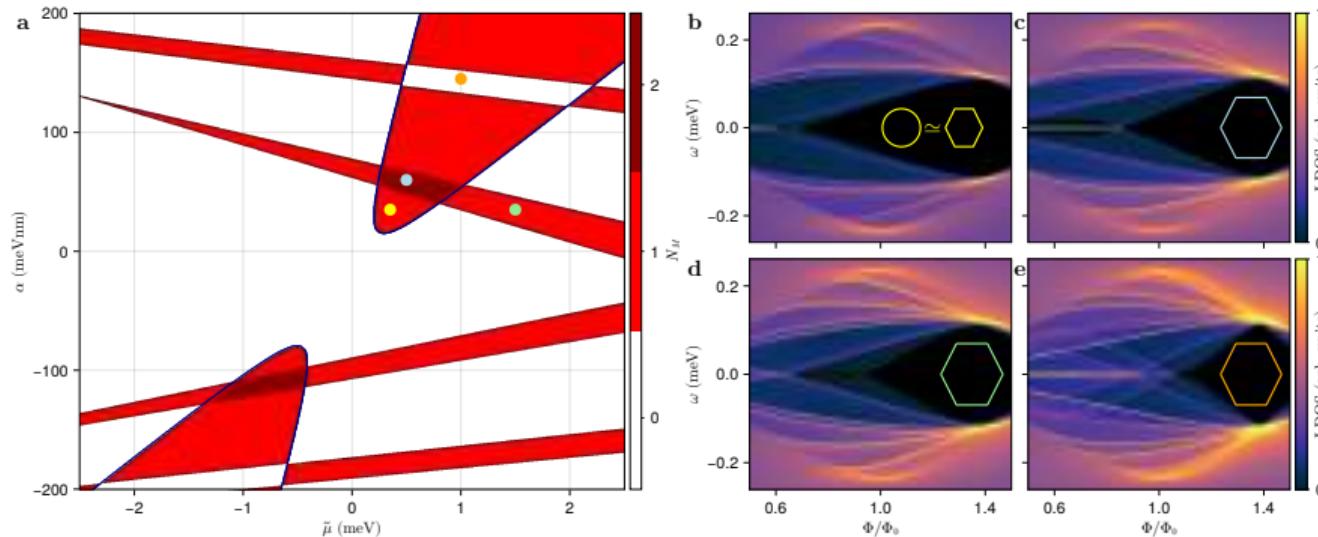
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Hexagonal wave-function



- ▶ New red stripes. Hexagon has $\ell = 6$.
- ▶ Upper stripe: $m_J = 0$ mixes with $m_J = \pm 6$.
- ▶ Lower stripe: $m_J = 3$ mixes with $m_J = -3$.
- ▶ The MZM coming from $m_J = \pm 3$ **cannot** interact with $m_J = 0 \Rightarrow$ they overlap.
- ▶ The $m_J = \pm 6$ MZM annihilates the $m_J = 0$ MZM.

Hexagonal wave-function



- Except for the new topological stripes and a region where the MZM splits, the system is equivalent to the cylinder.

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