Neural Networks

Rodrigo Gonzalez, PhD

Hello!

I am Rodrigo Gonzalez, PhD

You can find me at rodralez@gmail.com

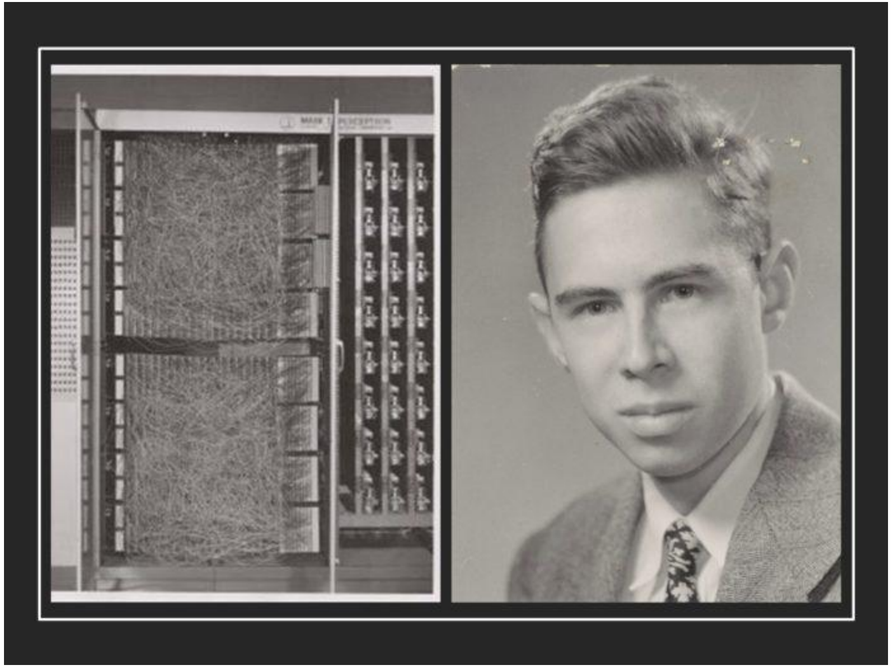
Summary

1. A brief history of neural networks
2. Neural networks architecture
3. Activation functions
4. Loss function
5. Optimizer
6. Forward and backward propagation
7. Deep learning

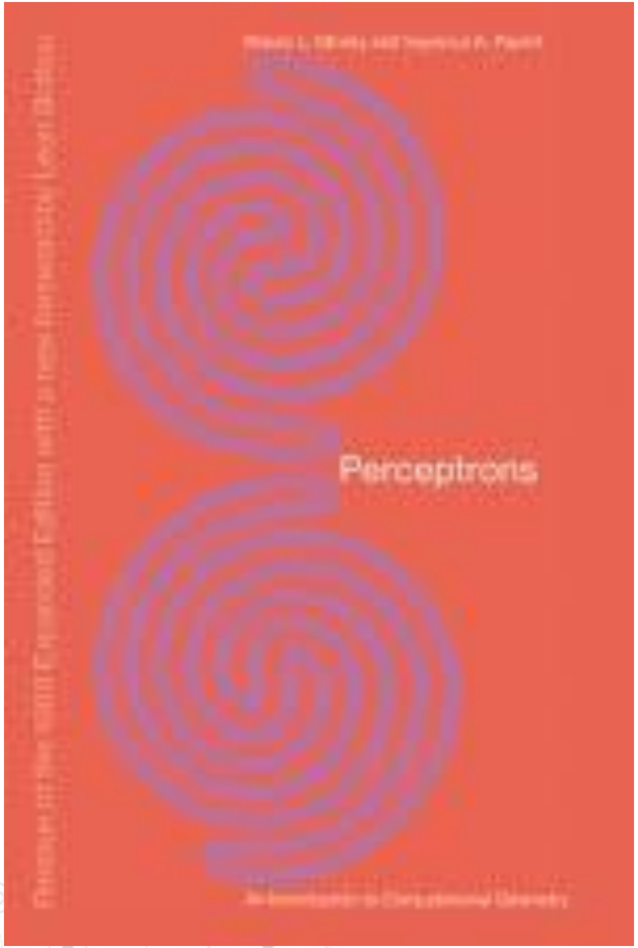
1.

A brief history of neural networks

A brief history of misconceptions and preconceptions



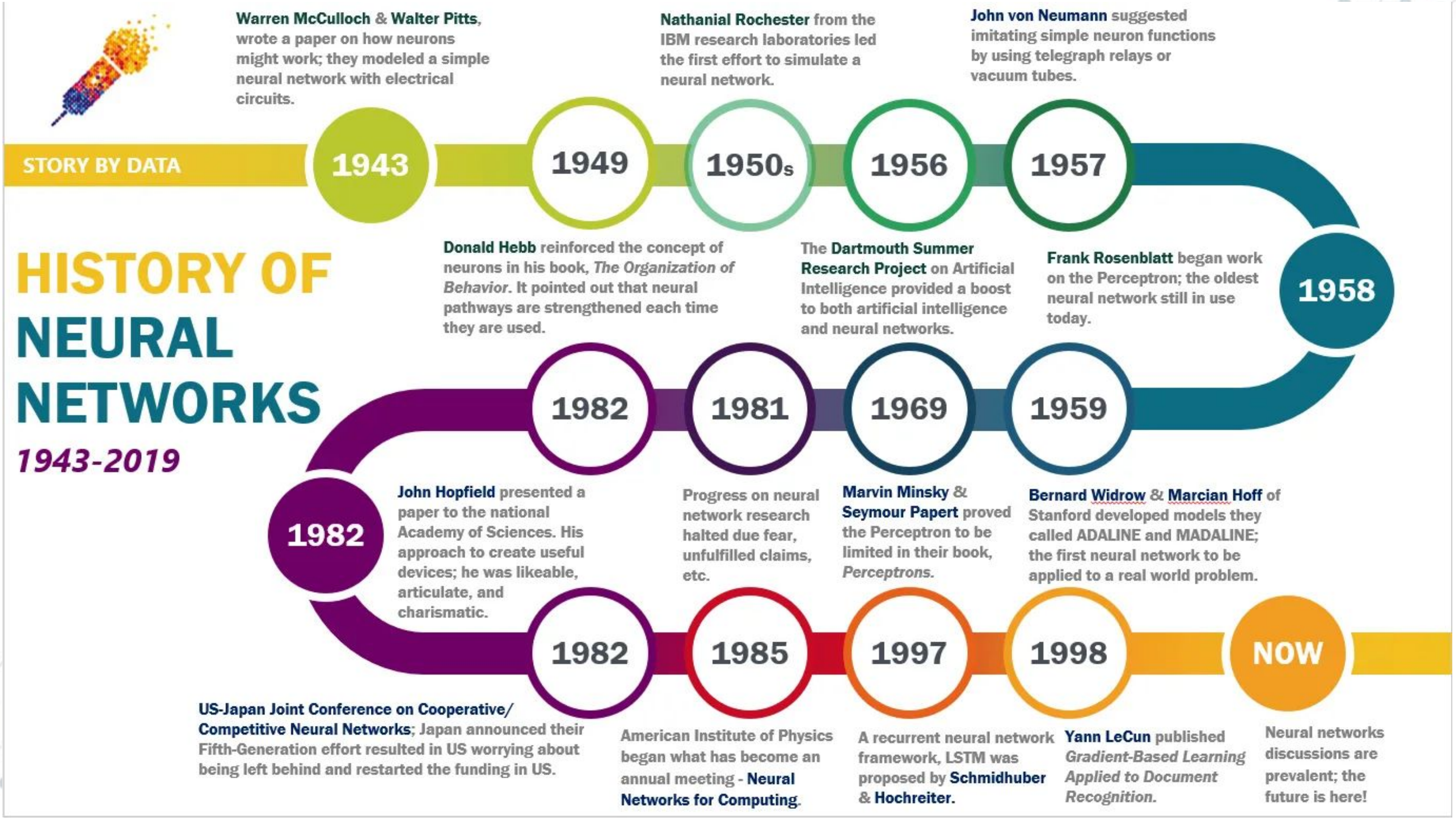
Perceptron
Frank Rosenblatt
1958



Perceptron
Seymour Papert
Marvin Minsky
1969

El invierno de la Inteligencia Artificial

A brief history of neural network

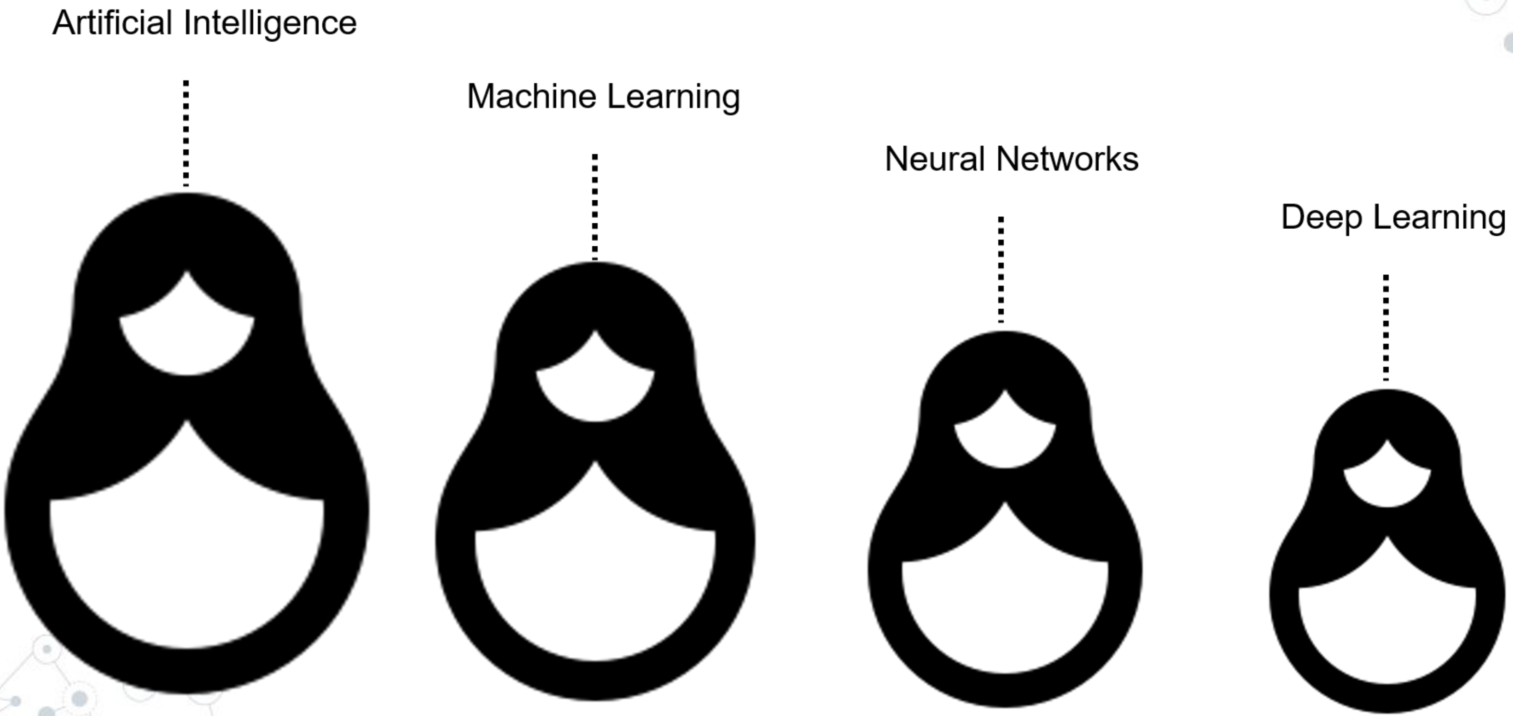


2.

Neural Networks

Architecture

Neural Network in the context of AI

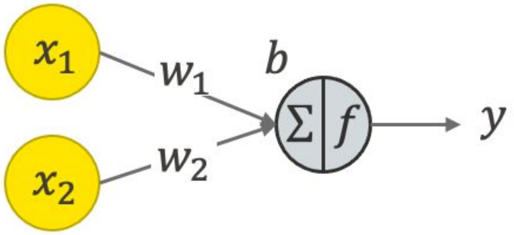


Neural Networks main characteristics

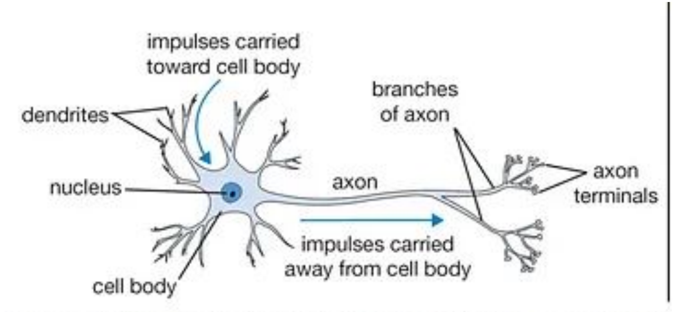
1. (Artificial) Neural networks are set of algorithms inspired by the functioning of human brian.
2. Neural networks (NN) are **universal function approximators** so that means neural networks can learn an approximation of any function f() such that,

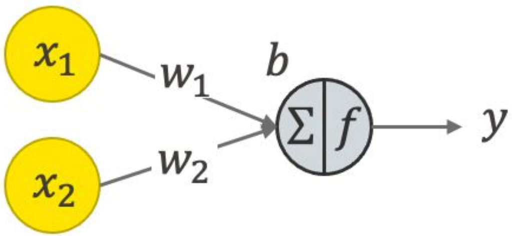
$$y = f(x)$$

3. NN learn by example (supervised learning).
4. Used for regression and classification problems.



A single neuron





- Input nodes: x1, x2
- Weights: w1, w2.
- Bias: b
- Sum: ∑
- Activation function: f()
- Output: y
- Loss function
- Optimizer

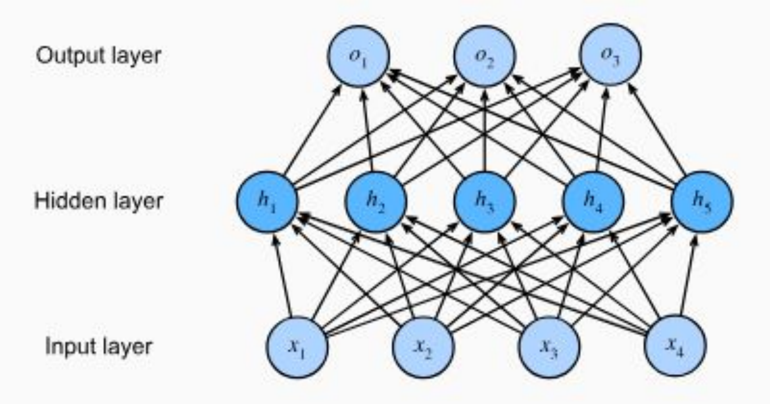
Step 1: Each input is multiplied by the associated weight.

$$a = x1 * w1 + x2 * w2 + b$$

Step 2: An activation function f() converts the result into the neuron output.

$$y = f(a)$$

Network layers



$$\mathbf{H} = \sigma(\mathbf{X}\,\mathbf{W}^{(1)} + \mathbf{b}^{(1)})$$

$$\mathbf{O} = \mathbf{H}\,\mathbf{W}^{(2)} + \mathbf{b}^{(2)}$$

Input units: The activity of the input units represents the raw information that is fed into the network.

Hidden units: The activity of each hidden unit is determined by the activities of the input units and the weights on the connections between the input and the hidden units.

Output units: The behaviour of the output units depends on the activity of the hidden units and the weights between the hidden and output units.

3.

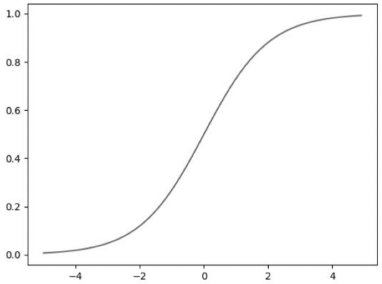
Activation functions

Activation functions

Activation function provides the possibility to learn non-linear functions

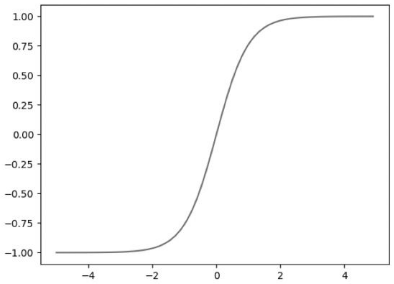
https://www.desmos.com/calculator/plevozbz1o

Sigmoid



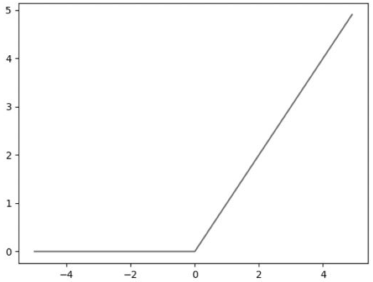
$$f(a) = \frac{1}{1 + e^{-ha}}$$

Tanh



$$f(a) = \frac{e^{2ha} - 1}{e^{2ha} + 1}$$

Rectified Linear Unit (ReLU)



$$f(a) = max\{0, ha\}$$

4.

Loss functions

Loss function names

- **Objective function**: In the context of an optimization algorithm, the function used to evaluate a candidate solution (i.e. a set of weights).
- **Cost function**.
- **Loss function**... or just **loss**.

$$\mathbf{w}^*, b^* = \underset{\mathbf{w}, b}{\operatorname{argmin}} \ L(\mathbf{w}, b).$$

Types of Loss Functions

Regression problems: given an input value, the model predicts a corresponding output value .

MSE (Mean Squared Error):

$$L(\mathbf{w}, b) = \frac{1}{n}\sum_{i=1}^{n} l^{(i)}(\mathbf{w}, b) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}\left(\mathbf{w}^\top \mathbf{x}^{(i)} + b - y^{(i)}\right)^2.$$

$$\mathbf{w}^*, b^* = \underset{\mathbf{w}, b}{\operatorname{argmin}}\ L(\mathbf{w}, b).$$

Binary classification problems: given an input, the neural network produces a vector of probabilities of the input belonging to two pre-set categories

Logarithmic loss or Cross-Entropy:

$$CE\ Loss = \frac{1}{n}\sum_{i=1}^{N} -\ (y_i \cdot log(p_i) + (1 - y_i) \cdot log(1 - p_i)$$

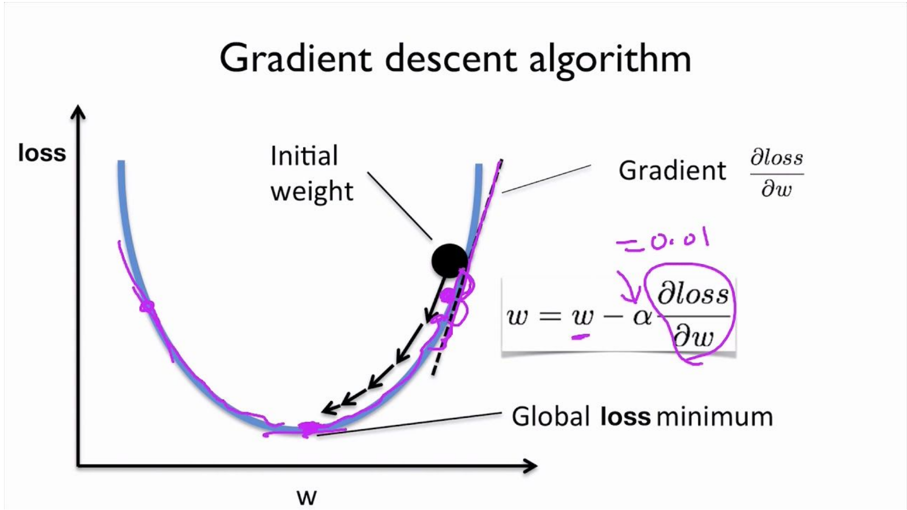
Multi-class classification problems: given an input, the neural network produces a vector of probabilities of the input belonging to various pre-set categories

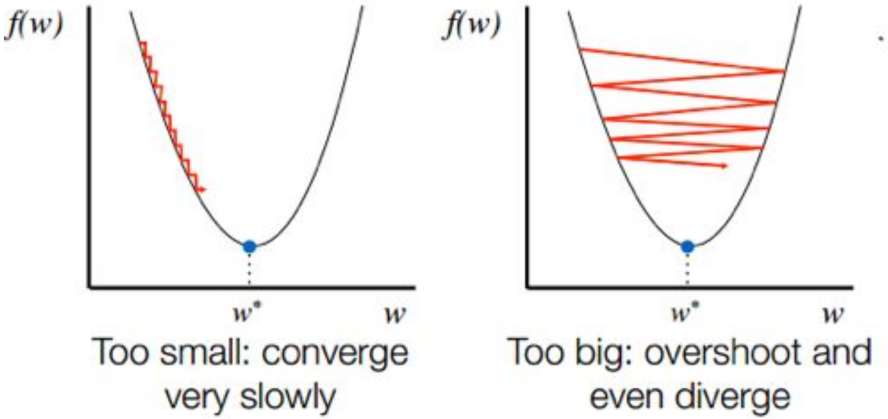
Softmax

5.

Optimizer

Gradient descent





Most common optimizers:

Stochastic gradient descent

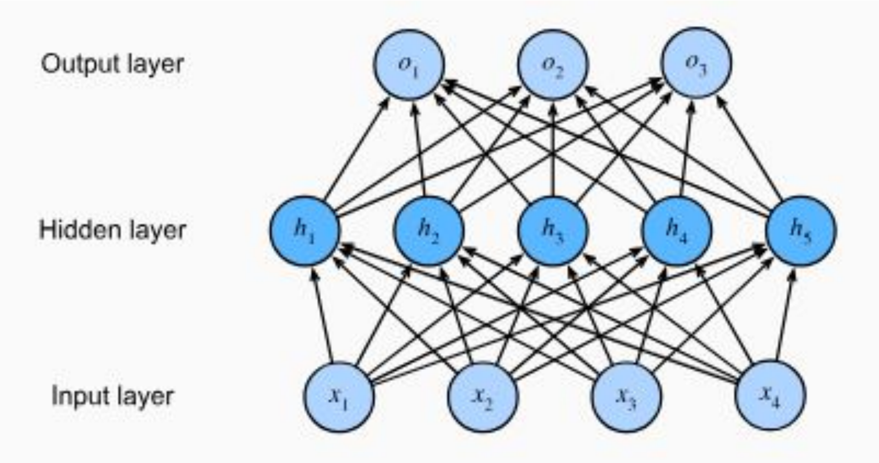
Adam

RMSProp

6.

Forward and backward propagation

Forward propagation



$$y = W_1X + b_1$$

$$h = \sigma(y,a)$$

$$o = W_2h + b_2$$

$$Loss(o)$$

Backward propagation

$$y = W_1 x + b_1$$

$$h = \sigma(y, a)$$

$$o = W_2 h + b_2$$

$$Loss(o)$$

$$\partial_{W1}\, Loss(o)$$

$$\partial_{b1}\, Loss(o)$$

$$\partial_{a}\, Loss(o)$$

$$\partial_{W2}\, Loss(o)$$

$$\partial_{b2}\, Loss(o)$$

$$y = W_1 x + b_1$$

$$h = \sigma(y, a)$$

$$o = W_2 h + b_2$$

$$\mathrm{Loss}(o)$$

$\partial_{W1}\, y$

$\partial_{b1}\, y$

$\partial_{y}\, h$

$\partial_{a}\, h$

$\partial_{h}\, o$

$\partial_{W2}\, o$

$\partial_{b2}\, o$

$\partial_{o}\, \mathrm{Loss}$

$\partial_{W1}\, \mathrm{Loss}$

$\partial_{b1}\, \mathrm{Loss}$

$\partial_{a}\, \mathrm{Loss}$

$\partial_{W2}\, \mathrm{Loss}$

$\partial_{b2}\, \mathrm{Loss}$

$y = W_1 x + b_1$

$h = \sigma(y, a)$

$o = W_2 h + b_2$

$\mathrm{Loss}(o)$

$\partial_{W1}\, y$

$\partial_{b1}\, y$

$\partial_{y}\, h$

$\partial_{a}\, h$

$\partial_{h}\, o$

$\partial_{W2}\, o$

$\partial_{b2}\, o$

$\partial_{o}\, \mathrm{Loss}$

$\partial_{W1}\, \mathrm{Loss}$

$\partial_{b1}\, \mathrm{Loss}$

$\partial_{a}\, \mathrm{Loss}$

$\partial_{W2}\, \mathrm{Loss}$

$\partial_{b2}\, \mathrm{Loss}$

Chain rule

$$g[f(x)]$$

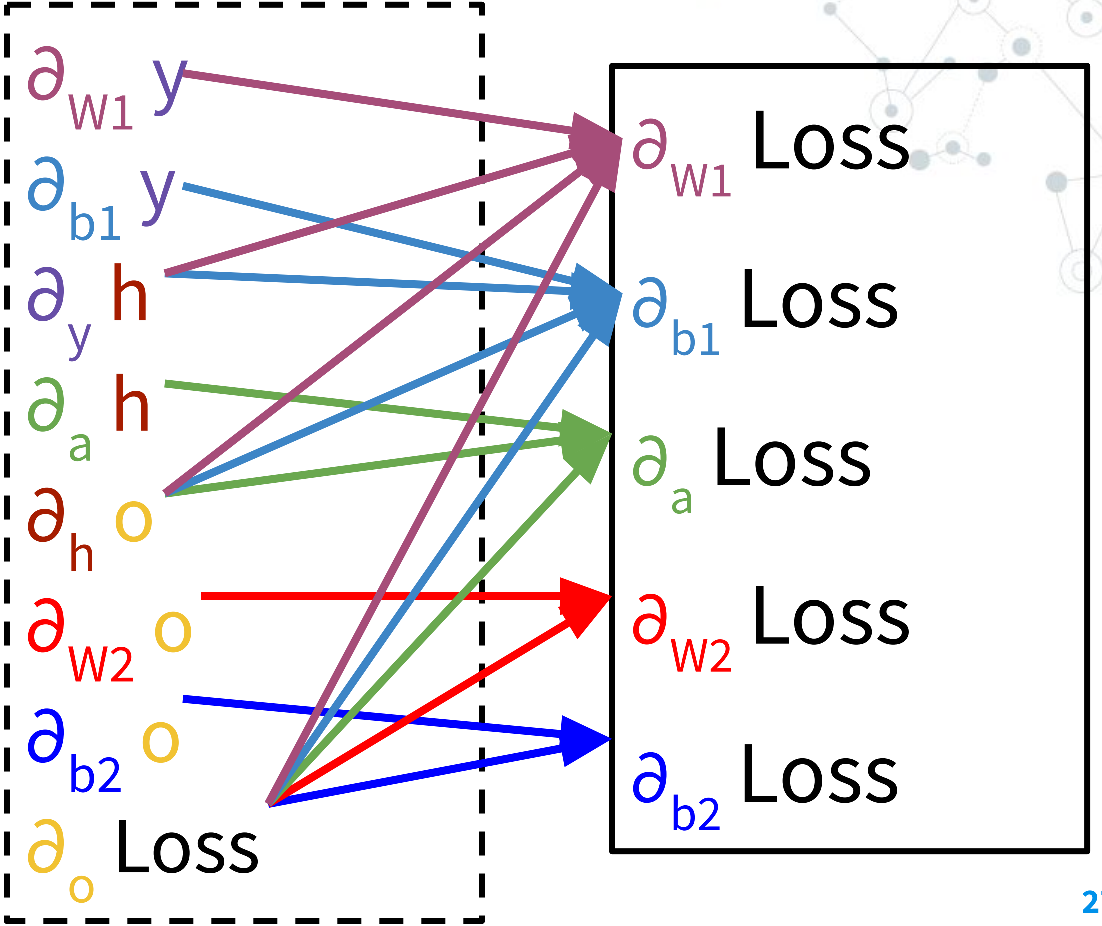
$$\partial_x g = \partial_f g \cdot \partial_x f$$

$$y = W_1 x + b_1$$

$$h = \sigma(y, a)$$

$$o = W_2 h + b_2$$

$$\mathrm{Loss}(o)$$



$\partial_{W1}\, y$

$\partial_{b1}\, y$

$\partial_{y}\, h$

$\partial_{a}\, h$

$\partial_{h}\, o$

$\partial_{W2}\, o$

$\partial_{b2}\, o$

$\partial_{o}\, \text{Loss}$

$$\partial_{W1}\, \text{Loss} = \partial_{o}\, \text{Loss} \cdot \partial_{h}\, o \cdot \partial_{y}\, h \cdot \partial_{W1}\, y$$

$$\partial_{b1}\, \text{Loss} = \partial_{o}\, \text{Loss} \cdot \partial_{h}\, o \cdot \partial_{y}\, h \cdot \partial_{b1}\, y$$

$$\partial_{a}\, \text{Loss} = \partial_{o}\, \text{Loss} \cdot \partial_{h}\, o \cdot \partial_{a}\, h$$

$$\partial_{W2}\, \text{Loss} = \partial_{o}\, \text{Loss} \cdot \partial_{W2}\, o$$

$$\partial_{b2}\, \text{Loss} = \partial_{o}\, \text{Loss} \cdot \partial_{b2}\, o$$

Chain rule

$$y = W_1 x + b_1$$

$$h = \sigma(y, a)$$

$$o = W_2 h + b_2$$

$$\mathrm{Loss}(o)$$

$$\partial_{W1} \mathrm{Loss} = \partial_o \mathrm{Loss} \cdot \partial_h o \cdot \partial_y h \cdot \partial_{W1} y$$

$$\partial_{b1} \mathrm{Loss} = \partial_o \mathrm{Loss} \cdot \partial_h o \cdot \partial_y h \cdot \partial_{b1} y$$

$$\partial_a \mathrm{Loss} = \partial_o \mathrm{Loss} \cdot \partial_h o \cdot \partial_a h$$

$$\partial_{W2} \mathrm{Loss} = \partial_o \mathrm{Loss} \cdot \partial_{W2} o$$

$$\partial_{b2} \mathrm{Loss} = \partial_o \mathrm{Loss} \cdot \partial_{b2} o$$

Chain rule

$$y = W_1 x + b_1$$

$$h = \sigma(y, a)$$

$$o = W_2 h + b_2$$

$$\mathrm{Loss}(o)$$

$$\partial_{W1}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_y\, h \cdot \partial_{W1}\, y$$

$$\partial_{b1}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_y\, h \cdot \partial_{b1}\, y$$

$$\partial_{a}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_a\, h$$

$$\partial_{W2}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_{W2}\, o$$

$$\partial_{b2}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_{b2}\, o$$

Chain rule

$$y = W_1 x + b_1$$

$$h = \sigma(y, a)$$

$$o = W_2 h + b_2$$

$$\mathrm{Loss}(o)$$

$$\partial_{W1}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_y\, h \cdot \partial_{W1}\, y$$

$$\partial_{b1}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_y\, h \cdot \partial_{b1}\, y$$

$$\partial_a\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_a\, h$$

$$\partial_{W2}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_{W2}\, o$$

$$\partial_{b2}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_{b2}\, o$$

~~Chain rule~~ Backpropagation

$$y = W_1 x + b_1$$

$$h = \sigma(y, a)$$

$$o = W_2 h + b_2$$

$$\mathrm{Loss}(o)$$

$$\partial_{W1}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_y\, h \cdot \partial_{W1}\, y$$

$$\partial_{b1}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_y\, h \cdot \partial_{b1}\, y$$

$$\partial_{a}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_h\, o \cdot \partial_a\, h$$

$$\partial_{W2}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_{W2}\, o$$

$$\partial_{b2}\,\mathrm{Loss} = \partial_o\,\mathrm{Loss} \cdot \partial_{b2}\, o$$

		Scalar	Vector	Matrix
		x $(1,)$	$\mathbf{x}$ $(n,1)$	$\mathbf{X}$ (n,k)
Scalar	y $(1,)$	$\frac{\partial y}{\partial x}$ $(1,)$	$\frac{\partial y}{\partial \mathbf{x}}$ $(1,n)$	$\frac{\partial y}{\partial \mathbf{X}}$ (k,n)
Vector	$\mathbf{y}$ $(m,1)$	$\frac{\partial \mathbf{y}}{\partial x}$ $(m,1)$	$\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$ (m,n)	$\frac{\partial \mathbf{y}}{\partial \mathbf{X}}$ (m,k,n)
Matrix	$\mathbf{Y}$ (m,l)	$\frac{\partial \mathbf{Y}}{\partial x}$ (m,l)	$\frac{\partial \mathbf{Y}}{\partial \mathbf{x}}$ (m,l,n)	$\frac{\partial \mathbf{Y}}{\partial \mathbf{X}}$ (m,l,k,n)

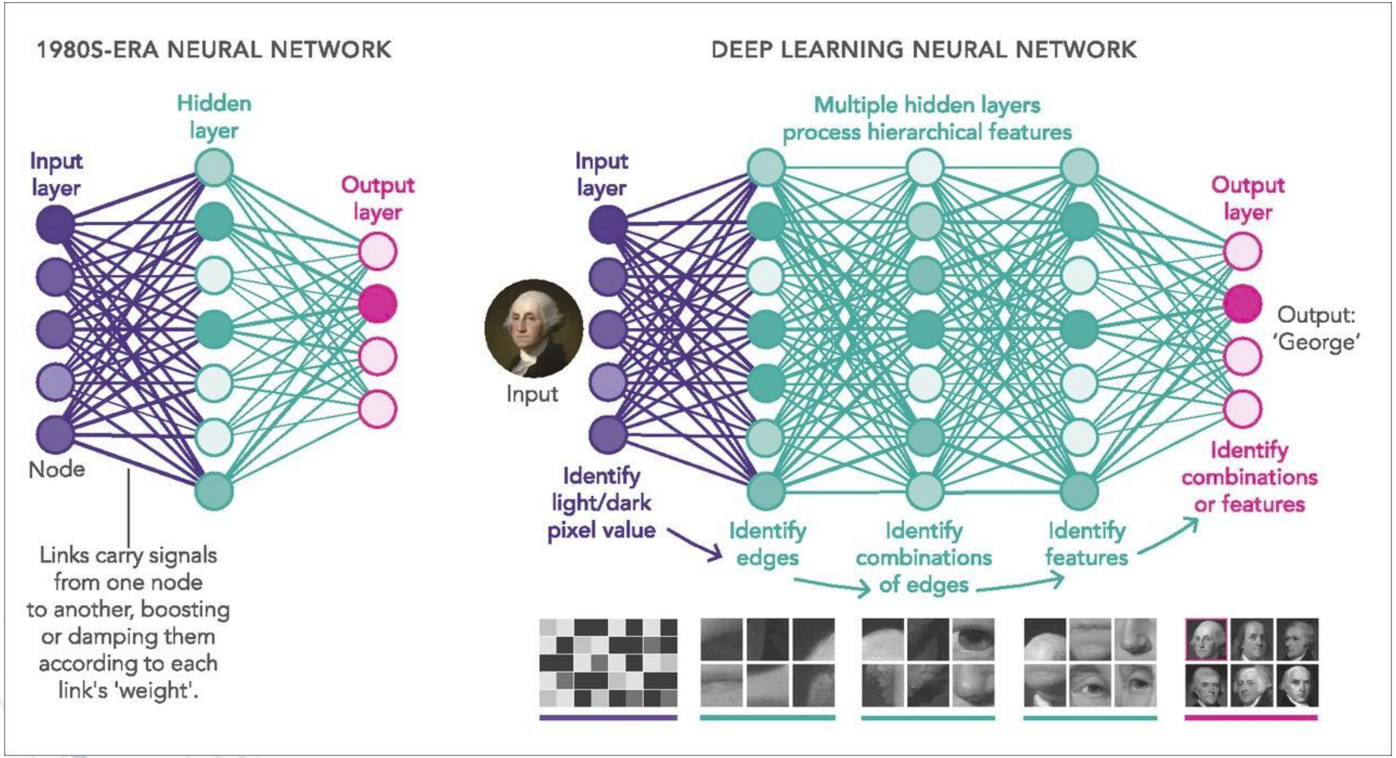
7.

Deep learning

Not enough layers!

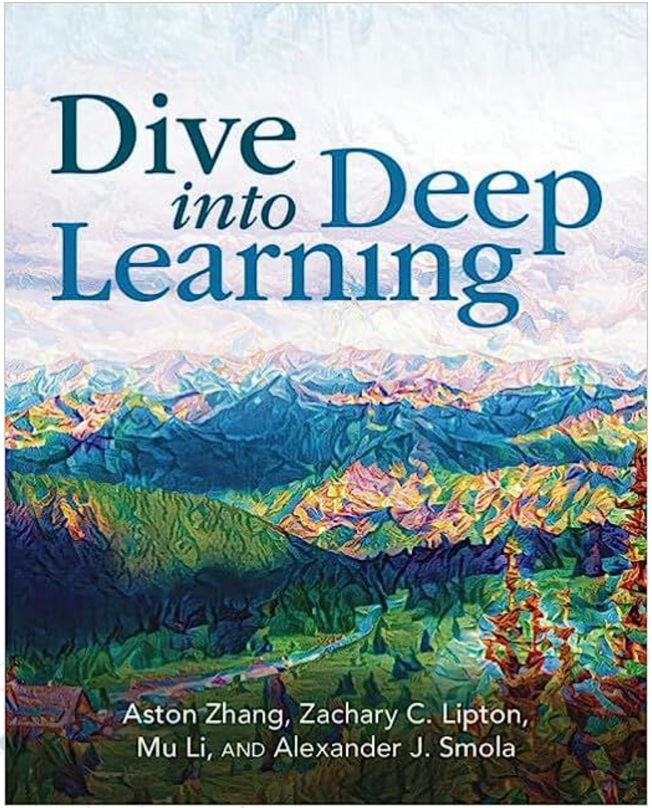
Deep learning is just a good name

Deep learning has become a powerful tool in various domains, including computer vision, natural language processing, and speech recognition.



8.
Books

Book



Dive into Deep Learning

https://d2l.ai/index.html

Thanks!

Any questions?

You can find me at:

rodralez@gmail.com