5.1 Least Squares Global Localization

Least Squares Positioning is a well-known algorithm for estimating the robot localization x given a set of known landmarks in a map. Least Squares is akin to find the best pose $\hat{x}$ by solving a system of equations of the form:

$$z_{m\times 1} = H_{m\times n} \cdot x_{n\times 1}$$

where:

- n is the length of the pose ($n = 3$ in our case, position plus orientation),
- m represents the number of observations, and
- H is the matrix that codifies the observation model relating the measurement z with the robot pose x.

This simple concept, nevertheless, has to be modified in order to be used in real scenarios:

5.1.1 Pseudo-inverse

Generally, to solve an equation system, we only need as many equations as variables. In the robot localization problem, each observation z sets an equation, while the variables are the components of the state/pose, x.

In such a case, where $n = m$, a direct attempt to this problem exists:

$$x = H^{-1} z$$

So a unique solution exists if H is invertible, that is, H is a square matrix with $det(H) \neq 0$.

However, in real scenarios typically there are available more observations than variables. An approach to address this could be to drop some of the additional equations, but given that observations z are inaccurate (they have been affected by some noise), we may use the additional information to try to mitigate such noise. However, by doing that H is no a squared matrix anymore, hence not being invertible.

Two tools can help us at this point. The first one is the utilization of **Least Squares** to find the closest possible $\hat{x}$, i.e. the one where the the error ($e = Hx - z$) is minimal:

$$\hat{x} = \arg\min_x e^T e = [(z - Hx)^T (z - Hx)] = \arg\min_x ||z - Hx||^2$$

which has a close form solution using the **pseudo-inverse** of a matrix:

$$\hat{x} = \underbrace{(H^T H)^{-1} H^T}_{\textit{pseudo-inverse } (H^+)} z$$

The **pseudo-inverse**, in contrast to the normal inverse operation, can be used in non-square matrices!

In [1]:
```
#%matplotlib widget
#%matplotlib inline

# IMPORTS

import math

import numpy as np
from numpy import linalg
import matplotlib
matplotlib.use('TkAgg')
import matplotlib.pyplot as plt
import scipy
from scipy import stats

import sys
sys.path.append("..")
from utils.PlotEllipse import PlotEllipse
from utils.DrawRobot import DrawRobot
from utils.tcomp import tcomp
from utils.tinv import tinv, jac_tinv1 as jac_tinv
from utils.Jacobians import J1, J2
```

ASSIGNMENT 1: Playing with a robot in a corridor

The following code illustrates a simple scenario where a robot is in a corridor looking at a door, which is placed at the origin of the reference system (see Fig.1). The robot is equipped with a laser scanner able to measure distances, and takes a number of observations z. The robot is placed 3 meters away from the door, but this information is unknown for it. **Your goal is** to estimate the position of the robot in this 1D world using such measurements.

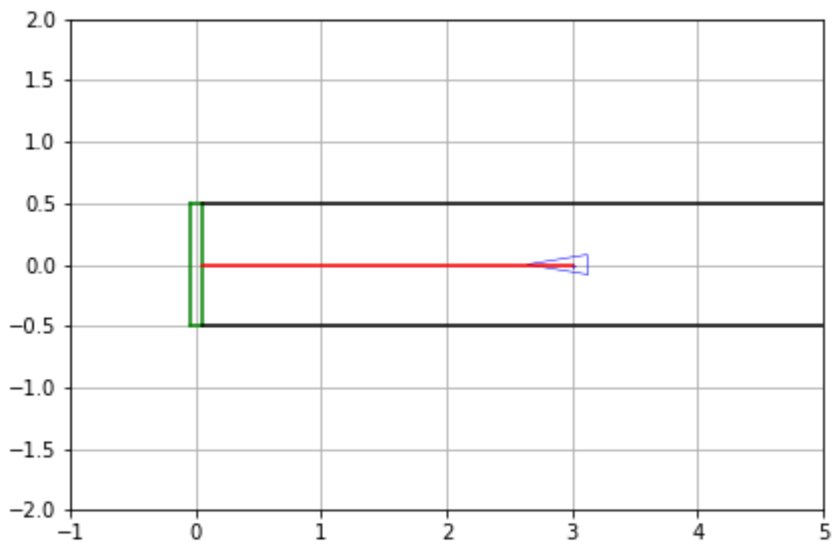


Fig. 1: Simple 1D scenario with a robot equipped with a laser scanner measuring distance to a door.

The following code cell shows the dimensions of all the actors involved in LS-positioning. Complete it for computing the robot pose x from the available information. *Recall* [`np.linalg.inv()` (https://numpy.org/doc/stable/reference/generated/numpy.linalg.inv.html)](https://numpy.org/doc/stable/reference/generated/numpy.linalg.inv.html).

In [2]:
```python
# Set the robot pose to unknown
x = np.vstack(np.array([None]))

# Sensor measurements to the door
z = np.vstack(np.array([3.7,2.9,3.6,2.5,3.5]))

# Observation model
H = np.ones(np.array([5,1]))

print ("Dimensions:")
print ("Pose x:           " + str(x.shape))
print ("Observations z: " + str(z.shape))
print ("Obs. model H:   " + str(H.shape))
print ("H.T@H:          " + str((H.T@H).shape))
print ("inv(H.T@H):     " + str((np.linalg.inv(H.T@H)).shape))
print ("H.T@z :         " + str((H.T@z).shape))

# Do Least Squares Positioning
x = np.linalg.inv(H.T @ H) @ H.T @ z

print('\nLS-Positioning')
print('x = ' + str(x[0]))
```

```
Dimensions:
Pose x:          (1, 1)
Observations z: (5, 1)
Obs. model H:   (5, 1)
H.T@H:          (1, 1)
inv(H.T@H):     (1, 1)
H.T@z :         (1, 1)

LS-Positioning
x = [3.24]
```

Expected output

```
x = [3.24]
```

5.1.2 Weighted measurements

In cases where multiple sensors affected by different noise profiles are used, or in those where the robot is using a sensor with a varying error (*e.g.* typically radial laser scans are more accurate while measuring distances to close objects), it is interesting to weight the contribution of such measurements while retrieving the robot pose. For example, we are going to consider a sensor whose accuracy drops the further the observed landmark is. Given a *covariance* matrix Q describing the error in the measurements, the equations above are rewritten as:

$$\hat{x} = \arg\min_x e^T Q^{-1} e = [(Hx - z)^T Q^{-1} (Hx - z)]$$

$$\hat{x} \leftarrow (H^T Q^{-1} H)^{-1} H^T Q^{-1} z \qquad \text{(1. Best estimation)}$$
$$\Sigma_{\hat{x}} \leftarrow (H^T Q^{-1} H)^{-1} \qquad \text{(2. Uncertainty of the estimation)}$$

Example with three measurements having different uncertainty ($\sigma_1^2, \sigma_2^2, \sigma_3^2$):

$$e^T Q^{-1} e = [e_1 \ e_2 \ e_3] \begin{bmatrix} 1/\sigma_1^2 & 0 & 0 \\ 0 & 1/\sigma_2^2 & 0 \\ 0 & 0 & 1/\sigma_3^2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \frac{e_1^2}{\sigma_1^2} + \frac{e_2^2}{\sigma_2^2} + \frac{e_3^2}{\sigma_3^2} = \sum_{i=1}^{m} \frac{e_i^2}{\sigma_i^2}$$

ASSIGNMENT 2: Adding growing uncertainty

We have new information! The manufacturer of the laser scanner mounted on the robot wrote an email telling us that the device is considerably more inaccurate for further distances. Concretely, such uncertainty is characterized by $\sigma^2 = e^z$ (the laser is not so accurate, being polite).

With this new information, implement the computation of the weighted LS-positioning so you can compare the previously estimated position with the new one.

In [3]:
```
# Sensor measurements to the door
z = np.vstack(np.array([3.7,2.9,3.6,2.5,3.5]))

# Uncertainty of the measurements
Q = np.eye(5)*np.exp(z)

# Observation model
H = np.ones(np.array([5,1]))

# Do Least Squares Positioning
x = np.linalg.inv(H.T @ H) @ H.T @ z

# Do Weighted Least Squares Positioning
x_w = np.linalg.inv(H.T @ np.linalg.inv(Q) @ H) @ H.T @ np.linalg.in

print('\nLS-Positioning')
print('x = ' + str(x[0]))

print('\nWeighted-LS-Positioning')
print('x = ' + str(np.round(x_w[0],2)))
```

```
LS-Positioning
x = [3.24]

Weighted-LS-Positioning
x = [3.01]
```

Expected output

```
LS-Positioning
x = [3.24]

Weighted-LS-Positioning
x = [3.01]
```

5.1.3 Non-linear Least Squares

Until now we have assumed that $\hat{x}$ can be solved as a simple system of equations, i.e. H is a matrix. Nevertheless, typically observation models are non-linear, that is: $z = h(x)$, so the problem now becomes:

$$\hat{x} = \arg\min_{x} ||z - h(x)||^2$$

No close-form solutions exists for this new problem, but we can approximate it iteratively:

$$\textit{(Recall) Taylor expansion: } h(x) = h(x_0 + \delta) = h(x_0) + J_{h_0}\delta$$

$$||z - h(x)||^2 \cong ||\underbrace{z - h(x_0)}_{\textit{error vector } e} - J_{h_o}\delta||^2 = ||e - J_{h_0}\delta||^2 \leftarrow \delta \textit{ is unknown, } J_e = -J_{h_0}$$

So we can define the equivalent optimization problem:

$$\delta = \arg\min_{\delta} ||e + J_e\delta||^2 \rightarrow \underbrace{\delta}_{nx1} =$$

$$-\underbrace{(J_e^T J_e)^{-1}}_{nxn}\underbrace{J_e^T}_{nxm}\underbrace{e}_{mx1} \textit{ (}\delta\textit{ that makes the previous squared norm minimum)}$$

The weighted form of the δ computation results:

$$\delta = (J_e^T Q^{-1} J_e)^{-1} J_e^T Q^{-1} e$$

Where:

- Q is the measurement covariance (*weighted measurement*)
- J_e is the negative of the Jacobian of the observation model at $\hat{x}$, also known as $\nabla h_{\hat{x}}$
- e is the error of z against $h(\hat{x})$ (computed using the map information).

As commented, there is no closed-form solution for the problem, but we can iteratively approximate it using the **Gauss-Newton algorithm**:

$$\begin{aligned}
&\hat{x} \leftarrow (\ldots) && \text{(1. Initial guess)} \\
&\delta \leftarrow (J_e^T Q^{-1} J_e)^{-1} J_e^T Q^{-1} e && \text{(2. Evaluate delta/increment)} \\
&\hat{x} \leftarrow \hat{x} - \delta && \text{(3. Update estimation)} \\
&\textit{if } \delta > \textit{tolerance} \rightarrow \textit{goto (1.)} && \\
&\textit{else } \rightarrow \textit{return } \hat{x} && \text{(4. Exit condition)}
\end{aligned}$$

LS positioning in practice

Suppose that a mobile robot equipped with a range sensor aims to localize itself in a map consisting of a number of landmarks by means of Least Squares and Gauss-Newton optimization.

For that, **you are provided with** the class `Robot` that implements the behavior of a robot that thinks that is placed at `pose` (that's its initial guess, obtained by composing odometry commands), but that has a real position `true pose`. In addition, the variable `cov` models the uncertainty of its movement, and `var_d` represents the variance (noise) of the range measurements. Take a look at it below.

In [4]:
```
class Robot(object):
    """ Simulate a robot base and positioning.

        Attrs:
            pose: Position given by odometry (in this case true_pose
            true_pose: True position, selected by the mouse in this
            cov: Covariance for the odometry sensor. Used to add noi
            var_d: Covariance (noise) of each range measurement
    """
    def __init__(self,
                 pose: np.ndarray,
                 cov: np.ndarray,
                 desv_d: int = 0):
        # Pose related
        self.true_pose = pose
        self.pose = pose + np.sqrt(cov)@np.random.randn(3, 1)
        self.cov = cov

        # Sensor related
        self.var_d = desv_d**2

    def plot(self, fig, ax, **kwargs):
        DrawRobot(fig, ax, self.pose, color='red', label="Pose estim
        DrawRobot(fig, ax, self.true_pose, color="blue", label="Real
```

ASSIGNMENT 3a: Computing distances from the robot to the landmarks

Implement the following function to simulate how our robot observes the world. In this case, the landmarks in the map act as beacons: the robot can sense how far away they are without any information about angles. The robot uses a range sensor with the following observation model:

$$z_i = [d_i] = h(m_i, x) = \left[\sqrt{(x_i - x)^2 + (y_i - y)^2} \right] + w_i$$

where m_i stands for the i^{th} landmark, and w_i is a noise added by the sensor.

Consider two scenarios in the function implementation:

- The measurment is carried out with an ideal sensor, so no noise nor uncertainty exists (`cov_d = 0`).

- The measurement comes from a real sensor affected by a given noise (`cov_d != 0`). We are going to consider that the range sensor is more accurate measuring distances to close landmarks than to far away ones. To implement this, consider that the noise grows with the root of the distance to the landmark, so the resultant uncertainty can be retrieved by:

$$\sigma_{\text{dist}} = \sigma\sqrt{z}$$

that is, `np.sqrt(z)*np.sqrt(cov_d)` . Recall that the sensor noise is modeled as a gaussian distribution, so you have to define such distribution and take samples from it using the [`stats.norm()` (https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.norm.html)](https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.norm.html) and [`rvs()` (https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.rv_continuous.rvs.html)](https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.rv_continuous.rvs.html) functions.

In [5]:
```
def distance(pose: np.ndarray, m: np.ndarray, cov_d: int = 0) -> np.
    """ Get observations for every landmark in the map.

    In this case our observations are range only.
    If cov_d > 0 then add gaussian noise with var_d covariance

    Args:
        pose: pose (true or noisy) of the robot taking observation
        m: Map containing all landmarks
        cov_d: Covariance of the sensor

    Returns
        z: numpy array containing distances to all obs. It has shape
    """
    z = np.sqrt(np.power(m[0]-pose[0],2)+np.power(m[1]-pose[1],2)) #

    if cov_d > 0:
        z += stats.norm(loc=0, scale=np.sqrt(z)*np.sqrt(cov_d)).rvs(

    return z
```

Try your brand new function with the following code:

In [6]:
```
pose = np.vstack([2, 2, 0.35])
m = np.array([[-5,-15],[20,56],[54,-18]]).T
cov_d = 0

# Compute distances from the sensor to the landmarks
z = distance(pose,m,cov_d)

# Now consider a noisy sensor
cov_d = 0.5
np.random.seed(seed=0)
z_with_noise = distance(pose,m,cov_d)

# Show the results
print('Measurements without noise:' + str(z))
print('Measurements with noise:   ' + str(z_with_noise))
```

```
Measurements without noise:[18.38477631 56.92099788 55.71355311]
Measurements with noise:   [23.73319805 59.05577186 60.87928514]
```

Expected output

```
Measurements without noise:[18.38477631 56.92099788 55.71355
311]
Measurements with noise:   [23.73319805 59.05577186 60.87928
514]
```

ASSIGNMENT 3b: Implementing the algorithm

Finally, we get to implement the Least Squares algorithm for localization. We ask you to complete the gaps in the following function, which:

- Starts by initializing the Jacobian of the observation function (`Jh`) and takes as initial guess (`xEst`) the position at which the robot thinks it is as given by its odometry (`R1.pose`).
- Then, it enters into a loop until convergence is reached, where:
  1. The distances `zEst` to each landmark from the estimated position `xEst` are computed. Recall that the map (landmarks positions) are known (`w_map`).
  - The error is computed by substracting to the obsevations provided by the sensor `z` the distances `zEst` computed at the previous point. Then, the residual error is computed as $e_{residual} = \sqrt{e_x^2 + e_y^2}$.
  - The Jacobian of the observation model is evaluated at the estimated robot pose (`xEst`). This Jacobian has two columns and as many rows as observations to the landmarks:

$$jH = \begin{bmatrix} \frac{-1}{d_1}(x_1 - x) & \frac{-1}{d_1}(y_1 - y) \\ \frac{-1}{d_2}(x_2 - x) & \frac{-1}{d_2}(y_2 - y) \\ \cdots & \cdots \\ \frac{-1}{d_n}(x_n - x) & \frac{-1}{d_n}(y_n - y) \end{bmatrix}$$

    being $xEst = [x, y]$, $[x_i, y_i]$ the position of the i^{th} landmark in the map, and d the distance previously computed from the robot estimated pose $xEst$ to the landmarks. The jacobian of the error `jE` is just `-jH` .
  - Computes the increment δ (`incr`) and substract it to the estimated pose (`xEst`). *Note: recall that* $\delta = (J_e^T Q^{-1} J_e)^{-1} J_e^T Q^{-1} e$

In [7]:
```python
def LeastSquaresLocalization(R1: Robot,
                             w_map: np.ndarray,
                             z: np.ndarray,
                             nIterations=10,
                             tolerance=0.001,
                             delay=0.5) -> np.ndarray:
    """ Pose estimation using Gauss-Newton for least squares optimiz

        Args:
            R1: Robot which pose we must estimate
            w_map: Map of the environment
            z: Observation received from sensor

            nIterations: sets the maximum number of iterations (defa
            tolerance: Minimum error difference needed for stopping

            delay: Wait time used to visualize the different iterati

        Returns:
            xEst: Estimated pose

    """

    iteration = 0

    # Initialization of useful variables
    incr = np.ones((2, 1)) # Delta
    jH = np.zeros((w_map.shape[1], 2)) # Jacobian of the observation
    xEst = R1.pose #Initial estimation is the odometry position (usu

    # Let's go!
    while linalg.norm(incr) > tolerance and iteration < nIterations:
        #if plotting:
        plt.plot(xEst[0], xEst[1], '+r', markersize=1+math.floor((it
        # Compute the predicted observation (from xEst) and their re

        # 1) TODO: Compute distance to each landmark from xEst (esti
        #
        zEst = distance(xEst, w_map)

        # 2) TODO: error = difference between real observations and
        e = z - zEst
        residual = np.sqrt(e.T@e) #residual error = sqrt(x^2+y^2)

        # 3) TODO: Compute Jacobians with respect (x,y) (slide 13)
        # The jH is evaluated at our current guest (xEst) -> z_p

        jH = np.array([
            (-1/zEst[i]) *
            np.hstack([
                w_map[0,i]-xEst[0,0],
                w_map[1,i]-xEst[1,0]
            ])
            for i in range(w_map.shape[1])
        ])
        jE = -jH

        # The observation variances Q grow with the root of the dist
        Q = np.diag(R1.var_d*np.sqrt(z))
```

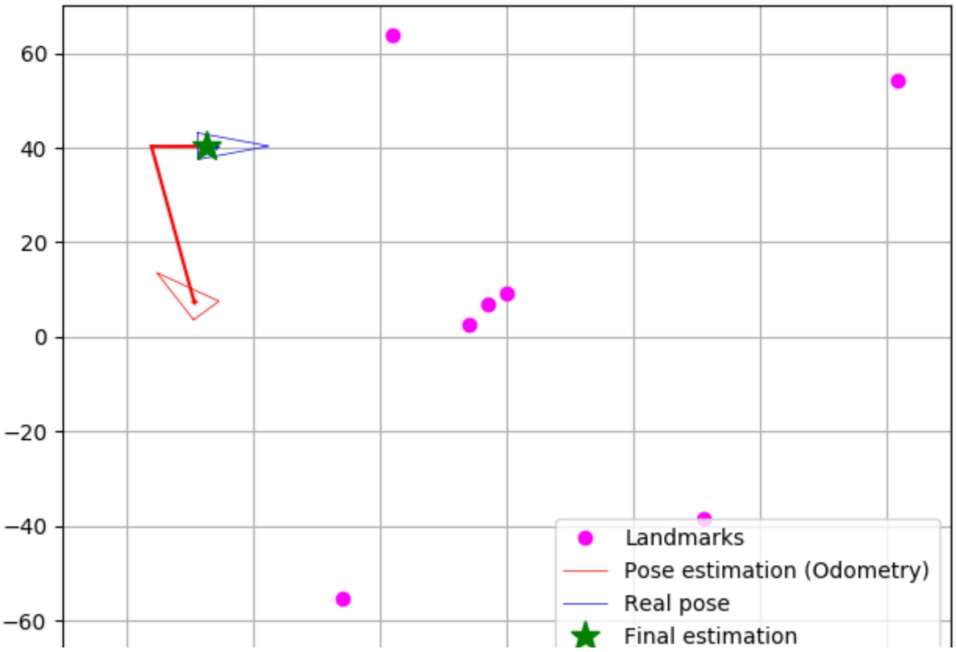
```
        # 4) TODO: Solve the equation --> compute incr
        invQ = np.linalg.inv(Q)
        incr = np.linalg.inv(jE.T @ invQ @ jE) @ jE.T @ invQ @ e

        plt.plot([xEst[0, 0], xEst[0, 0]-incr[0]], [xEst[1, 0], xEst
        xEst[0:2, 0] -= incr

        print ("Iteration :" + str(iteration))
        print ("  delta :    " + str(incr))
        print ("  residual: " + str(residual))

        iteration += 1

        plt.pause(delay)

    plt.plot(xEst[0, 0], xEst[1, 0], '*g', markersize=14, label="Fin

    return xEst
```

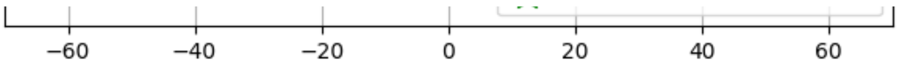
The next cell code launches our algorithm, so **we can try it!**. This is done according to the following steps:

1. The map `w_map` is built. In this case, the map consists of a number of landmarks (`nLandmarks`).
2. The program asks the user to set the true position of the robot (`xTrue`) by clicking with the mouse in the map.
3. A new pose is generated from it, `xOdom` , which represents the pose that the robot thinks it is in. This simulates a motion command from an arbitrary pose that ends up with the robot in `xTrue` , but it thinks that it is in `xOdom` .
4. Then the robot takes a (noisy) range measurement to each landmark in the map.
5. Finally, the robot employs a Least Squares definition of the problem and Gauss-Newton to iteratively optimize such a guess (`xOdom`), obtaining a new (and hopefully better) estimation of its pose `xEst` .

Example

The figure below shows an example of execution of this code (once completed).




−60
−40
−20
0
20
40
60

In [8]:
```python
def main(nLandmarks=7, env_size=140):
    # MATPLOTLIB
    fig, ax = plt.subplots()
    plt.xlim([-90, 90])
    plt.ylim([-90, 90])
    plt.grid()
    plt.ion()
    plt.tight_layout()

    fig.canvas.draw()

    # VARIABLES
    num_landmarks = 7 # number of landmarks in the environment
    env_size = 140 # A square environment with x=[-env_size/2,env_si

    # MAP CREATION AND VISUALIZATION
    w_map = env_size*np.random.rand(2, num_landmarks) - env_size/2 #
    ax.plot(w_map[0, :], w_map[1, :], 'o', color='magenta', label="L

    # ROBOT POSE AND SENSOR INITIALIZATION
    desv_d = 0.5 # standard deviation (noise) of the range measureme
    cov = np.diag([25, 30, np.pi*180])**2 # covariance of the motion
    xStart = np.vstack(plt.ginput(1)).T # get the robot starting poi
    robot_pose=np.vstack([xStart, 0]) # robot_pose

    R1 = Robot(robot_pose, cov, desv_d)
    R1.plot(fig, ax)

    # MAIN
    z = distance(R1.true_pose, w_map, cov_d=R1.var_d) # take (noisy)
    LeastSquaresLocalization(R1, w_map, z) # LS Positioning!

    # PLOTTING RESULTS
    plt.legend()
    fig.canvas.draw()

# RUN
main()
```

```
Iteration :0
  delta :   [62.76198308  2.01922692]
  residual: 97.99999116491452
Iteration :1
  delta :   [-5.06702922 15.08434138]
  residual: 37.57813235267521
Iteration :2
  delta :   [-1.02135505 -0.43253851]
  residual: 18.917617807277903
Iteration :3
  delta :   [-0.02513509 -0.04401566]
  residual: 18.681070995039338
Iteration :4
  delta :   [-0.00126247 -0.00286635]
  residual: 18.681657414395552
Iteration :5
  delta :   [-7.59565833e-05 -1.76843510e-04]
  residual: 18.681764931834394
```

Thinking about it (1)

Having completed this notebook above, you will be able to **answer the following questions**:

- What are the dimensions of the error residuals? Does them depend on the number of observations?

  El error residual es de dimension 1x1, es decir un escalar. No depende del numero de observaciones

- Why is Weighted LS obtaining better results than LS?

  Porque hay que tener en cuenta la precision de los sensores, teniendo mas peso los que son mas precisos

- Which is the minimum number of landmarks needed for localizing the robot? Why?

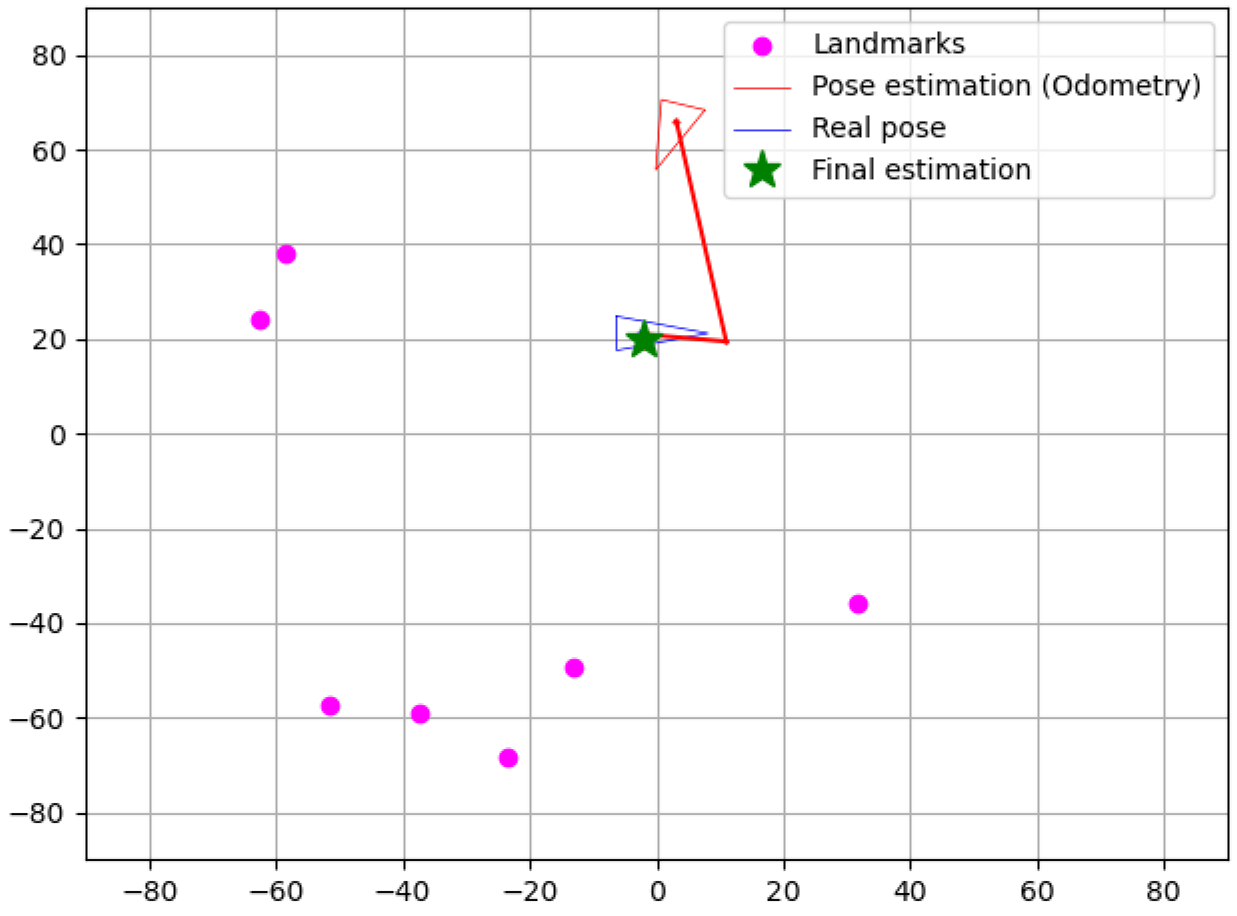
  Necesitamos como minimo tantas landmarks como elementos haya en nuestra pose, en este caso 3, $[x, y, \theta]$, porque si tuevieramos menos observaciones que elementos, $H^T H$ no seria invertible y no podriamos calcular el error minimo cuadratico

- Play with different “qualities” of the range sensor. Could you find a value for its variance so the LS method fails?

  Para valores altos de la varianza como 2 o 3, la estimacion es muy mala. Y para valores altos donde std > 8, el metodo falla

- Play also with different values for the odometry uncertainty. What does this affect?

  Esto afecta a la estimacion de la pose inicial realizada por la odometria, pero no afecta en gran medida a la estimacion final realizada por el metodo de minimos cuadrados



Landmarks
Pose estimation (Odometry)
Real pose
Final estimation
80
60
40
20
0
−20
−40
−60
−80
−80
−60
−40
−20
0
20
40
60
80

5.2 EKF Localization

The Kalman filter is one of the best studied techniques for filtering and prediction of linear systems. Among its virtues, it provides a way to overcome the occasional un-observability problem of the Least Squares approach. Nevertheless, it makes a strong assumption that the two involved process equations (state transition and observation) are linear.

Unfortunately, you should already know that our system of measurements (i.e. the observation function) and motion (i.e. pose composition) are non-linear. Therefore, this notebook focuses from the get-go on the **Extended Kalman Filter**, which is adapted to work with non-linear systems.

The EKF algorithm consists of 2 phases: **prediction** and **correction**.

$$
\begin{aligned}
&\texttt{def ExtendedKalmanFilter}(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t) : \\
&\qquad \textbf{Prediction.} \\
&\qquad \bar{\mu}_t = g(\mu_{t-1}, u_t) = \mu_{t-1} \oplus u_t && \text{(1. Pose prediction} \\
&\qquad \bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t && \text{(2. Uncertainty of prediction} \\
&\qquad \textbf{Correction.} \\
&\qquad K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1} && \text{(3. Kalman gain} \\
&\qquad \mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t)) && \text{(4. Pose estimation} \\
&\qquad \Sigma_t = (I - K_t H_t) \bar{\Sigma}_t && \text{(5. Uncertainty of estimation} \\
&\qquad \texttt{return } \mu_t, \Sigma_t
\end{aligned}
$$

Notice that R_t is the covariance of the motion u_t in the coordinate system of the predicted pose $(\bar{x}_t)$, then (Note: J_2 is our popular Jacobian for the motion command, you could also use J_1):

$$R_t = J_2 \Sigma_{u_t} J_2^T \quad \text{with} \quad J_2 = \frac{\partial g(\mu_{t-1}, u_t)}{\partial u_t}$$

Where:

- (μ_t, Σ_t) represents our robots pose.
- (u_t, Σ_{u_t}) is the motion command received, and its respective uncertainty.
- (z_t, Q_t) are the observations taken, and their covariance.
- G_t and H_t are the Jacobians of the motion model and the observation model respectively:

$$G_t = \frac{\partial g(\mu_{t-1}, u_t)}{\partial x_{t-1}}, \qquad H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t}$$

In this notebook we are going to play with the EKF localization algorithm using a map of landmarks and a sensor providing range and bearing measurements from the robot pose to such landmarks. Concretely, **we are going to**:

1. Implement a class modeling a **range and bearing sensor** able to take measurements to landmarks.
2. Complete a class that implements the robot behavior after completing **motion**

commands.
3. Implement the **Jacobian of the observation model**.
4. With the previous building blocks, implement our own **EKF filter** and see it in action.
5. Finally, we are going to consider a more **realistic sensor** with a given Field of View and a maximum operational range.

In [1]:
```
# IMPORTS
import numpy as np
from numpy import random
from numpy import linalg
import matplotlib
matplotlib.use('TkAgg')
from matplotlib import pyplot as plt
from IPython.display import display, clear_output
import time

import sys
sys.path.append("..")
from utils.AngleWrap import AngleWrapList
from utils.PlotEllipse import PlotEllipse
from utils.Drawings import DrawRobot, drawFOV, drawObservations
from utils.Jacobians import J1, J2
from utils.tcomp import tcomp
```

ASSIGNMENT 1: Getting an observation to a random landmark

We are going to implement the `Sensor()` class modelling a range and bearing sensor. Recall that the observation model of this type of sensos is:

$$z_i = \begin{bmatrix} d_i \\ \theta_i \end{bmatrix} = h(m_i, x) = \begin{bmatrix} \sqrt{(x_i - x)^2 + (y_i - y)^2} \\ atan\left(\frac{y_i - y}{x_i - x}\right) - \theta \end{bmatrix} + w_i$$

where $m_i = [x_i, y_i]$ are the landmark coordinates in the world frame, $x = [x, y, \theta]$ is the robot pose, and the noise w_i follows a Gaussian distribution with zero mean and covariance matrix:

$$\Sigma_S = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$

For that, complete the following methods:

- `observe()` : which, given a real robot pose (`from_pose`), returns the measurments to the landmarks in the map (`world`). If `noisy=true` , then a random gaussian noise with zero mean and covariance Σ_S (`cov`) is added to each measurement. *Hint you can use* [`random.randn()` (https://docs.scipy.org/doc/numpy-1.15.1/reference/generated/numpy.random.randn.html)](https://docs.scipy.org/doc/numpy-1.15.1/reference/generated/numpy.random.randn.html) *for that.*
- `random_observation()` : that, given again the robot pose (`from_pose`), randomly selects a landmark from the map (`world`) and returns an observation from the range-bearing sensor using the `observe()` method previously implemented. The `noisy` argument is just passed to `observe()` . *Hint: to randomly select a landmark, use* `randint()` (https://numpy.org/doc/stable/reference/random

/generated/numpy.random.randint.html).

In [2]:
```
class Sensor():
    def __init__(self, cov):
        """
        Args:
            cov: covariance of the sensor.
        """
        self.cov = cov

    def observe(self, from_pose, world, noisy=True, flatten=True):
        """Calculate observation relative to from_pose

        Args:
            from_pose: Position(real) of the robot which takes the o
            world: List of world coordinates of some landmarks
            noisy: Flag, if true then add noise (Exercise 2)

        Returns:
                Numpy array of polar coordinates of landmarks from t
                They are organised in a vertical vector ls = [d_0 ,
                Dims (2*n_landmarks, 1)
        """
        delta = world - from_pose[0:2]

        z = np.empty_like(delta)
        z[0, :] = np.sqrt(np.power(delta[0],2) + np.power(delta[1],2
        z[1, :] = np.arctan2(delta[1], delta[0]) - from_pose[2]
        z[1, :] = AngleWrapList(z[1, :])

        if noisy:
            z += np.sqrt(self.cov) @ random.randn(2, z.shape[1])

        if flatten:
            return np.vstack(z.flatten('F'))
        else:
            return z

    def random_observation(self, from_pose, world, noisy=True):
        """ Get an observation from a random landmark

            Args: Same as observe().

            Returns:
                z: Numpy array of obs. in polar coordinates
                landmark: Index of the randomly selected landmark in
                    Although it is only one index, you should return
                    a numpy array.
        """
        n_landmarks = world.shape[1]
        rand_idx = random.randint(0,n_landmarks)
        world = world[:, [rand_idx]]

        z = self.observe(from_pose, world, noisy)

        return z, np.array([rand_idx])
```

You can use the code cell below **to test your implementation**.

In [3]:
```python
# TRY IT!
seed = 0
np.random.seed(seed)

# Sensor characterization
SigmaR = 1 # Standard deviation of the range
SigmaB = 0.7 # Standard deviation of the bearing
Q = np.diag([SigmaR**2, SigmaB**2]) # Cov matrix

sensor = Sensor(Q)

# Map
Size = 50.0
NumLandmarks = 3
Map = Size*2*random.rand(2,NumLandmarks)-Size

# Robot true pose
true_pose = np.vstack([-Size+Size/3, -Size+Size/3, np.pi/2])

# Take a random measurement
noisy = False
z = sensor.random_observation(true_pose, Map, noisy)

noisy = True
noisy_z = sensor.random_observation(true_pose, Map, noisy)

# Take observations to every landmark in the map
zs = sensor.observe(true_pose, Map, noisy)

print('Measurement:\n' + str(z))
print('Noisy measurement:\n' + str(noisy_z))
print('Measurements to every landmark in the map:\n' + str(zs))
```

```
Measurement:
(array([[53.76652662],
       [-0.79056712]]), array([0]))
Noisy measurement:
(array([[64.73997127],
       [-0.81342958]]), array([2]))
Measurements to every landmark in the map:
[[ 5.51319938e+01]
 [-1.10770618e+00]
 [ 6.04762304e+01]
 [-1.46219661e+00]
 [ 6.23690518e+01]
 [-5.72010701e-02]]
```

Expected output

```
Measurement:
(array([[53.76652662],
       [-0.79056712]]), array([0]))
Noisy measurement:
```

ASSIGNMENT 2: Simulating the robot motion

In the robot motion chapter we commanded a mobile robot to follow a squared trajectory. We provide here the `Robot` class that implements:

- how the robot pose evolves after executing a motion command (`step()` method), and
- the functionality needed to graphically show its ideal pose (`pose`), true pose (`true_pose`) and estimated pose (`xEst`) in the `draw()` function.

Your mission is to complete the `step()` method by adding random noise to each motion command (`noisy_u`) based on the following covariance matrix, and update the true robot pose (`true_pose`):

$$\Sigma_{u_t} = \begin{bmatrix} \sigma^2_{\Delta x} & 0 & 0 \\ 0 & \sigma^2_{\Delta y} & 0 \\ 0 & 0 & \sigma^2_{\Delta \theta} \end{bmatrix}$$

Hint: Recall again the [`random.randn()` (https://docs.scipy.org/doc/numpy-1.15.1/reference/generated/numpy.random.randn.html)](https://docs.scipy.org/doc/numpy-1.15.1/reference/generated/numpy.random.randn.html) function.

In [4]:
```python
class Robot():
    def __init__(self, true_pose, cov_move):
        # Robot description (Starts as perfectly known)
        self.pose = true_pose
        self.true_pose = true_pose
        self.cov_move = cov_move

        # Estimated pose and covariance
        self.xEst = true_pose
        self.PEst = np.zeros((3, 3))

    def step(self, u):
        self.pose = tcomp(self.pose,u) # New pose without noise
        noise = np.sqrt(self.cov_move)@random.randn(3,1) # Generate
        noisy_u = noise + u #  Apply noise to the control action
        self.true_pose = tcomp(self.true_pose, noisy_u) #  New noisy

    def draw(self, fig, ax):
        DrawRobot(fig, ax, self.pose, color='r')
        DrawRobot(fig, ax, self.true_pose, color='b')
        DrawRobot(fig, ax, self.xEst, color='g')
        PlotEllipse(fig, ax, self.xEst, self.PEst, 4, color='g')
```

It is time **to test** your `step()` function!

In [5]:
```python
# Robot base characterization
SigmaX = 0.8 # Standard deviation in the x axis
SigmaY = 0.8 # Standard deviation in the y axis
SigmaTheta = 0.1 # Bearing standar deviation
R = np.diag([SigmaX**2, SigmaY**2, SigmaTheta**2]) # Cov matrix

# Create the Robot object
true_pose = np.vstack([2,3,np.pi/2])
robot = Robot(true_pose, R)

# Perform a motion command
u = np.vstack([1,2,0])
np.random.seed(0)
robot.step(u)

print('robot.true_pose.T:' + str(robot.true_pose.T) + '\'')
```

```
robot.true_pose.T:[[-0.32012577  5.41124188  1.66867013]]'
```

Expected output

```
robot.true_pose.T:[[-0.32012577  5.41124188  1.66867013]]'
```

ASSIGNMENT 3: Jacobians of the observation model

Given that the position of the landmarks in the map is known, we can use this information in a Kalman filter, in our case an EKF. For that we need to implement the **Jacobians of the observation model**, as required by the correction step of the filter.

Implement the function `getObsJac()` that given:

- the predicted pose in the first step of the Kalman filter,
- a number of observed landmarks, and
- the map,

returns such Jacobian. Recall that, for each observation to a landmark:

$$\nabla H = \frac{\partial h}{\partial \{x, y, \theta\}} = \begin{bmatrix} -\frac{x_i - x}{d} & -\frac{y_i - y}{d} & 0 \\ \frac{y_i - y}{d^2} & -\frac{x_i - x}{d^2} & -1 \end{bmatrix}_{2\times 3}$$

Recall that $[x_i, y_i]$ is the position of the i^{th} landmark in the map, $[x, y]$ is the robot predicted pose, and d the distance such to the landmark. This way, the resultant Jacobian dimensions are $(\#observed\_landmarks \times 2, 3)$, that is, the Jacobians are stacked vertically to form the matrix H.

In [6]:
```python
def getObsJac(xPred, lm, Map):
    """ Obtain the Jacobian for all observations.

        Args:
            xPred: Position of our robot at which Jac is evaluated.
            lm: Numpy array of observations to a number of landmarks
            Map: Map containing the actual positions of the observat

        Returns:
            jH: Jacobian matrix (2*n_landmaks, 3)
    """
    n_land = len(lm)
    jH = np.empty((2*n_land,3))

    for i in range(n_land):
        # Auxiliary variables
        dx = Map[0, lm[i]] - xPred[0,0]
        dy = Map[1, lm[i]] - xPred[1,0]
        d = np.sqrt(np.power(dx, 2)+np.power(dy, 2))
        d2 = np.power(d, 2)

        ii = 2*i

        # Build the Jacobian
        jH[ii:ii+2,:] = [
            [-dx/d, -dy/d, 0],
            [dy/d2, -dx/d2, -1]
        ]

    return jH
```

Time **to check** your function!

In [7]:
```python
# TRY IT!

observed_landmarks = np.array([0,2])
xPred = np.vstack([-Size+Size/3, -Size+Size/3, np.pi/2]) # Robot pre
jH = getObsJac(xPred, observed_landmarks, Map) # Retrieve the evalua

print ('Jacobian dimensions: ' + str(jH.shape) )
print ('jH:' + str(jH))
```

```
Jacobian dimensions: (4, 3)
jH:[[-0.71075232 -0.70344235  0.        ]
 [ 0.01308328 -0.01321923 -1.        ]
 [-0.67304061 -0.73960552  0.        ]
 [ 0.01141455 -0.01038723 -1.        ]]
```

Expected output:

```
Jacobian dimensions: (4, 3)
jH:[[-0.71075232 -0.70344235  0.        ]
 [ 0.01308328 -0.01321923 -1.        ]
 [-0.67304061 -0.73960552  0.        ]
 [ 0.01141455 -0.01038723 -1.        ]]
```

ASSIGNMENT 4: Completing the EKF

Congratulations! You now have all the building blocks needed to implement an EKF filter (both prediction and correction steps) for localizating the robot and show the estimated pose and its uncertainty.

For doing that, complete the `EKFLocalization()` function below, which returns:

- the estimated pose (`xEst`), and
- its associated uncertainty (`PEst`),

given:

- the previous estimations (`self.xEst` and `self.PEst` stored in `robot`),
- the features of the sensor (`sensor`),
- the motion command provided to the robot (`u`),
- the observations done (`z`),
- the indices of the observed landmarks (`landmark`), and
- the map of the environment (`Map`).

In [8]:
```
def EKFLocalization(robot, sensor, u, z, landmark, Map):
    """ Implement the EKF algorithm for localization

        Args:
            robot: Robot base (contains the state: xEst and PEst)
            sensor: Sensor of our robot.
            u: Motion command
            z: Observations received
            landmark: Indices of landmarks observed in z
            Map: Array with landmark coordinates in the map

        Returns:
            xEst: New estimated pose
            PEst: Covariance of the estimated pose
    """
    from scipy.linalg import block_diag

    # Prediction
    xPred = tcomp(robot.xEst, u)
    G = J1(xPred, robot.xEst)
    j2 = J2(xPred, u)
    PPred = G@robot.PEst@G.T + j2@robot.cov_move@j2.T

    # Correction (You need to compute the gain k and the innovation
    if landmark.shape[0] > 0:
        H = getObsJac(xPred, landmark, Map) # Observation Jacobian
        K = PPred @ H.T @ linalg.inv((H @ PPred @ H.T) + block_diag(
        xEst = xPred + (K @ (z - sensor.observe(xPred, Map[:,landmar
        PEst = (np.identity(H.shape[1]) - K @ H) @ PPred # New estim
    else:
        xEst = xPred
        PEst = PPred

    return xEst, PEst
```

You can **validate your code** with the code cell below.

In [9]:
```
# TRY IT!

np.random.seed(2)

# Create the map
Map=Size*2*random.rand(2,20)-Size

# Create the Robot object
true_pose = np.vstack([2,3,0])
R = np.diag([0.1**2, 0.1**2, 0.01**2]) # Cov matrix
robot = Robot(true_pose, R)

# Perform a motion command
u = np.vstack([10,0,0])
robot.step(u)

# Get an observation to a landmark
noisy = True
noisy_z, landmark_index = sensor.random_observation(true_pose, Map,

# Estimate the new robot pose using EKF!
robot.xEst, robot.PEst = EKFLocalization(robot, sensor, u, noisy_z,

# Show resutls!
print('robot.pose.T:' + str(robot.pose.T) + '\'')
print('robot.true_pose.T:' + str(robot.true_pose.T) + '\'')
print('robot.xEst.T:' + str(robot.xEst.T) + '\'')
print('robot.PEst:' + str(robot.PEst.T))
```

```
robot.pose.T:[[12.  3.  0.]]'
robot.true_pose.T:[[ 1.20000010e+01  3.05423526e+00 -3.13508197e-0
3]]'
robot.xEst.T:[[ 1.19586407e+01  2.96047951e+00 -1.48514185e-04]]'
robot.PEst:[[ 9.94877200e-03 -4.94253023e-05 -3.18283546e-08]
 [-4.94253023e-05  9.95211532e-03  3.29230513e-08]
 [-3.18283546e-08  3.29230513e-08  9.99795962e-05]]
```

Expected output:

```
robot.pose.T:[[12.  3.  0.]]'
robot.true_pose.T:[[ 1.20000010e+01  3.05423526e+00 -3.13508
197e-03]]'
robot.xEst.T:[[ 1.19586407e+01  2.96047951e+00 -1.48514185e-
04]]'
robot.PEst:[[ 9.94877200e-03 -4.94253023e-05 -3.18283546e-0
8]
 [-4.94253023e-05  9.95211532e-03  3.29230513e-08]
 [-3.18283546e-08  3.29230513e-08  9.99795962e-05]]
```

Playing with EKF

The following code helps you to see the EKF filter in action!. Press any key on the emerging window to send a motion command to the robot and check how the landmark it observes changes, as well as its ideal, true and estimated poses.

Notice that you can change the value of `seed` within the `main()` function to try different executions.

Example

The figure below shown an example of the execution of the EKF localization algorithm with the code implemented until this point.

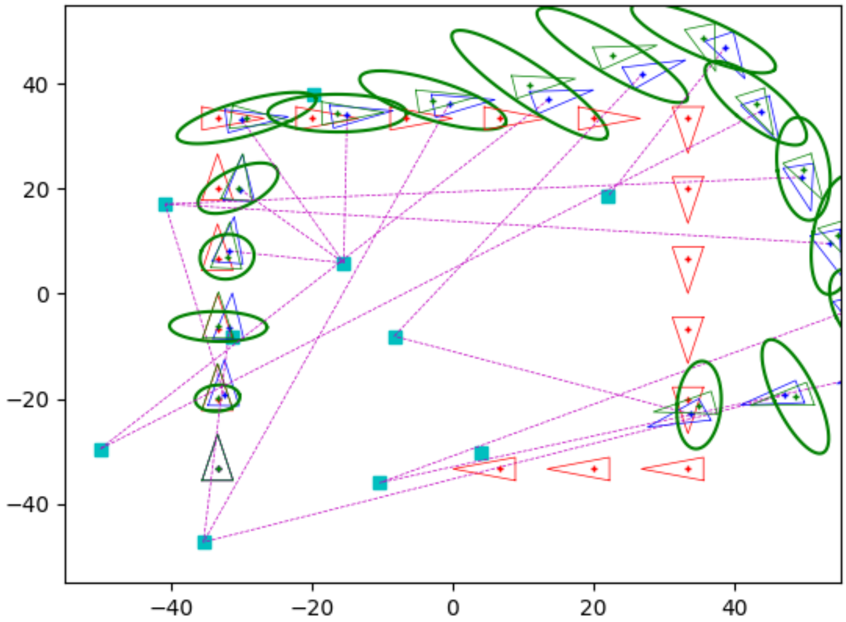


Fig. 1: Execution of the EKF algorithmn for localization,
it shows the true (in blue) and expected (in red) poses,
along the results from localization: pose and ellipse (in green),
the existing landmarks (in cyan),
and each observation made (dotted lines).

```
In [10]: def main(robot,
                  sensor,
                  mode='one_landmark',
                  visualization = 'non_stop',
                  nSteps=20, # Number of motions
                  turning=5, # Number of motions before turning (square path)
                  Size=50.0,
                  NumLandmarks=10):

             seed = 1
             np.random.seed(seed)

             #Create map
             Map=Size*2*random.rand(2,NumLandmarks)-Size

             # MATPLOTLIB
             if visualization == 'non_stop':
                 %matplotlib widget
             elif visualization == 'step_by_step':
                 #%matplotlib inline
                 matplotlib.use('TkAgg')
                 plt.ion()

             fig, ax = plt.subplots()
             plt.plot(Map[0,:],Map[1,:],'sc')
             plt.axis([-Size-15, Size+15, -Size-15, Size+15])
             plt.title(mode)

             robot.draw(fig, ax)
             fig.canvas.draw()

             # MAIN LOOP

             u = np.vstack([(2*Size-2*Size/3)/turning,0,0]) # control action

             if visualization == 'step_by_step':
                 plt.waitforbuttonpress(-1)

             for k in range(0, nSteps-3): # Main loop
                 u[2] = 0
                 if k % turning == turning-1: # Turn?
                     u[2] = -np.pi/2

                 robot.step(u)

                 # Get sensor observation/s
                 if mode == 'one_landmark':
                     # DONE (Exercise 4)
                     z, landmark = sensor.random_observation(robot.true_pose,
                     ax.plot(
                         [robot.true_pose[0,0], Map[0,landmark]],
                         [robot.true_pose[1,0], Map[1,landmark]],
                         color='m', linestyle="--", linewidth=.5)
                 elif mode == 'landmarks_in_fov':
                     # DONE (Exercise 5)
                     z, landmark = sensor.observe_in_fov(robot.true_pose, Map
                     drawObservations(fig, ax, robot.true_pose, Map[:, landma

                 robot.xEst, robot.PEst = EKFLocalization(robot, sensor, u, z
```

```
        # Drawings
        # Plot the FOV of the robot
        if mode == 'landmarks_in_fov':
            h = sensor.draw(fig, ax, robot.true_pose)
        #end

        robot.draw(fig, ax)
        fig.canvas.draw()

        if visualization == 'non_stop':
            clear_output(wait=True)
            display(fig)
        elif visualization == 'step_by_step':
            plt.waitforbuttonpress(-1)

        if mode == 'landmarks_in_fov':
            h.pop(0).remove()
        fig.canvas.draw()

    if visualization == 'non_stop':
        plt.close()
    elif visualization == 'step_by_step':
        plt.ioff()
```

In [12]:

```
# RUN
mode = 'one_landmark'
# mode = 'landmarks_in_fov'
visualization = 'non_stop'
visualization = 'step_by_step'

Size=50.0

# Robot base characterization
SigmaX = 0.8 # Standard deviation in the x axis
SigmaY = 0.8 # Standard deviation in the y axis
SigmaTheta = 0.1 # Bearing standar deviation
R = np.diag([SigmaX**2, SigmaY**2, SigmaTheta**2]) # Cov matrix

true_pose = np.vstack([-Size+Size/3, -Size+Size/3, np.pi/2])
robot = Robot(true_pose, R)

# Sensor characterization
SigmaR = 1 # Standard deviation of the range
SigmaB = 0.7 # Standard deviation of the bearing
Q = np.diag([SigmaR**2, SigmaB**2]) # Cov matrix

sensor = Sensor(Q)

main(robot, sensor, mode=mode, visualization=visualization, Size=Siz
```

ASSIGNMENT 5: Implementing the FoV of a sensor.

Sensors exhibit certain physical limitations regarding their field of view (FoV) and maximum operating distance (max. Range). Besides, these devices often do not deliver measurmenets from just one landmark each time, but from all those landmarks in the FoV.

The `FOVSensor()` class below extends the `Sensor()` one to implement this behaviour. Complete the `observe_in_fov()` method to consider that the sensor can only provide information from the landmkars in a limited range r_l and a limited orientation $\pm\alpha$ with respect to the robot pose. For that:

1. Get the observations to every landmark in the map. Use the `observe()` function previously implemented for that, but with the argument `flatten=False`. With that option the function returns the measurements as:
$$z = \begin{bmatrix} d_1 & \cdots & d_m \\ \theta_1 & \cdots & \theta_m \end{bmatrix}$$
2. Check which observations lay in the sensor FoV and maximum operating distance. *Hint: for that, you can use the [`np.asarray()` (https://docs.scipy.org/doc/numpy/reference/generated/numpy.asarray.html)](https://docs.scipy.org/doc/numpy/reference/generated/numpy.asarray.html) function with the conditions to be fulfilled by the valid measurements inside, and then filter the results with [`np.nonzero()` (https://docs.scipy.org/doc/numpy/reference/generated/numpy.nonzero.html)](https://docs.scipy.org/doc/numpy/reference/generated/numpy.nonzero.html).*
3. Flatten the resultant matrix `z` to be again a vector, so it has the shape $(2 \times \#Observed\_landmarks, 1)$. *Hint: take a look at [`np.ndarray.flatten()` (https://docs.scipy.org/doc/numpy/reference/generated/numpy.ndarray.flatten.html)](https://docs.scipy.org/doc/numpy/reference/generated/numpy.ndarray.flatten.html) and choose the proper argument.*

Notice that it could happen that any landmark exists in the field of view of the sensor, so the robot couldn't gather sensory information in that iteration. This, which is a problem using Least Squares Positioning, is not an issue with EKF. ***Hint: you can change the value of `seed` within the `main()` function to try different executions.***

In [38]:
```
class FOVSensor(Sensor):
    def __init__(self, cov, fov, max_range):
        super().__init__(cov)
        self.fov = fov
        self.max_range = max_range

    def observe_in_fov(self, from_pose, world, noisy=True):
        """ Get all observations in the fov

        Args:
            from_pose: Position(real) of the robot which takes the o
            world: List of world coordinates of some landmarks
            noisy: Flag, if true then add noise (Exercise 2)

        Returns:
            Numpy array of polar coordinates of landmarks from the p
            They are organised in a vertical vector ls = [d_0 , a_0,
            Dims (2*n_landmarks, 1)
        """
        # 1. Get observations to every landmark in the map WITHOUT N
        z = self.observe(from_pose, world, noisy=False, flatten=Fals

        # 2. Check which ones lay on the sensor FOV
        angle_limit = self.fov/2 # auxiliar variable
        feats_idx = np.nonzero((z[1]<angle_limit) & (z[1]>-angle_lim

        if noisy:
            # 1. Get observations to every landmark in the map WITH
            z = self.observe(from_pose, world, noisy=True, flatten=F

        z = z[:, feats_idx] # extracts the valid observations from z

        # 3. Flatten the  resultant vector of measurements so z=[d_1
        if z.size>0:
            z = np.vstack(z.flatten("F"))

        return z, feats_idx

    def draw(self, fig, ax, from_pose):
        """ Draws the Field of View of the sensor from the robot pos
        return drawFOV(fig, ax, from_pose, self.fov, self.max_range)
```

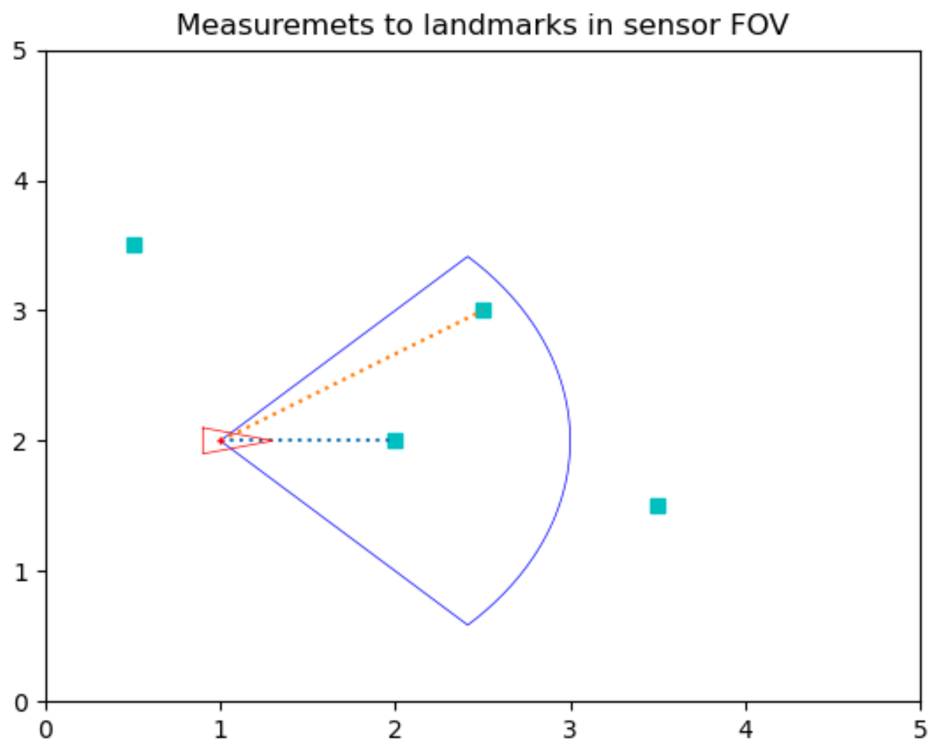
You can now **try** your new and more realistic sensor.

In [39]:
```
# TRY IT!
np.random.seed(0)

# Create the sensor object
cov = np.diag([0.1**2, 0.1**2]) # Cov matrix
fov = np.pi/2
max_range = 2
sensor = FOVSensor(cov, fov, max_range)

# Create a map with three landmarks
Map = np.array([[2., 2.5, 3.5, 0.5],[2., 3., 1.5, 3.5]])

# Take an observation of landmarks in FoV
robot_pose = np.vstack([1.,2.,0.])
z, feats_idx = sensor.observe_in_fov(robot_pose, Map)
print('z:' +str(z))

# Plot results
fig, ax = plt.subplots()
plt.axis([0, 5, 0, 5])
plt.title('Measuremets to landmarks in sensor FOV')
plt.plot(Map[0,:],Map[1,:],'sc')
sensor.draw(fig, ax, robot_pose)
drawObservations(fig, ax, robot_pose, Map[:, feats_idx])
DrawRobot(fig,ax,robot_pose)
```

```
z:[[1.17640523]
 [0.1867558 ]
 [1.84279136]
 [0.49027482]]
```

Out[39]:
```
[<matplotlib.lines.Line2D at 0x7faf5c1dcc70>]
```

Figure



Expected output:

```
z:[[1.17640523]
 [0.1867558 ]
 [1.84279136]
 [0.49027482]]
```

Playing with EKF and the new sensor

And finally, play with your own FULL implementation of the EKF filter with a more realistic sensor :)

Example

The figure below shows an example of the execution of EKF using information from all the landmarks within the FOV:

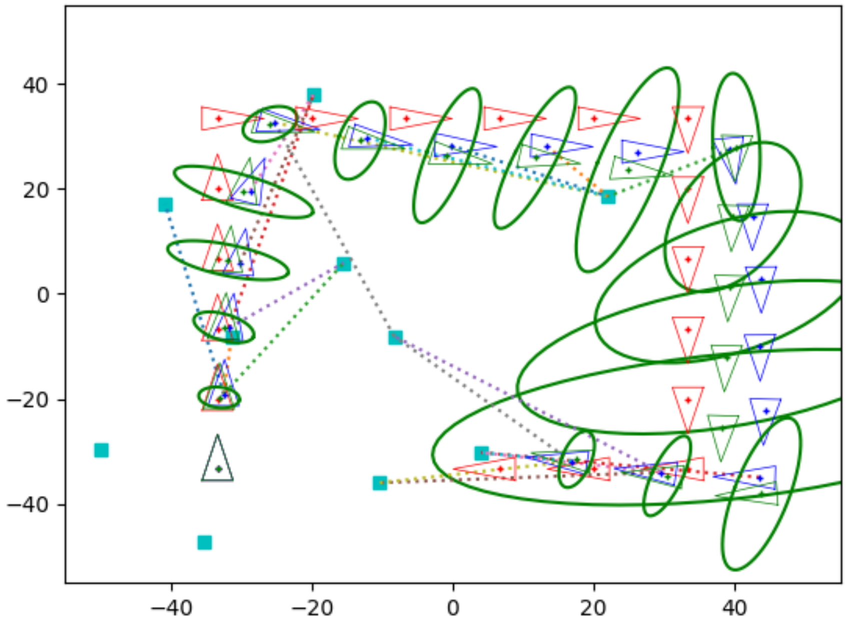
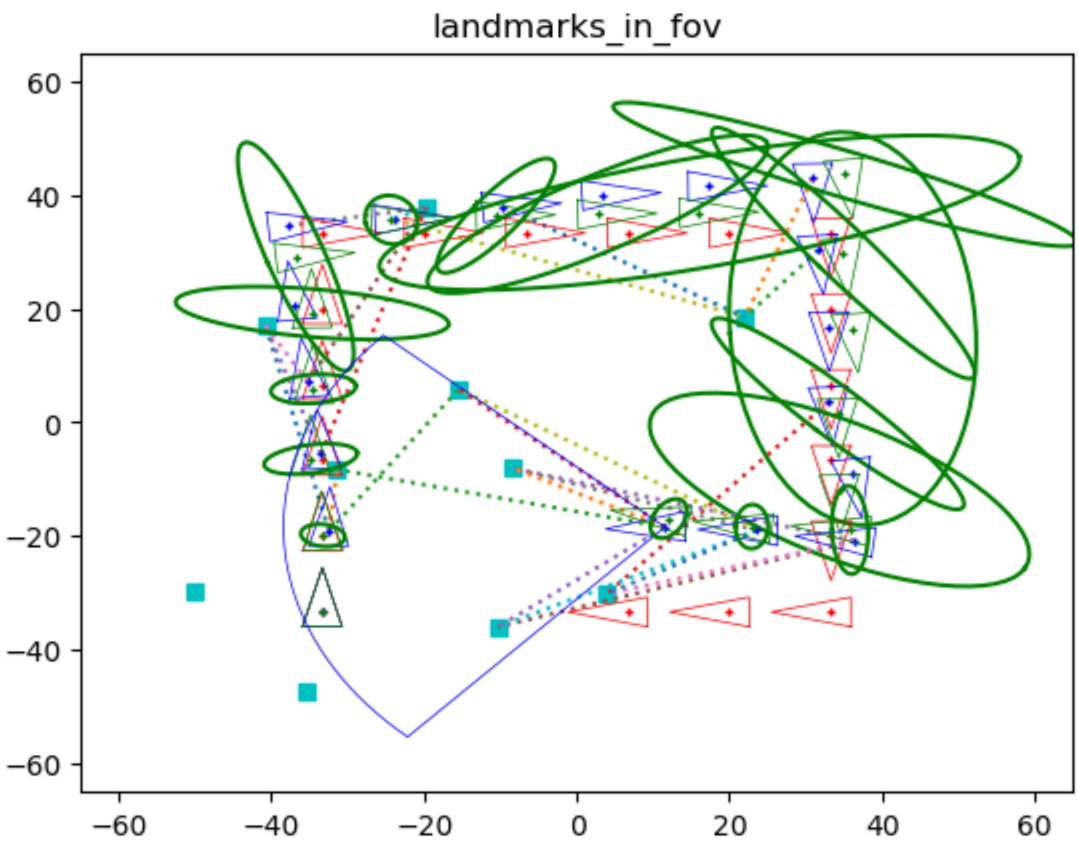


Fig. 2: Execution of the EKF algorithmn for localization. Same as in Fig. 1, except now our robot can observe every lanmark in its f.o.v.

```
In [41]: # RUN
#mode = 'one_landmark'
mode = 'landmarks_in_fov'
visualization = 'non_stop'
#visualization = 'step_by_step'
Size=50.0

# Robot base characterization
SigmaX = 0.8 # Standard deviation in the x axis
SigmaY = 0.8 # Standard deviation in the y axis
SigmaTheta = 0.1 # Bearing standar deviation
R = np.diag([SigmaX**2, SigmaY**2, SigmaTheta**2]) # Cov matrix

true_pose = np.vstack([-Size+Size/3, -Size+Size/3, np.pi/2])
robot = Robot(true_pose, R)

# Sensor characterization
SigmaR = 1 # Standard deviation of the range
SigmaB = 0.7 # Standard deviation of the bearing
Q = np.diag([SigmaR**2, SigmaB**2]) # Cov matrix
fov = np.pi/2 # field of view = 2*alpha
max_range = Size # maximum sensor measurement range

sensor = FOVSensor(Q, fov, max_range)

main(robot, sensor, mode=mode, visualization=visualization, Size=Siz
```



Thinking about it (1)

Having completed the EKF implementation, you are ready to **answer the following questions**:

- What are the dimensions of the Jacobians of the observation model (matrix H)? Why?

  Las dimensiones son de $(numero_d e_l andmarks * 2, 3)$, siendo el resultado de apilar todas las matrices 2x3 de los jacobianos de cada observacion. Cada jacobiano tiene dos filas, una para la distancia y otra para el angulo, y tres columnas para la x, y, θ
- Discuss the evolution of the ideal, true and estimated poses when executing the EKF filter (with the initial sensor).

  La pose ideal representa lo que queremos que el robot haga en una situacion ideal, cambiaria tal y como esperamos que haga. La pose real agrega ruido al movimiento ideal, haciendo que se desvie del movimento ideal. La estimacion de la pose es lo que hemos calculado con el EKF, dados los landmarks en nuestro mapa. La incertidumbre de la estimacion disminuye en ciertos pasos, gracias a las observaciones de dichos landmarks
- Discuss the evolution of the ideal, true and estimated poses when executing the EKF filter (with the sensor implementing a FOV). Pay special attention to their associated uncertainties.

  Cuando implementamos el FOV, nuestro robot solo puede observar los landmarks que estan dentro de su campo de vision, siendo asi, la correccion no podria hacerse tan frecuentemente como antes donde podiamos observar mucho mas landmarks. Como resultado las incertidumbres pueden aumentar mucho mas que en el caso anterior, pero tan pronto como observemos una landmark, dicha incertidumbre se podra reducir drasticamente
- What happens in the EKF filter when the robot performs a motion command, but it is unable to measure distances to any landmark, i.e. they are out of the sensor FOV?

  Dado que el robot no puede obtener ninguna informacion sensorial, la incertidumbre de nuestra prediccion empeora, pero no es un problema en EKF como si lo era en LS, y aun asi podra seguir calculando la pose del robot y su incertidumbre correctamente