设 $\rho > 0$, $\mathbf{v} \in \mathbb{R}^d$ 且 $|\mathbf{v}| < 1$, $\mathbf{B} \in \mathbb{R}^d$, $d \geq 1$ 为常数。定义:

$$
\begin{aligned}
&\mathbf{U} = (D, \mathbf{m}, \mathbf{B}, E)\\
&D = \frac{\rho}{\sqrt{1-|\mathbf{v}|^2}},\\
&\mathbf{m} = \frac{\rho \mathbf{v}}{1-|\mathbf{v}|^2} + |\mathbf{B}|^2 \mathbf{v} - (\mathbf{v} \cdot \mathbf{B})\mathbf{B},\\
&E = \frac{\rho}{1-|\mathbf{v}|^2} + \frac{1+|\mathbf{v}|^2}{2}|\mathbf{B}|^2 - \frac{(\mathbf{v} \cdot \mathbf{B})^2}{2}.
\end{aligned}
$$

设 $\rho_* > 0$, $\mathbf{v}_* \in \mathbb{R}^d$ 且 $|\mathbf{v}_*| < 1$, $\mathbf{B}_* \in \mathbb{R}^d$, $d \geq 1$ 为常数。定义:

$$
\begin{aligned}
&\mathbf{U}_* = (D_*, \mathbf{m}_*, \mathbf{B}_*, E_*),\\
&\mathbf{n}_* = (-\sqrt{1-|\mathbf{v}_*|^2}, -\mathbf{v}_*, -(1-|\mathbf{v}_*|^2)\mathbf{B}_* - (\mathbf{v}_* \cdot \mathbf{B}_*)\mathbf{v}_*, 1),\\
&D_* = \frac{\rho_*}{\sqrt{1-|\mathbf{v}_*|^2}},\\
&\mathbf{m}_* = \frac{\rho_* \mathbf{v}_*}{1-|\mathbf{v}_*|^2} + |\mathbf{B}_*|^2 \mathbf{v}_* - (\mathbf{v}_* \cdot \mathbf{B}_*)\mathbf{B}_*,\\
&E_* = \frac{\rho_*}{1-|\mathbf{v}_*|^2} + \frac{1+|\mathbf{v}_*|^2}{2}|\mathbf{B}_*|^2 - \frac{(\mathbf{v}_* \cdot \mathbf{B}_*)^2}{2}.
\end{aligned}
$$

Let $F = (U - U_*) \cdot n_*$

(1) If d = 1, *then prove that* $\frac{\partial F}{\partial \rho} > 0, \frac{\partial^2 F}{\partial \rho^2} > 0$

(2) If $d = 1$, *then prove that* $F|_{\rho=0} > 0$

(3) If $d = 1$, *then prove that* $F > 0$

(4) If $d = 3$, *then prove that* (1)(2)(3) still hold