Reach Bravo MKII
Kinematic and Dynamic Properties

Version 5

October 2024

1 Reach Bravo 7 Function

1.1 Kinematics

To describe the transformations between links the Standard Denavit-Hartenberg (DH) method has been used.

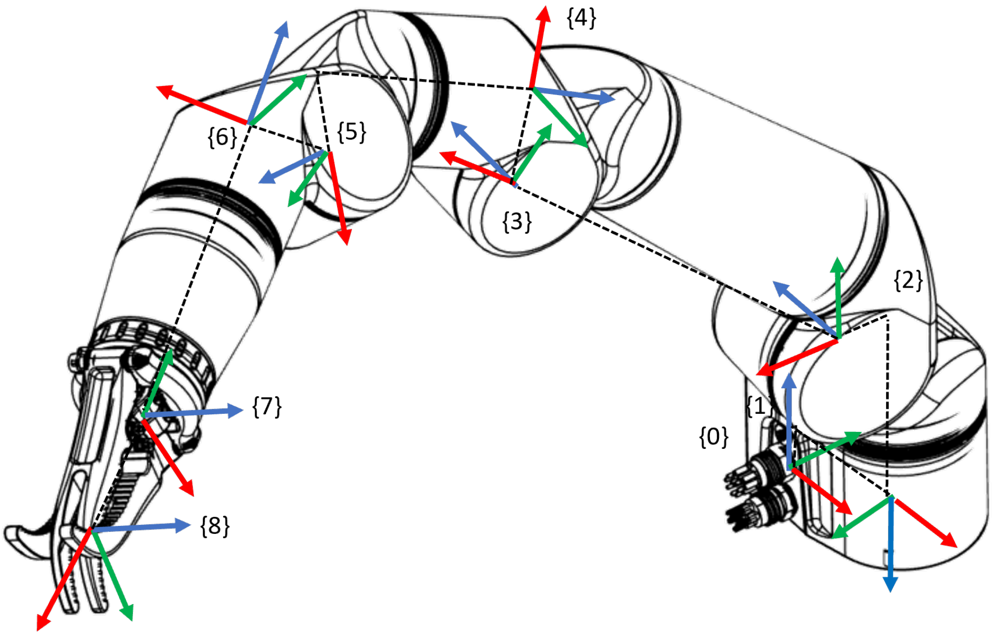

Figure 1: Reach Bravo 7 - DH Frame Location (x,y,z)

The DH parameters are given for the Reach Bravo 7 standard configuration in Table 1. These values define the frame locations shown in Figure 1. ***Note:*** *for manipulators with a force-torque sensor attachment an additional offset (42 mm) is required on frame {6} such that d = -266.16mm. Measurement center of the force-torque sensor is -127.6 from frame {6} in the z direction.*

Link	d (mm)	θ	a (mm)	α
0	41.45	0.0	67.5	π
1	-117.48	$\theta_1 + \pi$	46.0	$-\pi/2$
2	0.0	$\theta_2 - \pi/2 + \theta_s$	a_2	0.0
3	0.0	$\theta_3 - \pi/2 - \theta_s$	40.8	$-\pi/2$
4	-160.26	$\theta_4 + \pi$	40.8	$-\pi/2$
5	0.0	θ_5	40.8	$-\pi/2$
6	-224.16	θ_6	0.0	$\pi/2$
7	0.0	$-\pi/2$	120.0	0.0

Table 1: Reach Bravo 7 - DH Parameters, where $\theta_s = \tan^{-1}\left(\frac{5.2}{290.77}\right)$ and $a_2 = \sqrt{5.2^2 + 290.77^2}$

where d is offset along previous z to the common normal, θ is the angle about z_{i-1}, from x_{i-1} to x_i, a is the length of the common normal, α is the angle about the common normal, from z_{i-1} axis to z_i axis. The mapping between DH frames conveyed in Table 1 and the inertial frames is described in Section 1.2.

1.2 Dynamics

The inertial frames for Reach Bravo 7 are defined in Figure 2.

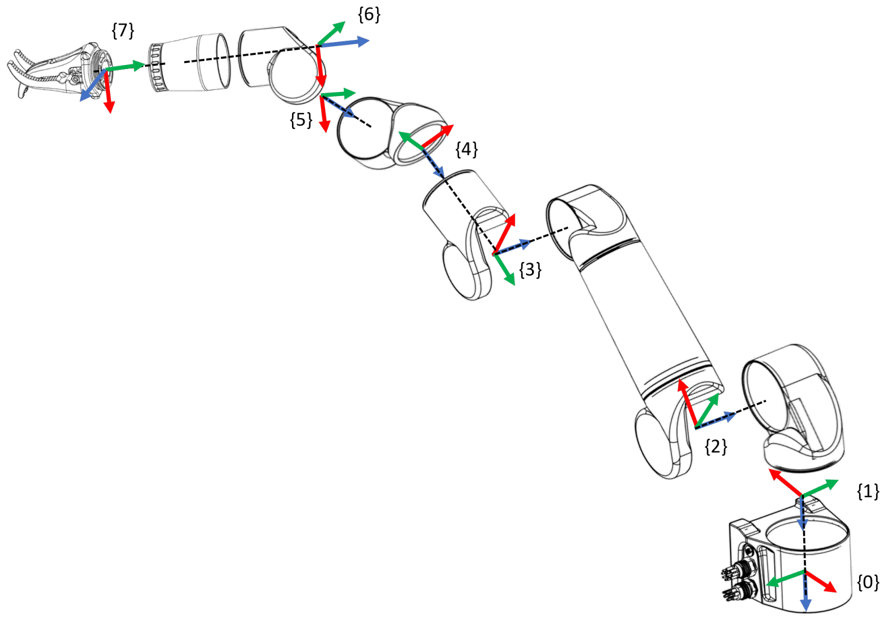

Figure 2: Reach Bravo 7 - Inertial Frame Locations

Such that, inertial frame {0} is the base frame located at the origin of DH frame {1}, before the rotation θ_0. Inertial frame {1} is located at the origin of DH frame {1} after the rotation θ_0. Inertial frame {2} is located at the origin of DH frame {2} after the rotation θ_1. Inertial frame {3} is located at the origin of DH frame {3} after the rotation θ_2. Inertial frame {4} is located at the origin of DH frame {4} after the rotation θ_3. Inertial frame {5} is located at the origin of DH frame {5} after the rotation θ_4, Inertial frame {6} is located at the origin of DH frame {6} after the rotation θ_4. Inertial frame {7} is located at the origin of DH frame {7}. ***Note:*** *the frames shown in Figure 2 are shown in exploded view for diagrammatic purposes and do not represent the actual frame locations.*

1.2.1 Inertial Properties

The inertial properties for Reach Bravo 7 are provided in Table 2. The center of mass (COM) offsets are given with respect to the frames described in Figure 2. For convenience, each inertia tensor is given for the link's output frame (I_{Link}) and the COM aligned with the output coordinate system (I_{COM}). ***Note:*** *the URDF standard expect the inertia tensor to be located at the COM and aligned with the output coordinate system. Additionally, the URDF standard assumes a negative product of inertia convention. The parameters provided in Table 2 use the SolidWorks convention and do not comply with this convention. See the documentation at wiki.ros.org/urdf/XML/link and www.mathworks.com/help/specify-custom-inertia for more details.*

Link	Mass (kg)	COM (mm)	I_{COM} ($\boldsymbol{kg.mm^2}$)	I_{Link} ($\boldsymbol{kg.mm^2}$)
0	1.85	$\begin{pmatrix} -15 & 1 & -2 \end{pmatrix}$	$\begin{pmatrix} 2528 & -20 & 23 \\ -20 & 2593 & 7 \\ 23 & 7 & 3312 \end{pmatrix}$	$\begin{pmatrix} 2535 & -53 & 65 \\ -53 & 2980 & 4 \\ 65 & 4 & 3697 \end{pmatrix}$
1	1.62	$\begin{pmatrix} 31 & 10 & -98 \end{pmatrix}$	$\begin{pmatrix} 3179 & 233 & -1024 \\ 233 & 4209 & -344 \\ -1024 & -344 & 2387 \end{pmatrix}$	$\begin{pmatrix} 18856 & 771 & -5936 \\ 771 & 21257 & -2039 \\ -5936 & -2039 & 4130 \end{pmatrix}$
2	2.12	$\begin{pmatrix} 176 & 22 & 4 \end{pmatrix}$	$\begin{pmatrix} 3420 & -1868 & 3150 \\ -1868 & 35166 & -79 \\ 3150 & -79 & 35981 \end{pmatrix}$	$\begin{pmatrix} 4439 & 6169 & 4586 \\ 6169 & 100570 & 98 \\ 4586 & 98 & 102343 \end{pmatrix}$
3	1.33	$\begin{pmatrix} 34 & -46 & -3 \end{pmatrix}$	$\begin{pmatrix} 1809 & -398 & 156 \\ -398 & 1557 & -197 \\ 156 & -197 & 2024 \end{pmatrix}$	$\begin{pmatrix} 4660 & -2512 & 21 \\ -2512 & 3143 & -15 \\ 21 & -15 & 6438 \end{pmatrix}$
4	1.33	$\begin{pmatrix} 28 & 12 & -145 \end{pmatrix}$	$\begin{pmatrix} 1983 & 180 & -641 \\ 180 & 2676 & -264 \\ -641 & -264 & 1526 \end{pmatrix}$	$\begin{pmatrix} 29892 & 624 & -6084 \\ 624 & 31470 & -2526 \\ -6084 & -2526 & 2780 \end{pmatrix}$
5	0.97	$\begin{pmatrix} 31 & -39 & -6 \end{pmatrix}$	$\begin{pmatrix} 1272 & -346 & 116 \\ -346 & 1205 & -189 \\ 116 & -189 & 1511 \end{pmatrix}$	$\begin{pmatrix} 2761 & -1534 & -53 \\ -1534 & 2204 & 20 \\ -53 & 20 & 3938 \end{pmatrix}$
6	1.09	$\begin{pmatrix} 0 & 0 & -127 \end{pmatrix}$	$\begin{pmatrix} 1977 & 0 & -15 \\ 0 & 1986 & 1 \\ -15 & 1 & 1035 \end{pmatrix}$	$\begin{pmatrix} 19498 & -1 & -35 \\ -1 & 19506 & 14 \\ -35 & 14 & 1035 \end{pmatrix}$
7	0.64	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 2237 & -18 & 0 \\ -18 & 309 & 1 \\ 0 & 1 & 2258 \end{pmatrix}$	$\begin{pmatrix} 2238 & -18 & 0 \\ -18 & 309 & 1 \\ 0 & 1 & 2259 \end{pmatrix}$

Table 2: Reach Bravo 7 - Inertial properties (Link 7 is for interlocking jaws in the closed position)

For units with a force-torque sensor attachment, the inertial parameters for link 6 should be replaced with table 3.

Link	Mass (kg)	COM (mm)	I_{COM} ($\boldsymbol{kg.mm^2}$)	I_{Link} ($\boldsymbol{kg.mm^2}$)
6	1.35	$\begin{pmatrix} 0 & 0 & -153 \end{pmatrix}$	$\begin{pmatrix} 3542 & -1 & -21 \\ -1 & 3549 & 4 \\ -21 & 4 & 1281 \end{pmatrix}$	$\begin{pmatrix} 35298 & -1 & -30 \\ -1 & 35305 & 13 \\ -30 & 13 & 1281 \end{pmatrix}$

Table 3: Force-Torque Assembly Inertial properties

Expressing inertia in different frames The inertia tensor can be expressed with respect to another reference frame through Equation 1. The relationship is defined by a similarity transform for the rotation and the tensor generalization of parallel axis theorem for the translation (MATLAB, 2021).

$$\mathbf{I_B} = \mathbf{R_{A/B}} \mathbf{I_A} \mathbf{R^T_{A/B}} + m[(\vec{P} \cdot \vec{P})\mathbf{E_3} - (\vec{P} \cdot \vec{P})^T] \tag{1}$$

where I_A is the inertia with respect to $\{A\}$, I_B is the inertia with respect to $\{B\}$, $\mathbf{R_{A/B}}$ is the rotation from $\{A\}$ to $\{B\}$, m is the mass, $\vec{P}$ is the translation vector between $\{A\}$ and $\{B\}$, and $\mathbf{E_3}$ denotes a 3x3 identity matrix. Using skew symmetric identities, this relationship can be stated more concisely as,

$$\mathbf{I_B} = \mathbf{R_{A/B}} \mathbf{I_A} \mathbf{R^T_{A/B}} + m(SKEW(\vec{P})^2) \tag{2}$$

where $SKEW(\vec{P})$ denotes the skew-symmetric matrix associated to the position vector $\vec{P}$.

1.2.2 Buoyancy

The Buoyancy parameters for Reach Bravo 7 are provided in Table 4. The Buoyancy parameters are given with respect to the reference frames in Figure 2.

Link	**Volume (L)**	**COB (mm)**
0	0.81	$\begin{pmatrix} -16 & 1 & -1 \end{pmatrix}$
1	0.74	$\begin{pmatrix} 27 & 9 & -101 \end{pmatrix}$
2	1.96	$\begin{pmatrix} 157 & 39 & 0 \end{pmatrix}$
3	0.44	$\begin{pmatrix} 38 & -48 & -6 \end{pmatrix}$
4	0.59	$\begin{pmatrix} 23 & 9 & -146 \end{pmatrix}$
5	0.39	$\begin{pmatrix} 36 & -42 & -9 \end{pmatrix}$
6	0.53	$\begin{pmatrix} 0 & 0 & -145 \end{pmatrix}$
7	0.16	$\begin{pmatrix} 1 & -27 & 0 \end{pmatrix}$

Table 4: Reach Bravo 7 - Internal Volume Displacement and Centre of Buoyancy (COB)

For units with a force-torque sensor attachment, the buoyancy parameters for link 6 should be replaced with the volume and COB from table 5.

Link	**Volume (L)**	**COB (mm)**
6	0.74	$\begin{pmatrix} 0 & 0 & -165 \end{pmatrix}$

Table 5: Force-Torque Assembly Volume Displacement and Centre of Buoyancy (COB)

1.2.3 Actuator Properties

The actuator properties for Reach Bravo 7 are provided in Table 6. The input friction F and damping B values are approximated from the current-velocity profile. All other properties are taken from the motor/gearbox manufacture data-sheets. ***Note:*** *actuators 1-6 are revolute joints rotating the reference frames described in Figure 2. Actuator 7 is the prismatic (linear) joint to actuate the end-effector.*

Actuator	**R [$\mu\Omega$]**	**L [H]**	**F [Nmm]**	**B [Ns/mm]**	$Kt[Nm/A_{RMA}]$	$Kv[V_{pk}/rad/s]$	**Gr**
1	1.0	43.2±20%	79±10%	1.43±10%	0.222	0.181	120.0
2	1.0	43.2±20%	79±10%	1.43±10%	0.222	0.181	120.0
3	2.3	64.8±20%	79±10%	1.43±10%	0.215	0.176	120.0
4	2.3	64.8±20%	79±10%	1.43±10%	0.215	0.176	120.0
5	2.3	64.8±20%	79±10%	1.43±10%	0.215	0.176	120.0
6	5.9	147±20%	79±10%	1.43±10%	0.209	0.171	120.0
7	5.9	147±20%	79±10%	0.89±10%	0.209	0.171	39.27

Table 6: Reach Bravo 7 - Actuator Properties

2 Reach Bravo 5 Function

2.1 Kinematics

To convey the transformations between links the Standard Denavit-Hartenberg (DH) method has been used.

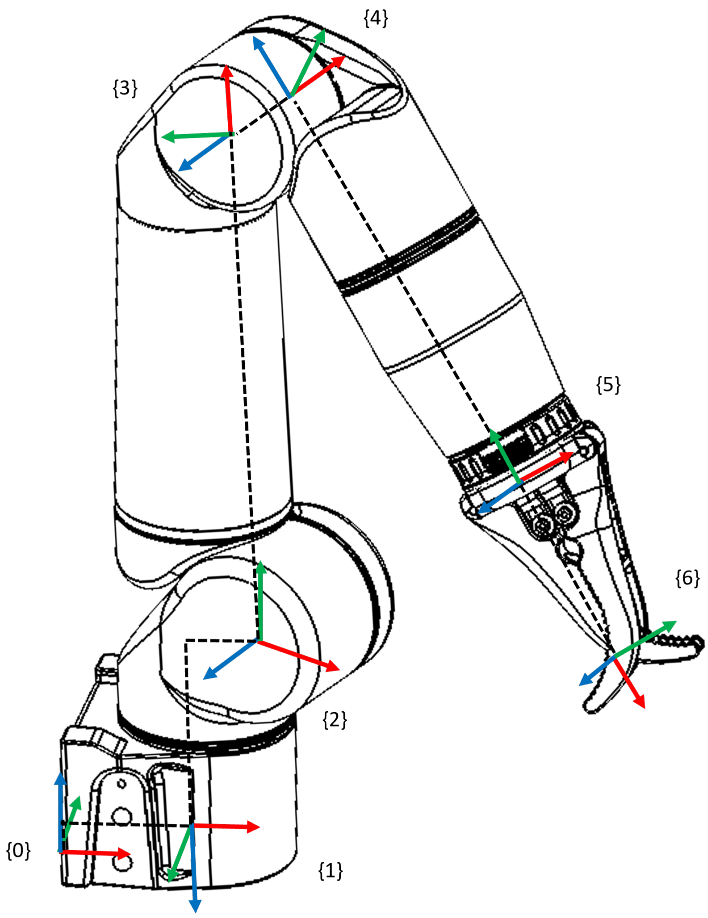

Figure 3: Reach Bravo 5 - DH Frame Location (x,y,z)

The DH parameters are given for the Reach Bravo 5 function standard configuration in Table 7. These values describe the frame locations shown in Figure 3. ***Note:*** *for manipulators with a force-torque sensor attachment an additional offset (42 mm) is required on frame {4} such that d = -266.16mm. Measurement center of the force-torque sensor is -127.6 from frame {4} in the z direction.*

Link	d (mm)	θ	a (mm)	α
0	41.0	0.0	67.5	π
1	-117.5	$\theta_1 + \pi$	46.0	$-\pi/2$
2	0.0	$\theta_2 - \pi/2 + \theta_s$	a_2	0.0
3	0.0	$\theta_3 - \pi/2 - \theta_s$	40.8	$-\pi/2$
4	-224.2	θ_4	0.0	$-\pi/2$
5	0.0	$-\pi/2$	120.0	0.0

Table 7: Reach Bravo 5 - DH Parameters, where $\theta_s = \tan^{-1}\left(\frac{5.2}{290.77}\right)$ and $a_2 = \sqrt{5.2^2 + 290.77^2}$

where d is offset along previous z to the common normal, θ is the angle about z_{i-1}, from x_{i-1} to x_i, a is the length of the common normal, α is the angle about the common normal, from z_{i-1} axis to z_i axis. The mapping between DH frames conveyed in Table 1 and the inertial frames is described in Section 2.2.

2.2 Dynamics

The inertial frames for Reach Bravo 5 are defined in Figure 4.

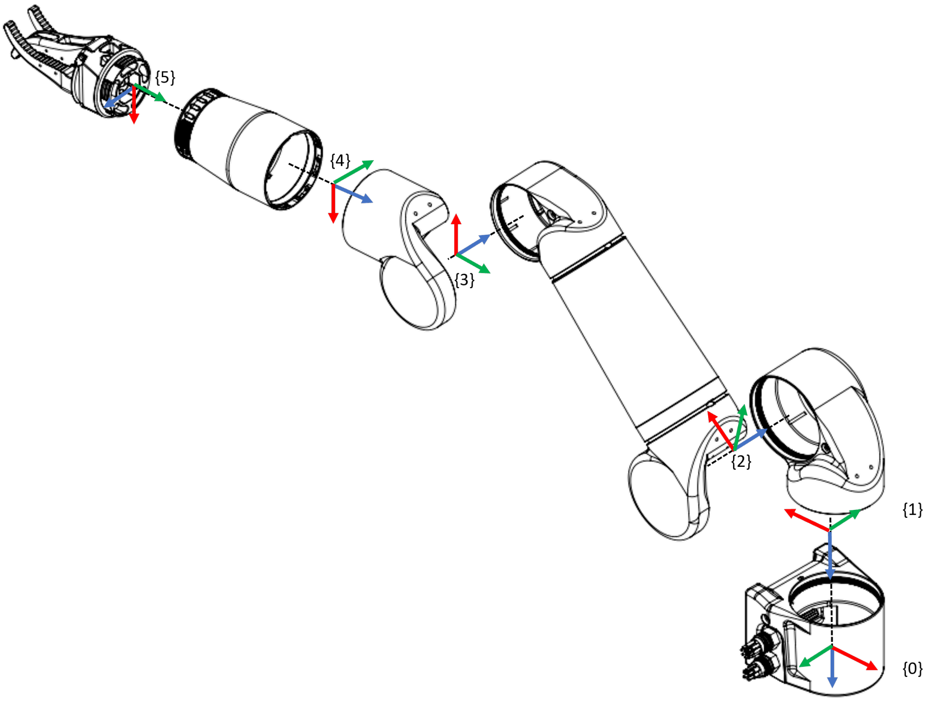

Figure 4: Reach Bravo 5 - Inertial Frame Locations

Such that, inertial frame {0} is the base frame located at the origin of DH frame {1}, before the rotation θ_0. Inertial frame {1} is located at the origin of DH frame {1} after the rotation θ_0. Inertial frame {2} is located at the origin of DH frame {2} after the rotation θ_1. Inertial frame {3} is located at the origin of DH frame {3} after the rotation θ_2. Inertial frame {4} is located at the origin of DH frame {4} after the rotation θ_3. Inertial frame {5} is located at the origin of DH frame {5}. ***Note:*** *the frames shown in Figure 2 are shown in exploded view for diagrammatic purposes and do not represent the actual frame locations.*

2.2.1 Inertial Properties

The inertial properties for Reach Bravo 5 are provided in Table 8. The center of mass (COM) offsets are given with respect to the frames described in Figure 4. For convenience, each inertia tensor is given for the link's output frame (I_{Link}) and the COM aligned with the output coordinate system (I_{COM}). ***Note:*** *the URDF standard expect the inertia tensor to be located at the COM and aligned with the output coordinate system. Additionally, the URDF standard assumes a negative product of inertia convention. The parameters provided in Table 8 use the SolidWorks convention and do not comply with this convention. See the documentation at wiki.ros.org/urdf/XML/link and www.mathworks.com/help/specify-custom-inertia for more details.*

Link	Mass (kg)	COM (mm)	I_{COM} ($\boldsymbol{kg.mm^2}$)	I_{Link} ($\boldsymbol{kg.mm^2}$)
0	1.85	$\begin{pmatrix} -15 & 1 & -2 \end{pmatrix}$	$\begin{pmatrix} 2528 & -20 & 23 \\ -20 & 2593 & 7 \\ 23 & 7 & 3312 \end{pmatrix}$	$\begin{pmatrix} 2535 & -53 & 65 \\ -53 & 2980 & 4 \\ 65 & 4 & 3697 \end{pmatrix}$
1	1.62	$\begin{pmatrix} 31 & 10 & -98 \end{pmatrix}$	$\begin{pmatrix} 3179 & 233 & -1024 \\ 233 & 4209 & -344 \\ -1024 & -344 & 2387 \end{pmatrix}$	$\begin{pmatrix} 18856 & 771 & -5936 \\ 771 & 21257 & -2039 \\ -5936 & -2039 & 4130 \end{pmatrix}$
2	2.12	$\begin{pmatrix} 176 & 22 & 4 \end{pmatrix}$	$\begin{pmatrix} 3420 & -1868 & 3150 \\ -1868 & 35166 & -79 \\ 3150 & -79 & 35981 \end{pmatrix}$	$\begin{pmatrix} 4439 & 6169 & 4586 \\ 6169 & 100570 & 98 \\ 4586 & 98 & 102343 \end{pmatrix}$
3	0.97	$\begin{pmatrix} 31 & -39 & -6 \end{pmatrix}$	$\begin{pmatrix} 1272 & -346 & 116 \\ -346 & 1205 & -189 \\ 116 & -189 & 1511 \end{pmatrix}$	$\begin{pmatrix} 2761 & -1534 & -53 \\ -1534 & 2204 & 20 \\ -53 & 20 & 3938 \end{pmatrix}$
4	1.09	$\begin{pmatrix} 0 & 0 & -127 \end{pmatrix}$	$\begin{pmatrix} 1977 & 0 & -15 \\ 0 & 1986 & 1 \\ -15 & 1 & 1035 \end{pmatrix}$	$\begin{pmatrix} 19498 & -1 & -35 \\ -1 & 19506 & 14 \\ -35 & 14 & 1035 \end{pmatrix}$
5	0.64	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 2237 & -18 & 0 \\ -18 & 309 & 1 \\ 0 & 1 & 2258 \end{pmatrix}$	$\begin{pmatrix} 2238 & -18 & 0 \\ -18 & 309 & 1 \\ 0 & 1 & 2259 \end{pmatrix}$

Table 8: Inertial properties for Bravo 5 (Link 5 is for interlocking jaws in the closed position)

For units with a force-torque sensor attachment, the inertial parameters for link 4 should be replaced with table 9.

Link	Mass (kg)	COM (mm)	I_{COM} ($\boldsymbol{kg.mm^2}$)	I_{Link} ($\boldsymbol{kg.mm^2}$)
4	1.35	$\begin{pmatrix} 0 & 0 & -153 \end{pmatrix}$	$\begin{pmatrix} 3542 & -1 & -21 \\ -1 & 3549 & 4 \\ -21 & 4 & 1281 \end{pmatrix}$	$\begin{pmatrix} 35298 & -1 & -21 \\ -1 & 35305 & 13 \\ -21 & 13 & 1281 \end{pmatrix}$

Table 9: Force-Torque Assembly Inertial properties

Expressing inertia in different frames The inertia tensor can be expressed with respect to another reference frame through Equation 3. The relationship is defined by a similarity transform for the rotation and the tensor generalization of parallel axis theorem for the translation (MATLAB, 2021).

$$\mathbf{I_B} = \mathbf{R_{A/B}I_AR^T_{A/B}} + m[(\vec{P}\cdot\vec{P})\mathbf{E_3} - (\vec{P}\cdot\vec{P})^T] \tag{3}$$

where I_A is the inertia with respect to $\{A\}$, I_B is the inertia with respect to $\{B\}$, $\mathbf{R_{A/B}}$ is the rotation from $\{A\}$ to $\{B\}$, m is the mass, $\vec{P}$ is the translation vector between $\{A\}$ and $\{B\}$, and $\mathbf{E_3}$ denotes a 3x3 identity matrix. Using skew symmetric identities, this relationship can be stated more concisely as,

$$\mathbf{I_B} = \mathbf{R_{A/B}I_AR^T_{A/B}} + m(SKEW(\vec{P})^2) \tag{4}$$

where $SKEW(\vec{P})$ denotes the skew-symmetric matrix associated to the position vector $\vec{P}$.

2.2.2 Buoyancy

The Buoyancy parameters for Reach Bravo 7 are provided in Table 10. The Buoyancy parameters are given with respect to the reference frames in Figure 4.

Link	Volume (L)	COB (mm)
0	0.81	$\begin{pmatrix} -16 & 1 & -1 \end{pmatrix}$
1	0.74	$\begin{pmatrix} 27 & 9 & -101 \end{pmatrix}$
2	1.96	$\begin{pmatrix} 157 & 39 & 0 \end{pmatrix}$
3	0.39	$\begin{pmatrix} 36 & -42 & -9 \end{pmatrix}$
4	0.53	$\begin{pmatrix} 0 & 0 & -145 \end{pmatrix}$
5	0.16	$\begin{pmatrix} 1 & -27 & 0 \end{pmatrix}$

Table 10: Reach Bravo 5 - Internal volume displacement and Centre of Buoyancy (COB)

For units with a force-torque sensor attachment, the buoyancy parameters for link 4 should be replaced with the volume and COB from table 11.

Link	**Volume (L)**	**COB (mm)**
4	0.74	$\begin{pmatrix} 0 & 0 & -165 \end{pmatrix}$

Table 11: Force-Torque Assembly Volume Displacement and Centre of Buoyancy (COB)

2.2.3 Actuator Properties

The actuator properties for Reach Bravo 5 are provided in Table 12. The input friction F and damping B values are approximated from the current-velocity profile. All other properties are taken from the motor/gearbox manufacture data-sheets. ***Note:*** *actuators 1-4 are revolute joints rotating the reference frames described in Figure 4. Actuator 5 is the prismatic (linear) joint to actuate the end-effector.*

Actuator	**R [$\mu\Omega$]**	**L [H]**	**F [Nmm]**	**B [Ns/mm]**	$Kt[Nm/A_{RMA}]$	$Kv[V_{pk}/rad/s]$	**Gr**
1	1.0	43.2±20%	79±10%	1.43±10%	0.222	0.181	120.0
2	1.0	43.2±20%	79±10%	1.43±10%	0.222	0.181	120.0
3	2.3	64.8±20%	79±10%	1.43±10%	0.215	0.176	120.0
4	5.9	147±20%	79±10%	1.43±10%	0.209	0.171	120.0
5	5.9	147±20%	79±10%	0.89±10%	0.209	0.171	39.27

Table 12: Reach Bravo 5 - Actuator Properties