Circuit Theory and Electronics Fundamentals

BSc Aerospace Engineering, Técnico, University of Lisbon

Lab 1: Circuit analysis methods

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Contents

1 Introduction

The objective of this laboratory assignment is to study a RC circuit containing linear components, such as resistors (R_i), dependent (rhombus shaped) voltage (V) and current (I) sources, a capacitor C and a an independent (circle shaped) voltage source, as seen in Figure 1, in which the nodes are ilustrated and the functions that define the sources are presented.

In Section 2, a theoretical analysis of the circuit is presented, using the *Octave*. This analysis includes determining the currents through the branches and the voltages in the nodes for $t < 0$, obtaining the equivalent resistance as seen from the capacitor terminals, determining the natural and forced solutions and superimposing them to determine the total solution for $t > 0$, and performing a frequency analysis. In Section 3, an analogous analysis is made through simulation, using *Ngspice*, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

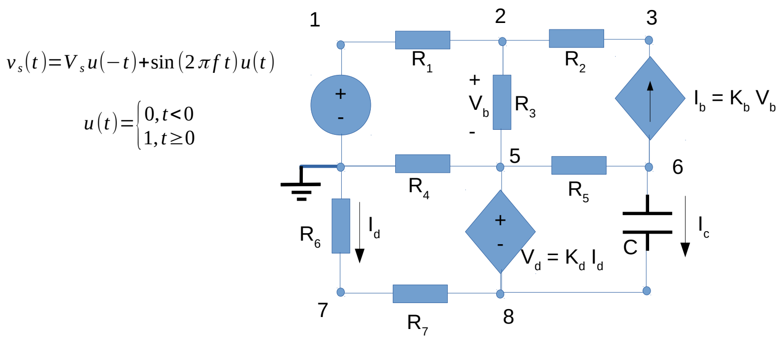


Figure 1: Original RC circuit.

2 Theoretical Analysis

2.1 Node Method

The circuit consists of 8 nodes: 0 (ground), 1, 2, 3, 5, 6, 7 and 8, respectively associated with nodal voltages V_0, V_1, V_2, V_3, V_5, V_6, V_7 and V_8 (seen in Figure 1).

Because we are considering a DC voltage source, there is no current variation, hence the current $I_c = 0$, so its consideres as being an open circuit between V_6 and V_8.

By first looking at the circuit we can assume the following equalities:

$$V_1 = V_s \tag{1}$$

$$I_c = 0 \tag{2}$$

$$K_b \cdot V_b - I_b = 0 \tag{3}$$

$$-V_d + K_d \cdot I_d = 0 \tag{4}$$

$$-V_b + V_2 - V_5 = 0 \tag{5}$$

$$(V_5 - V_8) + V_d = 0 \tag{6}$$

The other equations needed to solve this problem can be achived with Kirchoff's Current Law (KCL) and Ohm's Law:

Supernode 1

$$G_1 \cdot (V_1 - V_2) + G_4 \cdot V_5 + I_d = 0 \tag{7}$$

Node 2

$$G_1 \cdot (V_2 - V_1) + G_2 \cdot (V_2 - V_3) + G_3 \cdot (V_2 - V_5) = 0 \tag{8}$$

Node 3

$$G_2 \cdot (V_3 - V_2) + K_b \cdot (V_5 - V_2) = 0 \tag{9}$$

Node 6

$$K_b \cdot (V_2 - V_5) + G_5 \cdot (V_6 - V_5) = 0 \tag{10}$$

Node 7

$$G_7 \cdot (V_7 - V_8) + G_6 \cdot V_7 = 0 \tag{11}$$

In order to obtain the currents running through branches formed by resistors, Ohm's law is applied. Furthermore, given that this is yet a DC circuit and assuming its in a stationary state (a lot of time has passed and the capacitor is fully charged), there is no current in the capacitor's branch. All the other branches are formed by sources and all of them are associated with one node that is ”connected” to only one other branch, of which the current is known (they are resistor branches). Hence, applying KCL to these (”in common”) nodes, the last unknown currents are obtained.

In octave we get the values present in Tabel 1

Name	Value [A or V]
V1	5.083148
V2	4.851927
V3	4.368491
V5	4.885142
V6	5.612546
V7	-2.039502
V8	-3.074247
IR1	0.000224
IR2	0.000235
IR3	0.000011
IR4	0.001209
IR5	0.000235
IR6	0.000986
IR7	0.000986

Table 1: Values for the components using the node method.

2.2 Equivalent Resistance

In this subsection, it is intended to obtain the equivalent resistance of the circuit as seen from the capacitor terminals (at $t = 0$, the time in which the circuit will transform into an AC circuit). We need R_{eq} to obtain the natural solution to the differential equation that governs v_6. For this, the original independent voltage source is turned to zero, but the voltage drop in the capacitor should remain the same, because we assume the capacitor is fully charged (as mentioned in the previous subsection) and we need this voltage drop to obtain the current running through the capacitor (I_x) and, ultimately, R_{eq}, since $R_{eq} = \frac{V_x}{I_x}$ Therefore, $v_s = 0$ and we replace the capacitor for an independent voltage source $V_x = V_6 - V_8$.

To make the matrix, we used the following equations:

Node 0:

$$(V_1 - V_2)G_1 + (V_1 - V_5)G_4 + I_d = 0 \tag{12}$$

where we know that $V_1 = 0$

Node 2:

$$(V_2 - V_1)G_1 + (V_2 - V_3)G_2 + (V_2 - V_5)G_3 = 0 \tag{13}$$

Node 3:

$$(V_3 - V_2)G_2 - (V_2 - V_5)K_b = 0 \tag{14}$$

Node 3:

$$(V_3 - V_2)G_2 - (V_2 - V_5)K_b = 0 \tag{15}$$

Node 7:

$$V_7G_6 + (V_7 - V_8)G_7 = 0 \tag{16}$$

The equations that will complete this system are present in the previous section.

Once we know all the parameters in this system, we can calculate I_x using the Kirchoff Current Law (KCL):

Node 6:

$$I_x + (V_2 - V_5)K_b + (V_5 - V_6)G_6 = 0 \tag{17}$$

Once we have V_x and I_x, the R_{eq} is determined with:

Node 6:

$$R_{eq} = \frac{V_x}{I_x} \tag{18}$$

These calculations are usefull to find τ the time constant, which is equal to $R_{eq} \cdot C$, as in a simple in series RC circuit.

These calculations were done in Octave and it's values are present in Table 2

Name	**Value**
Vx	8.686793
Ix	0.002805
Req	3096.926423
Tau	0.003168

Table 2: Values for the equivelent resistence

2.3 Natural Solution $V_{6n}(t)$

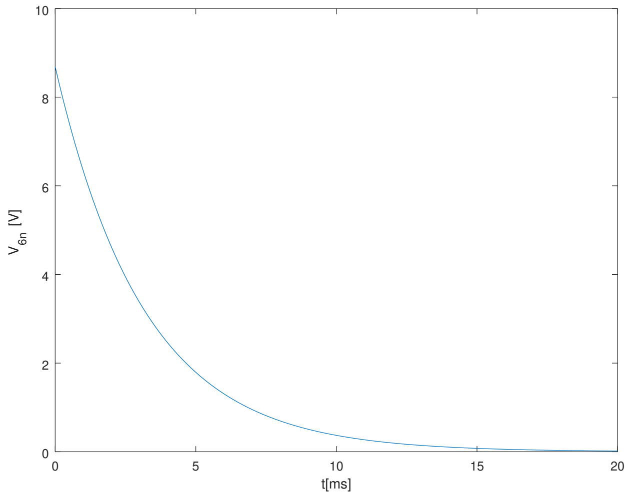
In this subsection, we want to determine $v_6(t)$ with $v_s(t) = 0$. As learned in theory classes, we can describe this using the equation 19.

$$v_{6n}(t) = K \cdot e^{\frac{-t}{RC}} \tag{19}$$

The constant K is V_6 determined for $t = 0$, obtained in the last subsection, which represents the initial condition.

This equation is computed in the time interval [0, 20] ms, using *Octave*, as presented in the Figure 2

Figure 2: Natural Solution in V_6



2.4 Forced Solution $v_{6f}(t)$

Additionally, the forced solution $v_{6f}(t)$ is calculated for $t > 0$. This time, $v_s(t)$ is a sinusoidal function, so phasors are used, as suggested, for practical purposes. To this end, we need to determine the phasor voltages in all the nodes and, consequently, the phasor V_6. The nodal method is used and it is an almost identical analysis as the one done in Subsection 2.1. Only now there is current flowing through the capacitor, which obeys equations 20 and 21

$$V_C = I_C \cdot Z_C \tag{20}$$

$$Z_C = \frac{1}{jwC} \tag{21}$$

in which Z_C is the capacitor impedance, with $w = 2\pi f$ and $f = 1KHz$. Additionally, we consider $Y_C = \frac{1}{Z_C}$

Moreover, the currents are also phasor (complex) currents, and the impedance of resistors is equal to their resistance ($Z_i = R_i = \frac{1}{G_i}$).

The matrix system we need to solve is therefore presented in 22.

$$\begin{bmatrix} -G_1 & G_1+G_2+G_3 & -G_2 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & -K_b-G_5 & G_5+Y_c & 0 & -Y_c \\ 0 & 0 & 0 & 0 & 0 & G_6+G_7 & -G_7 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & K_d \cdot G_6 & -1 & \\ G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{22}$$

The complex values in each node were calculated in octave, where amplitude for these results are presented in Table 3, while it's arguments is in Tabel 4

Name	Amplitude
V1	1.00000000000
V2	0.95451213476
V3	0.85940656896
V5	0.96104661213
V6	0.60656522123
V7	0.40122803814
V8	0.60479191683

Table 3: Amplitude values for the forced solution

Name	Argument [rad]
V1	0.00000000000e+00
V2	3.18830821905e-18
V3	9.99076334490e-16
V5	-5.80001699964e-17
V6	-2.99994331014
V7	3.14159265359
V8	3.14159265359

Table 4: Argument values for the forced solution

2.5 Total solution $v_6(t)$

The intent is now to obtain the total solution $v_6(t)$. For $t > 0$, we need to convert the phasor from the forced solution to a real time solution, as shown in equation 23

$$V_{6f} = V_6 \cdot sin(2\pi 1000t) \tag{23}$$

in which V_6 is the complex amplitude obtained in the previous subsection.

The total solution is obtained adding the natural solution calculated in Subsection 2.3 with equation 19 to the above presented equation 23. Using *Octave*, the signals $v_s(t)$ and $v_6(t)$ are plotted in time interval [-5, 20]ms, as presentend in the Figure 3.

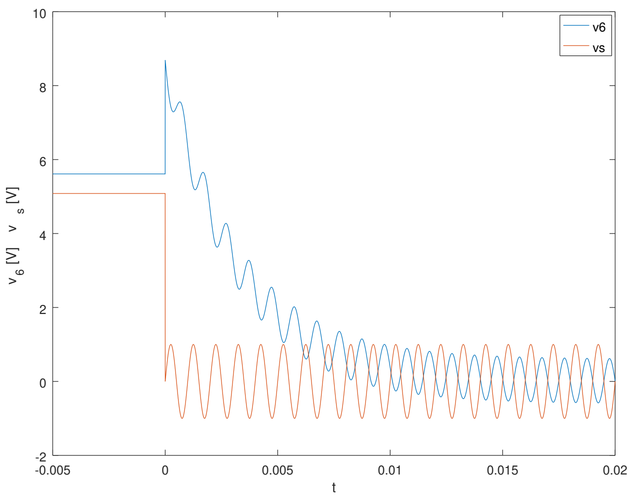


Figure 3: Total Solutions for V_6 and V_s

2.6 Frequency Analysis

In the last subsection of the Theoretical Analysis, we are answering the same question presented on Subsection 2.4, except for several frequencies. The voltage signals $v_c = v_6 - v_8$, v_6 and v_s are analysed as functions of frequency, ranging said frequency from $0.1Hz$ to $1MHz$. The voltages is analysed considering its magnituded in dBs and phase in degrees. The frequency is in the base 10 logarithmic scale.

v_s should not vary with frequency as it is the voltage source of the circuit and, hence, the source of the frequency variation. It should remain zero, since it is 1V. It should have no phase, because its sine function has no phase.

Considering the solution to simple RC circuits, it is also expected that magnitude and phase for V_C (and presumably for V_6) drops significantly, as they do in the following figures.

Bode diagrams are used to present the results of these analyses and they require transfer functions, in this case dependent on frequency. The argument of the function could be jw or jf, instead of the regular s (complex variable), since we are dealing with sinusoidal signals.

The bode diagram for magnitude and phase of the mentioned signals is presented in the next figures.

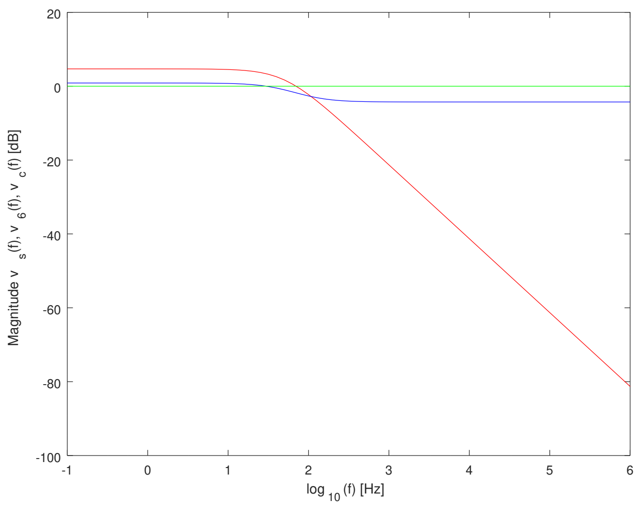


Figure 4: Magnitude bode diagram for $V_6(f)$, $V_s(f)$ and $V_c(f)$ in dBs

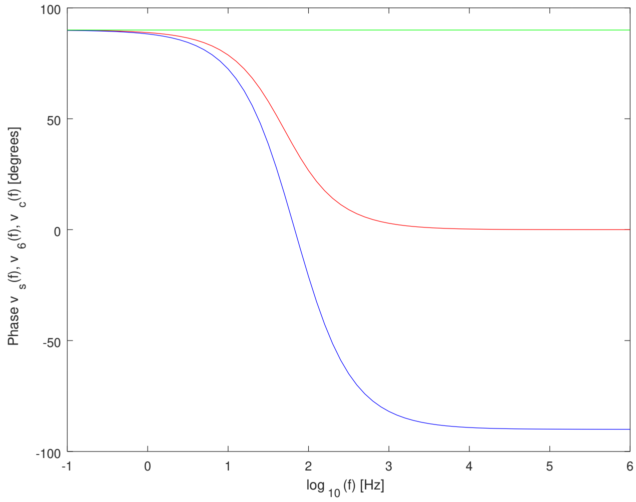


Figure 5: Phase bode diagram for $V_6(f)$, $V_s(f)$ and $V_c(f)$ in degrees

The signal $V_6(f)$ is represented in blue, $V_s(f)$ in green and $V_c(f)$ in red.

3 Simulation Analysis

To accurately simulate the circuit in *Ngspice*, precaution is needed with the positive and negative nodes associated with each component, since an error in these parameters would cause currents and voltage sources operating in the wrong direction.

It's important to add that due to a peculiarity of the program, we couldn't get the current in the resistance R_6 as a reference for the current dependent voltage source. To solve that, we created a "dummy" voltage source (valued 0) right below the ground node in that same branch, so that the program could get a valid read.

3.1 Operating Point Analysis for $t < 0$

Table 5 shows the simulated operating point results for the circuit under analysis at $t < 0$, analogous to the one in subsection 2.1

Name	Value
@c1[i]	0.000000e+00
@gib[i]	-2.34879e-04
@r1[i]	-2.23854e-04
@r2[i]	-2.34879e-04
@r3[i]	-1.10252e-05
@r4[i]	-1.20943e-03
@r5[i]	2.348793e-04
@r6[i]	9.855775e-04
@r7[i]	9.855775e-04
v1	5.083148e+00
v2	4.851927e+00
v3	4.368491e+00
v5	4.885142e+00
v6	5.612546e+00
v7	-2.03950e+00
v8	-3.07425e+00
v9	0.000000e+00

Table 5: Values for each component in operating point

3.2 Operating Point Analysis for $t = 0$

The purpose in simulating the operation point in a circuit analogous to the one in Subsection 2.2 is getting the boundary/initial conditions $V_6(t = 0)$ and $V_8(t = 0)$, which will be needed in the following subsections. The values obtained are in table 6

Name	Value
@gib[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.804972e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
v1	0.000000e+00
v2	0.000000e+00
v3	0.000000e+00
v5	0.000000e+00
v6	8.686793e+00
v7	0.000000e+00
v8	0.000000e+00
v9	0.000000e+00

Table 6: Values for each component in operating point

3.3 Transient Analysis for natural solution

In the present subsection, we perform a transient analysis to the original circuit in Figure 1 (adding the ”dummy” null voltage source mentioned in the beginning of this section) with $v_s = 0$, and using V_6 and V_8, obtained in the previous subsection, as boundary conditions, in order to obtain $v_{6n}(t)$ in the [0, 20]ms time interval.

The results are plotted in Figure 6.

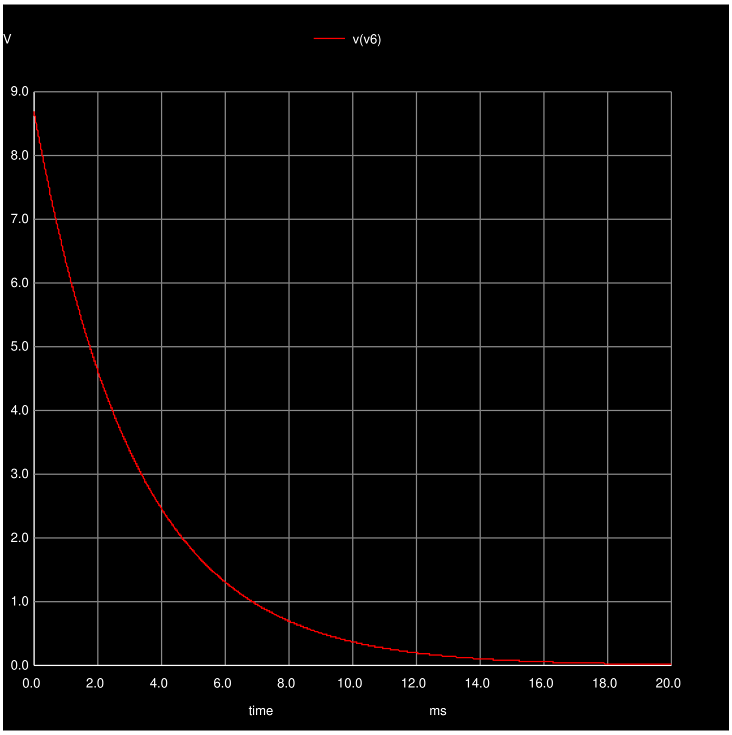


Figure 6: Natural Solution in V_6

3.4 Transient Analysis for total solution

In this subsection, we perform a transient analysis analogous to the last, except the voltage source is now the sinusoidal signal $sin(2\pi ft)$, being $f = 1kHz$.

In Figure 7, both the stimulus and $v_6(t)$ response are plotted for the time interval [0, 20]ms.

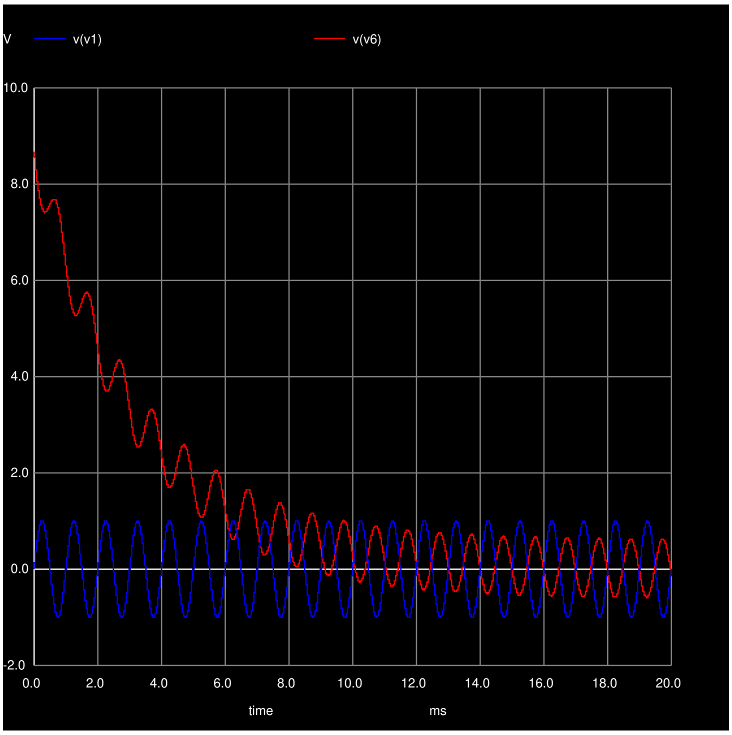


Figure 7: Total Solution for V_6 and V_s.

3.5 Frequency Analysis

Lastly, the voltage signals v_6 and v_s are analysed as functions of frequency, using units and intervals identical to the ones described in Subsection 2.6. The bode diagram computing the signals' magnitude and phase are, hence, presented in the following figures.

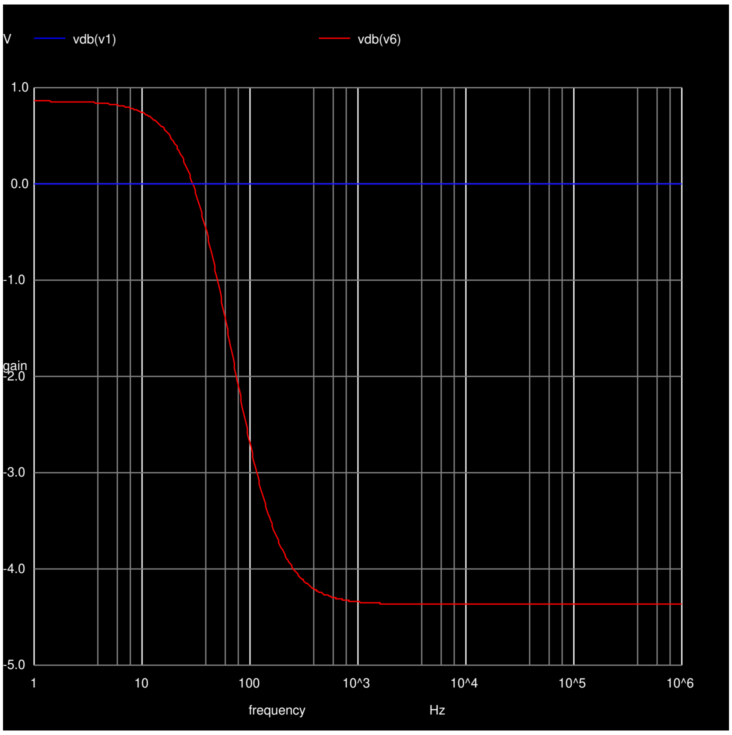


Figure 8: Magnitude bode diagram for $V_6(f)$ and $V_s(f)$ in dBs

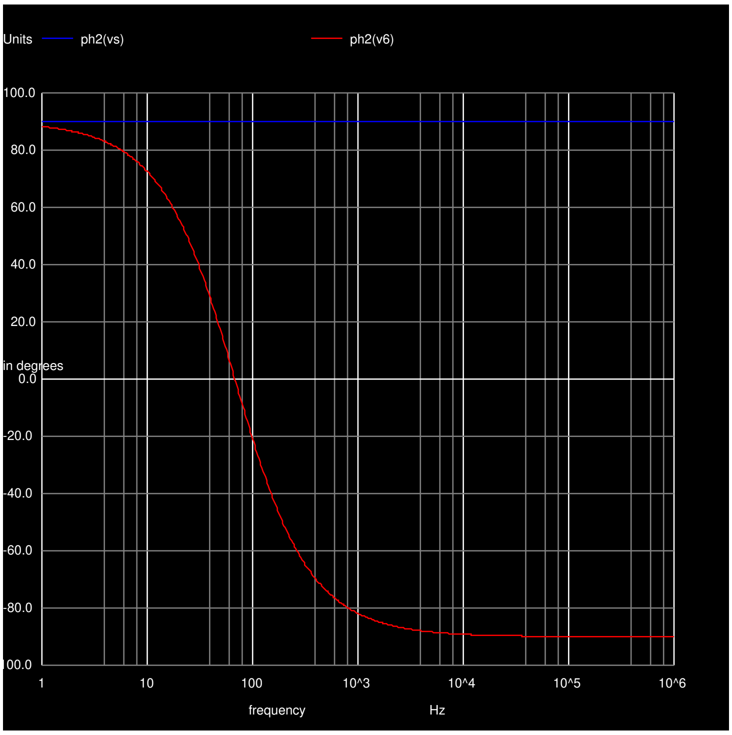


Figure 9: Phase bode diagram for $V_6(f)$ and $V_s(f)$ in degrees

3.6 Comparisons

In this last subsection, we present the tables and plots obtained from the Theoretical Analysis and the Simulations Analysis that refer to the same sets of evaluations, for visualization purposes, and comment on their similarities or discrepancies.

While analysing the mesh currents and nodal voltages in $t < 0$, the tables in Figure 11 were determined. Left and right tables are theoretical and simulation results for $t < 0$, respectively.

Name	Value in [V] or [A]
V1	5.083148
V2	4.851927
V3	4.368491
V5	4.885142
V6	5.612546
V7	-2.039502
V8	-3.074247
IR1	0.000224
IR2	0.000235
IR3	0.000011
IR4	0.001209
IR5	0.000235
IR6	0.000986
IR7	0.000986

Name	Value in [V] or [A]
@c1[i]	0.000000e+00
@gib[i]	-2.34879e-04
@r1[i]	-2.23854e-04
@r2[i]	-2.34879e-04
@r3[i]	-1.10252e-05
@r4[i]	-1.20943e-03
@r5[i]	2.348793e-04
@r6[i]	9.855775e-04
@r7[i]	9.855775e-04
v1	5.083148e+00
v2	4.851927e+00
v3	4.368491e+00
v5	4.885142e+00
v6	5.612546e+00
v7	-2.03950e+00
v8	-3.07425e+00
v9	0.000000e+00

Figure 10: Theoretical and simulation results for $t < 0$.

While analysing for the circuit for $t = 0$, the tables in Figure ?? were determined

Name	Value (SI units, not powers of ten)
Vx	8.686793
Ix	0.002805
Req	3096.926423
Tau	0.003168

Name	Value in [V] or [A]
@gib[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.804972e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
v1	0.000000e+00
v2	0.000000e+00
v3	0.000000e+00
v5	0.000000e+00
v6	8.686793e+00
v7	0.000000e+00
v8	0.000000e+00
v9	0.000000e+00

Figure 11: Theoretical and simulation results for $t < 0$.

Note that, since there is no current flowing through R_2, $I_{R_5} = I_x$.

Between all the aforementioned tables, it can be seen that the results are almost identical. The discrepancies begin, at worse, in the fifth decimal place. These inaccuracies can be attributed to the different approximations or types of variables the *Octave* math tool and *Ngspice* software may use.

In the following figures, plotted for time and frequency analysis, we can also notice the near perfect match. Considering this is a relatively simple linear circuit, this was to be expected.

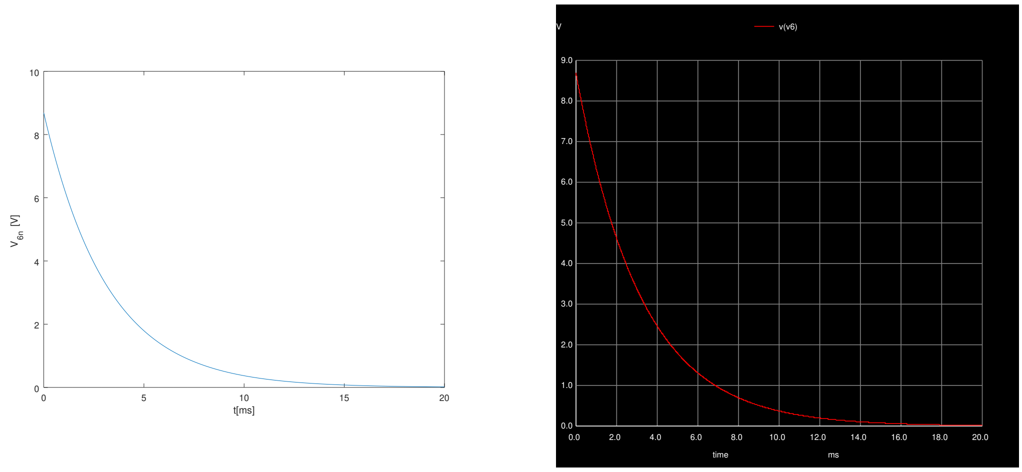


Figure 12: Natural solution.

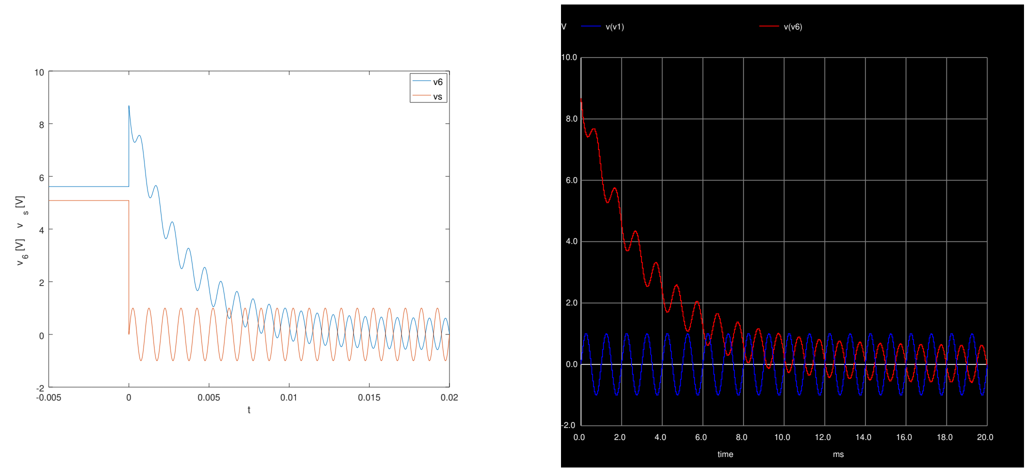


Figure 13: Total solution.

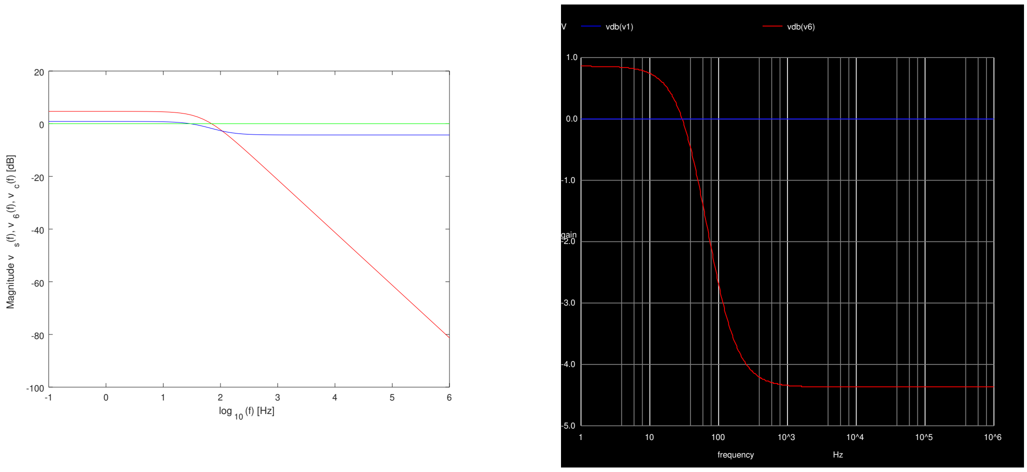


Figure 14: Magnitude with bode diagrams.

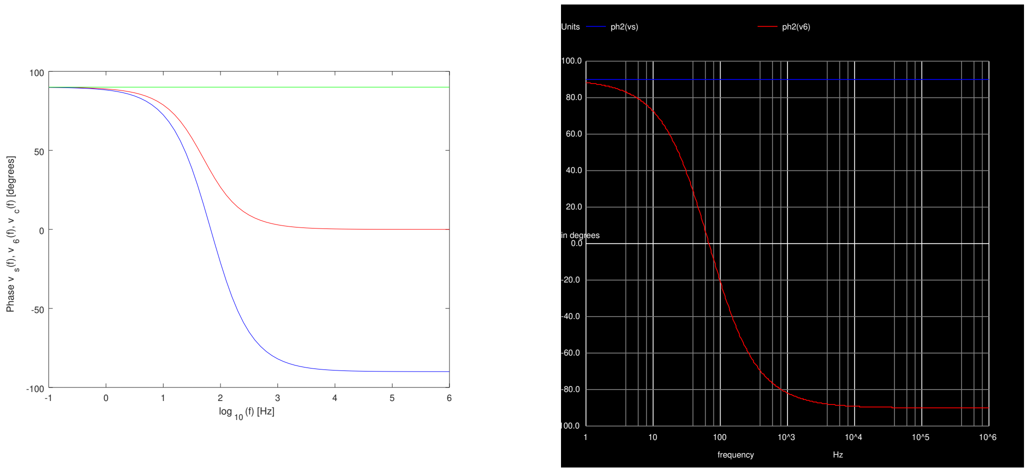


Figure 15: Phase with bode diagrams.

4 Conclusion

In this laboratory assignment, the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this near perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. In addition, the theoretical models used by Ngspice coincide with the models used by our group, therefore it wouldn't be expected a significant discrepancy in the obtained results. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.