Circuit Theory and Electronics Fundamentals

BSc Aerospace Engineering, Técnico, University of Lisbon

Lab 1: Circuit analysis methods

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Contents

1 Introduction

The objective of this laboratory assignment is to study a resistive circuit containing linear components, such as resistors (R_i), independent (circle shaped) and dependent (rhombus shaped) voltage (V) and current (I) sources, as seen in Figure 1. To this end, the currents in every branch and the nodal voltages of the circuit are evaluated, using both the mesh and the nodal methods. For illustration and analysis purposes, the nodes are identified by the red numbers, the elementary meshes by the red subscript letters (in I_i) and the arrows point in the conventionalized current way.

In Section 2, a theoretical analysis is presented, using said mesh (in Subsection 2.1) and nodal (in Subsection 2.2) methods. In Section 3, the circuit is analyzed by simulation, using the Ngspice software. The results are compared in Section 3. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

2.1 Mesh Method

The circuit consists of four elementary meshes: (a), (b), (c) and (d), respectively associated with mesh currents I_a, I_b, I_c and I_d (seen in Figure 1) and with equations Eq. (a), Eq. (b), Eq.

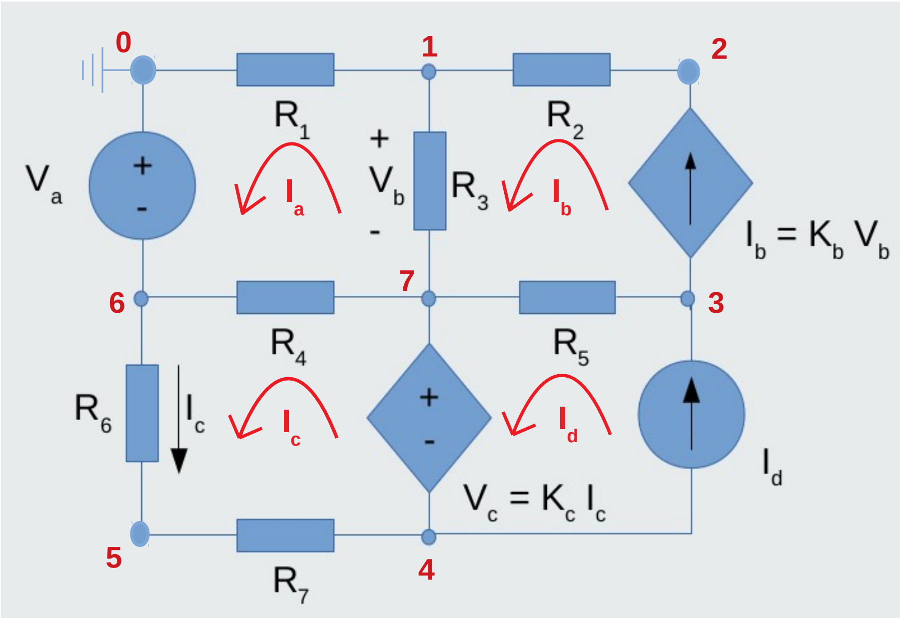

Figure 1: Resistive circuit.

(c) and Eq. (d): these are the result of the direct application of the mesh method and they are respectively presented as the first four equations in the system of linear equations 1.

Since the current in the dependent current source is a function of V_b, a four equation system would be undetermined. Hence, V_b is equated as a function of the mesh currents passing through R_3, in the fifth equation of the system 1.

$$\begin{cases} I_a \cdot R_1 + V_a + I_a \cdot R_4 - I_c \cdot R_4 + I_a \cdot R_3 - I_b \cdot R_3 = 0 \\ I_b = K_b \cdot V_b \\ I_c \cdot R_4 - I_a \cdot R_4 + I_c \cdot R_6 + I_c \cdot R_7 - K_c \cdot I_c = 0 \\ I_d = I_d \\ V_b = R_3 \cdot (I_b - I_a) \end{cases} . \tag{1}$$

This system can be presented in matrix form as in 2

$$\begin{bmatrix} (R_1 + R_4 + R_3) & -R_3 & -R_4 & 0 & 0 \\ 0 & 1 & 0 & 0 & -K_b \\ -R_4 & 0 & (R_4 + R_6 + R_7 - K_c) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ R_3 & -R_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \\ V_b \end{bmatrix} = \begin{bmatrix} -V_a \\ 0 \\ 0 \\ I_d \\ 0 \end{bmatrix} \tag{2}$$

and computed in Octave scripts, from which the Table 1 is obtained.

As learned in theory classes and understood from Figure 1, the current in a branch is a function of the mesh currents the branch is associated with. If the branch is associated with only one mesh current, then its current equals the mesh current. That is not the case for the branches formed by the R_3, R_4, R_5 and V_c components. To determined those currents, the equations 3, 4, 5, and 6 are used.

$$I_R 3 = I_a - I_b \tag{3}$$

Name	Value [A or V]
Ia	-0.000246
Ib	-0.000257
Ic	0.000968
Id	0.001004

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

in which $Ir3$ is the current flowing from node 7 to 1.

$$Ir4 = I_a - I_c \tag{4}$$

in which $Ir4$ is the current flowing from node 6 to 7.

$$Ir5 = I_d - I_b \tag{5}$$

in which $Ir5$ is the current flowing from node 3 to 7.

$$Ivc = I_d - I_c \tag{6}$$

in which Ivc is the current flowing from node 7 to 4.

The results obtained with Octave are presented in Table 2.

Name	Value [A or V]
Ir3	0.000011
Ir4	-0.001214
Ir5	0.001261
Ivc	0.000036

Table 2: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

2.2 Node Method

The circuit consists of 8 nodes: 0 (ground), 1, 2, 3, 4, 5, 6 and 7, respectively associated with nodal voltages V_0, V_1, V_2, V_3, V_4, V_5, V_6 and V_7 (seen in Figure 1).

The nodal equations obtained with the nodal method pertain to the mentioned nodes as follows.

Node 0:

$$V_0 = 0 \tag{7}$$

Node 1:

$$(V_1 - V_0) \cdot G_1 + (V_1 - V_2) \cdot G_2 + V_b \cdot G_3 = 0 \tag{8}$$

Node 2:

$$(V_2 - V_1) \cdot G_2 - K_b \cdot V_b = 0 \tag{9}$$

Node 3:

$$(V_3 - V_7) \cdot G_5 - I_d + K_b \cdot V_b = 0 \tag{10}$$

Node 4:

$$I_c - Ivc + I_d = 0 \tag{11}$$

$$V_4 = -V_c + V_7 \tag{12}$$

Node 5:

$$(V_5 - V_6) \cdot G_6 + (V_5 - V_4) \cdot G_7 = 0 \tag{13}$$

Node 6:

$$V_6 = -V_a \tag{14}$$

Node 7:

$$Ivc + (V_7 - V_3) \cdot G_5 + (V_7 - V_2) \cdot G_3 + (V_7 - V_6) \cdot G_4 = 0 \tag{15}$$

To obtain a determined linear system of equations, three other equations are added. Both V_b and I_c need to be written as a function of nodal voltages, as shown in equations 16 and 17. V_C should be equated as shown in Figure 1 and as written in equation 18.

$$V_b = V_1 - V_7 \tag{16}$$

$$V_5 + I_c \cdot R_6 = V_6 \Leftrightarrow I_c = (V_6 - V_5) \cdot G_6 \tag{17}$$

$$V_c = K_c \cdot I_c \tag{18}$$

Applying the added equations 16, 17 and 18 to the previously presented nodal equations, the system of linear equations (produced in matrix form) is 19

$$\begin{bmatrix} (G_1+G_2+G_3) & -G_2 & 0 & 0 & 0 & 0 & -G_3 & 0 \\ -G_2-K_b & G_2 & 0 & 0 & 0 & 0 & K_b & 0 \\ K_b & 0 & G_5 & 0 & 0 & 0 & (-K_b-G_5) & 0 \\ 0 & 0 & 0 & -G_7 & (G_6+G_7) & -G_6 & 0 & 0 \\ 0 & 0 & 0 & 1 & (-K_c \cdot G_6) & (K_c \cdot G_6) & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ I_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_d \\ 0 \\ 0 \\ -V_a \\ -I_d \\ 0 \end{bmatrix} \tag{19}$$

Lastly, an Octave script running 19 is used to obtain the results in Table 3.

Name	Value [A or V]
V1	-0.249911
V2	-0.778383
V3	3.700284
V4	-8.073949
V5	-7.067894
V6	-5.113245
V7	-0.214558
Ivc	0.000036

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3 Simulation Analysis

To accurately simulate the circuit in ngspice, precaution is needed with the positive and negative nodes associated with each component, since an error in these parameters would cause currents and voltage sources operating in the wrong direction.

It's important to add that due to a peculiarity of the program, we couldn't get the current in the resistance R_5 as a reference for the current dependent voltage source. To solve that, we created a dummy source(V_8 with value 0) in that branch so that the program could get a valid read. That is the reason why in the table 4 we have V_8 with the value that V_6.

Table 4 shows the simulated operating point results for the circuit under analysis were is present the values of current thru all components as well as the voltages in each node.

Name	Value [A or V]
@gb[i]	-2.57154e-04
@id[current]	1.003968e-03
@r1[i]	-2.45916e-04
@r2[i]	2.571543e-04
@r3[i]	-1.12383e-05
@r4[i]	1.213956e-03
@r5[i]	-1.26112e-03
@r6[i]	9.680395e-04
@r7[i]	9.680395e-04
v(1)	-2.49911e-01
v(2)	-7.78383e-01
v(3)	3.700284e+00
v(4)	-8.07395e+00
v(5)	-7.06789e+00
v(6)	-5.11325e+00
v(7)	-2.14558e-01
v(8)	-5.11325e+00

Table 4: Simulation results; A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Comparing these results with the previous tables given with octave, it can easily be seen that our results match, reasons from which will be mentioned in the next section. Diferences in the signal of some currents are due to assumptions previosly made in the direction of the current flow.

4 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.