

Tercer punto

$$\bullet \mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k} = x^i \hat{i}_i$$

$$\bullet \mathbf{a} = \mathbf{a}(\mathbf{r}) = \mathbf{a}(x, y, z) = a^i(x, y, z) \hat{i}_i \text{ y } b^i(x, y, z) \hat{i}_i$$

$$\phi = \phi(\mathbf{r}) = \phi(x, y, z) \text{ y } \psi = \psi(\mathbf{r}) = \psi(x, y, z)$$

$$a) \nabla(\phi\psi) = \phi \nabla\psi + \psi \nabla\phi$$

$$\begin{aligned} \nabla(\phi\psi) &= \partial_i(\phi\psi)_i = \phi \partial_i \psi + \psi \partial_i \phi \\ &= \phi \nabla\psi + \psi \nabla\phi \end{aligned}$$

$$b) \nabla \cdot (\nabla \times \mathbf{a}) \text{ ¿Que se puede decir de } \nabla \times (\nabla \cdot \mathbf{a})?$$

$$\begin{aligned} \nabla \cdot (\nabla \times \mathbf{a}) &= \partial_i (\epsilon^{ijk} \partial_j a_k)_i \\ &= \epsilon^{ijk} \partial_i \partial_j a_k = -[\epsilon^{jik} \partial_i \partial_j a_k] \end{aligned}$$

$$\begin{aligned} \nabla \times (\nabla \cdot \mathbf{a}) &= \epsilon^{ijk} \partial_i (\partial_k a_k) \\ &= -\epsilon^{jik} \partial_k \partial_i a_k \end{aligned}$$

$$\Rightarrow -[\epsilon^{jik} \partial_k \partial_i a_k] = -[\epsilon^{jik} \partial_k \partial_i a_k]$$

Son iguales

$$f) \nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$

$$\begin{aligned} \nabla \times (\epsilon^{ijk} \partial_j a_k) &\rightarrow \epsilon^{mni} \partial_n (\epsilon^{ijk} \partial_j a_k)_i \rightarrow \epsilon^{mil} \partial_l \epsilon_{ijk} \partial^j a^k \\ &= \epsilon^{mil} \epsilon_{ijk} (\partial_i \partial^j a^k) \\ &= \delta_j^m \delta_k^i - \delta_j^i \delta_k^m (\partial_i \partial^j a^k) \\ &= \partial_k \partial^m a^k - \partial_j \partial^i a^m \\ &= \partial^m (\partial_k a^k) - \partial_i^2 a^m \end{aligned}$$

$$= \nabla \cdot (\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$