Tercer punto

- $r = x\hat{\imath} + y\hat{\jmath} + z\hat{k} = x^i \hat{\imath}_i$

- $a = a(r) = a(x,y,z) = a^i(x,y,z)\,\hat{\imath}_i$ y $b^i(x,y,z)\,\hat{\imath}_i$

$\phi = \phi(r) = \phi(x,y,z)$ y $\psi = \psi(r) = \psi(x,y,z)$

a) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$\nabla(\phi\psi) = \partial_i(\phi\psi)_i = \phi\partial_i\psi + \psi\partial_i\phi$

$= \phi\nabla\psi + \psi\nabla\phi$

d) $\nabla\cdot(\nabla\times a)$ ¿Que se puede decirde $\nabla\times(\nabla\cdot a)$?

$\nabla\cdot(\nabla\times a) = \partial_i(\varepsilon^{ijk}\partial_j a_k)_i$

$= \varepsilon^{ijk}\partial_i\partial_j a_k = -[\varepsilon^{jik}\partial_j\partial_i a_k]$

$\nabla\times(\nabla\cdot a) = \varepsilon^{ijk}\partial_i(\partial_k a_k)$

$= -\varepsilon^{jik}\partial_k\partial_i a_k$

$\Rightarrow -[\varepsilon^{jik}\partial_K\partial_i a_k] = -[\varepsilon^{jik}\partial_k\partial_i a_k]$

Son iguales

f) $\nabla\times(\nabla\times a) = \nabla(\nabla\cdot a) - \nabla^2 a$

$\nabla\times(\varepsilon^{ijk}\partial_j a_k) \rightarrow \varepsilon^{mni}\partial_n(\varepsilon^{ijk}\partial_j a_k)_i \rightarrow \varepsilon^{mil}\partial_l\,\varepsilon_{ljk}\partial^j a^k$

$= \varepsilon^{mil}\varepsilon_{ljk}(\partial_i\partial^j a^k)$

$= \delta^m_j\delta^i_k - \delta^i_j\delta^m_k(\partial_i\partial^j a^k)$

$= \partial_k\partial_m a^k - \partial_j\partial^i a^m$

$= \partial_m(\partial_k a^k) - \partial_i^2 a^m$

$\downarrow$

$= \nabla\cdot(\nabla\cdot a) - \nabla^2 a$