Assignment 3

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Question 1

Part 1

The given distribution is: $f(x; \mu, b) = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}}$

Now we want to maximize the likelihood of the distribution, and this can be done as follows:

$$ln(\prod_{n=1}^{N} \frac{1}{2b} e^{-\frac{|x-\mu|}{b}}) = n * ln(2b) - \frac{1}{b} \sum_{i=1}^{n} |x_i - \mu|$$

Now we have to calculate the derivatives of this formula for both values, so we can get the maximum likelihood estimates.

For the value of μ:

$$\frac{d}{d\mu} = L(\mu, b | x_1, ..., x_n) = \frac{1}{b} \sum_{i=1}^{n} sgn(x_i - \mu) = 0$$

With the given hint, we get that the actual value of μ is the median of the data points, so that the value of the derivative reaches 0.

For the value of b:

$$\frac{d}{db} = L(\mu, b | x_1, ..., x_n) = - \frac{n}{b} + \frac{1}{b^2} \sum_{i=1}^{n} |x_i - \mu| = 0$$

From here, we can get the value of b as:

$$b = \frac{1}{n} \sum_{i=1}^{n} |x_i - \mu| = \frac{1}{n} \sum_{i=1}^{n} |x_i - median(x_1, ..., x_n)|$$

With the given hint, we get that the actual value of b is the mean absolute deviation for the data points, so that the value of the derivative reaches 0.

Part 2

First, we need to get the probabilities P(Y), and it is calculated as follows:

$$P(Y = 0) = \frac{5+1}{8+2} = \frac{6}{10}$$

$$P(Y = 1) = \frac{3+1}{8+2} = \frac{4}{10}$$

Now, we need to calculate the probabilities of the values X conditioned on the value of Y. This can be calculator from the given table as follows (λ is setted as 1):

$$P(X_1 = 0 | Y = 0) = \frac{3+1}{5+3} = \frac{4}{8} = 0.5$$

$$P(X_1 = - 1 | Y = 0) = \frac{2+1}{5+3} = \frac{3}{8}$$

$$P(X_1 = 1 | Y = 0) = \frac{0+1}{5+3} = \frac{1}{8}$$

$P(X_1 = 0|Y = 1) = \frac{2+1}{3+3} = \frac{3}{6} = 0.5$
$P(X_1 =- 1|Y = 1) = \frac{0+1}{3+3} = \frac{1}{6}$
$P(X_1 = 1|Y = 1) = \frac{1+1}{3+3} = \frac{1}{3}$

$P(X_2 = 0|Y = 0) = \frac{3+1}{5+2} = \frac{4}{7}$
$P(X_2 = 1|Y = 0) = \frac{2+1}{5+2} = \frac{3}{7}$
$P(X_2 = 0|Y = 1) = \frac{3+1}{3+2} = \frac{4}{5}$
$P(X_2 = 1|Y = 1) = \frac{0+1}{3+2} = \frac{1}{5}$

$P(X_3 = 0|Y = 0) = \frac{0+1}{5+2} = \frac{1}{7}$
$P(X_3 = 1|Y = 0) = \frac{5+1}{5+2} = \frac{6}{7}$
$P(X_3 = 0|Y = 1) = \frac{1+1}{3+2} = \frac{2}{5}$
$P(X_3 = 1|Y = 1) = \frac{2+1}{3+2} = \frac{3}{5}$

After calculating all probabilities, we have to calculate the probability corresponding to the values of X. This is calculated as:

$y(x_1) = P(Y = 0) * P(X_1 =- 1|Y = 0) * P(X_2 = 1|Y = 0) * P(X_3 = 0|Y = 0)$
$= \frac{6}{10} * \frac{3}{8} * \frac{3}{7} * \frac{1}{7} = 0.013$

$y(x_1) = P(Y = 1) * P(X_1 =- 1|Y = 1) * P(X_2 = 1|Y = 1) * P(X_3 = 0|Y = 1)$
$= \frac{4}{10} * \frac{1}{6} * \frac{1}{5} * \frac{2}{5} = 0.0053$

As we can see, given the values {-1,1,0} for X, the probability of Y=0 is the highest one. This means that the predicted label for Y in this case would be 0.