Assignment 3

Carlos Sanchez
ID: 21111910

Question 3

Part 1

We have the following matrix for the values of X1 and X2:

$$\begin{pmatrix} 1 & 1 & 4 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 4 \end{pmatrix}$$

If we compute the mean of each component, the result is:

$$\mu_{X1} = \frac{1+1+4+0+0+1+1+0}{8} = \frac{8}{8} = 1$$

$$\mu_{X2} = \frac{1+1+0+0+0+1+1+4}{8} = \frac{8}{8} = 1$$

To calculate the covariance matrix, we subtract first the mean of each component to the matrix, resulting in the following matrix:

$$\begin{pmatrix} 0 & 0 & 3 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 & 3 \end{pmatrix}$$

Now we have to multiply the previous matrix with its transposed matrix, and divide the result by the number of values taken. The result of doing this will be the covariance matrix. This is calculated as follows:

$$\frac{1}{8}\begin{pmatrix} 0 & 0 & 3 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 3 & -1 \\ -1 & -1 \\ -1 & -1 \\ 0 & 0 \\ 0 & 0 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

Now we have to compute the eigenvalues of the matrix. This process can be done as shown in the process:

$$Det\begin{pmatrix} \frac{3}{2} - \lambda & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} - \lambda \end{pmatrix} = 0$$

$$(\frac{3}{2} - \lambda)^2 - \frac{1}{4} = 0 \Rightarrow \frac{9}{4} - 3\lambda + \lambda^2 - \frac{1}{4} = 0 \Rightarrow \lambda^2 - 3\lambda + 2 = 0$$

$\lambda_1 = 1$

$\lambda_2 = 2$

Now we choose the greater eigenvalue of the ones calculated in the previous step, and we calculate the normalized eigenvector corresponding to that value. It is done as follows:

$(\frac{3}{2} - \lambda_2)v_1 - \frac{1}{2}v_2 = 0 \Rightarrow -\frac{1}{2}v_1 - \frac{1}{2}v_2 = 0$

$-\frac{1}{2}v_1 + (\frac{3}{2} - \lambda_2)v_2 = 0 \Rightarrow -\frac{1}{2}v_1 - \frac{1}{2}v_2 = 0$

The solution is given by

$$v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

And when normalizing this value for v we get:

$$v = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

The projection of the given data is calculated as:

$Projection = v^t X =$

$$\begin{pmatrix} 0 & 0 & -2\sqrt{2} & 0 & 0 & 0 & 0 & 2\sqrt{2} \end{pmatrix}$$

The reconstruction can be calculated as:

$Reconstruction = vX =$

$$\begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Now, the reconstruction error is calculated as following:

$\frac{1}{n}||X - Reconstruction||_F^2 = \frac{1}{8} * 8 = 1 = \lambda_1$