Optimization Techniques

Laboratory 10

Linear Programming

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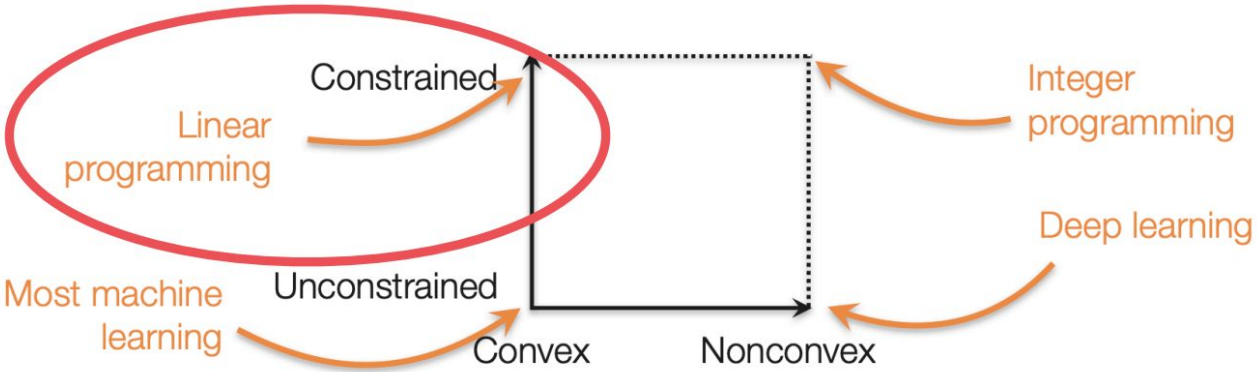
Linear Programming

Linear : the objective functions and constraints are linear
Programming : planning of activities to obtain an optimal result



$$\begin{aligned} \underset{x_1, x_2}{\text{maximize}} \quad & 2x_1 + x_2 \\ \text{subject to} \quad & 3x_1 + x_2 \leq 9 \\ & x_1 + 2x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Linear Programming

Slack Form

$$\begin{aligned} &\underset{x}{\text{minimize}} && c^T x \\ &\text{subject to} && Ax = b \\ &&& x \geq 0 \end{aligned}$$

$$\begin{aligned} &\underset{x_1, x_2}{\text{maximize}} && 2x_1 + x_2 \\ &\text{subject to} && 3x_1 + x_2 \leq 9 \\ &&& x_1 + 2x_2 \leq 6 \\ &&& x_1, x_2 \geq 0 \end{aligned} \longrightarrow \begin{aligned} &\underset{x_1, x_2, x_3, x_4}{\text{minimize}} && -2x_1 - x_2 \\ &\text{subject to} && 3x_1 + x_2 + x_3 = 9 \\ &&& x_1 + 2x_2 + x_4 = 6 \\ &&& x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

$$c = \begin{bmatrix} -2 \\ -1 \end{bmatrix}, A = \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

Linear Programming

Simplex Algorithm

Repeat:

1. Given index set $\mathcal{I}$ such that $x_{\mathcal{I}} = A_{\mathcal{I}}^{-1} b \geq 0$
2. Find j for which $\bar{c}_j = c_j - c_{\mathcal{I}}^T A_{\mathcal{I}}^{-1} A_j < 0$ (if none exists, return x)
3. Compute step direction $d_{\mathcal{I}} = -A_{\mathcal{I}}^{-1} A_j$ and determine index to remove (or return bounded if $d_{\mathcal{I}} \geq 0$)
$$i^\star = \underset{i \in \mathcal{I}: d_i < 0}{\operatorname{argmin}} -x_i / d_i$$
4. Update index set: $\mathcal{I} \leftarrow \mathcal{I} - \{i^\star\} \cup \{j\}$