

Trends in Mortality Decrease and Economic Growth

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Abstract The vast literature on extrapolative stochastic mortality models focuses mainly on the extrapolation of past mortality trends and summarizes the trends by one or more latent factors. However, the interpretation of these trends is typically not very clear. On the other hand, explanation methods are trying to link mortality dynamics with observable factors. This serves as an intermediate step between the two methods. We perform a comprehensive analysis on the relationship between the latent trend in mortality dynamics and the trend in economic growth represented by gross domestic product (GDP). Subsequently, the Lee-Carter framework is extended through the introduction of GDP as an additional factor next to the latent factor, which provides a better fit and better interpretable forecasts.

Keywords Longevity · GDP per capita · Lee-Carter model

Introduction

The twentieth century saw a remarkable increase in average human lifetime compared with previous centuries. For most developed countries, mortality rates have fallen dramatically at all ages. By the beginning of the twenty-first century, the average life span has reached about age 70, compared with 60 to 65 years in the middle of the twentieth century, and 40 to 45 years in the middle of the eighteenth century. The average lifetime among early humans is considered to have been between 20 and 30 years, as suggested by archaeological evidence (Pitacco et al. 2009). Roughly speaking, the

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average life span increased by 25 years in the 10,000 years before the middle of the eighteenth century and increased by another 25 years between the end of eighteenth century and the twentieth century. The recent longevity improvement is rather impressive. (For more details of the mortality trend, see, for example, Pitacco et al. (2009).)

The rapid increase of longevity has been accompanied by increasing attention to risk management for insurance companies and pension funds. For example, the Solvency II project, aimed at redesigning financial regulation of insurance companies in Europe, imposes a risk-based capital requirement. The tightening of regulation and supervision makes longevity risk a significant factor related to the sustainability of pensions and insurance companies, as well as the whole society.

In response to the increasing role of longevity risk and the demand for more accurate projections of future mortality rates, a vast literature on mortality forecasting has been produced during the recent decade. The mortality forecasting methods can be divided into three categories (Booth and Tickle 2008): expectation, extrapolation, and explanation. The expectation method is based on expert opinions, the explanation method tries to link mortality dynamics to some risk factors, and the extrapolation method assumes that past mortality trends will continue in the future.

Most of the models lie in the category of extrapolation. In general, these models focus on the historical mortality changes and then extract some latent factors from historical data. In general, each latent factor summarizes a trend in mortality rates along some dimension—for example, period or cohort. How these trends will behave in the future, however, is hard to determine as long as it is not fully understood what kind of forces are behind them. Most studies focus on an autoregressive integrated moving average (ARIMA) modeling of these latent factors; only few have confronted them with some observable socioeconomic variables. In contrast, we try to examine and understand the latent trends in mortality in terms of observable trends. One of the well-observed and heavily studied trends, accompanying the mortality decline in recent centuries, is the rapid growth in output.¹ This co-movement, which clearly seems to have lasted for centuries, is not likely to be a coincidence. In addition, the role of economic growth on the long-term mortality decline is widely documented (see, e.g., Brenner 2005). Even if there is not a strong direct link between mortality rates and economic levels, the trends in the two series might be affected by some similar underlying factors and are bundled in the long term.

The main goal of this article is to examine the equilibrium relationship between the trend in mortality and the trend in economic growth. In the first part of this article, we investigate the extent to which the trend in mortality, as quantified by the Lee and Carter (1992) model (hereafter referred to as the Lee-Carter model), is captured by the trend in economic growth, as represented by real GDP. More specifically, using data from 1950 to 2007 for six Organisation for Economic Co-operation and Development (OECD) countries—namely, the United States, the United Kingdom, the Netherlands, Canada, Australia, and Japan—the first part of the article compares the latent factor of the Lee-Carter model with real GDP per capita. Here, the main findings are as follows. First, Johansen cointegration tests indicate that the two series have a long-run relationship. Second, the two series have comparable performance in terms of fitting historical mortality rates. Third, the two series imply similar mortality projections. In short, the real GDP series is qualified to be a substitute for the latent factor in the Lee-Carter model. This part of our article is most

¹ See the section Economic Growth and Mortality Rates for an overview of related literature.

closely related to a study by Hanewald (2011). She mainly studied the relationship between mortality rates and the fluctuations in macroeconomics, and we present a comprehensive analysis on the trends, which can be associated with the long-term dynamics of these series.

Based on these findings, we propose a stochastic mortality model that includes both latent and observable factors, with the aim of better interpreting and predicting mortality dynamics. Forecasting mortality rates is a natural application of our model. Our mortality forecasting method can be seen as a combination of the explanation and extrapolation methods in Booth and Tickle (2008). First, from the perspective of explanation methods, we include real GDP per capita as an observable factor, which captures the correlation between long-term trends in mortality dynamics and economic growth. Second, from the perspective of the extrapolation methods, our model captures the trend in mortality rates and forecasts future mortality rates based on historical trends.

Literature Review

Extrapolative Mortality Modeling

One of the most well-known extrapolative mortality models is the Lee-Carter model, which we briefly discuss in this section. The central death rate with age x in year t is denoted by $m_{x,t} = D_{x,t} / E_{x,t}$, where $D_{x,t}$ is the number of deaths at age x and time t ; and $E_{x,t}$, the exposure-to-risk, is the number of person-years at age x and year t . Lee and Carter (1992) postulated a log bilinear form for the central death rate:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \quad (1)$$

with time-invariant parameters α_x and β_x , and a homoskedastic error term $\varepsilon_{x,t}$ with a mean of 0 and a variance of $\hat{\sigma}_\varepsilon^2$. The parameters in the Lee-Carter model cannot be identified without additional constraints. Lee and Carter (1992) imposed that $\sum_t \kappa_t = 0$ and $\sum_x \beta_x = 1$, where κ_t is summed over all periods, and β_x is summed over all available ages in the sample.

Equation (1) can be estimated using singular value decomposition (SVD) as in Lee and Carter (1992). Moreover, to match the estimated death rates with the observed number of deaths in a given year, $D(t) = \sum_x D_{x,t}$, Lee and Carter proposed to adjust κ_t after the first step estimation, such that

$$D(t) = \sum_x \left(E_{x,t} \exp(\hat{\alpha}_x + \hat{\beta}_x \kappa_t) \right), \quad (2)$$

where $\hat{\alpha}_x$ and $\hat{\beta}_x$ are estimated parameters from SVD.

The parameter κ_t , often labeled as the mortality index or mortality-reduction factor, is a one-dimensional time-dependent latent process that quantifies the variation in the level of mortality over time. The parameters α_x , by construction, represent the average log mortality at given ages, and the parameters β_x capture the sensitivity of the log central death rate at age x to variations in κ_t . Finally, the error terms $\varepsilon_{x,t}$ represent the age- and time-specific variations that are not captured by the systematic part.

Mortality prediction is based on the estimated parameters and projection of κ_t . In most studies, the latent variable κ_t is modeled as an ARIMA(p,d,q) process with best-fitting form $(p,d,q) = (0,1,0)$, which is a random walk with drift (Lee and Carter 1992; Lee and Miller 2001; Hanewald 2011). That is,

$$\kappa_t = \theta + \kappa_{t-1} + \delta_t, \quad (3)$$

where θ is a drift term, and δ_t is a white noise error term.

Many modifications of the original Lee-Carter model have been proposed (see, e.g., Booth et al. 2002; Cairns et al. 2006; Currie et al. 2004; Lee and Miller 2001; O'Hare and Li 2012; Plat 2009). For a review, see Booth and Tickle (2008) and the many references therein.

Despite of the popularity of these stochastic models with latent variables, they also have common limitations. Although the stochastic models can identify some historical trends in mortality rates, they are not aimed at explaining what underlies the historical trends nor whether these trends will continue. A more ambitious model might take into account the impact of some exogenous factors (such as biomedical, environmental, or socioeconomic factors) on mortality rates. Although it is not easy to identify all relevant variables and to comprehend their mechanisms, it is instructive to start with a few. In the following section, we discuss the relationship between trends in mortality rates and in one of the potential factors: economic growth. By choosing GDP as the proxy for economic growth, our approach can be associated with both the extrapolation and the explanation methods. The next section discusses the reasons.

Economic Growth and Mortality Rates

The relationship between economic growth and health and mortality has been studied for several decades. It is generally accepted that the two variables are closely linked, with causation often going in both directions. In this article, we use GDP to forecast mortality. However, there is also an extensive literature discussing an alternative pathway: for example, the impacts of health on economic outcomes at both the micro and the macro levels. The debate is still ongoing. For example, Bloom et al. (2004) found that good health has a positive and sizable effect on aggregate output. Bloom et al. (2004) also provided an overview of works that include health as a determinant of economic growth and the magnitude of the effect. In addition, both De la Croix and Licandro (1999) and Bhargava et al. (2001) claimed that the effect of life expectancy on growth is positive in low-income countries. On the other hand, Acemoglu and Johnson (2007) found no evidence that exogenous increase in life expectancy resulted in a significant increase in per capita economic growth. Although we acknowledge the controversy in the literature, our analysis is based on the hypothesis that in the long run, the trend in economic growth and the one in longevity growth should not greatly diverge from each other.

After emphasizing that studying the direction of causality is beyond the scope of this article, in the following, we continue to review the literature on the relationship between economic growth and health. Using cross-country and time series data on health and income per capita, Pritchett and Summers (1996) found a significant positive effect of income on health in the long run. Using individual-level data from several

surveys, Ettner (1996) documented that increases in income significantly improve mental and physical health, based on both ordinary and instrumental variables (IV) estimates. Brenner (2005) suggested that economic growth not only reduces poverty through an increase of real income but also stimulates the investments in new medicines, surgery and prosthetics, and hospital services, which may dramatically increase life expectancy. Applying time series analysis to U.S. data, Brenner (2005) also showed that GDP is strongly negatively related to mortality, over the medium to long term. Birchenall (2007) argued that improvements in economic conditions are an important force behind mortality decline. The author used income per capita to measure the economic condition, which is similar to the real GDP per capita used in our article. The effects of wealth on health are not constrained to developing countries. Swift (2011) applied cointegration analysis to investigate the relationship between health and GDP for 13 OECD countries over the last two centuries, finding that GDP per capita and total GDP have a significant impact on life expectancy for most countries. Nandi et al. (2012) found that monthly rates of death by suicide in New York City are negatively associated with levels of economic activity in New York State.

However, only a very limited number of studies have examined the role of macroeconomic variables and other observable factors in the context of stochastic mortality models such as the Lee-Carter model. Using data for six OECD countries spanning the period 1950 to 2006, Hanewald (2011) studied the impact of macroeconomic fluctuations on mortality dynamics in the Lee-Carter mortality forecasting model, finding a significant correlation between the mortality index κ , in this model and GDP levels for the considered periods and countries.

This article follows the idea that the latent factors in mortality might be related to some macroeconomic variables. However, instead of building a structural model to study causality, we first begin with a reduced-form approach to study the relationship between the latent factor and the observable factor. By doing so, we aim to provide an alternative perspective on understanding the existing stochastic models and pave the way for future developments of models including explanatory elements. Among a number of potential factors, we focus on the role of economic growth, where real GDP per capita is applied as proxy. In the long run, the trend in economic growth, as measured by real GDP per capita, is very likely to be associated with the trend in mortality reduction, which is the main component captured by many of the stochastic mortality models. The use of real GDP as a measure of economic growth is widely documented. In addition, GDP data have a number of merits in a forecasting model. First, GDP is relatively objective and easy to access, making the model more transparent. Second, the dynamics of the GDP process has been widely studied in the literature. Moreover, the trend in GDP may capture the trend in the overall economy.

Mortality and GDP

Data and Mortality Fitting by the Lee-Carter Model

Our analysis in this article is based on six industrialized countries—namely, the United States, the United Kingdom, the Netherlands, Canada, Australia, and Japan—whose mortality dynamics are often investigated in the literature and whose pension systems are exposed to increasing longevity risk. Annual death rates from 1950 to 2007 are

obtained from the Human Mortality Database (n.d.). The period that we select covers the recent mortality trends after World War II. Given that mortality data at very old ages are not very reliable, we set the maximum age in our sample to 99; thus, the total range of age investigated is 0–99.² The time series of real GDP per capita for each country of the corresponding period is obtained from the Maddison Data on the World Economy (Bolt and van Zanden 2013). To account for the exponential growth patterns, we take the natural logarithm of the real GDP series. Our data set is similar to that used by Hanewald (2011). However, we choose real GDP per capita instead of total GDP because the former, more closely related to individuals' purchasing power, seems to be a more appropriate measure of economic well-being.

The Lee-Carter model is estimated under the settings in Lee and Carter (1992) for each country and gender combination. Males and females are treated separately because they show different mortality patterns. In line with the previous literature, the mortality index, κ_t , shows a decreasing trend in each case and can explain a large amount of variance in historical mortality rates. We plot the estimated κ_t in Fig. 1. The proportion of the variance explained by the Lee-Carter model (R^2) is given in Table 1. Model fit differs somewhat across countries. Japan's mortality rates fit best, with more than 96 % of the variance being explained for both genders. The United States follows, and the Netherlands comes in last (85 %). This latter finding might not be surprising, given that the Netherlands has the smallest population size.

Compared with the other five countries, the mortality index (κ_t) of Japan also shows a deeper decline for each gender. On the whole, the mortality index shows a decreasing trend in each case and explains a large amount of variance in historical mortality rates, which is in line with the previous literature.

Long-Run Relationship Between Mortality and GDP

In this section, we examine the time series properties of the latent factor in the Lee-Carter model and the GDP process, with a focus on the long-run relationship between the two series.

We start with testing the stationarity of the mortality index (κ_t) and the real GDP per capita in logarithm (g_t) in each country. First, we apply the Phillips-Perron test (Perron 1988) in the most general setting, with the inclusion of a constant and a linear time trend.³ The results of the tests are shown in panel A of Table 2. For female κ_t , we can reject only the null of nonstationarity for the United Kingdom and Japan at the 1 % significance level. For male κ_t , the null of nonstationarity is rejected for Japan at the 5 % significance level. For GDP, we can reject only the null of nonstationarity for the United States and the United Kingdom at the 10 and the 5 % significance level, respectively. For those stationary series, we find that they are trend stationary. However, in a finite sample, it is very difficult to distinguish between difference-stationary and trend-stationary behavior, and the assumption of difference-stationarity might be more prudent. In general, the results indicate that most of

² In this regard, the probability of survival beyond age 99 will be set equal to 0. However, in this article, we focus on life expectancy at early ages so that the survival probability beyond age 99 has little impact.

³ Hanewald (2011) performed similar unit root tests, with two main procedural differences. First, she used the Brouhns et al. (2002) Poisson variant of the Lee-Carter model, whereas we maintain the original settings. Second, she used real GDP in levels, whereas our analysis is based on real GDP per capita in logarithm. However, the main results are similar.

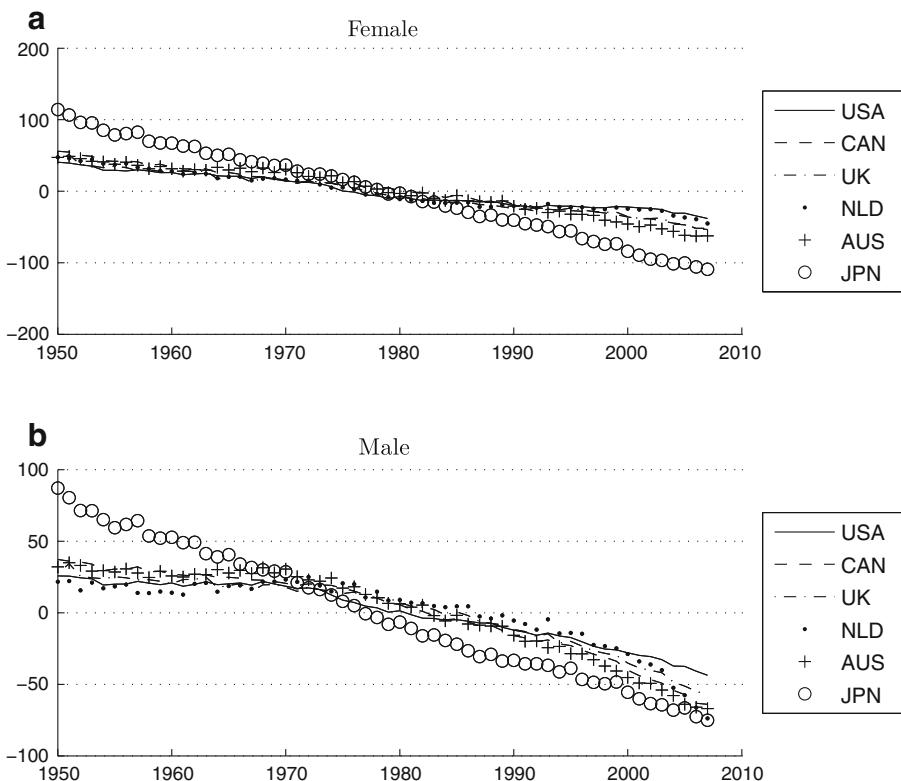


Fig. 1 Mortality index (K_t) of each country (1950–2007)

the series are not stationary. As a supplementary analysis, we perform the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test developed by Kwiatkowski et al. (1992) with the null hypothesis that the time series is trend-stationary, the results of which are shown in panel B of Table 2. The null of stationarity is rejected at the 1 % significance level without exception. Therefore, we proceed under the assumption of the presence of a unit root in the time series. We perform a similar analysis on the first differences of the series. Panel C of Table 2 gives the test statistics of the Phillips–Perron test. The null of nonstationarity is rejected at the 1 % significance level for each series. In brief, the preceding analysis indicates that the time series are integrated of order one (I(1)).

Table 1 Proportion of variance explained by Lee–Carter model (R^2)

	Female	Male
United States	.9561	.9379
Canada	.9258	.9256
United Kingdom	.9176	.9209
Netherlands	.8569	.8562
Australia	.9079	.9067
Japan	.9627	.9651

Table 2 Results of unit root tests: Test statistics

	United States	Canada	United Kingdom	Netherlands	Australia	Japan
Panel A: Levels of the Time Series (Phillips-Perron Test)						
$\kappa_{t, \text{female}}$	-7.06	-11.10	-28.67**	-7.98	-8.05	-32.74**
$\kappa_{t, \text{male}}$	-1.69	0.96	0.40	4.07	-1.23	-22.13*
g_t	-18.96 [†]	-4.84	-22.35*	-8.92	-9.18	-0.35
Panel B: Levels of the Time Series (KPSS Test)						
$\kappa_{t, \text{female}}$	0.40**	0.38**	0.43**	0.45**	0.53**	0.23**
$\kappa_{t, \text{male}}$	0.59**	0.68**	0.71**	0.64**	0.70**	0.39**
g_t	0.23**	0.54**	0.13 [†]	0.56**	0.31**	0.72**
Panel C: First Difference (Phillips-Perron Test)						
$\kappa_{t, \text{female}}$	-66.95**	-74.49**	-78.76**	-70.48**	-80.20**	-65.14**
$\kappa_{t, \text{male}}$	-66.68**	-69.98**	-76.40**	-74.50**	-78.04**	-63.10**
g_t	-50.28**	-42.00**	-35.55**	-43.18**	-40.42**	-30.64**

[†] $p < .10$; * $p < .05$; ** $p < .01$

The nonstationary series κ_t and g_t can be analyzed in first differences. However, as Hanewald (2011) also argued, such a transformation might miss the long-term properties of the data. A common strategy to study the long-run relationship among time series data is a cointegration analysis. Two or more nonstationary series are said to be cointegrated if they are integrated of the same order and a linear combination of them is stationary. The linear combination can be interpreted as a long-run equilibrium among the series.

Hanewald (2011) applied the Engle-Granger procedure (Engle and Granger 1987) to test the cointegration between the mortality index, κ_t , and real GDP level in six OECD countries and different age groups. Her findings were, however, mixed. Approximately one-quarter of her results indicated the existence of cointegration relationships, although the remaining results did not. As a complementary analysis, we directly study the long-run relationship between log central death rates and log real GDP per capita at each age by the Engle-Granger procedure. Formally, we are testing the stationarity of the estimated residuals, $\hat{\varepsilon}_{t,x}^g$, from the regression⁴

$$\ln(m_{x,t}) = \gamma_{0,x} + \gamma_{1,x} g_t + \varepsilon_{x,t}^g \quad (4)$$

for each age x , $x = 0, 1, \dots, 99$. As a comparison, we also study the long-run implications between the log central death rates and the mortality index κ_t . More specifically, we apply the Engle-Granger procedure to the model

$$\ln(m_{x,t}) = \beta_{0,x} + \beta_{1,x} \kappa_t + \varepsilon_{x,t}^\kappa \quad (5)$$

for each age x , $x = 0, 1, \dots, 99$.

Detailed results are presented in Online Resource 1. In general, the Engle-Granger test indicates mixed results. The mortality rates are found to be cointegrated with real GDP (g_t)

⁴ Based on results from the augmented Dickey-Fuller test, most of $\ln(m_{x,t})$ are I(1) processes.

or the mortality index (κ_t) only at certain ages. Besides, in terms of presenting cointegration relationships under the Engle-Granger procedure, neither κ_t nor g_t dominates the other.

In finite samples, the power of the cointegration analysis is often too small to discover a potential cointegration relationship. Additionally, the Engle-Granger procedure requires the specification of dependent and independent variables in the test, whereas a vector error correction model (VECM) does not have this concern. As a complement, we build a VECM for the mortality index and the macroeconomic indicator and perform the Johansen (1988) cointegration tests.⁵

The test results, shown in Table 3, support the existence of cointegration relationships between the mortality index and the macroeconomic indicator that are much stronger than the results presented by Hanewald (2011). The null of no cointegration ($r = 0$) is rejected in all cases at a significance level of 1 %, but the null of one cointegration vector ($r = 1$) cannot be rejected at a significance level of 5 % in all cases except for Dutch males. Above all, the analysis in this section indicates possible long-run relationships between the macroeconomic indicator, real GDP per capita, and the mortality rates—a finding that motivates us to compare their trends in more detail in the following sections.

Trend Comparison: Mortality and GDP

In this section, we begin with comparing the in-sample goodness of fit between the Lee-Carter model and a model with GDP per capita as the regressor. We then forecast mortality rates by extrapolating GDP per capita and compare the results with those from the Lee-Carter model. By doing this, we can compare the trend in mortality rates and the trend in economic growth from a new perspective.

First, we compare the performance of the two models on fitting historical data. The goodness of fit between the model based on the Lee-Carter mortality index and the model based on the real GDP per capita series are very similar. The goodness of fit is measured by R^2 from Eqs. (4) and (5) based on ordinary least squares (OLS). The results are available in Online Resource 1. In general, the mortality index and the macroeconomic indicator have comparable performance in fitting historical mortality data. We further study their implications on the trend and variance of future mortality rates. We fit historical mortality rates by the mortality index (κ_t) or real GDP per capital in logarithm (g_t), according to Eqs. (4) and (5) (see the previous section). The prediction of future mortality rates is based on the forecasts of κ_t or g_t , where we rely on ARIMA models. In most cases, the series is modeled most appropriately by a random walk model with drift. Following common practice, to correct for a jump-off bias in mortality fitting, we calculate the predicted changes of future mortality rates and base our projections on the actual rates in the final year of the estimated sample. This adjustment is performed for both the Lee-Carter model and the model based on GDP.

⁵ We multiply the test statistics by a factor $(T - pk) / T$, where p denotes the number of lags in the VAR model, and k denotes the number of variables. This is to correct small sample bias, as suggested in Ahn and Reinsel (1990).

Table 3 Johansen cointegration test statistic

	United States	Canada	United Kingdom	Netherlands	Australia	Japan
Females						
$r \leq 1$	5.12	6.54	8.94	4.53	3.58	5.52
$r = 0$	30.54	44.10	25.95	30.77	50.92	36.15
Males						
$r \leq 1$	4.50	5.91	5.40	14.30	6.78	4.44
$r = 0$	35.23	43.01	32.21	46.50	58.24	29.49

Notes: Critical values for the null of $r = 0$ are 13.73 for $p < .10$, 15.17 for $p < .05$, and 20.20 for $p < .01$. Critical values for the null of $r \leq 1$ are 7.52 for $p < .10$, 9.24 for $p < .05$, and 12.97 for $p < .01$.

Using historical data from 1950 to 2007, we forecast future mortality rates based on the procedures mentioned earlier. Given that life expectancy is an often-used summary of mortality, we project period life expectancies at birth 30 years ahead. The use of period life expectancies, instead of cohort life expectancies, is common in the literature and in practice.⁶ To account for the forecast uncertainty, we calculate 95 % confidence bounds for the ARIMA models for κ_t and g_t . To focus on the comparison between κ_t and g_t , we do not take into account uncertainties in other parts of the Lee-Carter model, such as $\varepsilon_{x,t}$ in Eq. (1).

Figure 2 shows the results for females. Results for males are shown in Online Resource 1. The forecasted mean values and confidence intervals by the Lee-Carter model and the model with GDP almost coincide in three countries (the United States, Canada, and the Netherlands). For the other countries, apart from the fact that the forecasts based on the Lee-Carter model have wider intervals, the mean predictions are still similar.⁷ The life expectancy at age 0 for females is expected to increase to approximately age 85 in most countries. The expected increase in life expectancy in Japan is more dramatic, with a figure about 5 years higher than the ones in other countries. The result implies that if the target is forecasting long-term mortality, we may replace the latent factor in the Lee-Carter model with an observable one and still achieve comparable results. Moreover, the forecast of GDP can be associated with future economic scenarios. For instance, the upper bound of the forecasted mortality rates in our model coincides with a lower economic growth prediction, and the lower bound is related to a higher economic growth prediction.

A Generalized Model

In this section, we study a stochastic mortality model that includes both latent and observable factors; in our case, GDP is the observable factor. This model can be

⁶ Furthermore, cohort life tables are based on the projections over a very long period. Thus, the forecast of cohort life tables is more sensitive to the underlying model and estimation methods.

⁷ When used to better fit the in-sample mortality rates at different ages, the Lee-Carter κ_t is more volatile than the GDP series, possibly yielding better out-of-sample point forecasts but at the expense of a larger prediction interval.

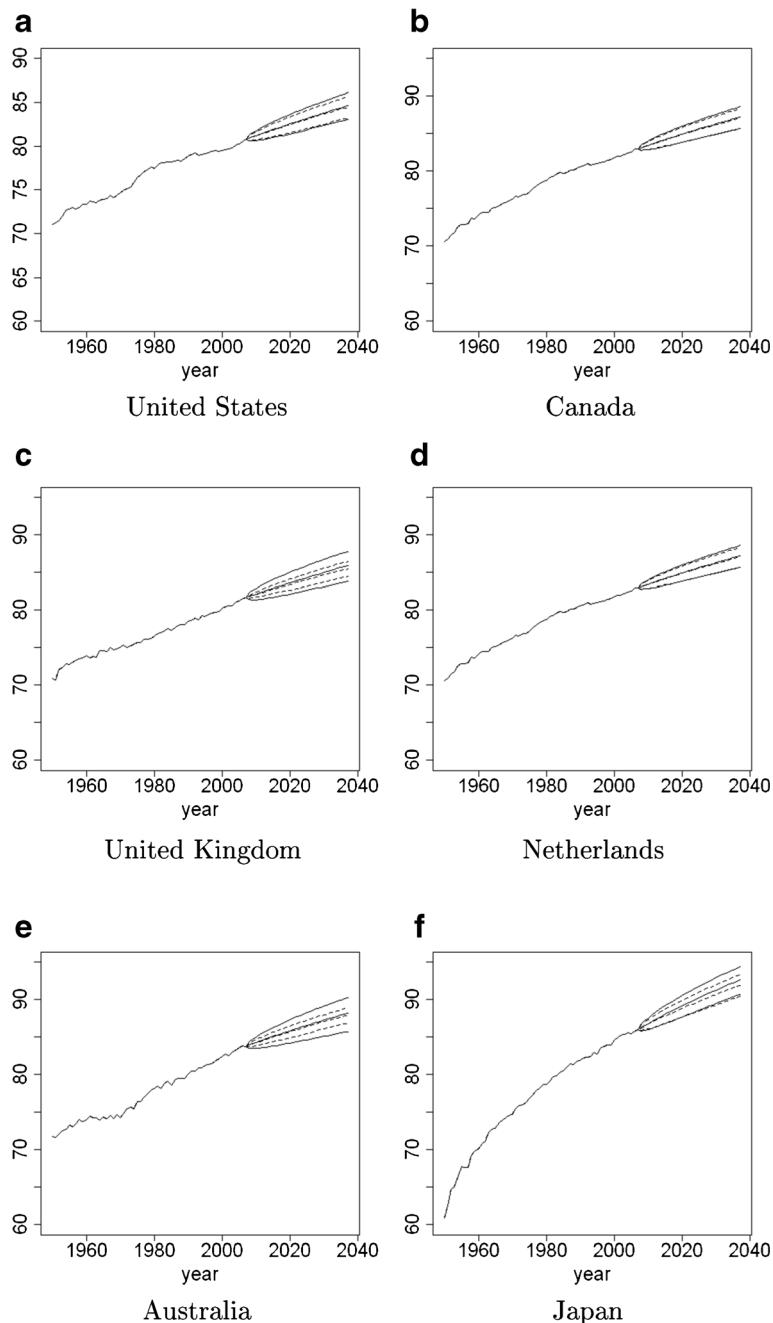


Fig. 2 Historical and forecasted life expectancy at birth (e_0), with 95 % intervals included (females). Solid lines represent the results from the Lee-Carter Model (1) and dashed lines from Model (4)

regarded as a generalized version of the original Lee-Carter method. We will begin with the model setup and estimation methods, then turn to the estimation results, and finally study the implied mortality projections.

Model Setup and Estimation Method

We postulate that the logarithm of the central mortality rate, $\ln(m_{x,t})$, has a linear form

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \gamma_x g_t + \varepsilon_{x,t}, \quad (6)$$

with time-invariant parameters α_x , β_x , and γ_x , and a time-varying latent variable κ_t . $\varepsilon_{x,t}$ is a error term that is uncorrelated with κ_t and g_t . α_x , β_x , and κ_t have similar interpretations as in the original Lee-Carter model. The newly included parameter, γ_x , measures the sensitivity of mortality rates at age x to the observable factor (here, real GDP per capita in logarithm). For a better interpretation of α_x , the observable factor g_t in the model is demeaned before estimation, implying that α_x will be the average log central death rate across time, just like in the original Lee-Carter model. The model that we propose includes as special cases two groups of models. If we let $\gamma_x = 0$, then we have the standard Lee-Carter model. Alternatively, if we assume that $\beta_x = 0$, we have Model (4) in the previous section.

Let $\mu_{x,t}$ denote the systematic part of $\ln(m_{x,t})$ and suppose that a set of parameters,

$$\theta = (\alpha_1, \dots, \alpha_X, \beta_1, \dots, \beta_X, \kappa_1, \dots, \kappa_T, \gamma_1, \dots, \gamma_X) \quad (7)$$

is given. The parameters are not identified without additional constraints because for any scalar c , any scalar e , and any scalar $d \neq 0$, it holds that

$$\begin{aligned} \mu_{x,t} &= \alpha_x + \beta_x \kappa_t + \gamma_x g_t \\ &= \alpha_x + \beta_x (\kappa_t - eg_t) + (\gamma_x + e\beta_x) g_t \\ &= (\alpha_x - \beta_x c) + \frac{\beta_x}{d} \{d(\kappa_t - eg_t + c)\} + (\gamma_x + e\beta_x) g_t \\ &= \tilde{\alpha}_x + \tilde{\beta}_x \tilde{\kappa}_t + \tilde{\gamma}_x g_t, \end{aligned} \quad (8)$$

where,

$$\tilde{\alpha}_x = \alpha_x - \beta_x c, \quad (9)$$

$$\tilde{\beta}_x = \beta_x / d, \quad (10)$$

$$\tilde{\kappa}_t = d(\kappa_t - eg_t + c), \quad (11)$$

and

$$\tilde{\gamma}_x = \gamma_x + e\beta_x. \quad (12)$$

We propose the following four normalization constraints:

$$\sum_t \kappa_t = 0, \quad (13)$$

$$\sum_x \beta_x = 1, \quad (14)$$

$$\text{cov}(\kappa_t, g_t) = 0, \quad (15)$$

and

$$\kappa = (\kappa_1, \dots, \kappa_T) \neq 0, \quad (16)$$

where the covariance in Eq. (15) will be calculated in sample.⁸ Following Nielsen and Nielsen (2010), the next theorem shows that our constraints identify the parameters uniquely. The proof is given in Online Resource 1.

Theorem 1: Let $\mu = (\mu_{x,t}, x = 1, \dots, X, t = 1, \dots, T)$, where $\mu = \mu(\theta)$ satisfies $\mu_{x,t} = \alpha_x + \beta_x \kappa_t + \gamma_x g_t$ for some θ as given by Eq. (7). Then the parametrization θ^o where $\sum_{x=1}^X \beta_x^o = 1$, $\sum_{t=1}^T \kappa_t^o = 0$, $\text{cov}(\kappa_t^o, g_t) = 0$ in sample, and $\kappa^o \neq 0$ satisfies the following:

- θ^o is a function of θ .
- μ is a function of θ through θ^o .
- The parametrization of μ by θ^o is exactly identified. That is, if $\theta^1 \neq \theta^2$ are two parameters satisfying the normalizing constraints, then $\mu(\theta^1) \neq \mu(\theta^2)$.

Model (6) is fitted to age-specific observed central mortality rates and time series of observable factors using the OLS approach. Specifically, the parameters are such that they minimize a quadratic loss function. A standard iteration optimization method is used to solve for the parameters. See, for example, Wilmoth (1993) for details.

Estimation Results

We use the same data set of the six OECD countries as in the previous section. The results for females are shown in Fig. 3. Results for males are presented in Online Resource 1. The patterns of average mortality (over time) across ages, measured by α_x , are quite similar in our sample countries. The values of β_x vary more between countries. For females, the values of β_x in the United Kingdom are relatively high at younger ages and low at old ages, which is contrary to the pattern in Japan. The parameters γ_x capture the sensitivity of mortality rates toward real GDP at different ages, which are relatively similar across

⁸ Compared with the Lee-Carter model, we add constraint (16): namely, $\kappa = (\kappa_1, \dots, \kappa_T) \neq 0$. Constraint (16) is needed to identify β_x . This constraint is also required in the Lee-Carter model to identify β_x . If Eq. (16) is not satisfied—that is, $\kappa = 0$ —then our model reduces to Model (4).

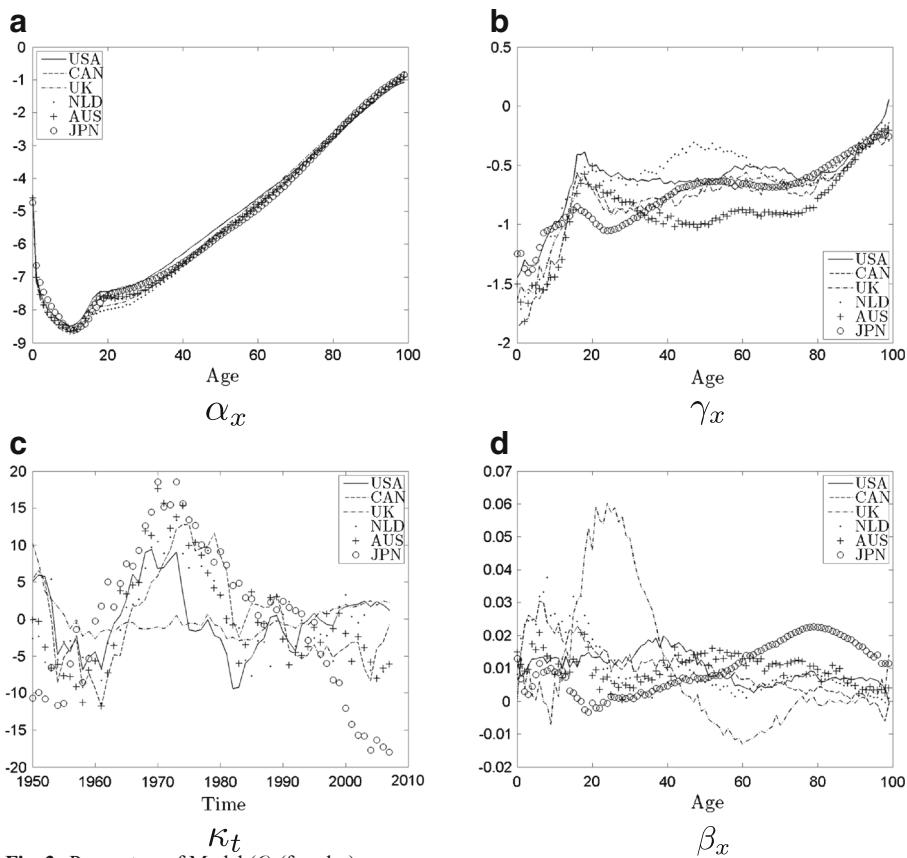


Fig. 3 Parameters of Model (6) (females)

countries. Mortality rates at very young ages are most closely related with economic growth. The γ_x s are negative in general because of the decreasing trend in mortality rates and the increasing trend in GDP. One striking difference between our method and the Lee-Carter model is the role of the latent factor. There is no longer a clear time trend in κ_t given that the time trend is now mostly captured by g_t . In addition, κ_t in our model, with average slope as a function of time being approximately -0.8 , is much flatter than the corresponding κ_t in the Lee-Carter model, whose slope is close to -4 . Moreover, κ_t s have large swings in our sample period, indicating that there might be nonlinear time effects captured by κ_t s. In general, β_x s are similar across ages and countries, except for females in the United Kingdom.

We use the Bayesian information criterion (BIC) to compare the goodness of fit between our model and the Lee-Carter model. The BIC is a popular criterion that takes into account the balance between parsimony and goodness of fit of a model. In our case, the BIC is defined as

$$BIC = N \log(\hat{\sigma}_\epsilon^2) + v \log N, \quad (17)$$

where $\hat{\sigma}_\epsilon^2$ is estimated variance of the error term, v is the difference between the number of parameters and the number of constraints,⁹ and N is the number of observations. Under the assumption that the errors are independent and identically distributed according to a normal distribution, $\log(\hat{\sigma}_\epsilon^2)$ is proportional to the maximum log-likelihood.

Values of BIC are shown in Table 4. A smaller (more negative) value indicates a better model fit. Our model outperforms the Lee-Carter model in every country and gender combination except for Australian females. The improvement of goodness of fit is mostly apparent in the data for Japanese females, as reflected in the large difference between the BIC values.

As mentioned earlier, the standard Lee-Carter model is a restricted version of our generalized model, when $\gamma_x = 0$ for all x . To examine whether the estimation based on our model provides more information than the standard Lee-Carter model, we test the null hypothesis that $\gamma_x = 0$ for each x based on bootstrapping the residuals of our model. To be specific, in each bootstrap simulation, we draw randomly with replacement from the residuals in Eq. (6) to construct a new sample and then reestimate the parameters. The results indicate that the estimated γ_x s are significantly different from 0 at all conventional significance levels.

To examine the relative contribution of different factors to the model fit, we present in Table 5 the proportion of the variance explained by the latent variable (κ_t) and GDP (g_t), respectively, as well as the total fraction of variance captured by Model (6). In all cases, GDP accounts for the major variance explained by Model (6), although κ_t explains only a marginal amount of the variance—less than 10 %—except for males in Canada and Japan. As a whole, the model that we propose can explain a large amount of the variance in the data.

Out-of-Sample Test

In this section, we compare the out-of-sample forecast accuracy between the Lee-Carter model and Eq. (6). We use 50 years of data on death rates (from 1950 to 1999) to fit the two models and then forecast death rates from 2000 to 2007 accordingly. The forecast of Model (6) is based on the projections of estimated g_t and the latent factors, whereas the forecast of the Lee-Carter model is based solely on the estimated latent factors. For latent factors in both models, we forecast them by standard AR models, with lags selected based on BIC. For the log real GDP per capita series, we use the random walk model with drift to forecast future values. We do not use vector autoregressive models because in our model, the latent factor is orthogonal to the GDP series by construction.

The forecast results are compared using the root mean squared forecast error (RMSFE). Specially, denote as y_{it} the true death rate at age i and year t , and denote as \hat{y}_{it} the corresponding forecasted death rates. The RMSFE is calculated according to the following formula:

$$RMSFE = \sum_t \sum_i (y_{it} - \hat{y}_{it})^2. \quad (18)$$

⁹ According to Eqs. (13), (14), (15), and (16), we have four constraints. However, constraint (16) is examined only *ex post* and is not included in the estimation, similar to the Lee-Carter model. Thus, the effective number of constraints is three.

Table 4 Fit of Model (6) and Lee-Carter model: Bayesian information criterion (BIC)

	Female		Male	
	Model (6)	Lee-Carter	Model (6)	Lee-Carter
United States	-32,056	-30,526	-31,555	-29,597
Canada	-25,017	-24,292	-26,400	-24,172
United Kingdom	-26,182	-25,297	-28,025	-25,357
Netherlands	-22,211	-21,816	-24,355	-19,068
Australia	-22,573	-22,743	-23,848	-22,833
Japan	-26,051	-16,931	-27,033	-23,553

Table 6 shows the out-of-sample test results for females. For each country, the specification with smaller RMSFE is indicated in bold. The model with GDP is better in four of the six cases. In particular, compared with the Lee-Carter model, our model reduces the forecasting errors by 17 % in Canada and by 64 % in Japan. Although the Lee-Carter model is better for the Netherlands and Australia, the two countries also have the smallest population size. Thus, for the two countries, there might be overfitting in our model because of the presence of additional κ_t to random fluctuations in the data.

Forecasting Using the Proposed Model

Forecasting future mortalities is a natural application of our model. In this section, we briefly illustrate the implications of this model forecast by both genders in each country.

Using historical data from 1950 to 2007, we forecast future mortality rates based on the time series models of g_t and κ_t , and estimated parameters for Model (6). We then project period life expectancies at birth 30 years ahead. We focus on two sources of uncertainty in forecasted mortality rates: the uncertainty in forecasted GDP and in forecasted κ_t . We construct the forecast intervals (2.5 and 97.5 % quantiles) by 1,000 simulations. To compare the uncertainty from different sources, we construct two types of intervals. The first intervals are based on the simulation of future g_t only, with the mean values of κ_t being projected. The second interval is based on the simulations of

Table 5 Proportion of variance explained by Model (6)

	Female			Male		
	κ_t	g_t	Total	κ_t	g_t	Total
United States	.0383	.9244	.9627	.0797	.8775	.9572
Canada	.0527	.8816	.9343	.1161	.8291	.9452
United Kingdom	.0314	.8969	.9283	.0622	.8810	.9431
Netherlands	.0557	.8119	.8676	.1580	.7415	.8996
Australia	.0441	.8689	.9131	.0920	.8317	.9237
Japan	.0415	.9410	.9825	.0475	.9292	.9767

Table 6 Out-of-sample test results: Root mean squared forecast error (RMSFE) for females: The Lee-Carter (LC) model, and the Lee-Carter model based on GDP (LC-GDP)

	United States	Canada	United Kingdom	Netherlands	Australia	Japan
LC	0.0070	0.0066	0.0045	0.0084	0.0043	0.0227
LC-GDP	0.0069	0.0055	0.0043	0.0088	0.0044	0.0081

Note: Values in bold indicate the specification with the smaller RMSFE for each country.

both g_t and κ_t . The results for females are shown in Fig. 4, and results for males are presented in Online Resource 1.

The life expectancies at birth for females are predicted to increase to approximately age 85 in 30 years in all countries except Japan, where the figure is as high as 90. The dash-dotted lines are the intervals based on the uncertainty in future GDP, which can be interpreted as different economic scenarios. The differences between life expectancies of lower and upper confidence intervals are within 5 years.

The uncertainty brought in by κ_t depends on its volatility. As shown in panel C of Fig. 3, the estimated female κ_t is least volatile in the United Kingdom, corresponding to the narrowest intervals in our forecasts, represented by dotted lines in Fig. 4; the estimated female κ_t is most volatile in Japan, corresponding to the widest intervals.

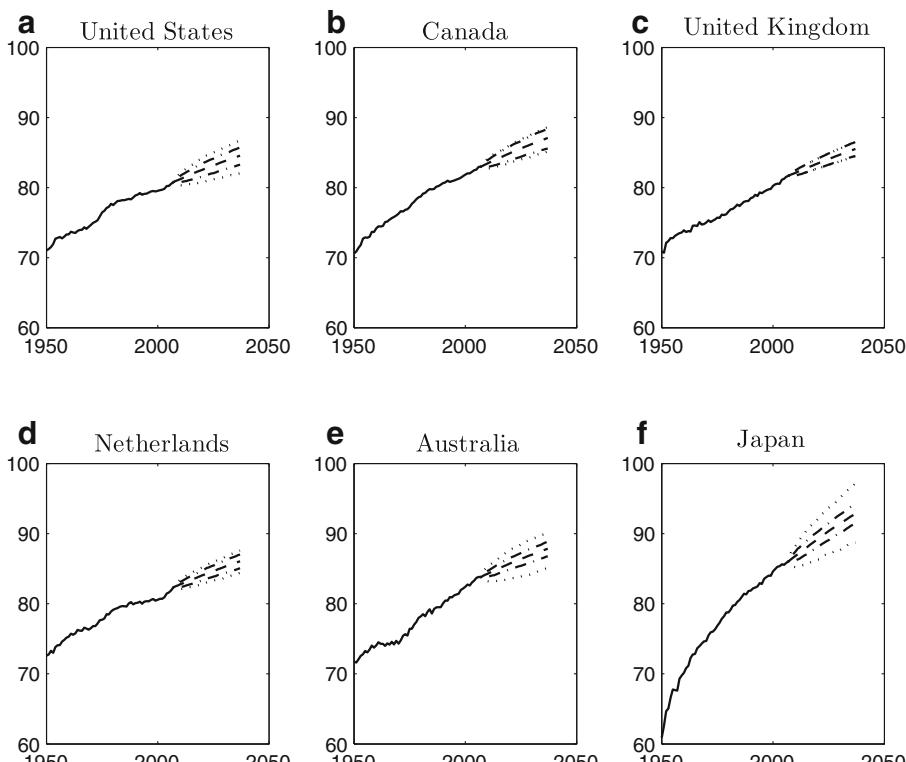


Fig. 4 Historical and forecasted life expectancy at birth (e_0), with 95 % intervals included (females). Solid lines represent the historical data, dashed lines the forecasted mean values from Model (6), dash-dotted lines the 95 % bounds including only the variations in g_t , and dotted lines the 95 % bounds including both g_t and κ_t

Conclusion

Based on data from 1950 to 2007 of six OECD countries—the United States, Canada, the United Kingdom, the Netherlands, Australia, and Japan—this article performs a comprehensive investigation of the relationship between the trends in mortality dynamics and economic growth. The former trend is represented by the latent mortality index κ_t in the Lee-Carter model, and the latter is represented by the real GDP per capita series g_t . We also present an extension of the Lee-Carter model that includes both latent and observable factors.

We compare the Lee-Carter model with a mortality model based solely on real GDP per capita in terms of both in-sample fitness and future projections. The results are quite similar in both aspects, indicating the similarity between the two trends of interest. Thus, the trend in economic growth might be an observable substitute to the latent variable. Nonetheless, this latent variable still explains a certain level of nonrandom variations in mortality rates, suggesting that economic growth might not be the only factor that is related to longevity. Based on the long-run relationship between economic growth and mortality decline, we augment the Lee-Carter model with an economic growth indicator and apply the model to mortality rates of six countries. Mortality forecasts in our model are based on projecting both the latent factor and the real GDP series. In this sense, our model integrates two major mortality forecast methods in the literature: namely, the extrapolation method and the explanation method. When both the economic growth indicator and the latent variable are included in our generalized mortality model, we have a better goodness of fit, and the role of latent factor is marginal, implying that the major trend in mortality rates is captured by real GDP data. Our model also can generate more interpretable scenarios about future longevity based on the forecast of future economic growth.

The role of economic growth on mortality dynamics deserves further investigation. As of this writing, a clear explanation for mortality decline is still lacking. However, the similarity between trends in mortality reduction factors and economic growth might shed light on possible directions toward explaining the trend. In addition, the method in our article can be extended to include other related factors, both latent variables and observable variables.

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