Task 6 - OLVHN

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Contents

1 Overview

This report contains the results and summary of the 12 step process described in Dr. Cizmas notes for computing the Operating Line. Also included in this report is the engine performance vartiatio with wheel speed, altitude and aircraft speed.

2 Methodology for Computing the Operating Line

As we determine the Operating Line of our engine, we first define our operating parameters:

$$\dot{m}_{air} = 1.64089 \left[\frac{kg}{s}\right] \tag{1}$$

$$w_{c_n} = 283.91025 \left[\frac{kJ}{kg}\right] \tag{2}$$

$$\eta_{compressor_n} = 0.90 \tag{3}$$

$$\sigma_{comb} = 0.90 \tag{4}$$

$$\eta_{turbine} = 0.94 \tag{5}$$

$$\pi_{c_n} = 9.2 \tag{6}$$

$$\lambda = 2.85377 \tag{7}$$

$$T^*_{1_n} = 288.16[K] \tag{8}$$

$$T^*_{3_n} = 1410[K] \tag{9}$$

$$p^*_{1_n} = 1.01325[bar] \tag{10}$$

$$p^*_{3_n} = 9.042243[bar] \tag{11}$$

$$h^*_{3_n} = 1574.3477 \left[\frac{kJ}{kg}\right] \tag{12}$$

$$N_n = 52612.464822[rpm] \tag{13}$$

$$\pi_D = 0.93 \tag{14}$$

$$\gamma_g = 1.304286 \tag{15}$$

$$\frac{A_{3.5}}{A_5} = 1.2 \tag{16}$$

$$h_1^* = 288.299988 \left[\frac{kJ}{kg}\right] \tag{17}$$

1. Calculate the compressor work w_c, given an angular speed calculated in Task2, as a function of nominal compressor work and nominal angular speed.

$$w_c = w_{c_n} \left(\frac{N}{N_n}\right)^x \tag{18}$$

Usually $x \in [1.9, 2.1]$, for convience we will start with $x = 2.1$.
Note:

$$N = 1.1 N_r \tag{19}$$

Recall when at nominal conditions:

$$N_r = \frac{N_n}{1.05} \tag{20}$$

N	w_c
55117.82029 rpm	313.0460185$\frac{kJ}{kg}$

2. Estimate the compressor efficiency η_c, given an angular speed calculated in Task2, as a function of nominal compressor efficiency and nominal angular speed. We start by calculated the pressure ratio π_c^*

$$\pi_c^* = \left[\left(\pi_{c_n}^{\frac{\gamma-1}{\gamma}}\right) \frac{\eta_c}{\eta_{c_n}} \left(\frac{N}{N_n}\right)^x + 1\right]^{\frac{\gamma}{\gamma-1}} \tag{21}$$

To begin the caluation we can start by assuming $\eta_c = \eta_{c_n}$. Once the pressure ratio is calculated, read that π_c^* from the compressor map, and find the corresponding $\dot{m}\frac{\sqrt{T_1^*}}{p_1^*}$. Once you have that, find the corresponding η_c from the compressor map. If $\eta_c \neq \eta_{c_n}$ then iterate until the change is less than a allowed tolerance. For our engine, we allowed a tolerance of 0.01.

π_c^*	η_c
10.4001622	0.878213

3. Calculate the T_3^* from:

$$\pi_c^* = \frac{1+f}{\sigma_{comb}} \left(\frac{p_3^*}{\dot{m}\sqrt{T_3^*}}\right)_n \sqrt{\frac{T_3^*}{T_1^*}} \frac{\dot{m}\sqrt{T_1^*}}{p_1^*} \tag{22}$$

Where:

$$\frac{1+f}{\sigma_{comb}} \left(\frac{p_3^*}{\dot{m}\sqrt{T_3^*}}\right)_n = constant \tag{23}$$

From here, calculate h_3^* from:

$$h_3^* = \left(\frac{1+minL}{1+\lambda minL}\right) h_{\lambda=1} + \left(\frac{(\lambda-1)minL}{1+\lambda minL}\right) h_{air} \tag{24}$$

Where:

- h_λ - enthalpy of the combustion products for λ excess air
- $h_{\lambda=1}$ - enthalpy of the combustion products for stoichiometric combustion
- h_{air} - enthalpy of the air

Check to see if the ratio $\frac{w_c}{h_3^*}$ is equal to the nominal ratio $\frac{w_{cn}}{h_{3_n}^*}$. If not, iterate x until it is, within a reasonable tolerance of about 1%.

After iterating x, we arrived to $x = 1.4$.

T_3^*	h_3^*
1552.103717 [K]	1753.74986 $\frac{kJ}{kg}$

4. We are now to find the critical conditions by:

$$\pi_{c_{cr}}^* = \frac{1}{\sigma_{comb}\pi_D} \left[\frac{\frac{\gamma_g+1}{2}}{1-\frac{w_c}{h_3^*}\frac{1}{\eta_{turbine}}}\right]^{\frac{\gamma_g}{\gamma_g-1}} \tag{25}$$

$$N_{cr} = N_n \sqrt{\frac{\eta_{c_n}}{\eta_{c_{cr}}} \frac{\pi_{c_{cr}}^{\frac{\gamma-1}{\gamma}}-1}{\pi_{c_n}^{\frac{\gamma-1}{\gamma}}-1}} \tag{26}$$

We are to then repeat steps (1)-(3) for three values of angular speed larger than the critical angular speed.

$\pi_{c_{cr}}^*$	N_{cr}	$\eta_{c_{cr}}$
5.462985	44711.24058 rpm	0.879

$\frac{w_c}{h_3^*}$	$\left(\frac{w_c}{h_3^*}\right)_n$	% diff
0.1785009519	0.180335162	1.0171116 %

Now we repeat steps (1)-(3) for three values of angular speed larger than the critical angular speed.

N	w_c	$\eta_{iteration}$	π_c^*	$\bar{m}$
46946.803	223.4949	0.881	6.17351	0.71
49182.365	246.4306	0.879	7.09622	0.785
51417.93	270.5425	0.9	8.5075	0.9

N	T_3^*	h_3^*	$\frac{w_c}{h_3^*}$	% diff
46946.803	1150.965	1251.825	0.17854	0.998
49182.365	1244.026485	1368.077	0.18013	0.1142
51417.93	1360.3106	1512.1349	0.1789	0.7879

5. Once we have reached the critical value, our ratio $\frac{w_c}{h_3^*}$ is no longer constant. If the flow isn't critical, then the variation of $\frac{w_c}{h_3^*}$ is given by:

$$\frac{w_c}{h_3^*} = \eta_{turbine}\left[1 - \left(\frac{p_H}{p_3^*}\right)^{\frac{\gamma_g - 1}{\gamma_g}} - K\left(\frac{A_{3.5}}{A_5}\right)^2\left(\frac{p_3^*}{p_H}\right)^{\frac{2}{\gamma_g}}\right] \tag{27}$$

Where:

$$K = \left(\frac{A_5}{A_{3.5}}\right)^2\left(\frac{p_H}{p_3^*}\right)^{\frac{2}{\gamma_g}}\left[1 - \left(\frac{p_H}{p_3^*}\right)^{\frac{\gamma_g - 1}{\gamma_g}} \frac{w_c}{h_3^*}\frac{1}{\eta_{turbine}}\right] \tag{28}$$

$$\frac{p_H}{p_3^*} = \frac{1}{\sigma^*_{combustion}\pi_c^*\pi_D} \tag{29}$$

Recall that π_D is given by:

$$\pi_D = \frac{p_1^*}{p_H} \tag{30}$$

When performing this step be sure to choose a π_c^* that is less than the critical value $\pi^*_{c_{cr}}$.

K	$\frac{p_H}{p_3^*}$	$\frac{w_c}{h_3^*}$
0.007177101	0.23020826	0.180335162

6. Now we are to choose an N value smaller than the critical value N_{cr}, and calculate w_c using equation (16) We chose a N value that was 1% less than the critical value.

$$N < N_{cr} \tag{31}$$

Now we use equation (16) to calculate w_c.

N	w_c
44500 rpm	199.73351054474077 $\frac{kJ}{kg}$

7. Now calculate h_3^* from equation (26) and $w_c = 171.430154\frac{kJ}{kg}$.

$$h_3^* = w_{c_n}\left(\frac{w_c}{h_3^*}\right)^{-1} \tag{32}$$

h_3^*
1107.56831 $\frac{kJ}{kg}$

8. Calculate T_3^* using the stoichiometric and air gas tables.

T_3^*
1028.878066 K

9. For known value of π_c^* and N read from the the compressor map the value of the corrected mass flow rate

$$\pi_c{}^* = 5.18983611172067 \tag{33}$$

Yields a corrected mass flow rate of:

$m_a \frac{\sqrt{T_1^*}}{p_1^*}$
$(0.63)(1.6087157)\frac{\sqrt{288}}{101325}$

10. Now determine T_3^* using equation (21).
11. Compare the values of T_3^* from step 8 to T_3^* from step 10, if they differ by more than 10 degrees K, then iterate until with different N value.

T_3^*
1033.097120243129 K

12. Now choose another value of π_c^* and go back to step 5 and repeat the process to get other points on the operating line at $\frac{p_H}{p_3^*}$

π_c^*	$\frac{p_H}{p_3^*}$	K	$\frac{w_c}{h_3^*}$	N
6.009283919	0.198816223	0.007124393	0.180335162	44500
7.101880995	0.168229112	0.006698233	0.180335162	40000
8.194478071	0.145798563	0.006163854	0.180335162	50000

w_c	h_3^*	T_3^*	T_3^*
199.73351054474077	1107.56831	1028.878066	1033.097120243129
159.6691061	885.4019626	839.0287095	833.1424783
255.1126073	1414.658156	1281.631036	1276.178685

3 Jet Engine Performance Variation with Wheel Speed

To determine the performance variation with wheel speed (N) we are to use the **Operating Line** from Section 2. We will choose wheel speeds that are different from the nominal wheel speed N_n, and then read from the Compressor Map the pressure ratios π_c^* and the efficiencies η_c.

N	π_c^*	η_c
40000 rpm	7.101880995	0.849
44500 rpm	6.009283919	0.877
49000 rpm	7.9900312	0.8325

Now similar to how we solved for compressor work, we make use of equation (17):

$$w_c = w_{c_n}\left(\frac{N}{N_n}\right)^x$$

Here we will use $x = 2$ following Dr. Cizmas' Example.

w_c
203.1064891
164.1057354
246.2611692

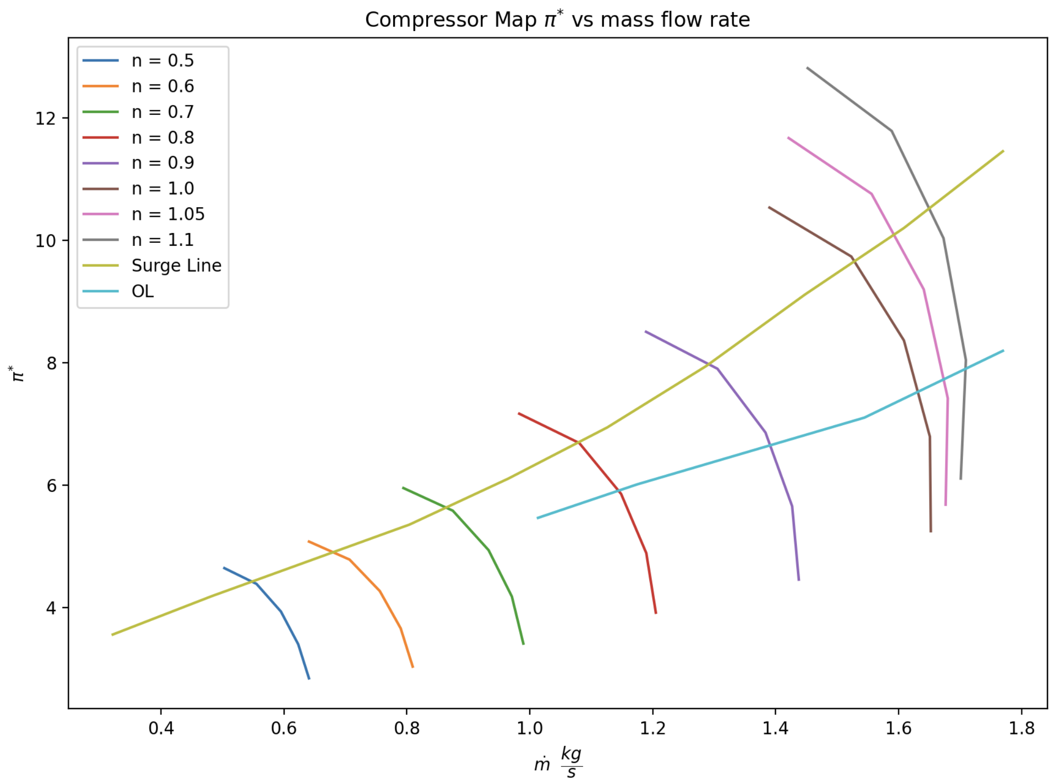

Figure 1: Compressor Map with Surge Line and Operating Line (OL)

The enthalpy at stage 02 is calculated:

$$h_2^* = h_1^* + w_c \tag{34}$$

h_2^*
491.4064771
452.4057234
534.5611572

The degree of dynamic compression in the inlet nozzle is recalculated as:

$$\pi_D = \sigma_{inlet}^* \left(1 + \frac{\gamma - 1}{2} M_H^2\right)^{\frac{\gamma}{\gamma-1}} \tag{35}$$

Recall $\sigma_{inlet}^* = 0.93$. Also here we are assuming $M_H = 0$, since we are at Take-off conditions.

π_D
0.93
0.93
0.93

The specific thrust is now recalculated as:

$$F_{sp} = \phi \sqrt{2h_3^* \left[1 - \frac{1}{\pi_D \pi_c^* \sigma_{comb}^*}\right]^{\frac{\gamma_g - 1}{\gamma_g}} - h_1^* \left(\frac{\pi_c^{\frac{\gamma-1}{\gamma}}}{\eta_c}\right)} \left(1 + \frac{1}{\lambda minL}\right) \tag{36}$$

F_{sp}	$\frac{p_H}{p_3^*}$	K	$\frac{w_c}{h_3^*}$	h_3^*
731.102889	0.198816223	0.007124393	0.180335162	1126.272254
656.888117	0.168229112	0.006698233	0.180335162	910.0040937
875.895256	0.149529804	0.006265198	0.180335162	1365.574893

The mass flow rate is recalculated as:

$$\dot{m}_{air} = \dot{m}_{air_n} \frac{\pi c^*}{\pi_{c_n}^*} \frac{N_n}{N} \tag{37}$$

$\dot{m}_{air}$
1.26719369
1.666071704
1.530139367

And the TSFC is recalculated as:

$$TSFC = \frac{3600}{F_{sp}} \frac{1}{\lambda minL} \tag{38}$$

TSFC
0.117699
0.130996
0.098242

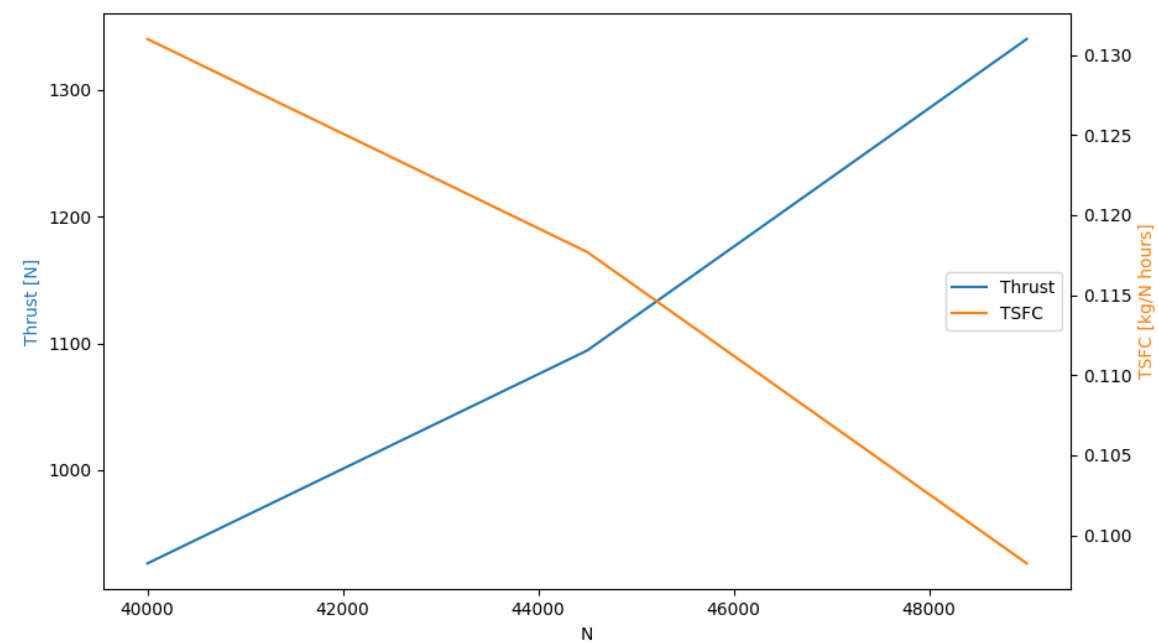

Figure 2: Engine performance variation with wheel speed

Above is the the plot of the engine performance variation with wheel speed N [rpm]

4 Jet Engine Performance Variation with Altitude and Speed

Up to this point, our engine has been performance has been calculate at take-off conditions. It is very important for us to determine how the performance varies with altitude and speed.

1. First we are to calculate the Mach number at H and V.

$$M_H = \frac{V}{\sqrt{\gamma R T_H}} \tag{39}$$

It is important for us to specify our altitude dependent properties and speed dependent properties.

H [m]	T_H [K]	p_H [Pa]	a_H [m/s]
0	288.15	101325	340.29
1000	281.65	89874.6	336.434
3000	268.65	70108.5	328.578
5000	255.65	54019.9	320.529
6000	249.15	47181	316.428
7000	242.65	41060.7	312.274
8000	236.15	35599.8	308.063
9000	229.65	30742.5	303.793
10000	223.15	26436	299.463

And our velocity values:

v [m/s]
0
100
200
300
350
400

2. Now one can calculate the degree of dynamic compression π_D.

$$\pi_D = \frac{p_1^*}{p_H} = \sigma_{inlet}^* \frac{p_H^*}{p_H} \tag{40}$$

This means that we can say:

$$\pi_D = \sigma_{inlet}^* \left(1 + \frac{\gamma - 1}{2} M_H^2\right)^{\frac{\gamma}{\gamma-1}} \tag{41}$$

Where σ_{inlet}^* is the inlet stagnation pressure loss in the inlet defined as the ratio between the stagnation pressure at the inlet in the compressor p_1^* and the stagnation pressure at the inlet of the engine p_H.

H **[km]**	0	1	3	5	6	7	8	9	1	v **[m/s]**
π_D	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0
π_D	0.9889	0.9887	0.9917	0.9949	0.9966	0.9985	1.0004	1.0024	1.0046	100
π_D	1.1809	1.1810	1.1943	1.2090	1.2170	1.2254	1.2344	1.2439	1.2541	200
π_D	1.5566	1.5586	1.5952	1.6361	1.6585	1.6823	1.7077	1.7348	1.7638	300
π_D	1.8411	1.8460	1.9023	1.9656	2.0004	2.0375	2.0772	2.1197	2.1654	350
π_D	2.2122	2.2226	2.3066	2.4019	2.4544	2.5105	2.5708	2.6356	2.7054	400

3. Now we calculate h_H^*.

$$h_H^* = h_H \left(1 + \frac{\gamma - 1}{2} M_H^2\right) \tag{42}$$

4. Now calculate the compressor pressure ratio π_c^* with variation of H and V.

$$\pi_c^* = \left[1 + \frac{h_0}{h_H^*}\left(\pi_{c_n}^{*\frac{\gamma-1}{\gamma}} - 1\right)\right]^{\frac{\gamma}{\gamma-1}} \tag{43}$$

Students can also calculate w_c:

$$w_c = h_2^* - h_H^* \tag{44}$$

5. Now oen can begin calculating the Specific Thrust F_{sp}

$$F_{sp} = \phi_5 \sqrt{2h_3^*\left[1 - \frac{1}{(\pi_D \pi_c^* \pi_{combust}^*)}\right] - h_1^* \frac{\pi_c^{\frac{\gamma-1}{\gamma}}}{\eta_c}} \left(1 + \frac{1}{\lambda minL}\right) - v \tag{45}$$

Recall, γ varies due to the variance in speed of sound a from the change in altitude.

$$\gamma = \frac{a_H^2}{R_{air} T_H} \tag{46}$$

We can calculate h_3^* similar to previous sections:

$$h_3^* = \left(\frac{w_c}{h_3^*}\right) w_c \tag{47}$$

Where w_c comes from equation (44) and $\frac{w_c}{h_3^*}$ comes from equations (27), (28), and (29).

$$\frac{w_c}{h_3^*} = \eta_{turbine}\left[1 - \left(\frac{p_H}{p_3^*}\right)^{\frac{\gamma_g - 1}{\gamma_g}} - K\left(\frac{A_{3.5}}{A_5}\right)^2 \left(\frac{p_3^*}{p_H}\right)^{\frac{2}{\gamma_g}}\right]$$

$$K = \left(\frac{A_5}{A_{3.5}}\right)^2 \left(\frac{p_H}{p_3^*}\right)^{\frac{2}{\gamma_g}} \left[1 - \left(\frac{p_H}{p_3^*}\right)^{\frac{\gamma_g - 1}{\gamma_g}} \frac{w_c}{h_3^*} \frac{1}{\eta_{turbine}}\right]$$

$$\frac{p_H}{p_3^*} = \frac{1}{\sigma_{combustion}^* \pi_c^* \pi_D}$$

Now all we have to solve for is the varying λ. We begin by making use of the energy conservation equation between the inlet and burner.

$$\dot{m}_{air} h_2^* + \dot{m}_{fuel}\left(h_{fuel} + \pi_{combust}^* LHV\right) = \dot{m}_{air} h_3^* \tag{48}$$

Recall $LHV = 43.5 \times 10^6 J/kg$ for standard fuel. We can make use of:

$$f = \frac{\dot{m}_{fuel}}{\dot{m}_{air}} = \frac{1}{L} = \frac{1}{\lambda minL} \tag{49}$$

Resulting in the following equation:

$$h_2^* + \frac{\pi_{combust}^* LHV}{\lambda minL} = \left[1 + \frac{1}{\lambda minL}\right] h_3^* \tag{50}$$

We have all the necessary variables to solve for λ, therfore equation (45) can be solved.

H **[km]**	0	1	3	5	6	7	8	9	1	v **[m/s]**
F_{sp}	1040	1002.6	1032	1060.84	1075.03	1089.1	1103	1116.8	1130.5	0
F_{sp}	938.35	900.6	930.9	960.5	975.2	989.6	1004.01	1018.3	1032.5	100
F_{sp}	830.5	792.0	824.4	856.4	872.2	888.0	903.7	919.3	934.8	200
F_{sp}	709.5	669.3	705.2	740.7	758.3	775.9	793.4	810.9	828.3	300
F_{sp}	641.0	599.4	637.4	675.0	693.7	712.3	730.8	749.4	767.9	350
F_{sp}	565.0	521.5	562.0	601.9	621.7	641.5	661.2	680.9	700.6	400

One can now calculate $TSFC$:

$$TSFC = (TSFC)_n \frac{F_{sp_n}}{F_{sp}} \frac{\lambda_n}{\lambda} \tag{51}$$

H **[km]**	0	1	3	5	6	7	8	9	1	v **[m/s]**
$TSFC$	0.1037	0.0985	0.1026	0.1066	0.1085	0.1104	0.1123	0.1142	0.1160	0
$TSFC$	0.1120	0.1066	0.1108	0.1149	0.1168	0.1187	0.1206	0.1225	0.1243	100
$TSFC$	0.1167	0.1109	0.1152	0.1192	0.1211	0.1230	0.1249	0.1267	0.1284	200
$TSFC$	0.1174	0.1110	0.1154	0.1194	0.1213	0.1232	0.1250	0.1267	0.1283	300
$TSFC$	0.1163	0.1094	0.1139	0.1180	0.1199	0.1218	0.1235	0.1252	0.1268	350
$TSFC$	0.1140	0.1066	0.1112	0.1154	0.1173	0.1192	0.1209	0.1225	0.1240	400

One can calculate mass flow rate by:

$$\dot{m}_a = \dot{m}_{a_n} \frac{\pi_c^*}{\pi_{c_n}^*} \frac{p_H}{p_0} \left(1 + \frac{\gamma - 1}{2} M_H^2\right)^{\frac{\gamma}{\gamma-1}} \frac{\sigma_{inlet}^*}{\sigma_{inlet_n}^*} \tag{52}$$

6. Finally calculate Thrust:

$$F = F_{sp} \dot{m}_a \tag{53}$$

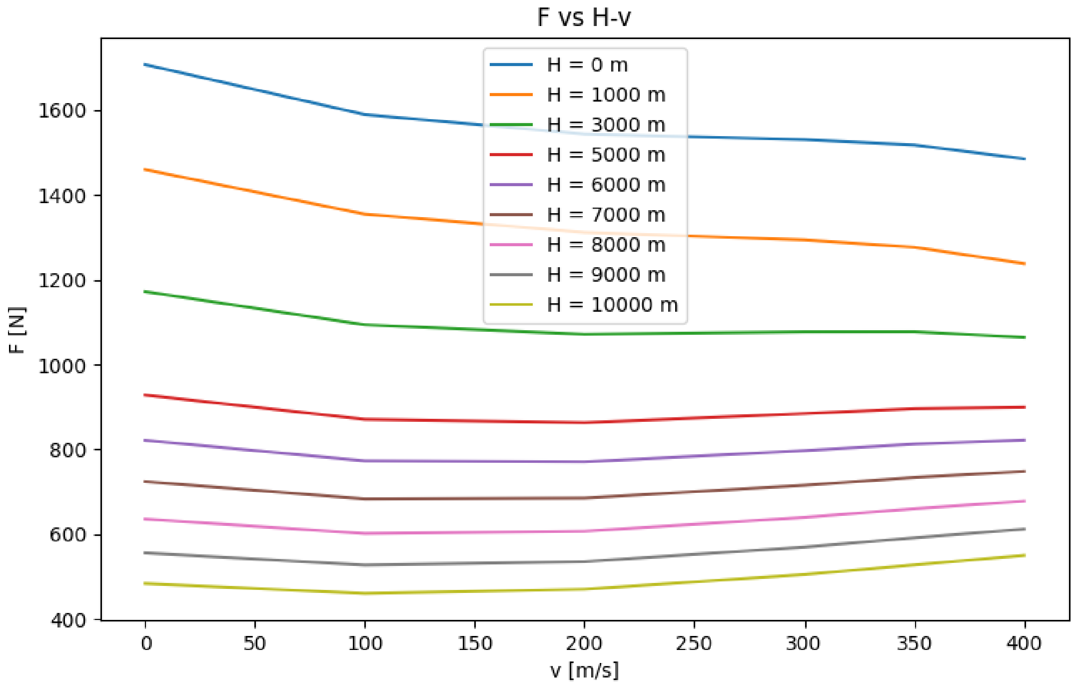

Figure 3: Thrust Performance with variation in speed and altitude

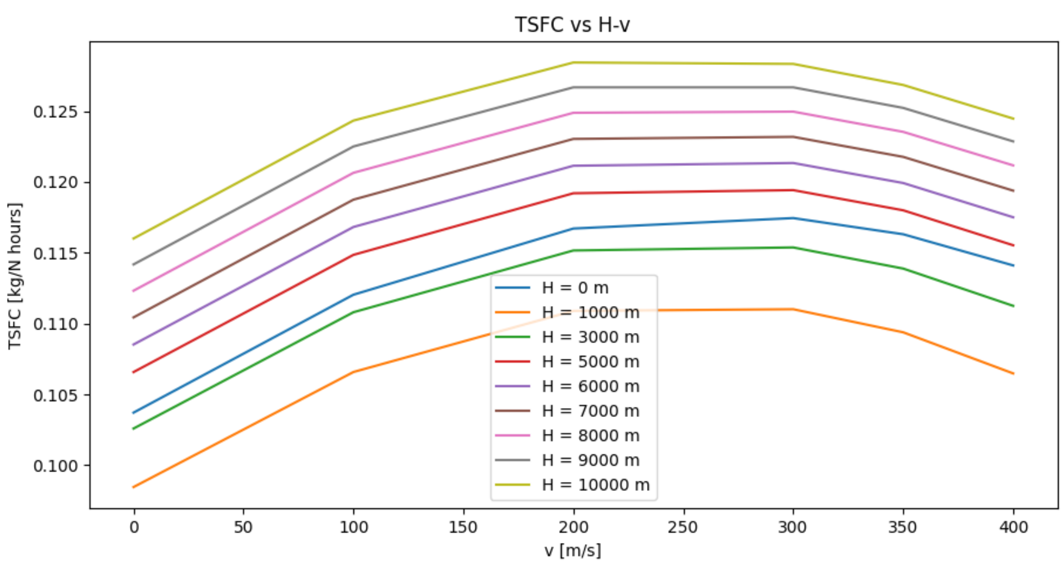

Figure 4: TSFC with variation in speed and altitude