

# Performance Analysis of the PT6A-114A Engine

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# 1 Preface

Dear Dr.Cizmas,

I want to thank you for the hard work you have demanded of me in order to be a successful student in both of the courses that I took under your instructioning. Our office hours meeting, following the Fall 2021 AERO 351 Exam 2 test grades release, still resonates in my mind. Up to that point, I had preformed poorly in the course, and it seemed as though I wasnt going to be able to pass the course. As you may recall, it was my third attempt at the course, and I was in danger of violating the "3-peat" rule. After that meeting, with your advice I studied harder than I had ever studied before, and I was able to perform well on the final exam and pass the course. But the more importantly that sucess continued and I went on to make the Dean's List the following semester (Spring 2022), a feat that I had never accomplished before. I took that same initiative into my career aspirations securing two internships, one in the Summer 2022 at Arkisys, and another in Fall 2022 (current) with Loopback Analytics. Upon graduation, I will be joining Loopback Analytics as a Machine Learning Engineer. While I may be leaving aerospace engineering for software engineering, I want to credit you for work ethic and drive you have instilled in me.

Thank you for everything,  
Carlos Meisel

## 2 Engine Introduction

As per the Pratt and Whitney website, the PT6A-114 is a turboprop engine that is used in a variety of applications. The PT6A class is the most popular engine in the world for its class and one of Pratt and Whitney's greatest success stories. The PT6A is used in a variety of applications, specifically for this project we will be looking at the PT6A-114A engine; which is primarily used by the Cessna 208/208B Caravan I.

## 3 Full Nominal Engine Cycle - Task 1

As stated earlier, the PT6A-114 engine is a Turboprop engine. A turboprop engine creates power from a shaft. Thrust is then produced from a combination of the propeller as well as the exhaust gas. The thrust produced from the propeller is far above the thrust provided by the exhaust gas. Given shaft horsepower, the efficiency of the compressor, and the inlet temperature of the turbine we find the specific thrust of the engine.

Below is a list of given engine parameters:

$$\begin{aligned} \text{Shaft Horsepower, } SHP &= 600 \text{ hp} \\ \text{Compressor Efficiency, } \eta_{comp} &= 0.90 \\ \text{Turbine Inlet Temperature, } TIT &= 1410 \text{ K} \\ \text{Compressor Ratio, } \pi_c &= 9.2 \\ \text{Turbine Efficiency, } \eta_{turb} &= 0.94 \\ \text{Combustor Efficiency, } \eta_{comb} &= 0.90 \\ \text{Free Turbine Efficiency, } \eta_{PT} &= 0.94 \\ \text{Nozzle Efficiency, } \phi_{nozzle} &= 0.98 \\ \text{Propeller Efficiency, } \eta_{prop} &= 0.8 \\ \text{Diffuser Efficiency, } \eta_d &= 0.99 \end{aligned} \tag{1}$$

### 3.1 Stage a

We begin at static conditions, specifically here we are calculating the static conditions at take-off. Since we are at take-off conditions, one can assume sea level atmospheric conditions.

$$\begin{aligned} \text{Static Pressure, } p_a &= 101.325 \text{ kPa} \\ \text{Static Temperature, } T_a &= 288.16 \text{ K} \end{aligned} \tag{2}$$

By knowing the Temperature  $T_a$  we are able to calculate enthalpy from the air gas tables:

$$h_a(T_a)_{Tables} = 288.299988 \frac{kJ}{kg} \tag{3}$$

And for entropy:

$$s_{a_{p=1bar}}(T_a)_{Tables} = 6.6570201 \frac{kJ}{kgK} \tag{4}$$

After reading from the tables for entropy, we will need to normalize to the static pressure of  $p_a$ .

$$s_a = s_{a_{p=1bar}}(T_a) - R \ln \left( \frac{p_a}{1 \text{ bar}} \right) \tag{5}$$

### 3.2 Stage 0a

In this stage we going to assume that stage 0a is equivalent to stage a.

$$\begin{aligned} p_{0a} &= 101.325 \text{ kPa} \\ T_{0a} &= 288.16 \text{ K} \\ h_{0a} &= 288.299988 \frac{kJ}{kg} \\ s_{0a} &= 6.6570201 \frac{kJ}{kgK} \end{aligned} \tag{6}$$

### 3.3 Stage 01

Here in this stage, we take into account the Diffuser.

$$p_{01} = p_{0a} \eta_d \tag{7}$$

$$T_{01} = T_{0a} \tag{8}$$

$$h_{01}(T_{01})_{Tables} = 288.299988 \frac{kJ}{kg} \tag{9}$$

$$s_{01_{p=1bar}}(T_{01})_{Tables} = 6.6570201 \frac{kJ}{kgK} \tag{10}$$

$$s_{01} = s_{01_{p=1bar}}(T_{01}) - R \ln \left( \frac{p_{01}}{1 \text{ bar}} \right) \tag{11}$$

### 3.4 Stage 02i

Here we are calculating the isentropic conditions of the compressor, stage 02i. At this point, we say that:

$$s_{02i} = s_{01} \tag{12}$$

Calculating the pressure, we use the pressure ratio of the compressor:

$$p_{02i} = p_{01} \pi_c \tag{13}$$

Recall, we need to normalize the entropy to 1 bar pressure to be used for the tables:

$$s_{02i_{p=1bar}} = s_{02i} + R \ln \left( \frac{p_{02i}}{1 \text{ bar}} \right) \quad (14)$$

We can use the entropy found, to use the H method, to find enthalpy:

$$h_{02i}(s_{02i_{p=1bar}})_{Tables} = 543.819 \frac{kJ}{kg} \quad (15)$$

We can now calculate the ideal work of the compressor:

$$w_{02i} = h_{02i} - h_{01} \quad (16)$$

### 3.5 Stage 02

The pressure at Stage 02 is the same as the pressure at Stage 02i. This allows for us to move forward through the engine cycle.

$$p_{02} = p_{02i} \quad (17)$$

$p_{02}$
9.228681 bar

Using the ideal work of the compressor, we can calculate the actual work of the compressor:

$$w_{02} = \frac{w_{02i}}{\eta_{comp}} \quad (18)$$

$w_{02}$
283.91025 kJ/kg

This will allow for us to easily calculate the enthalpy.

$$h_{02} = h_{01} + w_{02} \quad (19)$$

This enthalpy can now be used for the S method, to find entropy:

$$s_{02_{p=1bar}}(h_{02})_{Tables} = 6.7113694 \frac{kJ}{kgK} \quad (20)$$

Finally we normalize the entropy:

$$s_{02} = s_{02_{p=1bar}} - R \ln \left( \frac{p_{02}}{1 \text{ bar}} \right) \quad (21)$$

### 3.6 Stage 03

We were given a pressure drop in the combustor of 3%. Resulting in the following pressure:

$$p_{03} = p_{02} \left( 1 - \frac{3}{100} \right) \quad (22)$$

$p_{03}$
8.95182057 bar

We are also given a  $T_{03}$  of 1410 K.

$$h_{03_{air}}(T_{03})_{AirTables} = h_{03_{air}} \quad (23)$$

$$h_{03_{\lambda=1}}(T_{03})_{StoichTables} = h_{03_{\lambda=1}} \quad (24)$$

We then use the two above equations to calculate excess air:

$$\lambda = \frac{h_{03\lambda=1}(1 + \min L) - \eta_{comb}LHV - h_{03_{air}}\min L}{\min L(h_{02} - h_{03_{air}})} \quad (25)$$

Recall:

$$\begin{aligned} LHV &= 43.5 \frac{kJ}{kg} \\ \min L &= 14.66 \end{aligned} \quad (26)$$

$\lambda$
2.8537672

We can now calculate our weighting functions:

$$r = \frac{1 + \min L}{1 + \lambda \min L} \quad (27)$$

$$q = \frac{(\lambda - 1)\min L}{1 + \lambda \min L} \quad (28)$$

With the weighting functions, we can now calculate the enthalpy of the mixture:

$$h_{03} = r h_{03\lambda=1} + q h_{03_{air}} \quad (29)$$

Similar to how we calculated the enthalpies from the table, we will now calculate the entropy of the mixture:

$$s_{03_{air}, p=1bar}(T_{03})_{AirTables} = s_{03_{air}, p=1bar} \quad (30)$$

$$s_{03\lambda=1, p=1bar}(T_{03})_{StoichTables} = s_{03\lambda=1, p=1bar} \quad (31)$$

$$s_{03_{p=1\lambda}} = r s_{03\lambda=1, p=1bar} + q s_{03_{air}, p=1bar} \quad (32)$$

### 3.7 Stage 04i

The entropy of stage 04i is the same as the entropy of stage 03. This allows for us to move forward through the engine cycle.

$$s_{04i} = s_{03} \quad (33)$$

We make use of:

$$\mathcal{P}_C = \mathcal{P}_T \quad (34)$$

Which can be simplified to:

$$\dot{m}_g w_T = \dot{m}_a w_C \quad (35)$$

$$w_T = w_C \left( \frac{1}{1 + \frac{1}{\lambda \min L}} \right) \quad (36)$$

$w_T$
277.2824 kJ/kg

This will allow for us to calculate the ideal work of the turbine:

$$w_{T_i} = \frac{w_T}{\eta_T} \quad (37)$$

We will use the ideal work of the turbine to calculate the enthalpy of stage 04i:

$$h_{04i} = h_{03} - w_{T_i} \quad (38)$$

From here we will use the enthalpy  $h_{04i}$  to calculate the Temperature of stage 04i. To do so, one must iterate, such that the following equation is satisfied:

$$f(T) = h_{04i} - rh_{04i\lambda=1}(T) - qh_{04i\text{air}}(T) = 0 \quad (39)$$

Our group developed an iteration algorithm `Iterate_temp_h` to solve for the temperature. The algorithm is as follows:

$$\begin{aligned} i &= 1 \\ h_{\text{air}} &= h_{\text{input}} \\ T_{\text{air}_{i=1}}(h_{\text{input}})_{\text{AirTables}} &= T_{a_{i=1}} \\ h_{\lambda=1_{i=1}}(T_{a_{i=1}})_{\text{StoichTables}} &= h_{\lambda=1_{i=1}} \\ \\ i &= 2 \\ h_{\lambda=1_{i=2}} &= h_{\text{input}} \\ T_{\lambda=1_{i=2}}(h_{\text{input}})_{\text{StoichTables}} &= T_{\lambda=1_{i=2}} \\ h_{\text{air}_{i=2}}(T_{\lambda=1_{i=2}})_{\text{AirTables}} &= h_{\text{air}_{i=2}} \\ \\ f(T)_1 &= h_{\text{input}} - rh_{\lambda=1_{i=1}} - qh_{\text{air}_{i=1}} \\ f(T)_2 &= h_{\text{input}} - rh_{\lambda=1_{i=2}} - qh_{\text{air}_{i=2}} \\ \\ T &= \frac{T_{\text{air}_{i=1}} * f(T)_2 - T_{\lambda=1_{i=2}} * f(T)_1}{f(T)_2 - f(T)_1} \end{aligned}$$

After the `Iterate_temp_h(h04i, r, q)` algorithm is ran, we now have our temperature  $T_{04i}$ . We can now begin out entropy calculation.

$$s_{04i\text{air},p=1\text{bar}}(T_{04i})_{\text{AirTables}} = s_{04i\text{air},p=1\text{bar}} \quad (40)$$

$$s_{04i\lambda=1,p=1\text{bar}}(T_{04i})_{\text{StoichTables}} = s_{04i\lambda=1,p=1\text{bar}} \quad (41)$$

$$s_{04i_{p=1\lambda}} = rs_{04i\lambda=1,p=1\text{bar}} + qs_{04i\text{air},p=1\text{bar}} \quad (42)$$

$$s_{04i\lambda} = s_{04i_{p=1\lambda}} - R \ln \left( \frac{p_{04i}}{1\text{bar}} \right) \quad (43)$$

We can manipulate the above equation for pressure:

$$p_{04i} = e^{-\frac{s_{04i\lambda} - s_{04i_{p=1\lambda}}}{R}} \quad (44)$$

### 3.8 Stage 04

Moving from stage 04i to stage 04, we can assume no pressure change meaning:

$$p_{04} = p_{04i} \quad (45)$$

We can calculate the enthalpy of stage 04 using the work of the turbine:

$$h_{04} = h_{03} - w_T \quad (46)$$

From here we are to calculate the temperature by the iterations algorithm again:

$$\text{Iterate\_temp\_h}(h_{04}, r, q) = T_{04}$$

We can use the temperature to calculate the entropy of stage 04:

$$s_{04_{air,p=1bar}}(T_{04})_{AirTables} = s_{04_{air,p=1bar}} \quad (47)$$

$$s_{04_{\lambda=1,p=1bar}}(T_{04})_{StoichTables} = s_{04_{\lambda=1,p=1bar}} \quad (48)$$

$$s_{04_{p=1\lambda}} = r s_{04_{\lambda=1,p=1bar}} + q s_{04_{air,p=1bar}} \quad (49)$$

Finally we can calculate the entropy of stage 04, by normalizing the pressure:

$$s_{04_{\lambda}} = s_{04_{p=1\lambda}} - R \ln \left( \frac{p_{04}}{1bar} \right) \quad (50)$$

### 3.9 Stage 04.5i

One can assume that the transition from stage 04 to stage 04.5i is isentropic. This means that the entropy of stage 04.5i is equal to the entropy of stage 04:

$$s_{04.5i_{\lambda}} = s_{04_{\lambda}} \quad (51)$$

The specific work of the of the free turbine is:

$$w_{PT} = \frac{SHP}{\dot{m}_g} = \frac{SHP}{\dot{m}_a(1+f)} \quad (52)$$

Which can be simplified as:

$$w_{PT} = \frac{SHP}{\dot{m}_a \left( 1 + \frac{1}{\lambda_{min} L} \right)} \quad (53)$$

$w_{PT}$
262.83965 kJ/kg

Now we can calculate the ideal work of the free turbine, by making use of its efficiency:

$$w_{PT_i} = \frac{w_{PT}}{\eta_{PT}} \quad (54)$$

The enthalpy of stage 04.5i is:

$$h_{04.5i} = h_{04} - w_{PT_i} \quad (55)$$

We can now calculate the temperature of stage 04.5i by using the iterations algorithm:

$$\text{Iterate\_temp\_h}(h_{04.5i}, r, q) = T_{04.5i}$$

We can now calculate the entropy of stage 04.5i:

$$s_{04.5i_{air,p=1bar}}(T_{04.5i})_{AirTables} = s_{04.5i_{air,p=1bar}} \quad (56)$$

$$s_{04.5i_{\lambda=1,p=1bar}}(T_{04.5i})_{StoichTables} = s_{04.5i_{\lambda=1,p=1bar}} \quad (57)$$

$$s_{04.5i_{p=1\lambda}} = r s_{04.5i_{\lambda=1,p=1bar}} + q s_{04.5i_{air,p=1bar}} \quad (58)$$

We can use this to calculate pressure:

$$p_{04.5i} = e^{-\frac{s_{04.5i_{\lambda}} - s_{04.5i_{p=1\lambda}}}{R}} \quad (59)$$

$p_{04.5i}$
1.6164346 bar



### 3.10 Stage 04.5

The pressure at stage 04.5 is equal to the pressure at stage 04.5i:

$$p_{04.5} = p_{04.5i} \quad (60)$$

The enthalpy of stage 04.5 is:

$$h_{04.5} = h_{04} - w_{PT} \quad (61)$$

We will use the iterations algorithm to calculate the temperature of stage 04.5:

$$\text{Iterate\_temp\_h}(h_{04.5}, r, q) = T_{04.5}$$

We can now calculate the entropy of stage 04.5:

$$s_{04.5_{air}, p=1bar}(T_{04.5})_{AirTables} = s_{04.5_{air}, p=1bar} \quad (62)$$

$$s_{04.5_{\lambda=1}, p=1bar}(T_{04.5})_{StoichTables} = s_{04.5_{\lambda=1}, p=1bar} \quad (63)$$

$$s_{04.5_{p=1\lambda}} = r s_{04.5_{\lambda=1}, p=1bar} + q s_{04.5_{air}, p=1bar} \quad (64)$$

Normalize entropy by pressure:

$$s_{04.5_{\lambda}} = s_{04.5_{p=1\lambda}} - R \ln \left( \frac{p_{04.5}}{1bar} \right) \quad (65)$$

### 3.11 Stage 5i

The process from Stage 04.5 to Stage 5i is isentropic. This means that the entropy of stage 5i is equal to the entropy of stage 04.5:

$$s_{05i_{\lambda}} = s_{04.5_{\lambda}} \quad (66)$$

And the pressure of stage 5i goes to atmospheric pressure:

$$p_{05i} = p_a = p_{01} \quad (67)$$

Having determined the entropy of stage 5i and the pressure of stage 5i, we must now determine the temperature of stage 5i. We will use the iterations algorithm to calculate the temperature of stage 5i: The iterative process `Iterate_temp_ps` is similar to `Iterate_temp_h` algorithm however it solves:

$$f(T) = s_{5i} - q s_{5i_{air}}(T) - r s_{5i_{\lambda=1}}(T) = 0 \quad (68)$$

Where;

$$s_{5i_{air}} = s_{5i_{air}, p=1bar} - R \ln \left( \frac{p_{5i}}{1bar} \right) \quad (69)$$

$$s_{5i_{\lambda=1}} = s_{5i_{\lambda=1}, p=1bar} - R \ln \left( \frac{p_{5i}}{1bar} \right) \quad (70)$$

Ultimately, the algorithm will be solving:

$$\begin{aligned} s_{input} \\ s_{input_{p=1bar}} &= s_{input} + R \ln \left( \frac{p_{input}}{1bar} \right) \\ s_{algorithm} &= s_{input_{p=1bar}} \end{aligned}$$

$$i = 1$$

$$s_{air_{i=1}} = s_{algorithm}$$

$$T_{air_{i=1}}(s_{algorithm})_{AirTables} = T_{air_{i=1}}$$

$$s_{\lambda=1_{i=1}}(T_{air_{i=1}}) = s_{\lambda=1_{i=1}}$$

$$i = 2$$

$$s_{\lambda=1_{i=2}} = s_{algorithm}$$

$$T_{\lambda=1_{i=2}}(s_{algorithm})_{StoichTables} = T_{\lambda=1_{i=2}}$$

$$s_{air_{i=2}}(T_{\lambda=1_{i=2}}) = s_{air_{i=2}}$$

$$f(T)_1 = s_{algorithm} - r s_{\lambda=1_{i=1}} - q s_{air_{i=1}}$$

$$f(T)_2 = s_{algorithm} - r s_{\lambda=1_{i=2}} - q s_{air_{i=2}}$$

$$T_0 = \frac{T_{air_{i=1}} * f(T)_2 - T_{\lambda=1_{i=2}} * f(T)_1}{f(T)_2 - f(T)_1}$$

$$i = 3$$

$$s_{air_{i=3}}(T_0) = s_{air_{i=3}}$$

$$s_{\lambda=1_{i=3}}(T_0) = s_{\lambda=1_{i=3}}$$

$$f(T)_3 = s_{algorithm} - r s_{\lambda=1_{i=3}} - q s_{air_{i=3}}$$

$$T = T_0 - \frac{f(T_3)}{s_{air_{i=3}} - s_{\lambda=1_{i=3}}}$$

We can make use of the `Iterate_temp_ps` algorithm to calculate the temperature of stage 5i:

$$\text{Iterate\_temp\_ps}(s_{5i}, r, q) = T_{5i}$$

Once we have the temperature of stage 5i, we can calculate the enthalpy of stage 5i:

$$h_{5i} = q h_{5i_{air}}(T_{5i}) + r h_{5i_{\lambda=1}}(T_{5i}) \quad (71)$$

By making use of the exit enthalpies and assuming that the heat transfer is small compared to enthalpy variation, we can calculate the velocity of the fluid:

$$c_{5i} = \sqrt{2(h_{04.5} - h_{5i})} \quad (72)$$

### 3.12 Stage 5

The process from Stage 5i to Stage 5 is isentropic. This means that the entropy of stage 5 is equal to the entropy of stage 5i. As a matter of fact, we assume that stage 5 is equal to stage 5i, in most properties:

$$T_5 = T_{5i}$$

$$p_5 = p_{5i}$$

$$h_5 = h_{5i}$$

$$s_5 = s_{5i}$$

We can calculate the velocity of the fluid, by accounting for the nozzle efficiency:

$$c_5 = c_{5i} \phi_{nozzle} \quad (73)$$

The Specific Thrust is calculated by:

$$F_{sp} = \dot{m}_{air} \left( 1 + \frac{1}{\lambda_{minL}} \right) c_5 \quad (74)$$

$F_{sp}$
785.96769 N

The equivalent shaft power is calculated by:

$$\mathcal{P}_{es} = \eta_{prop} \mathcal{P}_s + \frac{F_{sp}}{11.1} \quad (75)$$

Recall  $\mathcal{P}_s = SHP$ .

One can calculate mass flow rate of the gases by:

$$\dot{m}_g = \dot{m}_{air} \left( \frac{1}{\lambda_{minL}} \right) \quad (76)$$

The equivalent brake specific fuel consumption is calculated by:

$$EBSFC = \frac{\dot{m}_g}{\mathcal{P}_{es}} \quad (77)$$

<i>EBSFC</i>
0.332946

### 3.13 Cycle Results

Stage	Pressure [bar]	Enthalpy [kJ/kg]	Entropy [kJ/kg-K]
a	1.01325	288.299988	6.6570201
0a	1.01325	288.299988	6.6570201
01	1.01325	288.299988	6.6570201
02i	9.228681	543.819214	6.6570201
02	9.228681	572.210239	6.7113694
03	8.9518206	1574.3477	7.8717827
04i	4.030023	1279.3664	7.8717827
04	4.030023	1297.06528	7.886781
045i	1.616434698	1017.44862	7.886781
045	1.616434698	1034.225618	7.904305
5i	1.0031175	920.29226	7.904305
5	1.0031175	920.29226	7.904305

$$\begin{aligned} w_c &= 283.910251 \left[ \frac{kJ}{kg} \right] \\ w_T &= 277.28244 \left[ \frac{kJ}{kg} \right] \\ w_{PT} &= 262.83966 \left[ \frac{kJ}{kg} \right] \\ \dot{m}_{air} &= 1.64089 \left[ \frac{kg}{s} \right] \\ F_{sp} &= 785.96769 [N] \\ EBSFC &= 0.332946 \end{aligned}$$

## 4 Axial Compressor Design - Task 2

The engine analyzed was a PT6A-114 which is a turboprop and thus contains a mixed compressor. Using values calculated in the previous assignment in which the engine's real cycle was calculated, here we are designing the axial component of the mixed compressor. Our design began with the process below in which the number of stages in the compressor was determined. Using the work in the compressor found from the previous assignment, this quantity is divided by two due to the fact that a mixed compressor contains both an axial and centrifugal component so it is efficient to divide the work of the compressor amongst these components evenly.

We can calculate the work per stage by:

$$w_{s_1} = 0.7 w_{stage} \quad (78)$$

$$w_{s_2} = 0.85w_{stage} \quad (79)$$

$$w_{s_n} = 0.9w_{stage} \quad (80)$$

$$w_{actual} = \frac{w - w_{s_1} - w_{s_2} - w_{s_n}}{n - 3} \quad (81)$$

Where  $w_{stage}$  is the work per stage,  $n$  is the number of stages. Also  $w$  is:

$$w = \frac{w_c}{2} \quad (82)$$

Once the number of stages was determined, next came the preliminary calculations that were needed in order to perform the calculations of desired quantities from each of the stages.

$$u_1^m = u_1^t \left( \frac{1 + \bar{d}_1}{2} \right) \quad (83)$$

Where  $\bar{d}_1 \in [0.5, 0.7]$  and  $u_1^t \in [330, 350] \frac{m}{s}$   
We can next find the flow coefficient by:

$$\frac{c_{1_a}^m}{u_1^m} \in [0.3, 0.6] \quad (84)$$

From here we can calculate  $\lambda$  number:

$$\lambda_{1_a}^m = \frac{c_{1_a}^m}{\sqrt{2 \left( \frac{\gamma-1}{\gamma+1} \right) h_{01}}} \quad (85)$$

$$\lambda_1^m = \frac{\lambda_{1_a}^m}{\sin(\tilde{\alpha}_1^m)} \quad (86)$$

Absolute velocity at the midpoint:

$$c_1^m = \lambda_1^m \sqrt{2 \left( \frac{\gamma-1}{\gamma+1} \right) h_{01}} \quad (87)$$

The transport component of the flow velocity:

$$c_{1_u}^m = \sqrt{(c_1^m)^2 - (c_{1_a}^m)^2} \quad (88)$$

The relative velocity in the  $u$  direction:

$$w_{1_u} = u_1^m - c_{1_u}^m \quad (89)$$

The relative flow velocity:

$$w_1^m = \sqrt{(w_{1_u})^2 + (c_{1_u})^2} \quad (90)$$

Mass flow rate function, as a function of  $\lambda$ :

$$q(\lambda_1^m) = \lambda_1^m \left[ \frac{\gamma+1}{2} \left( 1 - \frac{\gamma-1}{\gamma+1} (\lambda_1^m)^2 \right) \right] \quad (91)$$

We can now the Area of the compressor inlet:

$$A_1 = \frac{\dot{m} \sqrt{T_{01}}}{p_{01}} \frac{1}{q(\lambda_1^m) \sin(\tilde{\alpha}_1^m) \sqrt{\frac{\gamma}{R} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}} \quad (92)$$

We can calculate the Diameter of the Tip:

$$D_1^t = \sqrt{\frac{4A_1}{\pi(1 - \bar{d}_1)^2}} \quad (93)$$

Diameter of the Hub:

$$D_1^h = \bar{d}_1 D_1^t \quad (94)$$

Diameter of the Midpoint:

$$D_1^m = \frac{D_1^t + D_1^h}{2} \quad (95)$$

The number of blades:

$$n_1 = \frac{60u_1^m}{\pi D_1^m} \quad (96)$$

## 4.1 Stage 1

Calculating the axial velocity for the first stage:

$$c_{1_a}^{(1)} = c_1^{(1)} \sin(\tilde{\alpha}_1^{(1)}) \quad (97)$$

$$c_{1_u}^{(1)} = c_1^{(1)} \cos(\tilde{\alpha}_1^{(1)}) \quad (98)$$

The transport velocity:

$$U^{(1)} = \frac{2\pi D^{m(1)}}{2} \frac{n}{60} \quad (99)$$

$$\alpha^{(1)} = 90 - \left( \tan^{-1} \left( \frac{c_{1_u}^{(1)}}{c_{1_a}^{(1)}} \right) \right) \quad (100)$$

Relative velocity for the 1st stage in the  $u$  direction:

$$W_{1_u}^{(1)} = U^{(1)} - c_{1_u}^{(1)} \quad (101)$$

Change of  $C_u$  over the stage:

$$\Delta^{(stage)} C_u^{(1)} = \left[ \frac{\left( W_{1_u}^{(1)} - \frac{W_s^{(1)}}{2U^{(1)}} \right) \frac{w_s^{(1)}}{U^{(1)}}}{R^{(1)}} - c_{1_a}^{(1)} \Delta^{(stage)} C_a^{(1)} - w_s^{(1)} \right] \frac{1}{c_{1_u}^{(1)}} \quad (102)$$

Where in  $\Delta^{(stage)} C_u^{(1)}$  the values of  $R^{(1)}$  and  $\Delta^{(stage)} C_a^{(1)}$  were imposed which is also the case for the subsequent stages of the axial compressor. For the following stages, a similar process as shown above was computed in order to determine the desired quantities of  $R'$ ,  $w_s$ ,  $\Delta^{(stage)} C_a$ ,  $\Delta^{(stage)} C_u$ , and  $\alpha^{(1)}$ . For context, a generalized derivation of quantities is shown below in which  $(n)$  relates to the stage

## 4.2 Stage n

$$c_{1_a}^{(n)} = c_{1_a}^{(n-1)} - \Delta^{(stage)} C_a^{(n-1)} \quad (103)$$

$$c_{1_u}^{(n)} = c_{1_u}^{(n-1)} - \Delta^{(stage)} C_u^{(n-1)} \quad (104)$$

$$c_1^{(n)} = \sqrt{(c_{1_a}^{(n)})^2 + (c_{1_u}^{(n)})^2} \quad (105)$$

$$\alpha_1^{(n)} = 90 - \left( \tan^{-1} \left( \frac{c_{1_u}^{(n)}}{c_{1_a}^{(n)}} \right) \right) \quad (106)$$

The enthalpy at each stage:

$$h_{01}^{(n)} = h_1^{(n-1)} + w_s \quad (107)$$

The pressure ratio at each stage:

$$\pi_c^{(n-1)} = \left( 1 + \frac{\eta_{1_m} w_s}{h_{01}^{(n-1)}} \right)^{\frac{\gamma}{\gamma-1}} \quad (108)$$

The pressure at each stage:

$$p_{01}^{(n)} = p_{01}^{(n-1)} \pi_c^{(n-1)} \quad (109)$$

The critical speed of sound at each stage:

$$a_{cr}^{(n)} = \sqrt{2h_{01}^{(n)} \left( \frac{\gamma-1}{\gamma+1} \right)} \quad (110)$$

The  $\lambda$  number at each stage:

$$\lambda_1^{(n)} = \frac{c_1^{(n)}}{a_{cr}^{(n)}} \quad (111)$$

The mass flow rate as a function of  $\lambda$  at each stage:

$$q(\lambda_1^{(n)}) = \lambda_1^{(n)} \left[ \frac{\gamma+1}{2} \left( 1 - \frac{\gamma-1}{\gamma+1} (\lambda_1^{(n)})^2 \right) \right]^{\frac{1}{\gamma-1}} \quad (112)$$

The Area at each stage:

$$A_1^n = \frac{\dot{m} \sqrt{T_{01}^{(n)}}}{p_{01}^{(n)} \frac{1}{q(\lambda_1^{(n)}) \sin(\alpha_1^{(n)})} \sqrt{\left( \frac{\gamma}{R} \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}} \quad (113)$$

The Diameter at each stage:

$$D_{(n)}^t = D_{(n-1)}^t \quad (114)$$

$$D^{h(n)} = \sqrt{\left( D_{(n)}^t \right)^2 - \left( \frac{4A_1^{(n)}}{\pi} \right)^2} \quad (115)$$

$$D^{m(n)} = 0.5(D^{h(n)} + D^{h(n)}) \quad (116)$$

The transport velocity at each stage:

$$U^{(n)} = \frac{2\pi D^{m(n)}}{2} \frac{n}{60} \quad (117)$$

The relative velocity in the  $u$  direction at each stage:

$$W_{1_u^{(n)}} = U^{(n)} - c_{1_u^{(n)}} \quad (118)$$

The change in  $C_u$  at each stage:

$$\Delta^{(stage)} C_u^{(n)} = \left[ \frac{\left( W_{1_u^{(n)}} - \frac{W_s^{(n)}}{2U^{(n)}} \right) \frac{w_s^{(n)}}{U^{(n)}}}{R'^{(n)}} - c_{1_a^{(n)}} \Delta^{(stage)} C_a^{(n)} - w_s^{(n)} \right] \frac{1}{c_{1_u^{(n)}}} \quad (119)$$

### 4.3 Axial Design Results

Stage	$c_a$	$c_u$	$p$	$\alpha$	$R'$	$U$
1	157.5	27.771499	1.01325	1.39626340	0.68584	262.499
2	156.0	24.74814	1.401547	1.4134655	0.700413	286.6517
3	154.0	23.4701395	2.003553	1.4195566	0.679087	303.6294
4	151.5	20.22314	2.925198	1.438095	0.745890896837544	315.5214

$$\begin{aligned}
u_1^m &= 262.5 \left[ \frac{m}{s} \right] \\
c_{1_a} &= 157.5 \left[ \frac{m}{s} \right] \\
c_{1_u} &= 27.7715 \left[ \frac{m}{s} \right] \\
w_1 &= 236.36566 \left[ \frac{m}{s} \right] \\
w_{1_u} &= 234.59416 \left[ \frac{m}{s} \right] \\
D_1^m &= 0.0953 [m] \\
n &= 52612.4648 [rpm] \\
D_1^t &= 0.1270518 [m] \\
D_1^h &= 0.0635259 [m]
\end{aligned}$$

## 5 Radial Variation of a flow over Airfoil Geometry - Task 3

Flow parameters in a compressor stage vary with radial position. We will be making a few assumptions to simplify the problem.

1. No friction
2. Steady flow
3. Axisymmetric flow
4. Stream surfaces are cylindrical, on the same axis with rotational axis.
5. The number of blades (airfoils)  $n_b \rightarrow \infty$

### 5.1 Algorithm for Airfoil Radial Variation

1. Calculate  $\Delta w_U^m(w_{stage}, \omega)$

$$U^m = \frac{D_1^m}{2} \frac{2\pi n}{60} \quad (120)$$

Next we can use:

$$\Delta w_U^m = \frac{w_{stage}^m}{U^m} \quad (121)$$

2. Calculate  $\beta_1^m, \beta_2^m$ , where  $c_a^m, c_{U_1}^m$ , known from velocity diagrams

$$\Delta W_U = W_{U_2} - W_{U_1} \quad (122)$$

Where:

$$W_{U_1} = c_a \tan(\beta_1) \quad (123)$$

and similarly:

$$W_{U_2} = c_a \tan(\beta_2) \quad (124)$$

3. Now we calculate  $\bar{t}$ :

$$\bar{t}^m = 3 - \frac{1.15(\tilde{\beta}_2 - \tilde{\beta}_1)}{0.2\tilde{\beta}_2 - 2} \quad (125)$$

Recall we get angles  $\tilde{\beta}_1, \tilde{\beta}_2$  from the metal turning angles  $\beta_1, \beta_2$

$$\tilde{\beta}_1 = 90 - \beta_1 \quad (126)$$

$$\tilde{\beta}_2 = 90 - \beta_2 \quad (127)$$

4. Now we assume a variation of the blade chord with radius  $b = b(r)$ .

Say:

$$b^t = 0.5b^h \quad (128)$$

$$b^m = 0.75b^h \quad (129)$$

To derive the equation of a line we use the following:

$$A = \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

Where to find the equation of a line we will calculate the determinant of the matrix above.

$$y = \det A = b^h - \frac{xb^h}{2} \quad (130)$$

Finally we get an equation describing the variation of the chord with radius:

$$b = b^h \left(1 - \frac{r}{2}\right) \quad (131)$$

We can test the results of our calculation by testing:

$$\begin{aligned} b(0) &= b^h(1 - 0) = b^h \\ b(1) &= b^h(1 - \frac{1}{2}) = 0.5b^h \end{aligned}$$



5. Calculate  $\bar{t}^h$ ,  $t^h$ , and  $z_r$

$$\bar{t}^h = \bar{t}^m \frac{b^h}{b} \frac{D^h}{D^m} \quad (132)$$

From here we can find  $t^h$ :

$$t^h = \bar{t}^h b \quad (133)$$

To calculate the mean-camberline line, we must calculate  $\theta$ :

$$\theta_r = \Delta\tilde{\beta} = \tilde{\beta}_2 - \tilde{\beta}_1 \quad (134)$$

With  $\bar{a} = 0.4$  we can calculate the angles of the mean camberline:

$$\chi_1 = \frac{\theta_r}{2} [1 + 2(1 - 2\bar{a})] \quad (135)$$

$$\chi_2 = \frac{\theta_r}{2} [1 - 2(1 - 2\bar{a})] \quad (136)$$

We can calculate the mean camberline now:

$$z_r = \frac{r(b-r)}{rcot(\chi_1) + (b-r)cot(\chi_2)} \quad (137)$$

6. Now we assume a value for incidence  $i_n \in [1, 2]$  and a value of  $\bar{a}$ . We have already assumed  $\bar{a} = 0.4$ . For this project we chose  $i_n = 1.5$ .

7. Calculate  $\tilde{\beta}_{1_f}$  and  $\tilde{\beta}_{2_f}$ :

$$\tilde{\beta}_{1_f} = \tilde{\beta}_1 + i_n \quad (138)$$

$$\tilde{\beta}_{2_f} = \tilde{\beta}_2 + \frac{1}{\bar{t}^m} \quad (139)$$

8. Calculate  $\delta_n$ .

$$\delta_n = \tilde{\beta}_{2_f} - \tilde{\beta}_2 \quad (140)$$

We can calculate  $\delta_n$  from the following:

$$\delta_n = \frac{0.92\bar{a}^2 - 0.002\tilde{\beta}_{2_f} + 0.18}{\frac{1}{\theta\sqrt{t}} - 0.002} \quad (141)$$

9. Calculate  $\Delta\tilde{\beta}$ .

$$\Delta\tilde{\beta} = \tilde{\beta}_{2_n} - \tilde{\beta}_{1_n} \quad (142)$$

10. Now we are to calculate the stagger angle  $\alpha_s$ .

$$\theta_s = \tilde{\beta}_{2_n} - 0.4\theta_r \quad (143)$$

We can calculate the deviation angle  $\delta$ :

$$\delta = \frac{\beta_1 - \beta_2}{4\sqrt{\sigma}} \quad (144)$$

Where  $\sigma$  is the solidity:

$$\sigma = \frac{1}{t^m} \quad (145)$$

We can now calculate the  $\Delta\tilde{\alpha}$ :

$$\Delta\tilde{\alpha} = \theta_s - i_n + \delta \quad (146)$$

11. Now up to this point, we have calculated values at the midspan of the blade. Moving forward we will choose different radial locations along the blade.

12. At 5 different five different radial locations: hub, hub-mid (2), mid, mid-tip (4), and tip, calculate  $c_{U_1}(r), U_1(r), W_{U_1}(r), \tilde{\alpha}(r), \tilde{\beta}(r)$ .

13. Assume  $i \neq i(r)$  and calculate  $\beta_{1_f}(r)$

$$\beta_{1_f}(r) = \beta_1(r) + i \quad (147)$$

14. Calculate  $c_1(r), h_1(r), p_1(r), \rho_1(r)$

	h	2	m	4	t
$c_U[\frac{m}{s}]$	41.657	33.326	27.771	23.804	20.829
$U[\frac{m}{s}]$	175	218.75	262.5	306.250	350
$W_U[\frac{m}{s}]$	133.343	185.424	234.729	282.446	329.171
$c_a[\frac{m}{s}]$	157.5	157.5	157.5	157.5	157.5
$\tilde{\alpha}[deg]$	75.185	78.053	80	81.406	82.467
$\tilde{\beta}[deg]$	24.1	29.212	33.861	38.055	41.817
$\beta_{1_f}[deg]$	25.6	30.712	35.361	39.555	43.317
$c[\frac{m}{s}]$	162.916	160.987	159.3	159.289	158.871
$h[\frac{J}{kg}]$	288218.5301	288219.4944	288220.0232	288220.3437	288220.5524
$p[Pa]$	19676.1044	7537.9781	3429.78425	1757.55982	982.644356
$\rho[\frac{kg}{m^3}]$	3.2870588	2.103717	1.460915	1.073325	0.821765

15. Now calculate the above table for station 2, the exit of the rotor.

	h	2	m	4	t
$c_U[\frac{m}{s}]$	201.6573	161.3258	134.43817	115.2327	100.8286
$U[\frac{m}{s}]$	175	218.75	262.5	306.250	350
$W_U[\frac{m}{s}]$	-26.65725	57.424201	128.061834	191.0173	249.1714
$c_a[\frac{m}{s}]$	157.5	157.5	157.5	157.5	157.5
$\tilde{\alpha}[deg]$	52.01	45.6875	40.48326	36.19056	32.627
$\tilde{\beta}[deg]$	9.60641	20.03176	39.11425	50.4933	57.7032
$\beta_{2_f}[deg]$	101.10641	71.468241	52.38575	41.0067	33.79678
$c[\frac{m}{s}]$	255.8748	225.460	207.075	195.15335	187.01
$h[\frac{J}{kg}]$	288427.9254	288412.7181	288403.5253	288397.565	288393.493
$p[Pa]$	461089.909	176645.00873	80373.578	41186.664	23027.292
$\rho[\frac{kg}{m^3}]$	3.287059	2.103718	1.46091	1.07333	0.8217

16. Now calculate  $\rho_c(r)$ .

$$\rho_c(r) = 1 - (1 - \rho_c^m) \left( \frac{r^m}{r} \right)^2 \quad (148)$$

17. Calculate  $\bar{t}(r)$ .

$$\bar{t}(r) = \bar{t}^h \frac{D}{D^m} \frac{b^h}{b} \quad (149)$$

18. Calculate  $\delta_n(r)$ .

$$\delta_n(r) = \frac{0.92\bar{a}^2 - 0.002\tilde{\beta}_{2f} + 0.18}{\frac{1}{(\tilde{\beta}_{2f} - \tilde{\beta}_{1f})\sqrt{\bar{t}}} - 0.002} \quad (150)$$

19. Calculate  $\tilde{\beta}_{2n}(r)$ ,  $\theta_r(r)$ , and the stagger angle  $\theta_s(r)$ .

$$\tilde{\beta}_{2n}(r) = \tilde{\beta}_{2f}(r) - \delta_n(r) \quad (151)$$

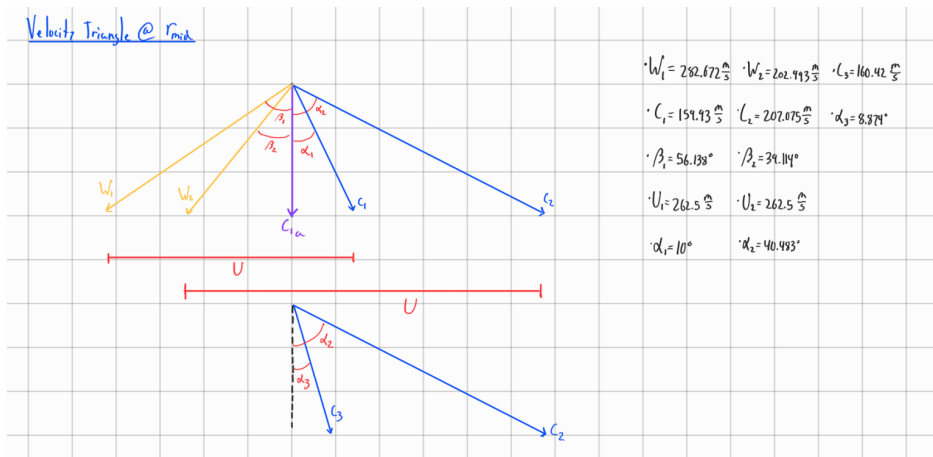
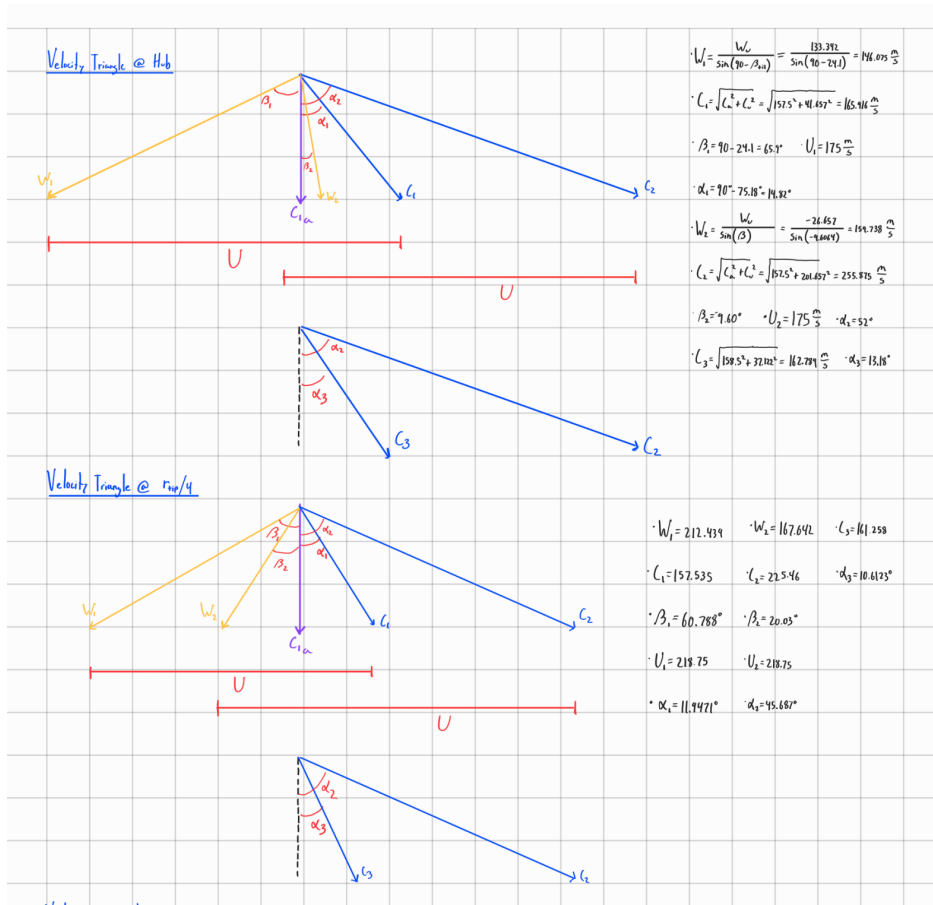
$$\theta_r(r) = \tilde{\beta}_{2f} - \tilde{\beta}_{1f} \quad (152)$$

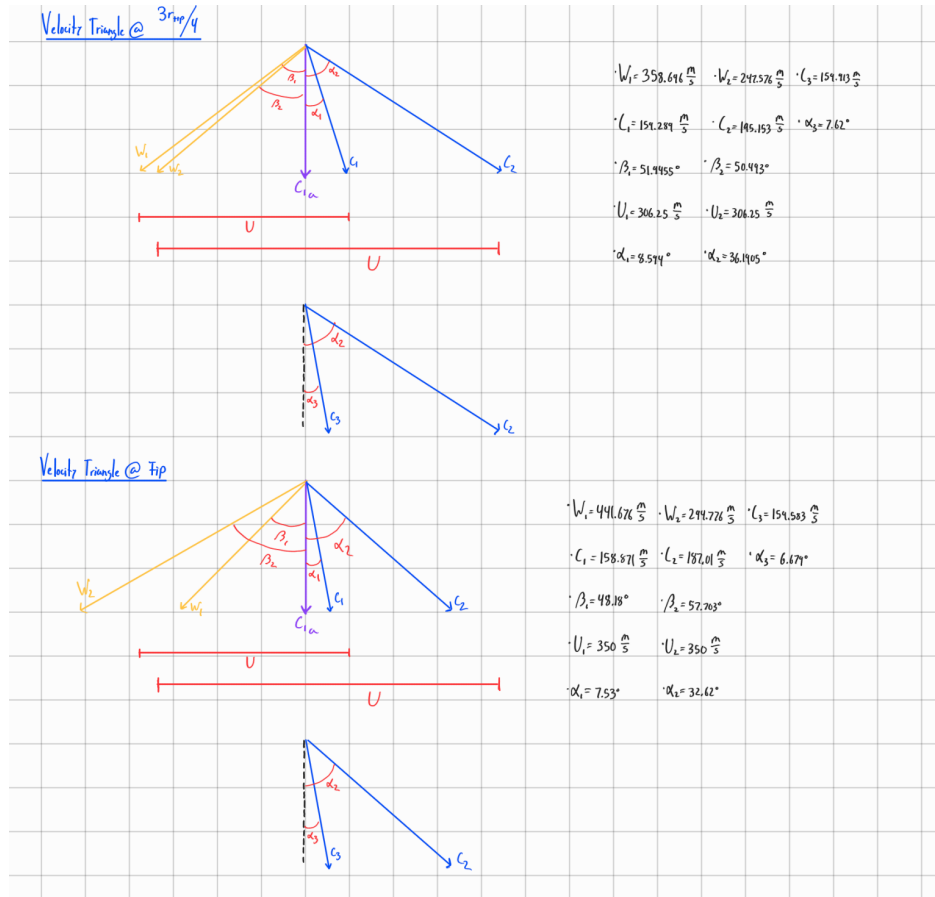
$$\theta_s(r) = \tilde{\beta}_{2n} - 0.4\theta_r \quad (153)$$

	h	2	m	4	t
$\rho_c$	0.29314	0.54761	0.6858	0.769189	0.82329
$\bar{t}$	0.60573	0.75716	0.908595	1.06003	1.21146
$\delta_n$	8.3232	7.03361	3.7306286	0.3677	2.664579
$\tilde{\beta}_{2n}[deg]$	101.10641	71.468241	52.38575	41.0067	33.79678
$\theta_r[deg]$	75.506	40.756	17.024614	1.4521716	9.520647
$\theta_s[deg]$	62.58075	48.1321445	41.845276	40.058145	40.26962

	h	2	m	4	t
$c_U[\frac{m}{s}]$	37.12220919	29.69776735	24.74813946	21.21269097	18.5611046
$U[\frac{m}{s}]$	175	218.75	262.5	306.250	350
$W_U[\frac{m}{s}]$	137.8777908	189.0522326	237.7518605	285.037309	331.4388954
$c_a[\frac{m}{s}]$	158.5	158.5	158.5	158.5	158.5
$\alpha[deg]$	13.18162287	10.61232774	8.874488881	7.622827236	6.67918768
$\beta[deg]$	41.01972799	50.02377919	56.31013941	60.92298402	64.44204029
$\beta_{3f}[deg]$	50.48027201	41.47622081	35.18986059	30.57701598	27.05795971
$c[\frac{m}{s}]$	161.8156618	160.275411	159.4324948	158.9220823	158.5899259
$h[\frac{J}{kg}]$	288380.8958	288380.1257	288379.7042	288379.449	288379.283
$p[Pa]$	15625.1994	5986.063516	2723.662254	1395.714423	780.3381059
$\rho[\frac{kg}{m^3}]$	3.287058799	2.10371763	1.460915022	1.073325322	0.8217647

20. Draw the velocity triangles for the rotor and the stator.





21. Draw the rotor and stator airfoil stack for the camber line.





## 6 Compressor Map - Task 4

Input values:

$$\dot{m} = 1.64089 \frac{kg}{s} \quad (154)$$

$$\pi_n^* = 9.2 \quad (155)$$

$$\eta = 0.89 \quad (156)$$

`task4_main.py` is the main file which calls the other files and runs the code. Readers are to run this code to perform the analysis.

`Table2.1.csv` contains the data from Table 2.1 in Dr. Cizmas' notes. This data is used to calculate the values for the compressor map. This table is read in by `task4_main.py` and is used to calculate the values for the compressor map. Table 2.1 is shown below:

$\bar{n}$	0.5	0.6	0.7	0.8	0.9	1.0	1.05	1.1
$\eta_{base}^-$	0.9	0.924	0.955	0.97	1.0	1.0	0.98	0.975
$m_{base}^-$	0.37	0.47	0.58	0.714	0.86	1.0	1.02	1.04
$\pi_{base}^-$	0.47	0.51	0.59	0.7	0.82	1.0	1.1	1.2

Where;

$$\begin{aligned}\bar{n} &= \frac{n}{n_{ref}} \\ \eta_{base}^- &= \frac{\eta_{base}}{\eta_{ref}} \\ m_{base}^- &= \frac{m_{base}}{m_{ref}} \\ \pi_{base}^- &= \frac{\pi_{base}^*}{\pi_{ref}^*}\end{aligned}$$

`Table2.3.csv` contains the data from Table 2.3 in Dr. Cizmas' notes. This data is used to calculate the values for the compressor map. This table is read in by `task4_main.py` and is used to calculate the values for the compressor map. Table 2.3 is shown below:

$\frac{\bar{C}_a}{C_{a_{base}}}$	0.8	0.9	1.0	1.1	1.2
$\frac{\eta}{\eta_{base}}$	0.92	0.98	1	0.97	0.88
$\frac{w}{w_{base}}$	1.25	1.12	1	0.9	0.82

Where;

$$w = \frac{h_1^*}{\eta} (\pi^{*\frac{\gamma-1}{\gamma}} - 1) \quad (157)$$

$$h_1^* = \frac{w\eta}{\pi^{*\frac{\gamma-1}{\gamma}} - 1} \quad (158)$$

Similarly we can say,

$$h_1^* = \frac{w_{base}\eta_{base}}{(\pi^{*\frac{\gamma-1}{\gamma}})_{base} - 1} \quad (159)$$

Making use of these equations, we can write the following:

$$\pi^* = \left[ 1 + ((\pi^{*\frac{\gamma-1}{\gamma}})_{base} - 1) \frac{w\eta}{w_{base}\eta_{base}} \right]^{\frac{\gamma}{\gamma-1}} \quad (160)$$

`steps.py` contains the 4 different steps that Dr. Cizmas has outlined in his notes. The steps are as follows:

## 6.1 Compressor Map Algorithm

1. Calculate  $\pi^* = \pi^*(\bar{n}, \frac{\bar{C}_a}{C_{a_{base}}})$  and  $\frac{\pi^*}{\pi_{base}^* = f(\bar{n}, \frac{\bar{C}_a}{C_{a_{base}}})}$ , where  $\bar{n} \in (0.5, 1.1)$  and  $\frac{\bar{C}_a}{C_{a_{base}}} \in (0.8, 1.2)$  producing a table as shown in Table 2.4.1 and Table 2.4.2. (Tables are in `Table_2_4_1.csv` and `Table_2_4_2.csv` respectively)



$\frac{\bar{C}_a}{C_{a_{base}}}$	0.8	0.9	1.0	1.1	1.2	$\bar{n}$
$\frac{\pi^*}{\pi^*}$	4.64045	4.38299	3.93091	3.39379	2.83897	0.5
$\pi^*$	5.07483	4.78063	4.26545	3.65607	3.03024	0.6
$\pi^*$	5.95028	5.57996	4.93454	4.17687	3.40661	0.7
$\pi^*$	7.16637	6.68649	5.85454	4.88616	3.91291	0.8
$\pi^*$	8.50639	7.90169	6.85818	5.65264	4.45332	0.9
$\pi^*$	10.5374	9.73710	8.36364	6.79104	5.24562	1.0
$\pi^*$	11.6748	10.7622	9.19999	7.41866	5.67803	1.05
$\pi^*$	12.8177	11.7906	10.0363	8.04337	6.10576	1.1
$\frac{\bar{C}_a}{C_{a_{base}}}$	0.8	0.9	1.0	1.1	1.2	$\bar{n}$
$\frac{\pi^*}{\pi^*}$	1.18050	1.11501	0.99999	0.86336	0.72222	0.5
$\frac{\pi^*}{\pi^*}$	1.18975	1.12078	0.99999	0.85713	0.71042	0.6
$\frac{\pi^*}{\pi^*}$	1.20584	1.13079	1.0	0.84646	0.69036	0.7
$\frac{\pi^*}{\pi^*}$	1.22407	1.14210	0.99999	0.83459	0.66835	0.8
$\frac{\pi^*}{\pi^*}$	1.24033	1.15215	0.99999	0.82422	0.64934	0.9
$\frac{\pi^*}{\pi^*}$	1.25991	1.16422	1.0	0.81197	0.62719	1.0
$\frac{\pi^*}{\pi^*}$	1.26899	1.16980	0.99999	0.80638	0.617177	1.05
$\frac{\pi^*}{\pi^*}$	1.27712	1.17479	0.99999	0.80142	0.608364	1.1

2. Calculate  $\frac{\bar{m}}{m_{base}} = f(\bar{n}, \frac{\bar{C}_a}{C_{a_{base}}})$ , by making use of:

$$\frac{\bar{m}}{m_{base}} = \frac{\bar{C}_a}{C_{a_{base}}} \left[ \frac{\pi^*}{\pi_{base}^*} \right]^{\frac{1}{3}} \quad (161)$$

Similar to step 1. Table 2.5 is produced. (Table is in Table\_2\_5\_csv)

$\frac{\bar{C}_a}{C_{a_{base}}}$	0.8	0.9	1.0	1.1	1.2	$\bar{n}$
$\frac{\bar{m}}{m_{base}}$	0.84549	0.93326	1.0	1.04743	1.07664	0.5
$\frac{\bar{m}}{m_{base}}$	0.84769	0.93487	0.99999	1.04490	1.07074	0.6
$\frac{\bar{m}}{m_{base}}$	0.85150	0.93764	1.0	1.04054	1.06057	0.7
$\frac{\bar{m}}{m_{base}}$	0.85577	0.94076	0.99999	1.03566	1.04918	0.8
$\frac{\bar{m}}{m_{base}}$	0.85955	0.94351	1.0	1.03135	1.03914	0.9
$\frac{\bar{m}}{m_{base}}$	0.86404	0.94679	1.0	1.02622	1.02718	1.0
$\frac{\bar{m}}{m_{base}}$	0.86612	0.94830	0.99999	1.02386	1.02169	1.05
$\frac{\bar{m}}{m_{base}}$	0.86796	0.94965	0.99999	1.02175	1.01680	1.1

3. Calculate  $\bar{\pi} = \bar{\pi}(\bar{n}, \frac{\bar{C}_a}{C_{a_{base}}})$  and  $\bar{m} = \bar{m}(\bar{n}, \frac{\bar{C}_a}{C_{a_{base}}})$ , by making use of:

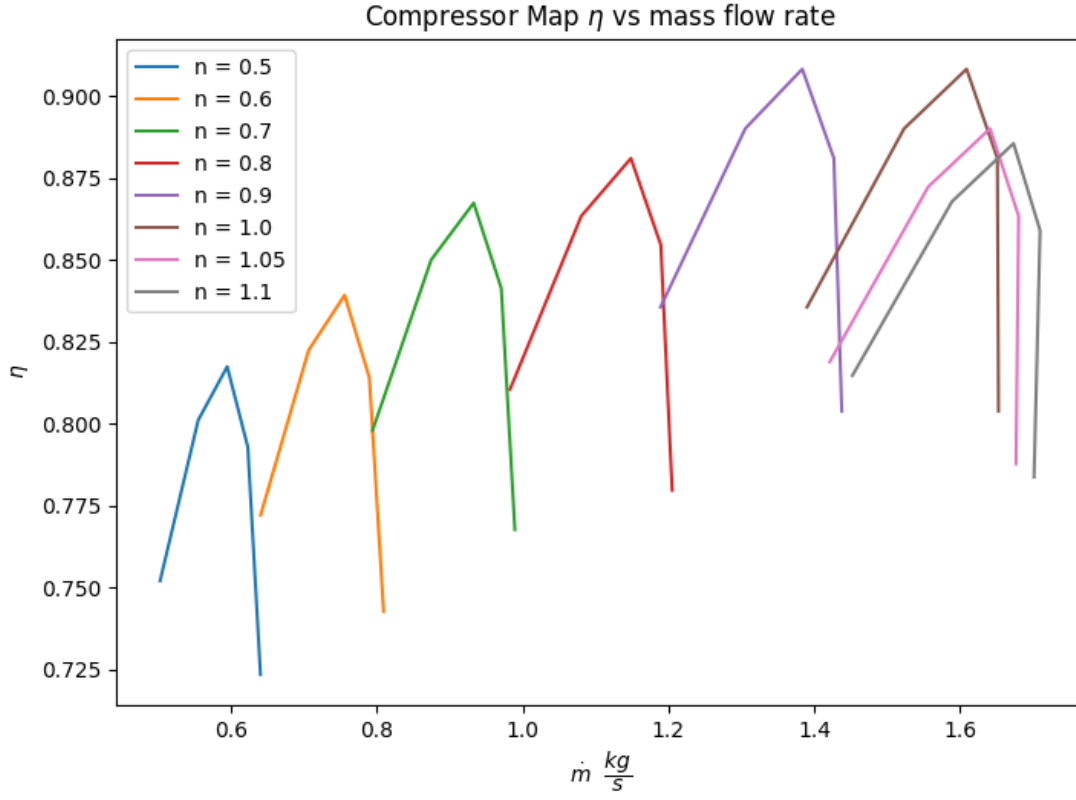
$$\bar{\pi} = \pi_{base} \frac{\pi^*}{\pi_{base}^*} \quad (162)$$

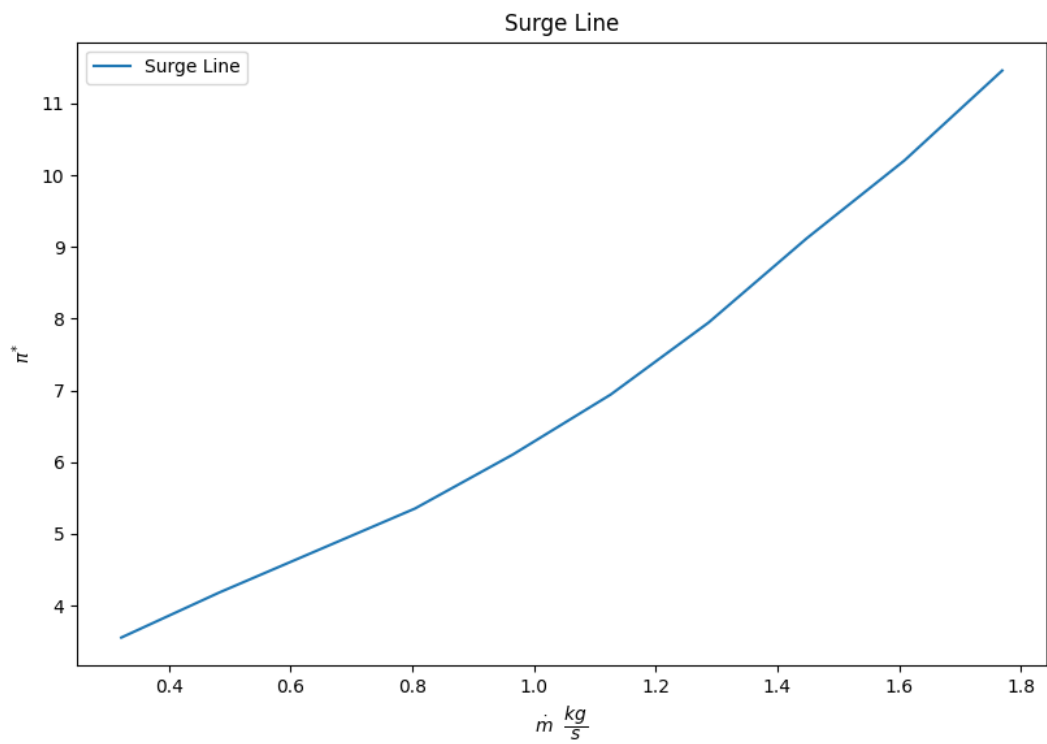
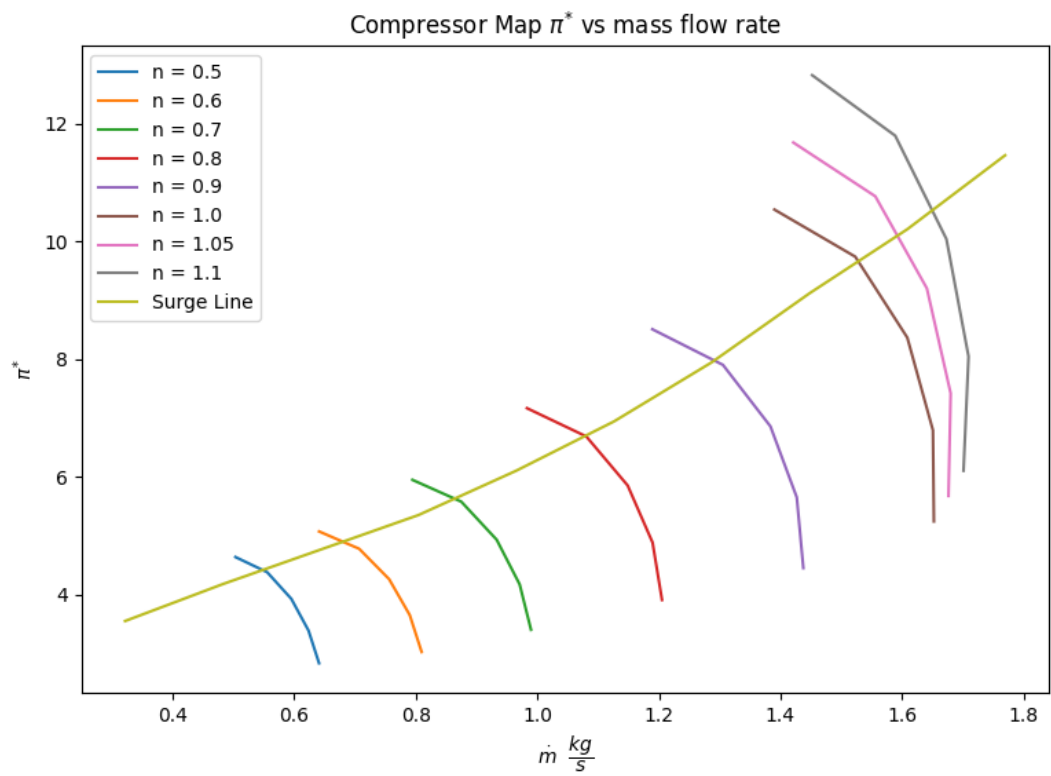
Where  $\pi_{base}$  comes from Table2.1.csv and  $\frac{\pi^*}{\pi_{base}^*}$  comes from Table\_2\_4\_2.csv. Similarly,

$$\bar{m} = m_{base} \frac{\bar{m}}{m_{base}} \quad (163)$$

Where  $m_{base}$  comes from Table2.1.csv and  $\frac{\bar{m}}{m_{base}}$  comes from Table\_2\_5.csv.

4. Calculate  $\eta = \eta(\bar{n}, \frac{\bar{C}_a}{C_{a_{base}}})$  using tables Table2.1.csv, Table2.3.csv.
5. Lastly we are to draw the Compressor map, with axes of  $\dot{m} \frac{\sqrt{T_1^*}}{p_1^*}$ ,  $\pi^*$  and  $\eta$ . We also provide a drawing of the surge line. The map is drawn:





## 7 Compressor Airfoil CFD - Task 5

### 7.1 Overview

This section contains the results of the CFD analysis performed on the compressor airfoil our team has chosen from Task 3.

### 7.2 Flow Conditions

As defined in the *T.L. User's Manual*, the flow through a cascade is defined by two parameters:

- The inlet flow Mach number  $M_{in}$
- inlet flow angle  $\alpha_{in}$

We have two non-dimensional parameters that define the flow through the cascade:

$$\rho_{in} = 1 \quad (164)$$

$$V_{in} = 1 \quad (165)$$

Where,  $\rho_{in}$  is the static density at the inlet and  $V_{in}$  is the velocity at the inlet. From here we can calculate the Mach number at the inlet:

$$M_{in} = \frac{V_{in}}{c_{in}} \quad (166)$$

Since  $V_{in}$  is a non-dimensional parameter, we need to non-dimensionalize the speed of sound  $c_{in}$ . We can do this by taking a look at how  $V_{in}$  is defined:

$$V_{in} = \frac{157.5}{157.5} \quad (167)$$

Applying the same logic to the speed of sound we get:

$$c_{in} = \frac{\sqrt{\gamma RT}}{157.5} \quad (168)$$

Where  $\gamma$  is the ratio of specific heats,  $R$  is the gas constant and  $T$  is the temperature. Since this all takes place in the compressor, the working fluid is air, meaning that  $\gamma = 1.4$  and  $R = 287.058 \frac{J}{kgK}$ .

Resulting in our Mach number to be:

$$M_{in} = 0.462823 \quad (169)$$

We are then able to calculate the static pressure at the inlet:

$$p_{in} = \frac{1}{\gamma M_{in}^2} \quad (170)$$

Readers may recall the standard isentropic relations for pressure and density from AERO 201:

$$p_0 = p \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad (171)$$

$$\rho_0 = \rho \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{1}{\gamma - 1}} \quad (172)$$

We can make use of these relations to calculate the total pressure and density at the inlet:

$$p_{0_{in}} = p_{in} \left( 1 + \frac{\gamma - 1}{2} M_{in}^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad (173)$$

$$\rho_{0_{in}} = \rho_{in} \left( 1 + \frac{\gamma - 1}{2} M_{in}^2 \right)^{\frac{1}{\gamma - 1}} \quad (174)$$

We calculated our total parameters to be:

$$p_{0_{in}} = 2.87924668 \quad (175)$$

$$\rho_{0_{in}} = 0.9004399 \quad (176)$$

We induced a flow angle of  $\alpha_{in} = 44.6$  degrees. Allowing for us to calculate the:

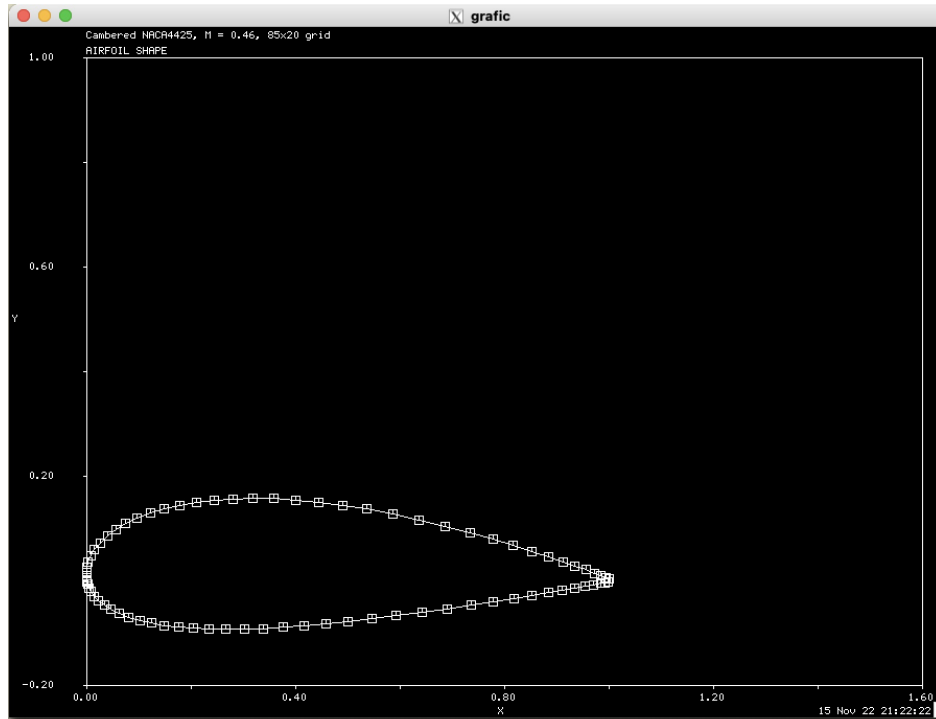
$$\text{FLUX} = V_{in} \cos \alpha_{in} \quad (177)$$

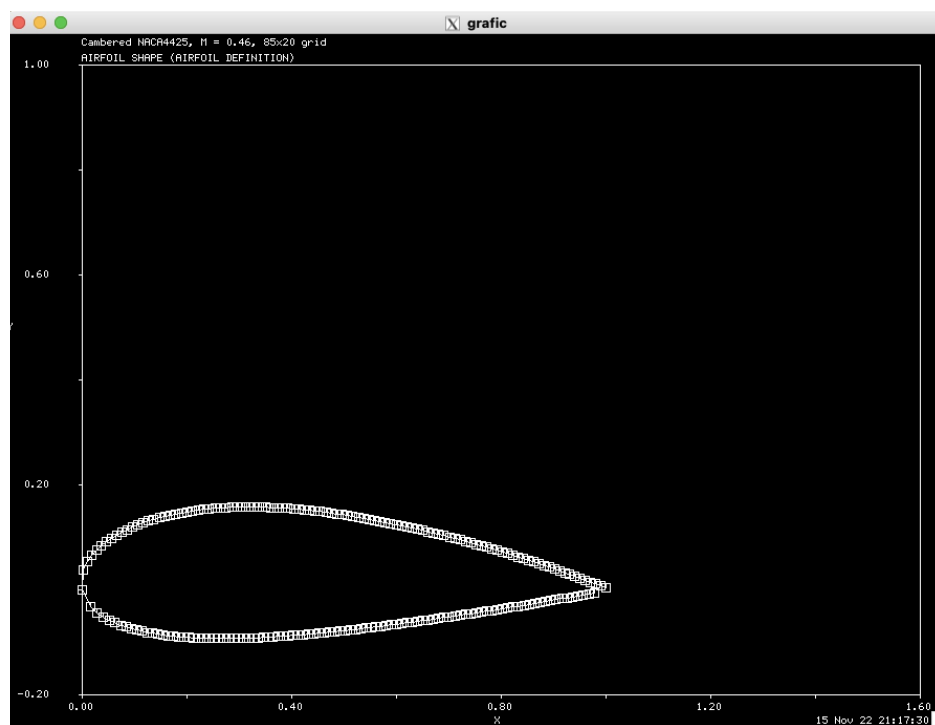
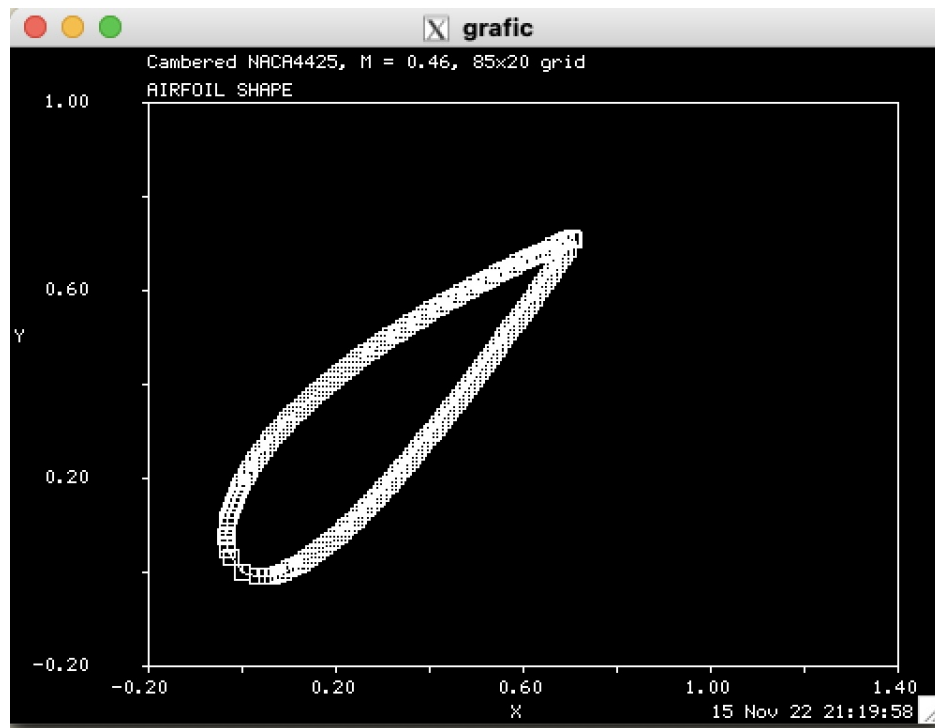
$$\text{VTAN} = V_{in} \sin \alpha_{in} \quad (178)$$

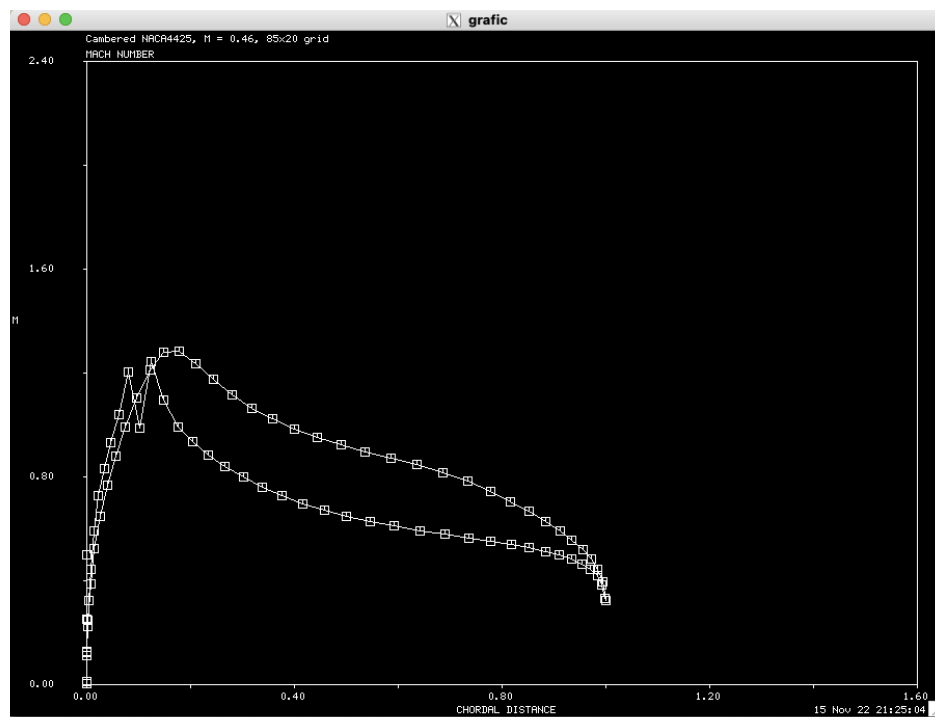
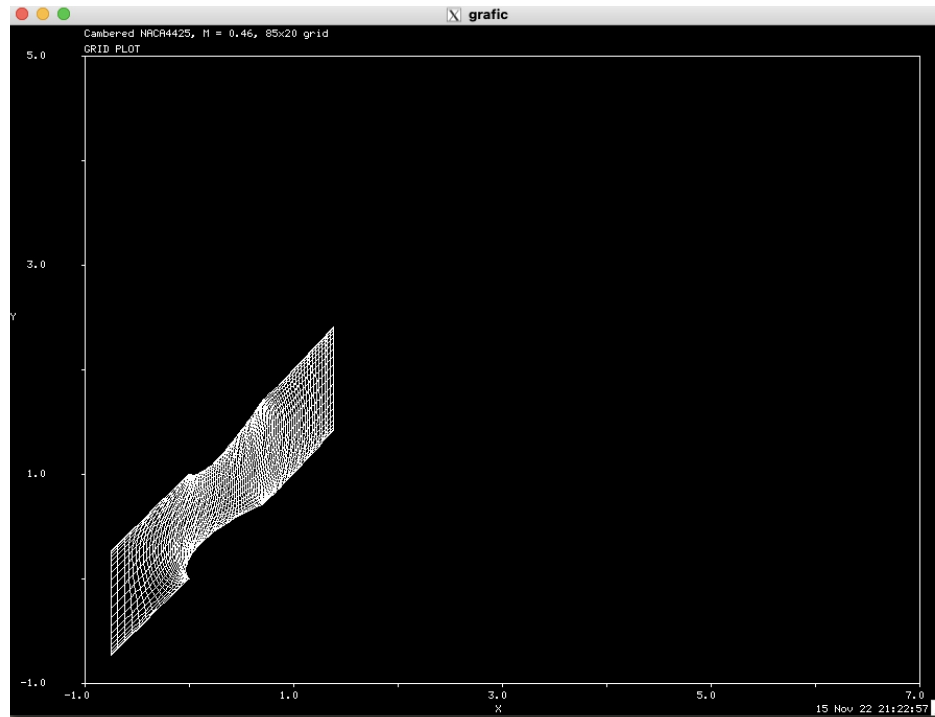
Resulting in:

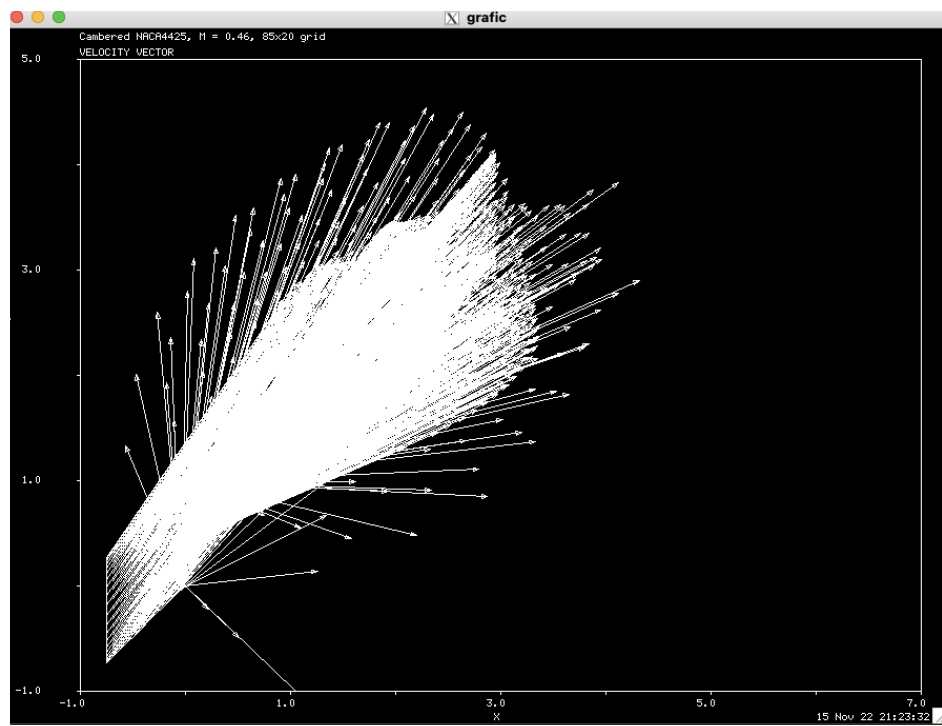
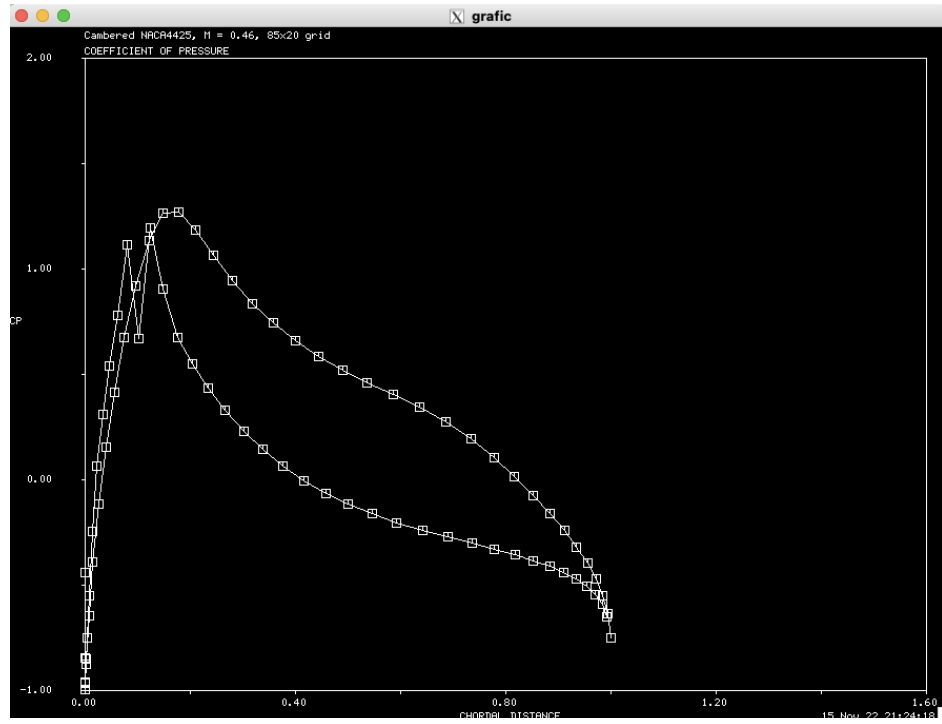
$$\begin{aligned} \text{FLUX} &= 0.712026045991 \\ \text{VTAN} &= 0.702153052995 \\ \text{UINIT} &= 0.712026045991 \\ \text{VINIT} &= 0.702153052995 \end{aligned}$$

### 7.3 Computational Fluid Dynamics Figures











## 8 OLVHN - Task6

### 8.1 Overview

This report contains the results and summary of the 12 step process described in Dr. Cizmas notes for computing the Operating Line. Also included in this report is the engine performance variatio with wheel speed, altitude and aircraft speed.

### 8.2 Methodology for Computing the Operating Line

As we determine the Operating Line of our engine, we first define our operating parameters:

$$\dot{m}_{air} = 1.64089 \left[ \frac{kg}{s} \right] \quad (179)$$

$$w_{c_n} = 283.91025 \left[ \frac{kJ}{kg} \right] \quad (180)$$

$$\eta_{compressor_n} = 0.90 \quad (181)$$

$$\sigma_{comb} = 0.90 \quad (182)$$

$$\eta_{turbine} = 0.94 \quad (183)$$

$$\pi_{c_n} = 9.2 \quad (184)$$

$$\lambda = 2.85377 \quad (185)$$

$$T_{1_n}^* = 288.16[K] \quad (186)$$

$$T_{3_n}^* = 1410[K] \quad (187)$$

$$p_{1_n}^* = 1.01325[bar] \quad (188)$$

$$p_{3_n}^* = 9.042243[bar] \quad (189)$$

$$h_{3_n}^* = 1574.3477 \left[ \frac{kJ}{kg} \right] \quad (190)$$

$$N_n = 52612.464822[rpm] \quad (191)$$

$$\pi_D = 0.93 \quad (192)$$

$$\gamma_g = 1.304286 \quad (193)$$

$$\frac{A_{3.5}}{A_5} = 1.2 \quad (194)$$

$$h_1^* = 288.299988 \left[ \frac{kJ}{kg} \right] \quad (195)$$

1. Calculate the compressor work  $w_c$ , given an angular speed calculated in Task2, as a function of nominal compressor work and nominal angular speed.

$$w_c = w_{c_n} \left( \frac{N}{N_n} \right)^x \quad (196)$$

Usually  $x \in [1.9, 2.1]$ , for convience we will start with  $x = 2.1$ .

Note:

$$N = 1.1N_r \quad (197)$$

Recall when at nominal conditions:

$$N_r = \frac{N_n}{1.05} \quad (198)$$

$N$	$w_c$
55117.82029 rpm	313.0460185 $\frac{kJ}{kg}$

2. Estimate the compressor efficiency  $\eta_c$ , given an angular speed calculated in Task2, as a function of nominal compressor efficiency and nominal angular speed. We start by calculated the pressure ratio  $\pi_c^*$

$$\pi_c^* = \left[ \left( \pi_{c_n}^{\frac{\gamma-1}{\gamma}} \right) \frac{\eta_c}{\eta_{c_n}} \left( \frac{N}{N_n} \right)^x + 1 \right]^{\frac{\gamma}{\gamma-1}} \quad (199)$$

To begin the caluation we can start by assuming  $\eta_c = \eta_{c_n}$ . Once the pressure ratio is calculated, read that  $\pi_c^*$  from the compressor map, and find the corresponding  $\dot{m} \frac{\sqrt{T_1^*}}{p_1^*}$ . Once you have that, find the corresponding  $\eta_c$  from the compressor map. If  $\eta_c \neq \eta_{c_n}$  then iterate until the change is less than a allowed tolerance. For our engine, we allowed a tolerance of 0.01.

$\pi_c^*$	$\eta_c$
10.4001622	0.878213

3. Calculate the  $T_3^*$  from:

$$\pi_c^* = \frac{1+f}{\sigma_{comb}} \left( \frac{p_3^*}{\dot{m} \sqrt{T_3^*}} \right)_n \sqrt{\frac{T_3^*}{T_1^*}} \frac{\dot{m} \sqrt{T_1^*}}{p_1^*} \quad (200)$$

Where:

$$\frac{1+f}{\sigma_{comb}} \left( \frac{p_3^*}{\dot{m} \sqrt{T_3^*}} \right)_n = constant \quad (201)$$

From here, calculate  $h_3^*$  from:

$$h_3^* = \left( \frac{1+minL}{1+\lambda minL} \right) h_{\lambda=1} + \left( \frac{(\lambda-1)minL}{1+\lambda minL} \right) h_{air} \quad (202)$$

Where:

- $h_\lambda$  - enthalpy of the combustion products for  $\lambda$  excess air
- $h_{\lambda=1}$  - enthalpy of the combustion products for stoichiometric combustion

- $h_{air}$  - enthalpy of the air

Check to see if the ratio  $\frac{w_c}{h_3^*}$  is equal to the nominal ratio  $\frac{w_{cn}}{h_{3n}^*}$ . If not, iterate  $x$  until it is, within a reasonable tolerance of about 1%.

After iterating  $x$ , we arrived to  $x = 1.4$ .

$T_3^*$	$h_3^*$
1552.103717 [K]	1753.74986 $\frac{kJ}{kg}$

4. We are now to find the critical conditions by:

$$\pi_{c_{cr}}^* = \frac{1}{\sigma_{comb}\pi_D} \left[ \frac{\frac{\gamma_g+1}{2}}{1 - \frac{w_c}{h_3^*} \frac{1}{\eta_{turbine}}} \right]^{\frac{\gamma_g}{\gamma_g-1}} \quad (203)$$

$$N_{cr} = N_n \sqrt{\frac{\eta_{cn}}{\eta_{c_{cr}}} \frac{\pi_{c_{cr}}^{\frac{\gamma-1}{\gamma}} - 1}{\pi_{cn}^{\frac{\gamma-1}{\gamma}} - 1}} \quad (204)$$

We are to then repeat steps (1)-(3) for three values of angular speed larger than the critical angular speed.

$\pi_{c_{cr}}^*$	$N_{cr}$	$\eta_{c_{cr}}$
5.462985	44711.24058 rpm	0.879

$\frac{w_c}{h_3^*}$	$\left(\frac{w_c}{h_3^*}\right)_n$	% diff
0.1785009519	0.180335162	1.0171116 %

Now we repeat steps (1)-(3) for three values of angular speed larger than the critical angular speed.

$N$	$w_c$	$\eta_{iteration}$	$\pi_c^*$	$\bar{m}$
46946.803	223.4949	0.881	6.17351	0.71
49182.365	246.4306	0.879	7.09622	0.785
51417.93	270.5425	0.9	8.5075	0.9

$N$	$T_3^*$	$h_3^*$	$\frac{w_c}{h_3^*}$	% diff
46946.803	1150.965	1251.825	0.17854	0.998
49182.365	1244.026485	1368.077	0.18013	0.1142
51417.93	1360.3106	1512.1349	0.1789	0.7879

5. Once we have reached the critical value, our ratio  $\frac{w_c}{h_3^*}$  is no longer constant. If the flow isn't critical, then the variation of  $\frac{w_c}{h_3^*}$  is given by:

$$\frac{w_c}{h_3^*} = \eta_{turbine} \left[ 1 - \left( \frac{p_H}{p_3^*} \right)^{\frac{\gamma_g-1}{\gamma_g}} - K \left( \frac{A_{3.5}}{A_5} \right)^2 \left( \frac{p_3^*}{p_H} \right)^{\frac{2}{\gamma_g}} \right] \quad (205)$$

Where:

$$K = \left( \frac{A_5}{A_{3.5}} \right)^2 \left( \frac{p_H}{p_3^*} \right)^{\frac{2}{\gamma_g}} \left[ 1 - \left( \frac{p_H}{p_3^*} \right)^{\frac{\gamma_g-1}{\gamma_g}} \frac{w_c}{h_3^*} \frac{1}{\eta_{turbine}} \right] \quad (206)$$

$$\frac{p_H}{p_3^*} = \frac{1}{\sigma_{combustion}^* \pi_c^* \pi_D} \quad (207)$$

Recall that  $\pi_D$  is given by:

$$\pi_D = \frac{p_1^*}{p_H} \quad (208)$$

When performing this step be sure to choose a  $\pi_c^*$  that is less than the critical value  $\pi_{c_{cr}}^*$ .

$K$	$\frac{p_H}{p_3^*}$	$\frac{w_c}{h_3^*}$
0.007177101	0.23020826	0.180335162

6. Now we are to choose an  $N$  value smaller than the critical value  $N_{cr}$ , and calculate  $w_c$  using equation (16) We chose a  $N$  value that was 1% less than the critical value.

$$N < N_{cr} \quad (209)$$

Now we use equation (16) to calculate  $w_c$ .

$N$	$w_c$
44500 rpm	199.73351054474077 $\frac{kJ}{kg}$

7. Now calculate  $h_3^*$  from equation (26) and  $w_c = 171.430154 \frac{kJ}{kg}$ .

$$h_3^* = w_{c_n} \left( \frac{w_c}{h_3^*} \right)^{-1} \quad (210)$$

$h_3^*$
1107.56831 $\frac{kJ}{kg}$

8. Calculate  $T_3^*$  using the stoichiometric and air gas tables.

$T_3^*$
1028.878066 K

9. For known value of  $\pi_c^*$  and  $N$  read from the the compressor map the value of the corrected mass flow rate

$$\pi_c^* = 5.18983611172067 \quad (211)$$

Yields a corrected mass flow rate of:

$m_a \frac{\sqrt{T_1^*}}{p_1^*}$
(0.63)(1.6087157) $\frac{\sqrt{288}}{101325}$

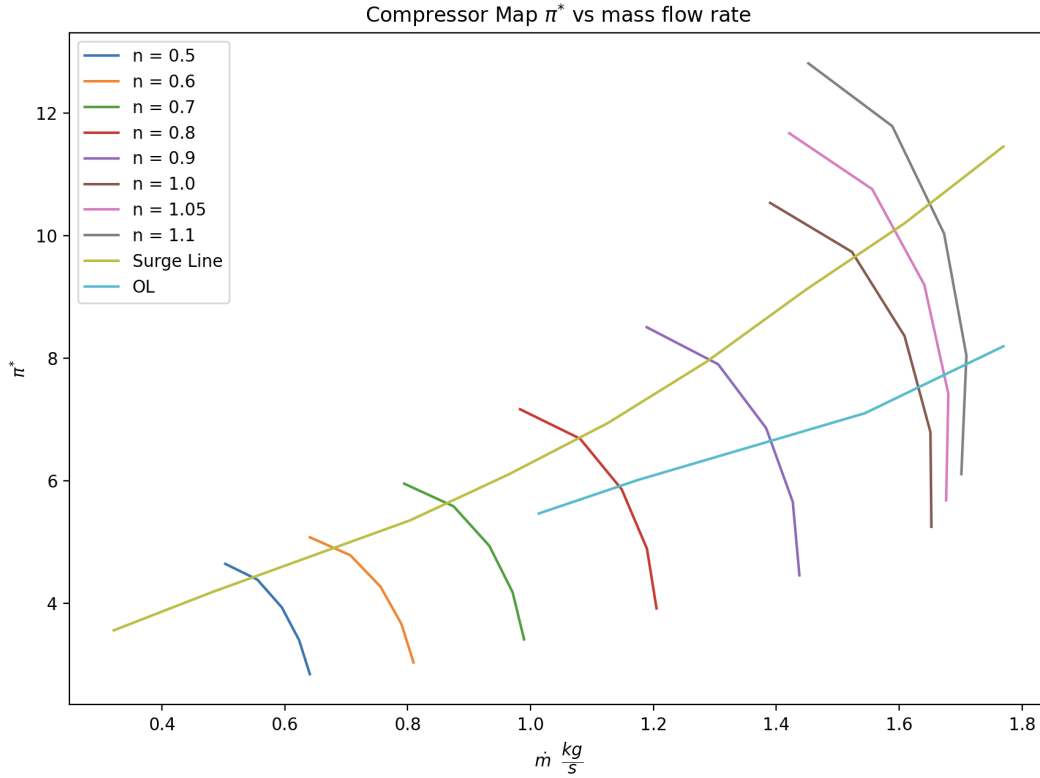
10. Now determine  $T_3^*$  using equation (21).
11. Compare the values of  $T_3^*$  from step 8 to  $T_3^*$  from step 10, if they differ by more than 10 degrees K, then iterate until with different  $N$  value.

$T_3^*$
1033.097120243129 K

12. Now choose another value of  $\pi_c^*$  and go back to step 5 and repeat the process to get other points on the operating line at  $\frac{p_H}{p_3^*}$

$\pi_c^*$	$\frac{p_H}{p_3^*}$	$K$	$\frac{w_c}{h_3^*}$	$N$
6.009283919	0.198816223	0.007124393	0.180335162	44500
7.101880995	0.168229112	0.006698233	0.180335162	40000
8.194478071	0.145798563	0.006163854	0.180335162	50000

$w_c$	$h_3^*$	$T_3^*$	$T_3^*$
199.73351054474077	1107.56831	1028.878066	1033.097120243129
159.6691061	885.4019626	839.0287095	833.1424783
255.1126073	1414.658156	1281.631036	1276.178685



### 8.3 Jet Engine Performance Variation with Wheel Speed

To determine the performance variation with wheel speed ( $N$ ) we are to use the **Operating Line** from Section 2. We will choose wheel speeds that are different from the nominal wheel speed  $N_n$ , and then read from the Compressor Map the pressure ratios  $\pi_c^*$  and the efficiencies  $\eta_c$ .

$N$	$\pi_c^*$	$\eta_c$
40000 rpm	7.101880995	0.849
44500 rpm	6.009283919	0.877
49000 rpm	7.9900312	0.8325

Now similar to how we solved for compressor work, we make use of equation (17):

$$w_c = w_{c_n} \left( \frac{N}{N_n} \right)^x$$

Here we will use  $x = 2$  following Dr. Cizmas' Example.

$w_c$
203.1064891
164.1057354
246.2611692

The enthalpy at stage 02 is calculated:

$$h_2^* = h_1^* + w_c \quad (212)$$

$h_2^*$
491.4064771
452.4057234
534.5611572

The degree of dynamic compression in the inlet nozzle is recalculated as:

$$\pi_D = \sigma_{inlet}^* \left( 1 + \frac{\gamma - 1}{2} M_H^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad (213)$$

Recall  $\sigma_{inlet}^* = 0.93$ . Also here we are assuming  $M_H = 0$ , since we are at Take-off conditions.

$\pi_D$
0.93
0.93
0.93

The specific thrust is now recalculated as:

$$F_{sp} = \phi \sqrt{2h_3^* \left[ 1 - \frac{1}{\pi_D \pi_c^* \sigma_{comb}^*} \right]^{\frac{\gamma_g - 1}{\gamma_g}} - h_1^* \left( \frac{\pi_c^{\frac{\gamma - 1}{\gamma}}}{\eta_c} \right) \left( 1 + \frac{1}{\lambda_{min} L} \right)} \quad (214)$$

$F_{sp}$	$\frac{p_H}{p_3^*}$	$K$	$\frac{w_c}{h_3^*}$	$h_3^*$
731.102889	0.198816223	0.007124393	0.180335162	1126.272254
656.888117	0.168229112	0.006698233	0.180335162	910.0040937
875.895256	0.149529804	0.006265198	0.180335162	1365.574893

The mass flow rate is recalculated as:

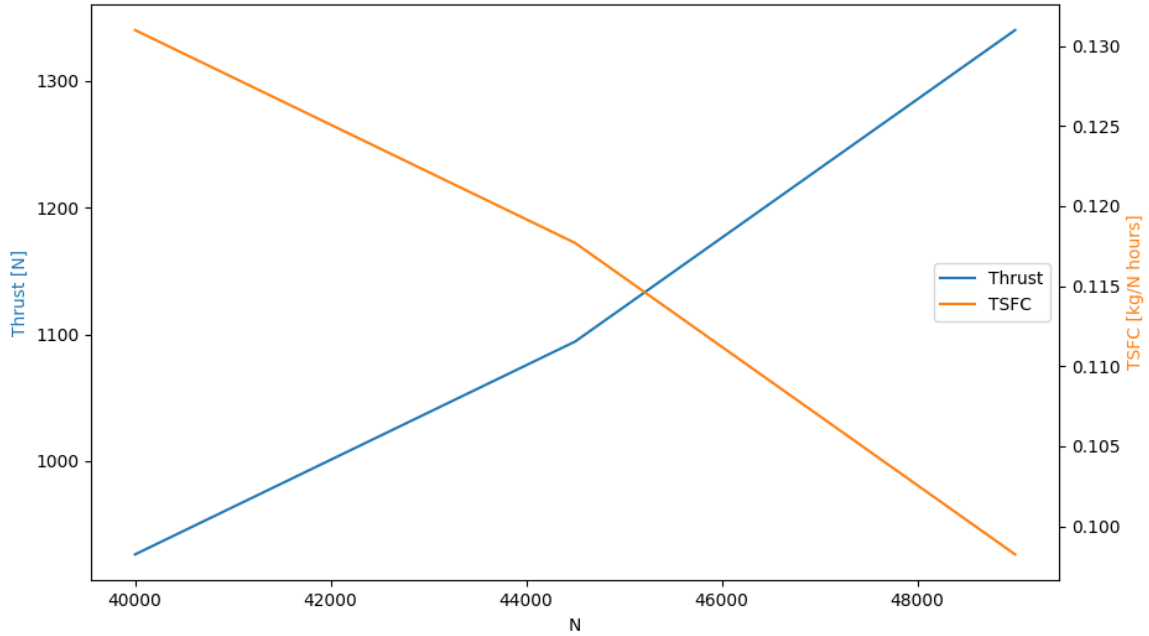
$$\dot{m}_{air} = \dot{m}_{air_n} \frac{\pi_c^* N_n}{\pi_{c_n}^* N} \quad (215)$$

$\dot{m}_{air}$
1.26719369
1.666071704
1.530139367

And the TSFC is recalculated as:

$$TSFC = \frac{3600}{F_{sp}} \frac{1}{\lambda min L} \quad (216)$$

TSFC
0.117699
0.130996
0.098242



Above is the the plot of the engine performance variation with wheel speed  $N$  [rpm]

## 8.4 Jet Engine Performance Variation with Altitude and Speed

Up to this point, our engine has been performance has been calculate at take-off conditions. It is very important for us to determine how the performance varies with altitude and speed.

1. First we are to calculate the Mach number at H and V.

$$M_H = \frac{V}{\sqrt{\gamma R T_H}} \quad (217)$$

It is important for us to specify our altitude dependent properties and speed dependent properties.

$H$ [m]	$T_H$ [K]	$p_H$ [Pa]	$a_H$ [m/s]
0	288.15	101325	340.29
1000	281.65	89874.6	336.434
3000	268.65	70108.5	328.578
5000	255.65	54019.9	320.529
6000	249.15	47181	316.428
7000	242.65	41060.7	312.274
8000	236.15	35599.8	308.063
9000	229.65	30742.5	303.793
10000	223.15	26436	299.463

And our velocity values:

$v$ [m/s]
0
100
200
300
350
400

2. Now one can calculate the degree of dynamic compression  $\pi_D$ .

$$\pi_D = \frac{p_1^*}{p_H} = \sigma_{inlet}^* \frac{p_H^*}{p_H} \quad (218)$$

This means that we can say:

$$\pi_D = \sigma_{inlet}^* \left( 1 + \frac{\gamma - 1}{2} M_H^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad (219)$$

Where  $\sigma_{inlet}^*$  is the inlet stagnation pressure loss in the inlet defined as the ratio between the stagnation pressure at the inlet in the compressor  $p_1^*$  and the stagnation pressure at the inlet of the engine  $p_H$ .

$H$ [km]	0	1	3	5	6	7	8	9	1	$v$ [m/s]
$\pi_D$	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0
$\pi_D$	0.9889	0.9887	0.9917	0.9949	0.9966	0.9985	1.0004	1.0024	1.0046	100
$\pi_D$	1.1809	1.1810	1.1943	1.2090	1.2170	1.2254	1.2344	1.2439	1.2541	200
$\pi_D$	1.5566	1.5586	1.5952	1.6361	1.6585	1.6823	1.7077	1.7348	1.7638	300
$\pi_D$	1.8411	1.8460	1.9023	1.9656	2.0004	2.0375	2.0772	2.1197	2.1654	350
$\pi_D$	2.2122	2.2226	2.3066	2.4019	2.4544	2.5105	2.5708	2.6356	2.7054	400

3. Now we calculate  $h_H^*$ .

$$h_H^* = h_H \left( 1 + \frac{\gamma - 1}{2} M_H^2 \right) \quad (220)$$

4. Now calculate the compressor pressure ratio  $\pi_c^*$  with variation of H and V.

$$\pi_c^* = \left[ 1 + \frac{h_0}{h_H^*} \left( \pi_{c_n}^{*\frac{\gamma-1}{\gamma}} - 1 \right) \right]^{\frac{\gamma}{\gamma-1}} \quad (221)$$

Students can also calculate  $w_c$ :

$$w_c = h_2^* - h_H^* \quad (222)$$



5. Now oen can begin calculating the Specific Thrust  $F_{sp}$

$$F_{sp} = \phi_5 \sqrt{2h_3^* \left[ 1 - \frac{1}{(\pi_D \pi_c^* \pi_{combust}^*)} \right]} - h_1^* \frac{\pi_c^{\frac{\gamma-1}{\gamma}}}{\eta_c} \left( 1 + \frac{1}{\lambda minL} \right) - v \quad (223)$$

Recall,  $\gamma$  varies due to the variance in speed of sound  $a$  from the change in altitude.

$$\gamma = \frac{a_H^2}{R_{air} T_H} \quad (224)$$

We can calculate  $h_3^*$  similar to previous sections:

$$h_3^* = \left( \frac{w_c}{h_3^*} \right) w_c \quad (225)$$

Where  $w_c$  comes from equation (44) and  $\frac{w_c}{h_3^*}$  comes from equations (27), (28), and (29).

$$\begin{aligned} \frac{w_c}{h_3^*} &= \eta_{turbine} \left[ 1 - \left( \frac{p_H}{p_3^*} \right)^{\frac{\gamma_g-1}{\gamma_g}} - K \left( \frac{A_{3.5}}{A_5} \right)^2 \left( \frac{p_3^*}{p_H} \right)^{\frac{2}{\gamma_g}} \right] \\ K &= \left( \frac{A_5}{A_{3.5}} \right)^2 \left( \frac{p_H}{p_3^*} \right)^{\frac{2}{\gamma_g}} \left[ 1 - \left( \frac{p_H}{p_3^*} \right)^{\frac{\gamma_g-1}{\gamma_g}} \frac{w_c}{h_3^*} \frac{1}{\eta_{turbine}} \right] \\ \frac{p_H}{p_3^*} &= \frac{1}{\sigma_{combustion}^* \pi_c^* \pi_D} \end{aligned}$$

Now all we have to solve for is the varying  $\lambda$ . We begin by making use of the energy conservation equation between the inlet and burner.

$$\dot{m}_{air} h_2^* + \dot{m}_{fuel} (h_{fuel} + \pi_{combust}^* LHV) = \dot{m}_{air} h_3^* \quad (226)$$

Recall  $LHV = 43.5 \times 10^6 J/kg$  for standard fuel. We can make use of:

$$f = \frac{\dot{m}_{fuel}}{\dot{m}_{air}} = \frac{1}{L} = \frac{1}{\lambda minL} \quad (227)$$

Resulting in the following equation:

$$h_2^* + \frac{\pi_{combust}^* LHV}{\lambda minL} = \left[ 1 + \frac{1}{\lambda minL} \right] h_3^* \quad (228)$$

We have all the necessary variables to solve for  $\lambda$ , therefore equation (45) can be solved.

$H$ [km]	0	1	3	5	6	7	8	9	1	$v$ [m/s]
$F_{sp}$	1040	1002.6	1032	1060.84	1075.03	1089.1	1103	1116.8	1130.5	0
$F_{sp}$	938.35	900.6	930.9	960.5	975.2	989.6	1004.01	1018.3	1032.5	100
$F_{sp}$	830.5	792.0	824.4	856.4	872.2	888.0	903.7	919.3	934.8	200
$F_{sp}$	709.5	669.3	705.2	740.7	758.3	775.9	793.4	810.9	828.3	300
$F_{sp}$	641.0	599.4	637.4	675.0	693.7	712.3	730.8	749.4	767.9	350
$F_{sp}$	565.0	521.5	562.0	601.9	621.7	641.5	661.2	680.9	700.6	400

One can now calculate  $TSFC$ :

$$TSFC = (TSFC)_n \frac{F_{spn}}{F_{sp}} \frac{\lambda_n}{\lambda} \quad (229)$$

$H$ [km]	0	1	3	5	6	7	8	9	1	$v$ [m/s]
$TSFC$	0.1037	0.0985	0.1026	0.1066	0.1085	0.1104	0.1123	0.1142	0.1160	0
$TSFC$	0.1120	0.1066	0.1108	0.1149	0.1168	0.1187	0.1206	0.1225	0.1243	100
$TSFC$	0.1167	0.1109	0.1152	0.1192	0.1211	0.1230	0.1249	0.1267	0.1284	200
$TSFC$	0.1174	0.1110	0.1154	0.1194	0.1213	0.1232	0.1250	0.1267	0.1283	300
$TSFC$	0.1163	0.1094	0.1139	0.1180	0.1199	0.1218	0.1235	0.1252	0.1268	350
$TSFC$	0.1140	0.1066	0.1112	0.1154	0.1173	0.1192	0.1209	0.1225	0.1240	400

One can calculate mass flow rate by:

$$\dot{m}_a = \dot{m}_{a_n} \frac{\pi_c^*}{\pi_{c_n}^*} \frac{p_H}{p_0} \left( 1 + \frac{\gamma-1}{2} M_H^2 \right)^{\frac{\gamma}{\gamma-1}} \frac{\sigma_{inlet}^*}{\sigma_{inlet_n}^*} \quad (230)$$

6. Finally calculate Thrust:

$$F = F_{sp} \dot{m}_a \quad (231)$$

