Performance Analysis of the PT6A-114A Engine

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1 Preface

2 Engine Introduction

As per the Pratt and Whitney website, the PT6A-114 is a turboprop engine that is used in a variety of applications. The PT6A class is the most popular engine in the world for its class and one of Pratt and Whitney's greatest success stories. The PT6A is used in a variety of applications, specifically for this project we will be looking at the PT6A-114A engine; which is primarily used by the Cessna 208/208B Caravan I.

3 Full Nominal Engine Cycle - Task 1

As stated earlier, the PT6A-114 engine is a Turboprop engine. A turboprop engine creates power from a shaft. Thrust is then produced from a combination of the propeller as well as the exhaust gas. The thrust produced from the propeller is far above the thrust provided by the exhaust gas. Given shaft horsepower, the efficiency of the compressor, and the inlet temperature of the turbine we find the specific thrust of the engine.

Below is a list of given engine parameters:

$$
\begin{aligned}
\text{Shaft Horsepower, } SHP &= 600 \text{ hp} \\
\text{Compressor Efficiency, } \eta_{comp} &= 0.90 \\
\text{Turbine Inlet Temperature, } TIT &= 1410 \text{ K} \\
\text{Compressor Ratio, } \pi_c &= 9.2 \\
\text{Turbine Efficiency, } \eta_{turb} &= 0.94 \\
\text{Combustor Efficiency, } \eta_{comb} &= 0.90 \\
\text{Free Turbine Efficiency, } \eta_{PT} &= 0.94 \\
\text{Nozzle Efficiency, } \phi_{nozzle} &= 0.98 \\
\text{Propeller Efficiency, } \eta_{prop} &= 0.8 \\
\text{Diffuser Efficiency, } \eta_d &= 0.99
\end{aligned} \tag{1}
$$

3.1 Stage a

We begin at static conditions, specifically here we are calculating the static conditions at take-off. Since we are at take-off conditions, one can assume sea level atmospheric conditions.

$$
\begin{aligned}
\text{Static Pressure, } p_a &= 101.325 \text{ kPa} \\
\text{Static Temperature, } T_a &= 288.16 \text{ K}
\end{aligned} \tag{2}
$$

By knowing the Temperature T_a we are able to calculate enthalpy from the air gas tables:

$$h_a(T_a)_{Tables} = 288.299988 \frac{kJ}{kg} \tag{3}$$

And for entropy:

$$s_{a_{p=1bar}}(T_a)_{Tables} = 6.6570201 \frac{kJ}{kgK} \tag{4}$$

After reading from the tables for entropy, we will need to normalize to the static pressure of p_a.

$$s_a = s_{a_{p=1bar}}(T_a) - R \ln \left(\frac{p_a}{1 \text{ bar}} \right) \tag{5}$$

3.2 Stage 0a

In this stage we going to assume that stage 0a is equivalent to stage a.

$$
\begin{aligned}
p_{0a} &= 101.325 \text{ kPa} \\
T_{0a} &= 288.16 \text{ K} \\
h_{0a} &= 288.299988 \frac{kJ}{kg} \\
s_{0a} &= 6.6570201 \frac{kJ}{kgK}
\end{aligned} \tag{6}
$$

3.3 Stage 01

Here in this stage, we take into account the Diffuser.

$$p_{01} = p_{0a} \eta_d \tag{7}$$

$$T_{01} = T_{0a} \tag{8}$$

$$h_{01}(T_{01})_{Tables} = 288.299988 \frac{kJ}{kg} \tag{9}$$

$$s_{01_{p=1bar}}(T_{01})_{Tables} = 6.6570201 \frac{kJ}{kgK} \tag{10}$$

$$s_{01} = s_{01_{p=1bar}}(T_{01}) - R \ln\left(\frac{p_{01}}{1 \text{ bar}}\right) \tag{11}$$

3.4 Stage 02i

Here we are calculating the isentropic conditions of the compressor, stage 02i. At this point, we say that:

$$s_{02i} = s_{01} \tag{12}$$

Calculating the pressure, we use the pressure ratio of the compressor:

$$p_{02i} = p_{01}\pi_c \tag{13}$$

Recall, we need to normalize the entropy to 1 bar pressure to be used for the tables:

$$s_{02i_{p=1bar}} = s_{02i} + R \ln\left(\frac{p_{02i}}{1 \text{ bar}}\right) \tag{14}$$

We can use the entropy found, to use the H method, to find enthalpy:

$$h_{02i}(s_{02i_{p=1bar}})_{Tables} = 543.819 \frac{kJ}{kg} \tag{15}$$

We can now calculate the ideal work of the compressor:

$$w_{02i} = h_{02i} - h_{01} \tag{16}$$

3.5 Stage 02

The pressure at Stage 02 is the same as the pressure at Stage 02i. This allows for us to move forward through the engine cycle.

$$p_{02} = p_{02i} \tag{17}$$

p_{02}
9.228681 bar

Using the ideal work of the compressor, we can calculate the actual work of the compressor:

$$w_{02} = \frac{w_{02i}}{\eta_{comp}} \tag{18}$$

w_{02}
283.91025 kJ/kg

This will allow for us to easily calculate the enthalpy.

$$h_{02} = h_{01} + w_{02} \tag{19}$$

This enthalpy can now be used for the S method, to find entropy:

$$s_{02_{p=1bar}}(h_{02})_{Tables} = 6.7113694 \frac{kJ}{kgK} \tag{20}$$

Finally we normalize the entropy:

$$s_{02} = s_{02_{p=1bar}} - R \ln\left(\frac{p_{02}}{1 \text{ bar}}\right) \tag{21}$$

3.6 Stage 03

We were given a pressure drop in the combustor of 3%. Resulting in the following pressure:

$$p_{03} = p_{02}\left(1 - \frac{3}{100}\right) \tag{22}$$

p_{03}
8.95182057 bar

We are also given a T_{03} of 1410 K.

$$h_{03_{air}}(T_{03})_{AirTables} = h_{03_{air}} \tag{23}$$

$$h_{03_{\lambda=1}}(T_{03})_{StoichTables} = h_{03_{\lambda=1}} \tag{24}$$

We then use the two above equations to calculate excess air:

$$\lambda = \frac{h_{03_{\lambda=1}}(1 + minL) - \eta_{comb}LHV - h_{03_{air}}minL}{minL(h_{02} - h_{03_{air}})} \tag{25}$$

Recall:

$$\begin{aligned} LHV &= 43.5\frac{kJ}{kg} \\ minL &= 14.66 \end{aligned} \tag{26}$$

λ
2.8537672

We can now calculate our weighting functions:

$$r = \frac{1 + minL}{1 + \lambda minL} \tag{27}$$

$$q = \frac{(\lambda - 1)minL}{1 + \lambda minL} \tag{28}$$

With the weighting functions, we can now calculate the enthalpy of the mixture:

$$h_{03} = rh_{03_{\lambda=1}} + qh_{03_{air}} \tag{29}$$

Similar to how we calculated the enthalpies from the table, we will now calculate the entropy of the mixture:

$$s_{03_{air,p=1bar}}(T_{03})_{AirTables} = s_{03_{air,p=1bar}} \tag{30}$$

$$s_{03_{\lambda=1,p=1bar}}(T_{03})_{StoichTables} = s_{03_{\lambda=1,p=1bar}} \tag{31}$$

$$s_{03_{p=1\lambda}} = rs_{03_{\lambda=1,p=1bar}} + qs_{03_{air,p=1bar}} \tag{32}$$

3.7 Stage 04i

The entropy of stage 04i is the same as the entropy of stage 03. This allows for us to move forward through the engine cycle.

$$s_{04i} = s_{03} \tag{33}$$

We make use of:

$$\mathcal{P}_C = \mathcal{P}_T \tag{34}$$

Which can be simplified to:

$$\dot{m}_g w_T = \dot{m}_a w_C \tag{35}$$

$$w_T = w_C \left(\frac{1}{1 + \frac{1}{\lambda minL}} \right) \tag{36}$$

w_T
277.2824 kJ/kg

This will allow for us to calculate the ideal work of the turbine:

$$w_{T_i} = \frac{w_T}{\eta_T} \tag{37}$$

We will use the ideal work of the turbine to calculate the enthalpy of stage 04i:

$$h_{04i} = h_{03} - w_{T_i} \tag{38}$$

From here we will use the enthalpy h_{04i} to calculate the Temperature of stage 04i. To do so, one must iterate, such that the following equation is satsified:

$$f(T) = h_{04i} - r h_{04i_{\lambda=1}}(T) - q h_{04i_{air}}(T) = 0 \tag{39}$$

Our group developed an iteration algorithm `Iterate_temp_h` to solve for the temperature. The algorithm is as follows:

$$\begin{gathered} i = 1 \\ h_{air} = h_{input} \\ T_{air_{i=1}}(h_{input})_{AirTables} = T_{a_{i=1}} \\ h_{\lambda=1_{i=1}}(T_{a_{i=1}})_{StoichTables} = h_{\lambda=1_{i=1}} \end{gathered}$$

$$\begin{gathered} i = 2 \\ h_{\lambda=1_{i=2}} = h_{input} \\ T_{\lambda=1_{i=2}}(h_{input})_{StoichTables} = T_{\lambda=1_{i=2}} \\ h_{air_{i=2}}(T_{\lambda=1_{i=2}})_{AirTables} = h_{air_{i=2}} \end{gathered}$$

$$\begin{gathered} f(T)_1 = h_{input} - r h_{\lambda=1_{i=1}} - q h_{air_{i=1}} \\ f(T)_2 = h_{input} - r h_{\lambda=1_{i=2}} - q h_{air_{i=2}} \end{gathered}$$

$$T = \frac{T_{air_{i=1}} * f(T)_2 - T_{\lambda=1_{i=2}} * f(T)_1}{f(T)_2 - f(T)_1}$$

After the `Iterate_temp_h(h04i, r, q)` algorithm is ran, we now have our temperature T_{04i}. We can now begin out entropy calculation.

$$s_{04i_{air,p=1bar}}(T_{04i})_{AirTables} = s_{04i_{air,p=1bar}} \tag{40}$$

$$s_{04i_{\lambda=1,p=1bar}}(T_{04i})_{StoichTables} = s_{04i_{\lambda=1,p=1bar}} \tag{41}$$

$$s_{04i_{p=1\lambda}} = rs_{04i_{\lambda=1,p=1bar}} + qs_{04i_{air,p=1bar}} \tag{42}$$

$$s_{04i_{\lambda}} = s_{04i_{p=1\lambda}} - R\ln\left(\frac{p_{04i}}{1bar}\right) \tag{43}$$

We can manipulate the above equation for pressure:

$$p_{04i} = e^{-\frac{s_{04i_{\lambda}} - s_{04i_{p=1\lambda}}}{R}} \tag{44}$$

3.8 Stage 04

Moving from stage 04i to stage 04, we can assume no pressure change meaning:

$$p_{04} = p_{04i} \tag{45}$$

We can calculate the enthalpy of stage 04 using the work of the turbine:

$$h_{04} = h_{03} - w_T \tag{46}$$

From here we are to calculate the temperature by the iterations algorithm again:

$$\texttt{Iterate\_temp\_h(h04, r, q)} = T_{04}$$

We can use the temperature to calculate the entropy of stage 04:

$$s_{04_{air,p=1bar}}(T_{04})_{AirTables} = s_{04_{air,p=1bar}} \tag{47}$$

$$s_{04_{\lambda=1,p=1bar}}(T_{04})_{StoichTables} = s_{04_{\lambda=1,p=1bar}} \tag{48}$$

$$s_{04_{p=1\lambda}} = rs_{04_{\lambda=1,p=1bar}} + qs_{04_{air,p=1bar}} \tag{49}$$

Finally we can calculate the entropy of stage 04, by normalizing the pressure:

$$s_{04_{\lambda}} = s_{04_{p=1\lambda}} - R\ln\left(\frac{p_{04}}{1bar}\right) \tag{50}$$

3.9 Stage 04.5i

One can assume that the transition from stage 04 to stage 04.5i is isentropic. This means that the entropy of stage 04.5i is equal to the entropy of stage 04:

$$s_{04.5i_{\lambda}} = s_{04_{\lambda}} \tag{51}$$

The specific work of the of the free turbine is:

$$w_{PT} = \frac{SHP}{\dot{m}_g} = \frac{SHP}{\dot{m}_a(1+f)} \tag{52}$$

Which can be simplified as:

$$w_{PT} = \frac{SHP}{\dot{m}_a\left(1 + \frac{1}{\lambda minL}\right)} \tag{53}$$

w_{PT}
262.83965 kJ/kg

Now we can calculate the ideal work of the free turbine, by making use of its efficiency:

$$w_{PT_i} = \frac{w_{PT}}{\eta_{PT}} \tag{54}$$

The enthalpy of stage 04.5i is:

$$h_{04.5i} = h_{04} - w_{PT_i} \tag{55}$$

We can now calculate the temperature of stage 04.5i by using the iterations algorithm:

`Iterate_temp_h(h04.5i, r, q)` $= T_{04.5i}$

We can now calculate the entropy of stage 04.5i:

$$s_{04.5i_{air,p=1bar}}(T_{04.5i})_{AirTables} = s_{04.5i_{air,p=1bar}} \tag{56}$$

$$s_{04.5i_{\lambda=1,p=1bar}}(T_{04.5i})_{StoichTables} = s_{04.5i_{\lambda=1,p=1bar}} \tag{57}$$

$$s_{04.5i_{p=1\lambda}} = rs_{04.5i_{\lambda=1,p=1bar}} + qs_{04.5i_{air,p=1bar}} \tag{58}$$

We can use this to calculate pressure:

$$p_{04.5i} = e^{-\frac{s_{04.5i_{\lambda}} - s_{04.5i_{p=1\lambda}}}{R}} \tag{59}$$

$p_{04.5i}$
1.6164346 bar

3.10 Stage 04.5

The pressure at stage 04.5 is equal to the pressure at stage 04.5i:

$$p_{04.5} = p_{04.5i} \tag{60}$$

The enthalpy of stage 04.5 is:

$$h_{04.5} = h_{04} - w_{PT} \tag{61}$$

We will use the iterations algorithm to calculate the temperature of stage 04.5:

`Iterate_temp_h(h04.5, r, q)` $= T_{04.5}$

We can now calculate the entropy of stage 04.5:

$$s_{04.5_{air,p=1bar}}(T_{04.5})_{AirTables} = s_{04.5_{air,p=1bar}} \tag{62}$$

$$s_{04.5_{\lambda=1,p=1bar}}(T_{04.5})_{StoichTables} = s_{04.5_{\lambda=1,p=1bar}} \tag{63}$$

$$s_{04.5_{p=1\lambda}} = rs_{04.5_{\lambda=1,p=1bar}} + qs_{04.5_{air,p=1bar}} \tag{64}$$

Normalize entropy by pressure:

$$s_{04.5_{\lambda}} = s_{04.5_{p=1\lambda}} - R\ln\left(\frac{p_{04.5}}{1bar}\right) \tag{65}$$

3.11 Stage 5i

The process from Stage 04.5 to Stage 5i is isentropic. This means that the entropy of stage 5i is equal to the entropy of stage 04.5:

$$s_{05i_\lambda} = s_{04.5_\lambda} \tag{66}$$

And the pressure of stage 5i goes to atmospheric pressure:

$$p_{05i} = p_a = p_{01} \tag{67}$$

Having determined the entropy of stage 5i and the pressure of stage 5i, we must now determine the temperature of stage 5i. We will use the iterations algorithm to calculate the temperature of stage 5i: The iterative process `Iterate_temp_ps` is similar to `Iterate_temp_h` algorithm however it solves:

$$f(T) = s_{5_i} - qs_{5_{i_{air}}}(T) - rs_{5_{i_{\lambda=1}}}(T) = 0 \tag{68}$$

Where;

$$s_{5_{i_{air}}} = s_{5_{i_{air,p=1bar}}} - R\ln\left(\frac{p_{5_i}}{1bar}\right) \tag{69}$$

$$s_{5_{i_{\lambda=1}}} = s_{5_{i_{\lambda=1,p=1bar}}} - R\ln\left(\frac{p_{5_i}}{1bar}\right) \tag{70}$$

Ultimately, the algorithm will be solving:

$$\begin{gathered} s_{input} \\ s_{input_{p=1bar}} = s_{input} + R\ln\left(\frac{p_{input}}{1bar}\right) \\ s_{algorithm} = s_{input_{p=1bar}} \end{gathered}$$

$$\begin{gathered} i = 1 \\ s_{air_{i=1}} = s_{algorithm} \\ T_{air_{i=1}}(s_{algorithm})_{AirTables} = T_{air_{i=1}} \\ s_{\lambda=1_{i=1}}(T_{air_{i=1}}) = s_{\lambda=1_{i=1}} \end{gathered}$$

$$\begin{gathered} i = 2 \\ s_{\lambda=1_{i=2}} = s_{algorithm} \\ T_{\lambda=1_{i=2}}(s_{algorithm})_{StoichTables} = T_{\lambda=1_{i=2}} \\ s_{air_{i=2}}(T_{\lambda=1_{i=2}}) = s_{air_{i=2}} \end{gathered}$$

$$\begin{gathered} f(T)_1 = s_{algorithm} - rs_{\lambda=1_{i=1}} - qs_{air_{i=1}} \\ f(T)_2 = s_{algorithm} - rs_{\lambda=1_{i=2}} - qs_{air_{i=2}} \end{gathered}$$

$$T_0 = \frac{T_{air_{i=1}} * f(T)_2 - T_{\lambda=1_{i=2}} * f(T)_1}{f(T)_2 - f(T)_1}$$

$$\begin{gathered} i = 3 \\ s_{air_{i=3}}(T_0) = s_{air_{i=3}} \\ s_{\lambda=1_{i=3}}(T_0) = s_{\lambda=1_{i=3}} \end{gathered}$$

$$f(T)_3 = s_{algorithm} - rs_{\lambda=1_{i=3}} - qs_{air_{i=3}}$$

$$T = T_0 - \frac{f(T_3)}{s_{air_{i=3}} - s_{\lambda=1_{i=3}}}$$

We can make use of the `Iterate_temp_ps` algorithm to calculate the temperature of stage 5i:

$$\texttt{Iterate\_temp\_ps(s5i, r, q)} = T_{5i}$$

Once we have the temperature of stage 5i, we can calculate the enthalpy of stage 5i:

$$h_{5i} = qh_{5i_{air}}(T_{5i}) + rh_{5i_{\lambda=1}}(T_{5i}) \tag{71}$$

By making use of the exit enthalpies and assuming that the heat transfer is small compared to enthalpy variation, we can calculate the velocity of the fluid:

$$c_{5_i} = \sqrt{2(h_{04.5} - h_{5_i})} \tag{72}$$

3.12 Stage 5

The process from Stage 5i to Stage 5 is isentropic. This means that the entropy of stage 5 is equal to the entropy of stage 5i. As a matter of fact, we assume that stage 5 is equal to stage 5i, in most properties:

$$\begin{aligned} T_5 &= T_{5i} \\ p_5 &= p_{5i} \\ h_5 &= h_{5i} \\ s_5 &= s_{5i} \end{aligned}$$

We can calculate the velocity of the fluid, by accounting for the nozzle efficiency:

$$c_5 = c_{5_i}\phi_{nozzle} \tag{73}$$

The Specific Thrust is calculated by:

$$F_{sp} = \dot{m}_{air}\left(1 + \frac{1}{\lambda minL}\right)c_5 \tag{74}$$

F_{sp}
785.96769 N

The equivalent shaft power is calculated by:

$$\mathcal{P}_{es} = \eta_{prop}\mathcal{P}_s + \frac{F_{sp}}{11.1} \tag{75}$$

Recall $\mathcal{P}_s = SHP$.

One can calculate mass flow rate of teh gases by:

$$\dot{m}_g = \dot{m}_{air}\left(\frac{1}{\lambda minL}\right) \tag{76}$$

The equivalent brake specific fuel consumption is calculated by:

$$EBSFC = \frac{\dot{m}_g}{\mathcal{P}_{es}} \tag{77}$$

$EBSFC$
0.332946

3.13 Cycle Results

Stage	Pressure [bar]	Enthalpy [kJ/kg]	Entropy [kJ/kg-K]
a	1.01325	288.299988	6.6570201
0a	1.01325	288.299988	6.6570201
01	1.01325	288.299988	6.6570201
02i	9.228681	543.819214	6.6570201
02	9.228681	572.210239	6.7113694
03	8.9518206	1574.3477	7.8717827
04i	4.030023	1279.3664	7.8717827
04	4.030023	1297.06528	7.886781
045i	1.616434698	1017.44862	7.886781
045	1.616434698	1034.225618	7.904305
5i	1.0031175	920.29226	7.904305
5	1.0031175	920.29226	7.904305

$$w_c = 283.910251[\tfrac{kJ}{kg}]$$
$$w_T = 277.28244[\tfrac{kJ}{kg}]$$
$$w_{PT} = 262.83966[\tfrac{kJ}{kg}]$$
$$\dot{m}_{air} = 1.64089[\tfrac{kJ}{s}]$$
$$F_{sp} = 785.96769[N]$$
$$EBSFC = 0.332946$$

4 Axial Compressor Design - Task 2

The engine analyzed was a PT6A-114 which is a turboprop and thus contains a mixed compressor. Using values calculated in the previous assignment in which the engine's real cycle was calculated, here we are designing the axial component of the mixed compressor. Our design began with the process below in which the number of stages in the compressor was determined. Using the work in the compressor found from the previous assignment, this quantity is divided by two due to the fact that a mixed compressor contains both an axial and centrifugal component so it is efficient to divide the work of the compressor amongst these components evenly.

We can calculate the work per stage by:

$$w_{s_1} = 0.7 w_{stage} \tag{78}$$

$$w_{s_2} = 0.85 w_{stage} \tag{79}$$

$$w_{s_n} = 0.9 w_{stage} \tag{80}$$

$$w_{actual} = \frac{w - w_{s_1} - w_{s_2} - w_{s_n}}{n-3} \tag{81}$$

Where w_{stage} is the work per stage, n is the number of stages. Also w is:

$$w = \frac{w_c}{2} \tag{82}$$

Once the number of stages was determined, next came the preliminary calculations that were needed in order to perform the calculations of desired quantities from each of the stages.

$$u_1^m = u_1^t \left(\frac{1 + \bar{d}_1}{2} \right) \tag{83}$$

Where $\bar{d}_1 \in [0.5, 0.7]$ and $u_1^t \in [330, 350] \frac{m}{s}$

We can next find the flow coefficient by:

$$\frac{c_{1_a}^m}{u_1^m} \in [0.3, 0.6] \tag{84}$$

From here we can calculate λ number:

$$\lambda_{1_a}^m = \frac{c_{1_a}^m}{\sqrt{2\left(\frac{\gamma-1}{\gamma+1}\right)h_{01}}} \tag{85}$$

$$\lambda_1^m = \frac{\lambda_{1_a}^m}{sin(\tilde{\alpha}_1^m)} \tag{86}$$

Absolute velocity at the midpoint:

$$c_1^m = \lambda_1^m \sqrt{2\left(\frac{\gamma-1}{\gamma+1}\right)h_{01}} \tag{87}$$

The transport component of the flow velocity:

$$c_{1_u}^m = \sqrt{(c_1^m)^2 - (c_{1_a}^m)^2} \tag{88}$$

The relative velocity in the u direction:

$$w_{1_u} = u_1^m - c_{1_u}^m \tag{89}$$

The relative flow velocity:

$$w_1^m = \sqrt{(w_{1_u})^2 + (c_{1_u})^2} \tag{90}$$

Mass flow rate function, as a function of λ:

$$q(\lambda_1^m) = \lambda_1^m \left[\frac{\gamma+1}{2}\left(1 - \frac{\gamma-1}{\gamma+1}(\lambda_1^m)^2\right)\right] \tag{91}$$

We can now the Area of the compressor inlet:

$$A_1 = \frac{\dot{m}\sqrt{T_{01}}}{p_{01}} \frac{1}{q(\lambda_1^m) sin(\tilde{\alpha}_1^m)\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}} \tag{92}$$

We can calculate the Diameter of the Tip:

$$D_1^t = \sqrt{\frac{4A_1}{\pi(1-\bar{d}_1)^2}} \tag{93}$$

Diameter of the Hub:

$$D_1^h = \bar{d}_1 D_1^t \tag{94}$$

Diameter of the Midpoint:

$$D_1^m = \frac{D_1^t + D_1^h}{2} \tag{95}$$

The number of blades:

$$n_1 = \frac{60u_1^m}{\pi D_1^m} \tag{96}$$

4.1 Stage 1

Calculating the axial velcoity for the first stage:

$$c_{1_a}^{(1)} = c_1^{(1)} sin(\tilde{\alpha}_1^{(1)}) \tag{97}$$

$$c_{1_u}^{(1)} = c_1^{(1)} cos(\tilde{\alpha}_1^{(1)}) \tag{98}$$

The transport velocity:

$$U^{(1)} = \frac{2\pi D^{m(1)}}{2}\frac{n}{60} \tag{99}$$

$$\alpha^{(1)} = 90 - \left(tan^{-1}\left(\frac{c_{1_{[u]}}^{(1)}}{c_{1_a}^{(1)}}\right)\right) \tag{100}$$

Relative velocity for the 1st stage in teh u direction:

$$W_{1_u^{(1)}} = U^{(1)} - c_{1_u^{(1)}} \tag{101}$$

Change of C_u over the stage:

$$\Delta^{(stage)} C_u^{(1)} = \left[\frac{\left(W_{1_u}^{(1)} - \frac{W_s^{(1)}}{2U^{(1)}}\right)\frac{w_s^{(1)}}{U^{(1)}}}{R'^{(1)}} - c_{1_a}^{(1)}\Delta^{(stage)} C_a^{(1)} - w_s^{(1)}\right]\frac{1}{c_{1_u^{(1)}}} \tag{102}$$

Where in $\Delta^{(stage)} C_u^{(1)}$ the values of $R'^{(1)}$ and $\Delta^{(stage)} C_a^{(1)}$ were imposed which is also the case for the subsequent stages of the axial compressor. For the following stages, a similar process as shown above was computed in order to determine the desired quantities of R', w_s, $\Delta^{(stage)} C_a$, $\Delta^{(stage)} C_u$, and $\alpha^{(1)}$. For context, a generalized derivation of quantities is shown below in which (n) relates to the stage

4.2 Stage n

$$c_{1_a}^{(n)} = c_{1_a}^{(n-1)} - \Delta^{(stage)} C_a^{(n-1)} \tag{103}$$

$$c_{1_u}^{(n)} = c_{1_u}^{(n-1)} - \Delta^{(stage)} C_u^{(n-1)} \tag{104}$$

$$c_1^{(n)} = \sqrt{(c_{1_a}^{(n)})^2 + (c_{1_u}^{(n)})^2} \tag{105}$$

$$\alpha_1^{(n)} = 90 - \left(tan^{-1}\left(\frac{c_{1_u}^{(n)}}{c_{1_a}^{(n)}}\right)\right) \tag{106}$$

The enthalpy at each stage:

$$h_{01}^{(n)} = h_1^{(n-1)} + w_s \tag{107}$$

The pressure ratio at each stage:

$$\pi_c^{(n-1)} = \left(1 + \frac{\eta_{1_m} w_s}{h_{01}^{(n-1)}}\right)^{\frac{\gamma}{\gamma-1}} \tag{108}$$

The pressure at each stage:

$$p_{01}^{(n)} = p_{01}^{(n-1)}\pi_c^{(n-1)} \tag{109}$$

The critical speed of sound at each stage:

$$a_{cr}^{(n)} = \sqrt{2h_{01}^{n}\left(\frac{\gamma-1}{\gamma+1}\right)} \tag{110}$$

The λ number at each stage:

$$\lambda_1^{(n)} = \frac{c_1^{(n)}}{a_{cr}^{(n)}} \tag{111}$$

The mass flow rate as a function of λ at each stage:

$$q(\lambda_1^{(n)}) = \lambda_1^{(n)}\left[\frac{\gamma+1}{2}\left(1-\frac{\gamma-1}{\gamma+1}(\lambda_1^{(n)})^2\right)\right]^{\frac{1}{\gamma-1}} \tag{112}$$

The Area at each stage:

$$A_1^n = \frac{\dot{m}\sqrt{T_{01}^{(n)}}}{p_{01}^{(n)}\frac{1}{q(\lambda_1^{(n)})sin(\alpha_1^{(n)})}\sqrt{\left(\frac{\gamma}{R}\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}} \tag{113}$$

The Diameter at each stage:

$$D_{(n)}^t = D_{(n-1)}^t \tag{114}$$

$$D^{h(n)} = \sqrt{\left(D_{(n)}^t\right)^2 - \left(\frac{4A_1^{(n)}}{\pi}\right)} \tag{115}$$

$$D^{m(n)} = 0.5(D^{h(n)} + D^{h(n)}) \tag{116}$$

The transport velocity at each stage:

$$U^{(n)} = \frac{2\pi D^{m(n)}}{2}\frac{n}{60} \tag{117}$$

The relative velocity in the u direction at each stage:

$$W_{1_u^{(n)}} = U^{(n)} - c_{1_u^{(n)}} \tag{118}$$

The change in C_u at each stage:

$$\Delta^{(stage)}C_u^{(n)} = \left[\frac{\left(W_{1_u}^{(n)} - \frac{W_s^{(n)}}{2U^{(n)}}\right)\frac{w_s^{(n)}}{U^{(n)}}}{R'^{(n)}} - c_{1_a}^{(n)}\Delta^{(stage)}C_a^{(n)} - w_s^{(n)}\right]\frac{1}{c_{1_u^{(n)}}} \tag{119}$$

4.3 Axial Design Results

Stage	c_a	c_u	p	α	R'	U
1	157.5	27.771499	1.01325	1.39626340	0.68584	262.499
2	156.0	24.74814	1.401547	1.4134655	0.700413	286.6517
3	154.0	23.4701395	2.003553	1.4195566	0.679087	303.6294
4	151.5	20.22314	2.925198	1.438095	0.745890896837544	315.5214

$$u_1^m = 262.5[\frac{m}{s}]$$
$$c_{1_a} = 157.5[\frac{m}{s}]$$
$$c_{1_u} = 27.7715[\frac{m}{s}]$$
$$w_1 = 236.36566[\frac{m}{s}]$$
$$w_{1_u} = 234.59416[\frac{m}{s}]$$
$$D_1^m = 0.0953[m]$$
$$n = 52612.4648[rpm]$$
$$D_1^t = 0.1270518[m]$$
$$D_1^h = 0.0635259[m]$$

5 Radial Variation of a flow over Airfoil Geometry - Task 3