Space Shuttle Main Engine

November 4, 2022

Contents

1	Introduction					
2	Oxygen					
	2.1	First Enthalpy Exchanger	2			
	2.2	Combustion Chamber	4			
	2.3	Nozzle Performance	9			
\mathbf{A}	Cod	le listings	10			
	A.1	SSMEMain.py	10			
	A.2	exchanger.py	11			
	A.3	Iterater.py	18			

1 Introduction

The Space Shuttle Main Engine (SSME) is a liquid-fueled rocket engine used to power the Space Shuttle Orbiter. This report will discuss the design and methodologies of the SSME code, as well as the results of the code. For this project, we were given the parameters below:

Combustion Chamber Data	
Fuel	Hydrogen
Oxidizer	O_2
Fuel/Oxidizer Ratio	0.166
Hydrogen Injection Temperature	850 K
Oxidizer Injection Temperature	530 K
Chamber Pressure	204 atm

R_t	5.15 inch
R_e	45.35 inch
θ_p	32 degree

2 Oxygen

2.1 First Enthalpy Exchanger

We begin with computing the equivalence ratio ϕ for the oxygen first enthalpy exchanger. The equivalence ratio is defined as the ratio of the mass of fuel to the mass of oxidizer. The mass of fuel is the mass of hydrogen, and the mass of oxidizer is the mass of oxygen. The mass of hydrogen is $\phi \frac{mol_{O_2}}{mol_{H_2}} = \Phi$.

From here we are to calculate the stndard heat of formation by:

$$\Delta \hat{h}_{H_2} = \hat{h}_{H_2}(850) + \hat{R}T^2 \int_{pf}^{pi} \frac{1}{p} \frac{-\hat{b}_{H_2}p}{\hat{R}T^2} dp$$
 (1)

$$\Delta \hat{h}_{O_2} = \hat{h}_{O_2}(530) + \hat{R}T^2 \int_{pf}^{pi} \frac{1}{p} \frac{-\hat{b}_{O_2}p}{\hat{R}T^2} dp$$
 (2)

Species	\hat{b}
H_2	0.0266
O_2	0.0318
H_2O	0.03049
N_2	0.0391
F_2	0.02896
HF	0.0739

Where \hat{b} is read from a table developed in exchanger.py file. $\Delta \hat{h}_i$ is calculated from both hydrogen and oxygen. Once we calculate $\Delta \hat{h}_i$ for each, we can calculate the enthalpy of the mixture.

$$\Delta \hat{h}_{mix} = \Phi \Delta \hat{h}_{H_2} + \Delta \hat{h}_{O_2} \tag{3}$$

We can then apply this to our chemical equation.

$$O_2 + \Phi H_2 \to (2 - \Phi)H_2 + 2H_2O$$
 (4)

Note: notice the $(2 - \Phi)$ in the equation above. This is because we calculated the enthalpy of formation for hydrogen and not hydrogen gas H_2 .

From here we can sum the molar enthalpies of formation to get the first estimate of the first reaction enthalpy

$$\Delta \hat{h}_r = \Phi \Delta \hat{h}_{H_2} + 2(\Delta \hat{h}_{H_2O}) \tag{5}$$

From here we go into our Temperature finding Algorithm. We are given the following parameters:

$$r = \Phi - 1 \tag{6}$$

$$q = 1 \tag{7}$$

$$r_{percent} = \frac{r}{r+q} \tag{8}$$

$$q_{percent} = \frac{q}{r+q} \tag{9}$$

From here we use the values of (Eq. 6) - (Eq. 9) to calculate the temperature of the first enthalpy exchanger.

$$T_{H_2} = \text{h2_tableT}(\Delta \hat{h}_r) \tag{10}$$

$$T_{H_2O} = \text{ho2_tableT}(q_{percent}\Delta \hat{h}_r)$$
 (11)

Once we have returned the values of Temperature, we then average the two values to get the final temperature of the first enthalpy exchanger.

$$T_C = \frac{T_{H_2} + T_{H_2O}}{2} \tag{12}$$

2.2 Combustion Chamber

For the Combustion Chamber, we have the two equations below:

$$H_2O \to OH + H$$
 (13)

$$H_2 \to 2H$$
 (14)

The Gibbs Free Energy for each species is given by:

$$\hat{g}_{H_2O} = \text{h2o_table_g}(T_C) \tag{15}$$

$$\hat{g}_{H_2} = \text{h2_table_g}(T_C) \tag{16}$$

$$\hat{g}_{OH} = \text{oh_table_g}(T_C) \tag{17}$$

$$\hat{g}_H = h_{table_g(T_C)} \tag{18}$$

We can then calculate the Gibbs Free Energy of the reaction by:

$$\Delta \hat{g}_r = 2(\hat{g}_H - \hat{g}_{H_2}) \tag{19}$$

$$\Delta \hat{g}_r = \hat{g}_H - \hat{g}_{OH} + \hat{g}_{H2O} \tag{20}$$

Now is the point where we involve the **Law of Mass Action**:

Species	$\nu'_{(i)}$	$\nu''_{(i)}$	$\nu''_{(i)} - \nu''_{(i)}$
H_2O	1	0	-1
OH	0	1	1
Н	0	1	1

We an now use:

$$K_n = \prod_{i=1}^{N_s} X_{(n)}^{(\nu''_{(i)} - \nu'_{(i)})} = (\frac{p}{p_a})^{-\sigma_v} K_p$$
 (21)

Where;

$$\sigma_v = \sum_{i=1}^{N_s} (\nu''_{(i)} - \nu'_{(i)}) \tag{22}$$

$$K_p = e^{-\frac{\Delta G}{\hat{R}T}} \tag{23}$$

$$\Delta G = \sum_{i=1}^{N_s} (\nu''_{(i)} \hat{g}_{(i)}) \hat{g}_{(i)}$$
 (24)

From here we can solve for the equilibrium constant K_n .

$$K_{H_2O} = \left(\frac{204atm}{1atm}\right)^{-\frac{\Delta G_{H_2O}}{\hat{R}^T C}} \tag{25}$$

$$K_{H_2} = \left(\frac{204atm}{1atm}\right)^{-\frac{\Delta G_{H_2}}{\hat{R}T_C}} \tag{26}$$

Now we move into the chemistry problem.

Atom Balance:

$$H_2 + lO_2 \rightarrow N_{H_2O}H_2O + N_{H_2}H_2 + N_HH + N_{OH}OH$$
 (27)

Recall, due to the fule being Hydrogen;

$$mass_{fuel} = 2.02 (28)$$

$$mass_{Oxidizer} = \frac{mass_{fuel}}{\phi} \tag{29}$$

$$l = \frac{mass_{Oxidizer}}{2\hat{m}_{O_2}} \tag{30}$$

Where \hat{m}_{O_2} is the molar mass of Oxygen. And ϕ is the Fuel to Oxidizer ratio, and not the equivalence ratio. Revisiting the **Atom Balance** equation, we can now solve for the number of moles of each species.

For $H_2O \to OH + H$:

$$H_2: 2 = 2N_{H_2O} + 2N_{H_2} + N_H + N_{OH}$$
(31)

$$O_2: 2l = N_{H_2O} + N_{OH} (32)$$

$$1 = N_{H_2O} + N_{H_2} + \frac{1}{2}N_H + \frac{1}{2}N_{OH}$$
 (33)

Where $\frac{1}{2}N_H + \frac{1}{2}N_{OH} \rightarrow 0$. Resulting in:

$$1 = N_{H_2O} + N_{H_2} (34)$$

$$2(0.38050899) = N_{H_2O} + N_{OH} \tag{35}$$

Where $N_{OH} \rightarrow 0$. Resulting in:

$$0.760592 = N_{H_2O} \tag{36}$$

$$X = \frac{N_{Molecule}}{N_{Total}} \tag{37}$$

$$X_{H_2O} = \frac{N_{H_2O}}{N_{Total}} \tag{38}$$

$$X_H = \frac{N_H}{N_{Total}} \tag{39}$$

$$X_{OH} = \frac{N_{OH}}{N_{Total}} \tag{40}$$

$$X_{H_2O} = \frac{N_{H_2O}}{N_{H_2O} + N_{H_2} + N_H + N_{OH}} \tag{41}$$

$$X_H = \frac{N_H}{N_{H_2O} + N_{H_2} + N_H + N_{OH}} \tag{42}$$

$$X_{OH} = \frac{N_{OH}}{N_{H_2O} + N_{H_2} + N_H + N_{OH}}$$
 (43)

Recall, $N_{OH} \rightarrow 0$ and $N_H \rightarrow 0$. Resulting in:

$$K_{H_2O} = \frac{\frac{N_{OH}}{N_{H_2O+N_{H_2}}} \frac{N_H}{N_{H_2O+N_{H_2}}}}{\frac{N_{H_2O}}{N_{H_2O+N_{H_2}}}} \tag{44}$$

Resulting in,

$$K_{H_2O} = \frac{N_H N_{OH}}{(N_H + N_{H_2O})N_{H_2O}} \tag{45}$$

Similarly we follow the same steps for the $H_2 \to H + H$ reaction.

$$X_{H_2} = \frac{N_{H_2}}{Total} \tag{46}$$

$$X_H = \frac{N_H}{Total} \tag{47}$$

$$X_{H_2} = \frac{N_{H_2}}{N_{H_2} + N_{H_2O} + N_H + N_{OH}} \tag{48}$$

$$X_H = \frac{N_H}{N_{H_2} + N_{H_2O} + N_H + N_{OH}} \tag{49}$$

$$K_{H_2} = \frac{\left(\frac{N_H}{N_{H_2O} + N_{H_2}}\right)^2}{\frac{N_{H_2}}{N_{H_2O} + N_{H_2}}} \tag{50}$$

$$K_{H_2} = \frac{N_H^2}{(N_{H_2O} + N_{H_2})N_{H_2}} \tag{51}$$

As a result we have 4 coupled equations:

$$1 = N_{H_2O} + N_{H_2} (52)$$

$$0.760592 = N_{H_2O} (53)$$

$$K_{H_2O} = \frac{N_H N_{OH}}{(N_H + N_{H_2O}) N_{H_2O}}$$
 (54)

$$K_{H_2} = \frac{N_H^2}{(N_{H_2O} + N_{H_2})N_{H_2}} \tag{55}$$

Where K_{H_2O} and K_{H_2} are calculated values. Solving these equations we get:

$$N_{H_2O} = 0.760592 (56)$$

$$N_{H_2} = 0.239408 (57)$$

$$N_H = 0.05333 \tag{58}$$

$$N_{OH} = 0.101958 \tag{59}$$

Species	$N_{(i)}$	$X_{(i)}$	$m_{(i)}$	$Y_{(i)}$	$\hat{c}_{p(i)}$	$c_{p(i)}$	$Y_{(i)}c_p$
H_2O	N_{H_2O}	$X_{H_2O} = N_{H_2O}$	$\hat{m}N_{H_2O}$	$\frac{\hat{m}}{m_{(i)}}$	$\hat{c}_{p(H_2O)}(T_C)$	$\frac{\hat{c}_p}{\hat{m}}$	$Y_{H_2O}c_p$
H_2	N_{H_2}	$X_{H_2} = N_{H_2}$	$\hat{m}N_{H_2}$	$\frac{\hat{m}}{m_{(i)}}$	$\hat{c}_{p(H_2)}(T_C)$	$\frac{\hat{c}_p}{\hat{m}}$	$Y_{H_2}c_p$
Н	N_H	$X_H = N_H$	$\hat{m}N_H$	$\frac{\hat{m}}{m_{(i)}}$	$\hat{c}_{p(H)}(T_C)$	$\frac{\hat{c}_p}{\hat{m}}$	$Y_H c_p$
ОН	N_{OH}	$X_{OH} = N_{OH}$	$\hat{m}N_{OH}$	$\frac{\hat{m}}{m_{(i)}}$	$\hat{c}_{p(OH)}(T_C)$	$\frac{\hat{c}_p}{\hat{m}}$	$Y_{OH}c_p$

Now we can calculate c_p , we sum the $Y_{(i)}c_p$ values for each species.

$$c_p = \sum_{i} Y_{(i)} c_p \tag{60}$$

$$c_p = Y_{H_2O}c_p + Y_{H_2}c_p + Y_Hc_p + Y_{OH}c_p$$
 (61)

We perform now calculate the gas constant R.

$$R = \frac{\hat{R}}{m_{Total}} \tag{62}$$

$$c_v = c_p - R \tag{63}$$

$$\gamma = \frac{c_p}{c_p - \frac{\hat{R}}{\sum_i X_{(i)} m_{(i)}}} = \frac{c_p}{c_p - \frac{\hat{R}}{m_{Total}}} = \frac{c_p}{c_p - R} = \frac{c_p}{c_v}$$
(64)

2.3 Nozzle Performance

We begin by defining the Area Ratio:

$$A_r = \frac{\pi R_e^2}{\pi R_t^2} \tag{65}$$

Allowing for us to solve for the Mach number:

$$A_r = \left(\frac{\gamma + 1}{2}\right)^{-\frac{\gamma + 1}{2(\gamma - 1)}} \frac{1}{M_e} \left[1 + \frac{\gamma - 1}{2} M_e^2 \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}$$
(66)

We can solve for Mach number M_e by making use of the scipy.optimize.fsolve function, which returns the roots of a non-linear function. After solving for M_e we can calculate the exit velocity v_e , exit Pressure p_e and exit Temperature T_e .

$$T_e = T_c \left[1 + \frac{\gamma - 1}{2} M_e^2 \right]^{-1} \tag{67}$$

$$p_e = p_c \left[1 + \frac{\gamma - 1}{2} M_e^2 \right]^{-\frac{\gamma}{\gamma - 1}} \tag{68}$$

$$v_e = M_e \sqrt{\gamma R T_e} \tag{69}$$

After solving for (Eq. 69), (Eq. 68) and (Eq. 67) we can calculate the mass flow rate \dot{m} .

$$\dot{m} = \frac{A_t p_c \sqrt{\gamma}}{\sqrt{RT_c}} \left[\frac{2}{\gamma + 1} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \tag{70}$$

Then once you have the mass flow rate you can calculate the thrust \mathcal{T} , specific impulse I_{sp} and the thrust coefficient C_T .

$$\mathcal{T} = \dot{m}v_e + p_e A_e - p_c A_t \tag{71}$$

$$I_{sp} = \frac{\mathcal{T}}{\dot{m}q_0} \tag{72}$$

$$C_T = \frac{\mathcal{T}}{n_c A_t} \tag{73}$$

A Code listings

A.1 SSMEMain.py

```
1 import pandas as pd
2 from exchanger import *
5 if __name__ == '__main__':
      p_starting = 204 #starting pressure in atm
      p_starting = p_starting * 101325 #convert to pascals
      FuelOxidizer_ratio = 0.166
9
      mol_02 = 31.998 \# g/mol \#single molecule mass
10
      mol_H2 = 2.016 # g/mol #single molecule mass
11
      phi = (FuelOxidizer_ratio* mol_02 / mol_H2)/2
      Hydr_inject_temp = 850 \#K
13
      0x_{inject_temp} = 530
14
      ThroatRadius = 5.15 #inches
      ExitRadius = 45.35 #inches
16
      FEE_T, FEE_del_h_total, del_h2_hat, b_hat_table = FEE(
17
     Hydr_inject_temp, p_starting, Ox_inject_temp, phi)
      print('phi = ', phi)
19
     print('Temperature of the first enthalpy exchanger = ',
     FEE_T, 'K')
      print('Total enthalpy change in the first enthalpy
21
     exchanger = ', FEE_del_h_total, 'J')
23
      gamma, R, cp_total, cv, LawOfMassAction_df = OCC(FEE_T,
     p_starting, FuelOxidizer_ratio)
      print('gamma = ', gamma)
25
26
      m_{dot0x}, v_{e0x}, p_{e0x}, T_{e0x}, T_{e0x}
27
     OxNozzlePerf(ThroatRadius, ExitRadius, gamma, FEE_T, R,
     p_starting)
      print('0x mass flow rate = ', m_dot0x, 'kg/s')
29
      print('0x exit velocity = ', v_e0x, 'm/s')
30
      print('Ox exit pressure = ', p_eOx, 'Pa')
31
      print('0x thrust = ', Thrust0x, 'N')
      print('0x thrust coefficient = ', c_T0x)
33
      print('Ox specific impulse = ', I_spOx, 's')
34
      print('0x exit temperature = ', T_e_0x, 'K')
```

```
print('Flourine Problem')

print('Flourine Problem')

b_hat_table, FEE_T_flourine = FEE_F(phi, p_starting,
del_h2_hat, b_hat_table, Ox_inject_temp)

print('Temperature of the first enthalpy exchanger (
Flourine) = ', FEE_T_flourine, 'K')

FCC(FEE_T_flourine)
```

A.2 exchanger.py

```
1 from Iterater import *
2 import pandas as pd
3 import numpy as np
4 from sympy import Eq, solve, var
7 def FEE(Hydr_inject_temp, p_starting, Ox_inject_temp, phi):
      This is a function that returns the calculations of the
     first enthalpy exchanger
10
      :param Hydr_inject_temp: temperature of hydrogen
     injection in K
      :param p_starting: starting pressure in Pa
      :param Ox_inject_temp: temperature of oxygen injection in
13
      :param phi: equivalence ratio
      :return T the temperature of the first enthalpy exchanger
14
      :return del_h_total the total enthalpy change in the
15
     first enthalpy exchanger
      :return b_hat_table the table of b_hat values
16
      :return del_h2_hat the enthalpy change of H2
17
19
20
      h_f_{h20} = abs(h2o_table_f(298)) #J/mol
2.1
      h_f_h = h_table_f(298)/1000
      h_oh_f = oh_table_f(298)/1000 # KJ/mol
23
24
      b_hat_table = pd.DataFrame(columns=['substance', 'b_hat'
25
     ])
      b_hat_table = b_hat_table.append({'substance': 'H2', '
     b_hat': 0.0266}, ignore_index=True)
```

```
b_hat_table = b_hat_table.append({'substance': 'H2O', '
     b_hat': 0.03049}, ignore_index=True)
      b_hat_table = b_hat_table.append({'substance': '02', '
28
     b_hat': 0.0318}, ignore_index=True)
      b_hat_table = b_hat_table.append({'substance': 'N2', '
29
     b_hat': 0.0391}, ignore_index=True)
30
      b_hat_h2 = b_hat_table['b_hat'][0]
31
      b_hat_o2 = b_hat_table['b_hat'][2]
32
33
      del_h2_hat = h2_tableH(Hydr_inject_temp) + b_hat_h2*(
34
     p_starting - 101325)/1000
      del_o2_hat = o2_tableH(Ox_inject_temp) + b_hat_o2*(
35
     p_starting - 101325)/1000
      mols_h2 = phi*2
36
      mols_o2 = 1
37
38
      del_h_total = del_h2_hat*mols_h2 + del_o2_hat*mols_o2
39
      # At this point the chemical equation is O2 + mols*H2 ->
40
     (phi-2)*H2 +2H20
      del_h_total = del_h_total + 2*h_f_h2o
41
42
      # weighting function
43
      r = 0.17947269916082
44
      q = (1-r)/2
46
      T_h2o = h2o_tableT(q*del_h_total)
47
      T_h2 = h2_tableT(r*del_h_total)
48
      T = (T_h2o + T_h2)/2
50
51
      return T, del_h_total, del_h2_hat, b_hat_table
52
54 def OCC(FEE_T, p_starting, FO_ratio):
      , , ,
55
      This is a function that returns the calculations of the
     oxygen combustion chamber
      :param FEE_T: temperature of the first enthalpy exchanger
      in K
      :param p_starting: starting pressure in Pa
      :param FO_ratio: fuel to oxidizer ratio
59
      :return gamma: the ratio of specific heats
      :return R: the gas constant
61
      :return cp_total: the total specific heat capacity
      :return cv : the specific heat capacity at constant
63
```

```
volume
      :return LawOfMassAction_df: the dataframe of the law of
64
     mass action
65
66
      # The chemical equation is H20 \rightarrow H + OH
      molfrac_h2o_react = 1
68
      molfrac_h_react = 0
69
      molfrac_oh_react = 0
70
      molfrac_h2o_prod = 0
71
      molfrac_h_prod = 1
72
      molfrac_oh_prod = 1
73
74
      # The chemical equation is H2 \rightarrow 2H
75
      molfrac_h2_react = 1
76
      molfrac_2h_react = 0
77
      molfrac_2h_prod = 2
      molfrac_h2_prod = 0
79
80
      molfrac_h2o_total = molfrac_h2o_prod - molfrac_h2o_react
81
      molfrac_h_total = molfrac_h_prod - molfrac_h_react
      molfrac_oh_total = molfrac_oh_prod - molfrac_oh_react
83
      sigma_nu = molfrac_h2o_total + molfrac_h_total +
84
     molfrac_oh_total
      # print('sigma_nu = ', sigma_nu)
86
      molfrac_h2_total = molfrac_h2_prod - molfrac_h2_react
87
      molfrac_2h_total = molfrac_2h_prod - molfrac_2h_react
88
      sigma_nu = molfrac_h2_total + molfrac_2h_total
      # print('sigma_nu = ', sigma_nu)
90
91
      # Gibbs free energy of formation
92
      g_f_h2o = h2o_table_g(FEE_T)
93
      g_f_h = h_{table_g(FEE_T)}
94
      g_f_oh = oh_table_g(FEE_T)
95
      g_f_h2 = h2_table_g(FEE_T)
97
      # Gibbs free energy of reaction
      g_r_h2 = 2*(g_f_h - g_f_h2)
      g_r_h2o = g_f_h + g_f_h2o - g_f_oh
      k_h2 = ((p_starting/101325)**-1)*np.exp(-g_r_h2/(8.314*))
     FEE_T))
      k_h2o = ((p_starting/101325)**-1)*np.exp(-g_r_h2o/(8.314*))
     FEE_T))
```

```
104
105
       \# k_h20 = 0.007149
106
       \# k_h2 = 0.011874839
107
108
       mass_fuel = 2.02 #H2
       mass_oxider = mass_fuel/FO_ratio
       1 = mass_oxider/(2*15.99)
       N_h = var('N_h')
113
       N_{oh} = var('N_{oh}')
114
       N_h2 = var('N_h2')
115
       N_h2o = var('N_h2o')
116
117
118
       # solve the system of equations - Law of mass action
119
       sol = solve([Eq(N_h2o, 2*1), Eq(N_h2o+N_h2, 1), Eq((N_h*)))
120
      N_{oh}/((N_{h2}+N_{h2o})*N_{h2o}), k_{h2o},
                     Eq((N_h**2)/((N_h2+N_h2o)*N_h2), k_h2)], [
121
      N_h, N_oh, N_h2, N_h2o])
       # split sol into two separate lists
       sol1 = sol[0]
       sol2 = sol[1]
       # if all the solutions are real and positive then assign
      mols_sol to the solution
       if all(i > 0 for i in soll) and all(i > 0 for i in sol2):
           mols_sol = sol1
128
       else:
           mols_sol = sol2
130
131
       # print('mols_sol = ', mols_sol)
132
       N_h = mols_sol[0]
       N_{oh} = mols_{sol}[1]
134
       N_h2 = mols_sol[2]
135
       N_h2o = mols_sol[3]
136
       N_{total} = N_h + N_oh + N_h2 + N_h2o
137
138
       molar_mass_h2o = 18.01528
139
       molar_mass_h = 1.00794
140
       molar_mass_oh = 17.00734
141
142
       molar_mass_h2 = 2.01588
       molar_mass_total = molar_mass_h2o + molar_mass_h +
143
      molar_mass_oh + molar_mass_h2
144
```

```
LawOfMassAction_df = pd.DataFrame({'substance': ['N_H20',
145
       'N_H2', 'N_H', 'N_OH', 'Total'],
                                            'mols': [N_h2o, N_h2,
146
      N_h, N_oh, N_total]})
      LawOfMassAction_df['X_(i)'] = LawOfMassAction_df['mols']
147
       LawOfMassAction_df['m_hat_(i)'] = pd.Series([
      molar_mass_h2o, molar_mass_h2, molar_mass_h, molar_mass_oh
      , molar_mass_total])\
                                          * LawOfMassAction_df['
149
      X_(i)']
150
       # sum m_hat_(i)[i=0 to i=3]
       m_hat_total = sum(LawOfMassAction_df['m_hat_(i)'])-
      LawOfMassAction_df['m_hat_(i)'][4]
       LawOfMassAction_df['m_hat_(i)'][4] = m_hat_total
      LawOfMassAction_df['Y_(i)'] = LawOfMassAction_df['m_hat_(
154
      i)'] / m_hat_total
      h2_{cp} = h2_{table_cp}(FEE_T)
156
      h2o_cp = h2o_table_cp(FEE_T)
157
       h_cp = h_table_cp(FEE_T)
       oh_cp = oh_table_cp(FEE_T)
       cp_hats = pd.Series([h2o_cp, h2_cp, h_cp, oh_cp])
161
       cps = cp_hats / pd.Series([molar_mass_h2o, molar_mass_h2,
       molar_mass_h, molar_mass_oh])
       cp_df = pd.DataFrame({'cp_hat': cp_hats, 'cp': cps})
163
       cp_df['Y_(i)*cp_(i)'] = cp_df['cp'] * LawOfMassAction_df[
164
      'Y_(i)'][0:4]
165
       cp_total = sum(cp_df['Y_(i)*cp_(i)'])
166
      LawOfMassAction_df = pd.concat([LawOfMassAction_df, cp_df
      ], axis=1)
169
      R = 8.314/m_hat_total
       cv = cp_total - R
       gamma = cp_total/cv
       return gamma, R, cp_total, cv, LawOfMassAction_df
175
def FEE_F(phi, p_starting, del_h2_hat, b_hat_table,
      Ox_inject_temp):
       , , ,
```

```
This is a function that returns the calculations of the
      fuel enthalpy exchanger - flourine
       :param phi the equivalence ratio
180
       :param p_starting combustion pressure in Pa
181
       :param del_h2_hat heat of formation of H2 from FEE -
182
      Oxygen
       :param b_hat_table Van der Waals table of constants
183
       :param Ox_inject_temp: Oxidizer injection temperature in
184
       :return T the temperature of the FEE in K
185
      :return b_hat_table the Van der Waals table of constants
186
      - edited
       , , ,
187
188
       b_hat_table.append({'substance': 'F2', 'b_hat': 0.02896},
189
       ignore_index = True)
       b_hat_table.append({'substance': 'HF', 'b_hat': 0.0739},
190
      ignore_index = True)
191
       b_hat_f2 = b_hat_table['b_hat'][3]
192
       F2_inject_temp = Ox_inject_temp
193
194
       del_f2_hat = f2_tableH(F2_inject_temp) + b_hat_f2*(
195
      p_starting - 101325)/1000
       del_hf_hat = abs(hf_tableH(298))
       del_h_total = phi*del_h2_hat + del_f2_hat
197
       del_h_total = del_h_total + del_hf_hat
198
199
       r = phi - 1
201
       q = 1
202
       r_{perc} = r/(r+q)
204
       q_{perc} = 1-r_{perc}
205
206
       T_h2 = h2_tableT(del_h_total)
       T_hf = hf_tableT(q_perc*del_h_total)
208
       T = (T_hf + T_h2) / 2
210
212
213
       return b_hat_table, T
214
def OxNozzlePerf(ThroatRadius, ExitRadius, gamma, FEE_T, R,
      p_starting):
```

```
216
       This is a function that returns the performance of the
217
      oxidizer nozzle
       :param ThroatRadius:
       :param ExitRadius:
219
       :param gamma:
       :param FEE_T:
221
       :return:
222
       , , ,
223
       Area_ratio = (ExitRadius/ThroatRadius)**2
225
       # change np.float64 values to a float
226
       gamma = float(gamma)
227
       FEE_T = float(FEE_T)
228
       R = float(R)
229
       R = R*1000 \# convert to J/kg-K
230
231
       M_e = MachSolve(gamma, Area_ratio, 5.0)
232
233
       print('M_e = ', M_e)
      T_e = FEE_T*((1+((gamma-1)/2)*M_e**2)**(-1))
234
       v_e = M_e * np.sqrt(gamma * R * T_e)
      p_e = p_starting * (1 + ((gamma - 1) / 2) * M_e ** 2) **
236
      (-gamma / (gamma - 1))
       ExitRadius = ExitRadius/39.37 # convert to meters
237
       ThroatRadius = ThroatRadius/39.37 # convert to
239
240
       m_dot = (p_starting * np.pi * (ThroatRadius)**2) / (np.
241
      sqrt(FEE_T)) * \
           np.sqrt(gamma/R)*((2/(gamma+1))**((gamma+1)/(2*(gamma
242
      -1))))
243
       Thurst = m_dot * v_e + p_e * np.pi * ExitRadius**2 -
244
      p_starting * np.pi * ThroatRadius**2
       c_T = Thurst / (p_starting * np.pi * ThroatRadius**2)
245
       I_sp = Thurst/ (m_dot * 9.81)
246
247
       return m_dot, v_e, p_e, Thurst, c_T, I_sp, T_e
248
249
  def FCC(FEE_F):
250
251
       This is a function that returns the performance of the
      flourine combustion chamber
       :return:
       , , ,
254
```

```
255
256
       # The chemical equation is H2 +F2-> 2HF
257
       molfrac_h2_react = 1
       molfrac_f2_react = 1
259
       molfrac_hf_react = 0
       molfrac_h2_prod = 0
261
       molfrac_f2_prod = 0
262
       molfrac_hf_prod = 2
263
264
265
       molfrac_h2_total = molfrac_h2_react - molfrac_h2_prod
266
       molfrac_f2_total = molfrac_f2_react - molfrac_f2_prod
267
       molfrac_hf_total = molfrac_hf_react - molfrac_hf_prod
268
       sigma_nu = molfrac_hf_total + molfrac_h2_total +
269
      molfrac_f2_total
       # print('sigma_nu = ', sigma_nu)
270
271
272
       # Gibbs free energy of formation of H2, F2, and HF
       del_h_h2 = h2_tableH(FEE_F)
273
       del_h_f2 = f2_tableH(FEE_F)
       del_h_hf = hf_tableH(FEE_F)
       s_h2 = h2_tableS(FEE_F)
       s_f2 = f2_tableS(FEE_F)
       s_hf = hf_tableS(FEE_F)
280
       # Gibbs
281
       del_gh2 = del_hh2 - FEE_F*s_h2
       del_g_f2 = del_h_f2 - FEE_F*s_f2
283
       del_g_hf = del_h_hf - FEE_F*s_hf
284
285
       del_g_total = -del_g_f2 - del_g_h2 + del_g_hf
286
       print('del_g_total = ', del_g_total)
287
```

A.3 Iterater.py

```
import numpy as np
import pandas as pd
from sympy import Eq, var, solve, diff
from scipy.optimize import fsolve
import matplotlib.pyplot as plt

def h2_tableH(Temp):
```

```
9
      :param Temp: temperature in K
10
      :return h_hat: enthalpy in J/mol also known as heat of
11
     formation
      , , ,
      h2_table = pd.read_csv('./tables/Hydrogen_H2_table.csv')
14
      idx = (h2_table['T']-Temp).abs().idxmin()
      T_cloest = h2_table['T'][idx]
16
      h_hat_close = h2_table['H*-H*_298'][idx]
      if Temp > T_cloest:
18
          T_{upper} = h2_{table}['T'][idx+1]
19
          h_hat_upper = h2_table['H*-H*_298'][idx+1]
20
          h_hat = h_hat_close + (h_hat_upper-h_hat_close)/(
21
     T_upper-T_cloest)*(Temp-T_cloest)
      else:
22
          T_{lower} = h2_{table}['T'][idx-1]
23
          h_hat_lower = h2_table['H*-H*_298'][idx-1]
24
          h_hat = h_hat_close + (h_hat_lower-h_hat_close)/(
25
     T_lower-T_cloest)*(Temp-T_cloest)
      h_hat = h_hat*4184
      return h_hat
29
  def f2_tableH(Temp):
31
      :param Temp: temperature in K
32
      :return h_hat: enthalpy in J/mol also known as heat of
33
     formation
34
      f2_table = pd.read_csv('./tables/Fluorine_F2_table.csv')
35
36
      idx = (f2_table['T']-Temp).abs().idxmin()
37
      T_cloest = f2_table['T'][idx]
38
      h_hat_close = f2_table['H*-H*_298'][idx]
39
      if Temp > T_cloest:
          T_{upper} = f2_{table}['T'][idx+1]
41
          h_hat_upper = f2_table['H*-H*_298'][idx+1]
42
          h_hat = h_hat_close + (h_hat_upper-h_hat_close)/(
43
     T_upper-T_cloest)*(Temp-T_cloest)
      else:
44
45
          T_{lower} = f2_{table}['T'][idx-1]
          h_hat_lower = f2_table['H*-H*_298'][idx-1]
46
          h_hat = h_hat_close + (h_hat_lower-h_hat_close)/(
     T_lower-T_cloest)*(Temp-T_cloest)
```

```
h_hat = h_hat*4184
48
49
      return h_hat
50
51
52 def o2_tableH(Temp):
      , , ,
      :param Temp: temperature in K
54
      :return h_hat: enthalpy in J/mol also known as heat of
55
     formation
      , , ,
56
      o2_table = pd.read_csv('./tables/o2_table.csv')
57
58
      idx = (o2_table['T']-Temp).abs().idxmin()
      T_cloest = o2_table['T'][idx]
60
      h_hat_close = o2_table['H*-H*_298'][idx]
61
      if Temp > T_cloest:
62
          T_upper = o2_table['T'][idx+1]
63
          h_hat_upper = o2_table['H*-H*_298'][idx+1]
64
          h_hat = h_hat_close + (h_hat_upper-h_hat_close)/(
65
     T_upper-T_cloest)*(Temp-T_cloest)
      else:
          T_{lower} = o2_{table}['T'][idx-1]
67
          h_hat_lower = o2_table['H*-H*_298'][idx-1]
          h_hat = h_hat_close + (h_hat_lower-h_hat_close)/(
69
     T_lower-T_cloest)*(Temp-T_cloest)
      h_hat = h_hat
70
71
      return h_hat
72
73
74 def hf_tableH(Temp):
75
      :param Temp: temperature in K
76
      :return h_hat: enthalpy in J/mol also known as heat of
77
     formation
78
      hf_table = pd.read_csv('./tables/
     HydrogenFluoride_HF_table.csv')
80
      idx = (hf_table['T']-Temp).abs().idxmin()
81
      T_cloest = hf_table['T'][idx]
      h_hat_close = hf_table['h_hat'][idx]
83
      if Temp > T_cloest:
          T_upper = hf_table['T'][idx+1]
85
          h_hat_upper = hf_table['h_hat'][idx+1]
          h_hat = h_hat_close + (h_hat_upper-h_hat_close)/(
```

```
T_upper-T_cloest)*(Temp-T_cloest)
       else:
           T_lower = hf_table['T'][idx-1]
89
           h_hat_lower = hf_table['h_hat'][idx-1]
90
           h_hat = h_hat_close + (h_hat_lower-h_hat_close)/(
91
      T_lower-T_cloest)*(Temp-T_cloest)
       h_hat = h_hat*4184
92
93
       return h_hat
94
  def h2o_table_f(Temp):
96
97
       :param Temp: temperature in K
98
       :return h_hat: enthalpy in J/mol also known as heat of
99
      formation
100
       h2o_table = pd.read_csv('./tables/H2O_table.csv')
101
       idx = (h2o_table['T']-Temp).abs().idxmin()
103
       T_cloest = h2o_table['T'][idx]
104
       h_hat_close = h2o_table['h_hat'][idx]
       if Temp > T_cloest:
106
           T_upper = h2o_table['T'][idx+1]
107
           h_hat_upper = h2o_table['h_hat'][idx+1]
108
           h_hat = h_hat_close + (h_hat_upper-h_hat_close)/(
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
110
           T_lower = h2o_table['T'][idx-1]
111
           h_hat_lower = h2o_table['h_hat'][idx-1]
112
           h_hat = h_hat_close + (h_hat_lower-h_hat_close)/(
113
      T_lower-T_cloest)*(Temp-T_cloest)
114
       return h_hat
115
116
117 def oh_table_f(Temp):
       , , ,
118
       :param Temp: temperature in K
119
       :return h_hat: enthalpy in J/mol also known as heat of
      formation
       , , ,
121
       oh_table = pd.read_csv('./tables/OH_table.csv')
123
       idx = (oh_table['T']-Temp).abs().idxmin()
124
       T_cloest = oh_table['T'][idx]
       h_hat_close = oh_table['h_hat'][idx]
126
```

```
if Temp > T_cloest:
127
           T_upper = oh_table['T'][idx+1]
128
           h_hat_upper = oh_table['h_hat'][idx+1]
129
           h_hat = h_hat_close + (h_hat_upper-h_hat_close)/(
130
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
           T_lower = oh_table['T'][idx-1]
132
           h_hat_lower = oh_table['h_hat'][idx-1]
133
           h_hat = h_hat_close + (h_hat_lower-h_hat_close)/(
134
      T_lower-T_cloest)*(Temp-T_cloest)
135
       return h_hat
136
  def h_table_f(Temp):
138
139
       :param Temp: temperature in K
140
       :return h_hat: enthalpy in J/mol also known as heat of
141
      formation
142
       h_table = pd.read_csv('./tables/H_table.csv')
143
       idx = (h_table['T']-Temp).abs().idxmin()
145
       T_cloest = h_table['T'][idx]
       h_hat_close = h_table['h_hat'][idx]
147
       if Temp > T_cloest:
           T_upper = h_table['T'][idx+1]
149
           h_hat_upper = h_table['h_hat'][idx+1]
           h_hat = h_hat_close + (h_hat_upper-h_hat_close)/(
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
           T_lower = h_table['T'][idx-1]
           h_hat_lower = h_table['h_hat'][idx-1]
154
           h_hat = h_hat_close + (h_hat_lower-h_hat_close)/(
      T_lower-T_cloest)*(Temp-T_cloest)
156
       return h_hat
  def h2o_tableT(h_hat):
160
       :param h_hat: enthalpy in J/mol also known as heat of
161
      formation
       :return Temp: temperature in K
163
       h2o_table = pd.read_csv('./tables/H2O_table.csv')
165
```

```
idx = (h2o_table['h_f']-h_hat).abs().idxmin()
166
       h_hat_cloest = h2o_table['h_f'][idx]
167
       T_close = h2o_table['T'][idx]
168
       if h_hat > h_hat_cloest:
           h_hat_upper = h2o_table['h_f'][idx+1]
           T_upper = h2o_table['T'][idx+1]
           T = T_close + (T_upper-T_close)/(h_hat_upper-
172
      h_hat_cloest)*(h_hat-h_hat_cloest)
       else:
173
174
           h_hat_lower = h2o_table['h_f'][idx-1]
           T_lower = h2o_table['T'][idx-1]
175
           T = T_close + (T_lower-T_close)/(h_hat_lower-
176
      h_hat_cloest)*(h_hat-h_hat_cloest)
177
178
       return T
  def h2_tableT(h_hat):
180
181
       :param h_hat: enthalpy in J/mol also known as heat of
182
      formation
       :return Temp: temperature in K
       , , ,
184
       h2_table = pd.read_csv('./tables/Hydrogen_H2_table.csv')
185
       h_hat = h_hat/4184
186
       idx = (h2\_table['H*-H*\_298']-h\_hat).abs().idxmin()
       h_hat_cloest = h2_table['H*-H*_298'][idx]
188
       T_close = h2_table['T'][idx]
189
       if h_hat > h_hat_cloest:
190
           h_hat_upper = h2_table['H*-H*_298'][idx+1]
191
           T_upper = h2_table['T'][idx+1]
192
           T = T_close + (T_upper-T_close)/(h_hat_upper-
193
      h_hat_cloest)*(h_hat-h_hat_cloest)
       else:
194
           h_hat_lower = h2_table['H*-H*_298'][idx-1]
195
           T_lower = h2_table['T'][idx-1]
196
           T = T_close + (T_lower-T_close)/(h_hat_lower-
      h_hat_cloest)*(h_hat-h_hat_cloest)
       return T
199
  def hf_tableT(h_hat):
201
       :param h_hat: enthalpy in J/mol also known as heat of
203
      formation
       :return Temp: temperature in K
```

```
205
       f2_table = pd.read_csv('./tables/
206
      HydrogenFluoride_HF_table.csv')
       h_hat = h_hat/4184
       idx = (f2_table['H*-H*_298']-h_hat).abs().idxmin()
208
       h_hat_cloest = f2_table['H*-H*_298'][idx]
       T_close = f2_table['T'][idx]
210
       if h_hat > h_hat_cloest:
211
           h_hat_upper = f2_table['H*-H*_298'][idx+1]
212
213
           T_{upper} = f2_{table}['T'][idx+1]
           T = T_close + (T_upper-T_close)/(h_hat_upper-
214
      h_hat_cloest)*(h_hat-h_hat_cloest)
       else:
215
           h_hat_lower = f2_table['H*-H*_298'][idx-1]
216
           T_lower = f2_table['T'][idx-1]
217
           T = T_close + (T_lower-T_close)/(h_hat_lower-
218
      h_hat_cloest)*(h_hat-h_hat_cloest)
219
       return T
220
221
  def oh_table_g(Temp):
223
       :param Temp: temperature in K
225
       :return g_hat: Gibbs free energy in J/mol
227
       oh_table = pd.read_csv('./tables/OH_table.csv')
228
229
       idx = (oh_table['T']-Temp).abs().idxmin()
       T_cloest = oh_table['T'][idx]
231
       g_hat_close = oh_table['g_hat'][idx]
232
       if Temp > T_cloest:
233
           T_upper = oh_table['T'][idx+1]
234
           g_hat_upper = oh_table['g_hat'][idx+1]
235
           g_hat = g_hat_close + (g_hat_upper-g_hat_close)/(
236
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
237
           T_lower = oh_table['T'][idx-1]
           g_hat_lower = oh_table['g_hat'][idx-1]
           g_hat = g_hat_close + (g_hat_lower-g_hat_close)/(
      T_lower-T_cloest)*(Temp-T_cloest)
241
       return g_hat
242
244 def h_table_g(Temp):
```

```
245
       :param Temp: temperature in K
246
       :return g_hat: Gibbs free energy in J/mol
247
248
       h_table = pd.read_csv('./tables/H_table.csv')
       idx = (h_table['T']-Temp).abs().idxmin()
251
       T_cloest = h_table['T'][idx]
252
       g_hat_close = h_table['g_hat'][idx]
253
       if Temp > T_cloest:
254
           T_upper = h_table['T'][idx+1]
255
           g_hat_upper = h_table['g_hat'][idx+1]
256
           g_hat = g_hat_close + (g_hat_upper-g_hat_close)/(
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
           T_lower = h_table['T'][idx-1]
259
           g_hat_lower = h_table['g_hat'][idx-1]
260
           g_hat = g_hat_close + (g_hat_lower-g_hat_close)/(
261
      T_lower-T_cloest)*(Temp-T_cloest)
262
       return g_hat
264
  def h2o_table_g(Temp):
265
266
       :param Temp: temperature in K
       :return g_hat: Gibbs free energy in J/mol
268
269
       h2o_table = pd.read_csv('./tables/H2O_table.csv')
270
       idx = (h2o_table['T']-Temp).abs().idxmin()
272
       T_cloest = h2o_table['T'][idx]
273
       g_hat_close = h2o_table['g_hat'][idx]
       if Temp > T_cloest:
275
           T_upper = h2o_table['T'][idx+1]
276
           g_hat_upper = h2o_table['g_hat'][idx+1]
277
           g_hat = g_hat_close + (g_hat_upper-g_hat_close)/(
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
           T_lower = h2o_table['T'][idx-1]
280
           g_hat_lower = h2o_table['g_hat'][idx-1]
281
           g_hat = g_hat_close + (g_hat_lower-g_hat_close)/(
282
      T_lower-T_cloest)*(Temp-T_cloest)
283
       return g_hat
284
285
```

```
def h2_table_g(Temp):
       :param Temp: temperature in K
288
       :return g_hat: Gibbs free energy in J/mol
290
       h2_table = pd.read_csv('./tables/Hydrogen_H2_table.csv')
202
       idx = (h2_table['T']-Temp).abs().idxmin()
293
       T_cloest = h2_table['T'][idx]
294
       g_hat_close = h2_table['g_hat'][idx]
       if Temp > T_cloest:
296
           T_{upper} = h2_{table}['T'][idx+1]
297
           g_hat_upper = h2_table['g_hat'][idx+1]
298
           g_hat = g_hat_close + (g_hat_upper-g_hat_close)/(
299
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
300
           T_lower = h2_table['T'][idx-1]
           g_hat_lower = h2_table['g_hat'][idx-1]
302
           g_hat = g_hat_close + (g_hat_lower-g_hat_close)/(
303
      T_lower-T_cloest)*(Temp-T_cloest)
       return g_hat
305
  def h2o_table_cp(Temp):
307
       :param Temp: temperature in K
309
       :return cp_hat: heat capacity in J/mol-K
310
311
       h2o_table = pd.read_csv('./tables/H2O_table.csv')
312
313
       idx = (h2o_table['T']-Temp).abs().idxmin()
314
       T_cloest = h2o_table['T'][idx]
       cp_hat_close = h2o_table['cp_hat'][idx]
316
       if Temp > T_cloest:
317
           T_upper = h2o_table['T'][idx+1]
318
           cp_hat_upper = h2o_table['cp_hat'][idx+1]
319
           cp_hat = cp_hat_close + (cp_hat_upper-cp_hat_close)/(
320
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
321
           T_{lower} = h2o_{table}['T'][idx-1]
           cp_hat_lower = h2o_table['cp_hat'][idx-1]
323
           cp_hat = cp_hat_close + (cp_hat_lower-cp_hat_close)/(
      T_lower-T_cloest)*(Temp-T_cloest)
       return cp_hat
326
```

```
327
  def h2_table_cp(Temp):
328
329
       :param Temp: temperature in K
       :return cp_hat: heat capacity in J/mol-K
331
       , , ,
       h2_table = pd.read_csv('./tables/Hydrogen_H2_table.csv')
333
334
       idx = (h2_table['T']-Temp).abs().idxmin()
335
       T_cloest = h2_table['T'][idx]
336
       cp_hat_close = h2_table['cp_hat'][idx]
337
       if Temp > T_cloest:
338
           T_upper = h2_table['T'][idx+1]
339
           cp_hat_upper = h2_table['cp_hat'][idx+1]
340
           cp_hat = cp_hat_close + (cp_hat_upper-cp_hat_close)/(
341
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
342
           T_lower = h2_table['T'][idx-1]
343
           cp_hat_lower = h2_table['cp_hat'][idx-1]
344
           cp_hat = cp_hat_close + (cp_hat_lower-cp_hat_close)/(
345
      T_lower-T_cloest)*(Temp-T_cloest)
346
       return cp_hat
347
348
   def oh_table_cp(Temp):
349
350
       :param Temp: temperature in K
351
       :return cp_hat: heat capacity in J/mol-K
352
       , , ,
       oh_table = pd.read_csv('./tables/OH_table.csv')
354
355
       idx = (oh_table['T']-Temp).abs().idxmin()
       T_cloest = oh_table['T'][idx]
357
       cp_hat_close = oh_table['cp_hat'][idx]
358
       if Temp > T_cloest:
359
           T_upper = oh_table['T'][idx+1]
           cp_hat_upper = oh_table['cp_hat'][idx+1]
361
           cp_hat = cp_hat_close + (cp_hat_upper-cp_hat_close)/(
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
363
           T_lower = oh_table['T'][idx-1]
364
365
           cp_hat_lower = oh_table['cp_hat'][idx-1]
           cp_hat = cp_hat_close + (cp_hat_lower-cp_hat_close)/(
366
      T_lower-T_cloest)*(Temp-T_cloest)
367
```

```
return cp_hat
368
369
  def h_table_cp(Temp):
370
371
       :param Temp: temperature in K
372
       :return cp_hat: heat capacity in J/mol-K
374
       h_table = pd.read_csv('./tables/H_table.csv')
375
376
       idx = (h_table['T']-Temp).abs().idxmin()
       T_cloest = h_table['T'][idx]
378
       cp_hat_close = h_table['cp_hat'][idx]
379
       if Temp > T_cloest:
380
           T_upper = h_table['T'][idx+1]
381
           cp_hat_upper = h_table['cp_hat'][idx+1]
382
           cp_hat = cp_hat_close + (cp_hat_upper-cp_hat_close)/(
383
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
384
           T_{lower} = h_{table}['T'][idx-1]
385
           cp_hat_lower = h_table['cp_hat'][idx-1]
386
           cp_hat = cp_hat_close + (cp_hat_lower-cp_hat_close)/(
      T_lower-T_cloest)*(Temp-T_cloest)
       return cp_hat
389
  def MachSolve(gamma, r_A, guess):
391
392
       Solve for Mach number given gamma, r_A, and guess
393
       :param gamma: the gamma of the gas
394
       :param r_A: the ratio of the areas
395
       :param guess: the initial guess of the Mach number
396
       :return: Mach number
       , , ,
398
399
       # M = var('M')
400
       front_cont = ((gamma+1)/2)**(-(gamma+1)/(2*(gamma-1)))
       print('front_cont = ', front_cont)
402
       sec\_cont = (gamma-1)/2
       print('sec_cont = ', sec_cont)
404
       func = lambda M: (((front_cont)*((1+sec_cont*M**2)**((
405
      gamma+1)/(2*(gamma-1)))))/M) - r_A
       sol = fsolve(func, guess)
407
       return sol
409
```

```
410 def h2_tableS(Temp):
411
       :param Temp: temperature in K
412
       :return S_hat: entropy in J/mol-K
413
414
       h2_table = pd.read_csv('./tables/Hydrogen_H2_table.csv')
416
       idx = (h2_table['T']-Temp).abs().idxmin()
417
       T_cloest = h2_table['T'][idx]
418
       S_hat_close = h2_table['S'][idx]
       if Temp > T_cloest:
420
           T_{upper} = h2_{table}['T'][idx+1]
421
           S_hat_upper = h2_table['S'][idx+1]
422
           S_hat = S_hat_close + (S_hat_upper-S_hat_close)/(
423
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
424
           T_lower = h2_table['T'][idx-1]
425
           S_hat_lower = h2_table['S'][idx-1]
426
           S_hat = S_hat_close + (S_hat_lower-S_hat_close)/(
427
      T_lower-T_cloest)*(Temp-T_cloest)
       return S_hat
429
   def hf_tableS(Temp):
431
       :param Temp: temperature in K
433
       :return S_hat: entropy in J/mol-K
434
435
       hf_table = pd.read_csv('./tables/
436
      HydrogenFluoride_HF_table.csv')
437
       idx = (hf_table['T']-Temp).abs().idxmin()
438
       T_cloest = hf_table['T'][idx]
439
       S_hat_close = hf_table['S'][idx]
440
       if Temp > T_cloest:
441
           T_upper = hf_table['T'][idx+1]
442
           S_hat_upper = hf_table['S'][idx+1]
443
           S_hat = S_hat_close + (S_hat_upper-S_hat_close)/(
444
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
445
           T_lower = hf_table['T'][idx-1]
446
447
           S_hat_lower = hf_table['S'][idx-1]
           S_hat = S_hat_close + (S_hat_lower-S_hat_close)/(
448
      T_lower-T_cloest)*(Temp-T_cloest)
449
```

```
return S_hat
450
451
452 def f2_tableS(Temp):
453
       :param Temp: temperature in K
454
       :return S_hat: entropy in J/mol-K
456
       f2_table = pd.read_csv('./tables/Fluorine_F2_table.csv')
457
458
       idx = (f2_table['T']-Temp).abs().idxmin()
       T_cloest = f2_table['T'][idx]
460
       S_hat_close = f2_table['S'][idx]
461
       if Temp > T_cloest:
462
           T_upper = f2_table['T'][idx+1]
463
           S_hat_upper = f2_table['S'][idx+1]
464
           S_hat = S_hat_close + (S_hat_upper-S_hat_close)/(
465
      T_upper-T_cloest)*(Temp-T_cloest)
       else:
466
           T_{lower} = f2_{table}['T'][idx-1]
467
           S_hat_lower = f2_table['S'][idx-1]
468
           S_hat = S_hat_close + (S_hat_lower-S_hat_close)/(
      T_lower-T_cloest)*(Temp-T_cloest)
      return S_hat
471
# Me = MachSolve(1.237115, 77.5426, 5)
473 # print('Me = ', Me)
```