Tarea final AC

Algoritmo de la división

Código

```
DIVISIÓN(f_1, ..., f_2, f):
q_1 := 0; ...; q_s := 0; r := 0 p := f
while p ≠ 0 do
    i := 1
    divisionoccurred=false
    while i ≤ s and divisionoccurred=false do
        if lt(f_i) divides lt(p) then
            q_i := q_i + lt(p)/ lt(f_i)
            p := p − (lt(p)/ op(f_i))f_i
            divisionoccurred=true
        else
            i := i + 1
    if divisionoccurred = false then
        r := r + lt(p) p := p − lt(p)
return q_1, ..., q_s, r
```

Ejemplo del libro

Sean $f_1 = xy - 1$, $f_2 = y^2 - 1$, con órden lexicográfico. Si dividimos $f = xy^2 - x$ por $F = (f_1, f_2)$ obtenemos:

$$xy^2 - x = y(xy - 1) + 0(y^2 - 1) + (-x + y)$$

Si tomamos $F = (f_2, f_1)$, obtenemos:

$$xy^2 - x = x(y^2 - 1) + 0(xy - 1) + 0$$

En python:

```
R.<x,y> = PolynomialRing(RR, 2, "xy", order = "lex")
f = x*y^2-x; f1=x*y-1; f2=y^2-1

[q1,q2], r = div_poly([f1,f2],f)
print("Caso 1:")
show(html(f"${f} = {q1} ({f1}) + {q2} ({f2}) + ({r})$"))
print("Caso 2:")
f = x*y^2-x
f1=x*y-1
f2=y^2-1

[q2,q1], r = div_poly([f2,f1],f)
show(html(f"${f} = {q2} ({f2}) + {q1} ({f1}) + {r}$"))
```

Ejemplo del libro

`Caso 1:`

$$x * y^{2} - x = x\left(y^{2} - 1.00000000000000\right) + 0(x * y - 1.00000000000000) + 0$$

`Caso 2:`

$$x * y^{2} - x = y(x * y - 1.00000000000000) + 0\left(y^{2} - 1.00000000000000\right) + (-x + y)$$

Aplicado a polinomios aleatorios

```
R.<x_1,x_2,x_3,x_4> = PolynomialRing(GF(7), order="lex")

f = R.random_element(degree=4, terms = 4)
F = [R.random_element(degree=1, terms=2) for _ in range(3)]
Q, r = div_poly(F, f, R = R)

show(html(f"""
$$
\\begin{{align*}}
f = &{f} \\\\
\\sum_i q_i f_i +r = &{
    sum([q*f for q,f in zip(Q,F)]) + r
    }\\\\
    f_1 = &{F[0]} \\quad \\quad q_1 = {Q[0]} \\\\
    f_2 = &{F[1]} \\quad \\quad q_2 = {Q[1]} \\\\
    f_3 = &{F[2]} \\quad \\quad q_3 = {Q[2]} \\\\
    r = &{r}
\\end{{align*}}
$$
"""))
```

Aplicado a polinomios aleatorios

$$f = 3 * x_1 * x_2 + x_3^2 + 3 * x_3$$

$$\sum_i q_i f_i + r = 3 * x_1 * x_2 + x_3^2 + 3 * x_3$$

$$f_1 = -x_2 + 3 * x_4 \qquad q_1 = -3 * x_1$$

$$f_2 = -2 * x_3 - 3 \qquad q_2 = 3 * x_3 + 1$$

$$f_3 = -x_2 - 1 \qquad q_3 = 0$$

$$r = 2 * x_1 * x_4 + 3$$