Ez:

$$A = \frac{1}{\mu_0} Dxf \cdot Dxb + \frac{1}{\mu_0} Dyf \cdot Dyb + \omega^2 \epsilon_0 \epsilon_r$$

$$E_z = A^{-1} b \qquad \frac{\partial A}{\partial \epsilon} = \omega^2 \epsilon_0 \delta$$

$$H_x = - Dyb \cdot E_z \frac{1}{\mu_0} \qquad \left(v^T \frac{\partial H_x}{\partial E_z}\right)^T = -\frac{1}{\mu_0} Dyb^T \cdot v$$

$$H_y = Dxb \cdot E_z \frac{1}{\mu_0} \qquad \left(v^T \frac{\partial H_y}{\partial E_z}\right)^T = \frac{1}{\mu_0} Dxb^T \cdot v$$

Hz:

$$A = Dxf \frac{1}{\epsilon_0 \epsilon_r} Dxb + Dyf \frac{1}{\epsilon_0 \epsilon_r} Dyb + \omega^2 \mu_0 I$$

$$H_z = A^{-1} b \qquad \frac{\partial A}{\partial \epsilon} = - Dxf \cdot \frac{1}{\epsilon} \cdot \frac{\delta}{\epsilon_0} \cdot \frac{1}{\epsilon} \cdot Dxb - Dyf \cdot \frac{1}{\epsilon} \frac{\delta}{\epsilon_0} \frac{1}{\epsilon} Dyb$$

$$E_x = -\frac{1}{\epsilon_r \epsilon_0} \cdot Dyb \cdot H_z, \quad \left(v^T \frac{\partial E_x}{\partial H_z}\right)^T = - Dyb^T \, diag\left(\frac{1}{\epsilon}\right) \cdot v / \epsilon_0$$

$$E_y = \frac{1}{\epsilon_r \epsilon_0} Dxb \cdot H_z, \quad \left(v^T \frac{\partial E_y}{\partial H_z}\right) = + Dxb^T \, diag\left(\frac{1}{\epsilon}\right) \cdot v / \epsilon_0$$

$$-v^T A^{-1} \frac{\partial A}{\partial \epsilon} A^{-1} b = \underbrace{-v^T Dxf \frac{1}{\epsilon}}_{\tilde{E}_y^T \cdot \epsilon_0} \frac{\delta}{\epsilon_0} \underbrace{\frac{1}{\epsilon} Dxb \cdot H_z}_{E_y \cdot \epsilon_0} + \overset{(y)}{\cdots}$$

$$\tilde{E}_y = -\frac{1}{\epsilon} Dxf^T v$$

$$= \epsilon_0 \tilde{E}_y \odot \tilde{E}_x + \epsilon_0 \tilde{E}_x \odot E_x$$

$$\left(v^T \frac{\partial E_x}{\partial \epsilon_r}\right)^T = \left(v^T + \frac{\delta}{\epsilon_r^2 \epsilon_0} Dyb \cdot H_z\right)^T = \frac{1}{\epsilon_0} H_z^T Dyb^T \, diag\left(\frac{1}{\epsilon_r^2}\right) \delta v$$