Collective Attacks in Assumption-Based Argumentation

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ABSTRACT

Conflicts in argumentation-based frameworks are usually described in terms of attacks of arguments, or sets of arguments, on specific counter-arguments. In this paper we consider (assumption-based) argumentation frameworks, in which attacks have a more general form: they are performed on a *collective* of arguments that cannot stand together with the attacking arguments. We show that not only that this generalized form of attacks increases the expressive power of the argumentation frameworks, but in certain cases it also allows more sensible patterns of reasoning with conflicting considerations. Along the way, we also provide a novel characterization of the grounded semantics in prioritized argumentation frameworks.

KEYWORDS

Assumption-based argumentation, non-monotonic reasoning, attack relations and conflicts handling.

ACM Reference Format:
Ofer Arieli and Jesse Heyninck. 2024. Collective Attacks in Assumption-Based Argumentation. In *The 39th ACM/SIGAPP Symposium on Applied Computing (SAC '24), April 8–12, 2024, Avila, Spain.* ACM, New York, NY, USA, 8 pages. https://doi.org/10.1145/3605098.3635916

1 INTRODUCTION

Assumption-based argumentation frameworks (ABFs, for short) [4] are deductive rule-based systems, where assumptions and their contraries are incorporated for capturing different paradigms of non-monotonic reasoning. It has been shown that these frameworks are adequate for modeling reasoning with conflicting and dynamic information (see, e.g., [9, 14, 24] for some relevant tutorials). The basic idea in assumption-based argumentation is that sets of formulas (in some language) may 'attack' each other, where – intuitively – one set attacks another set, if formulas in the attacking set entail (according to some underlying inference relation) the contrary of a formula in the attacked set.

The purpose of this paper is to generalize the concept of attacks in ABFs, for better capturing cases in which conflicts may arise, and so refine and improve the decision making process in various conflicting situations. The idea is, instead of attacking a particular assumption, to allow attacks on a *collective* of assertions, the contrary of which is inferred by the attacking set. We call this *collective attacks*.[1] To illustrate the benefit of using collective attacks, we first recall in the next section some basic notations and definitions that are related to our setting. Then, in Section 3 we motivate and define collective attacks. In Section 4 we extend the setting with preferences among the assumptions, and in Section 5 we provide some results on the properties and the inference relations that are induced by our formalism. In Section 6 we point to some related work and conclude.

2 SIMPLE CONTRAPOSITIVE ABFS

Our starting point is a *logic* $\mathfrak{L} = \langle \mathsf{L}, \vdash \rangle$, where L is a (propositional) language and $\vdash$ is a consequence relation for L. Atomic formulas in L are denoted by p, q, r, compound formulas are denoted by ψ, ϕ, σ, and sets of formulas in L are denoted by Γ, Δ, Θ (all may be primed or indexed). The consequence relation $\vdash$ is a binary relation between sets of formulas and formulas in L, satisfying the following conditions:

- *Reflexivity*: if $\psi \in \Gamma$ then $\Gamma \vdash \psi$.
- *Monotonicity*: if $\Gamma \vdash \psi$ and $\Gamma \subseteq \Gamma'$, then $\Gamma' \vdash \psi$.
- *Transitivity*: if $\Gamma \vdash \psi$ and $\Gamma', \psi \vdash \phi$, then $\Gamma, \Gamma' \vdash \phi$.

As usual, the $\vdash$-transitive closure of a set Γ of L-formulas is $Cn_{\vdash}(\Gamma) = \{\psi \mid \Gamma \vdash \psi\}$. We say that a formula ψ is $\vdash$-*tautological* if $\psi \in Cn_{\vdash}(\emptyset)$, and that Γ is $\vdash$-*consistent* if $Cn_{\vdash}(\Gamma) \neq \mathsf{L}$.

We shall assume that the language L contains at least the following connectives and constant:

$\vdash$-*negation* $\neg$, satisfying: $p \nvdash \neg p$ and $\neg p \nvdash p$ (for every atomic p).
$\vdash$-*conjunction* $\wedge$, satisfying: $\Gamma \vdash \psi \wedge \phi$ iff $\Gamma \vdash \psi$ and $\Gamma \vdash \phi$.
$\vdash$-*disjunction* $\vee$, satisfying: $\Gamma, \phi \vee \psi \vdash \sigma$ iff $\Gamma, \phi \vdash \sigma$ and $\Gamma, \psi \vdash \sigma$.
$\vdash$-*implication* $\supset$, satisfying: $\Gamma, \phi \vdash \psi$ iff $\Gamma \vdash \phi \supset \psi$.
$\vdash$-*falsity* F, satisfying: $\mathsf{F} \vdash \psi$ for every formula ψ.

We abbreviate $\{\neg\gamma \mid \gamma \in \Gamma\}$ by $\neg\Gamma$, and when Γ is finite we denote by $\wedge\Gamma$ (respectively, by $\vee\Gamma$), the conjunction (respectively, the disjunction) of all the formulas in Γ.

In the sequel, we shall assume that the logics at hand are:

- *explosive*: $\Gamma, \psi, \neg\psi \vdash \phi$, and
- *contrapositive*: $\Gamma, \psi \vdash \phi$ iff $\Gamma, \neg\phi \vdash \neg\psi$.

Intuitively, this respectively means that every conclusion follows from a contradictory set of assumption, and that the formulas may be replaced by their negated formulas when switched between the two sides of the consequence relations.[2]

[*]Partially supported by the Israel Science Foundation, grant No. 550/19

ACM ISBN 979-8-4007-0243-3/24/04.
https://doi.org/10.1145/3605098.3635916

[1]This notion should be distinguished from a similar notion considered in [22], where the term 'collective' refers to multiple attackers (in abstract argumentation frameworks). While the latter is 'built in' in our approach (allowing sets of assumptions as premises in the attack consideration), we further extend attacks to multiple *attacked* formulas.
[2]Many well-known logics satisfy these properties, including classical logic (CL), intuitionistic logic, and standard modal logics.

Now we can define assumption-based argumentation frameworks [4]. The following family of these frameworks is shown in [15] to be a useful setting for argumentative reasoning.

Definition 2.1. An *assumption-based framework* (ABF, for short) is a tuple $\mathsf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$, where:

- $\mathfrak{L} = \langle \mathsf{L}, \vdash \rangle$ is a propositional logic.
- Γ (the *strict assumptions*) and Ab (the *candidate/defeasible assumptions*) are distinct (countable) sets of L-formulas, where the former is assumed to be $\vdash$-consistent and the latter is assumed to be nonempty.
- $\sim : Ab \to 2^{\mathsf{L}}$ is a *contrariness operator*, assigning a finite set of L-formulas to every defeasible assumption in Ab, such that for every consistent and non-tautological formula $\psi \in Ab \setminus \{\mathsf{F}\}$ it holds that $\psi \nvdash \bigwedge \sim\psi$ and $\bigwedge \sim \psi \nvdash \psi$.

A *simple contrapositive* ABF is an assumption-based framework $\mathsf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$, where $\mathfrak{L}$ is an explosive and contrapositive logic, and for every $\psi \in Ab$ it holds that $\sim\psi = \{\neg\psi\}$.

Defeasible assertions in an ABF may be attacked in the presence of a counter defeasible information. This is described in the next definition.

Definition 2.2. Let $\mathsf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ be an assumption-based framework, $\Delta, \Theta \subseteq Ab$, and $\psi \in Ab$. We say that Δ *attacks* ψ iff $\Gamma, \Delta \vdash \phi$ for some $\phi \in \sim\psi$. Accordingly, Δ attacks Θ if Δ attacks some $\psi \in \Theta$.

Definition 2.2 gives rise to the following adaptation to ABFs of the usual Dung-style semantics [13] for abstract argumentation frameworks.

Definition 2.3. ([4]) Let $\mathsf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ be an assumption-based framework, and let Δ be a set of defeasible assumptions. Below, maximum and minimum are taken with respect to set inclusion.

- Δ is *conflict-free* (in ABF) iff there is no $\Delta' \subseteq \Delta$ that attacks some $\psi \in \Delta$.
- Δ *defends* (in ABF) a set $\Delta' \subseteq Ab$ iff for every set Θ that attacks Δ' there is $\Delta'' \subseteq \Delta$ that attacks Θ.
- Δ is *admissible* (in ABF) iff it is conflict-free and defends every $\Delta' \subseteq \Delta$.
- Δ is *complete* (in ABF) iff it is admissible and contains every $\Delta' \subseteq Ab$ that it defends.
- Δ is *well-founded* (in ABF) iff $\Delta = \bigcap\{\Theta \subseteq Ab \mid \Theta \text{ is complete}\}$.
- Δ is *grounded* (in ABF) iff it is minimally complete (i.e., no $\Delta' \subsetneq \Delta$ is complete).
- Δ is *preferred* (in ABF) iff it is maximally admissible (i.e., there is no admissible $\Delta' \subseteq Ab$ such that $\Delta \subsetneq \Delta'$).
- Δ is *stable* (in ABF) iff it is conflict-free and attacks every $\psi \in Ab \setminus \Delta$.

NOTE 1. According to standard definitions of semantics of ABFs (see, e.g., [4, 23]), the set Δ in Definition 2.3 is required to be *closed*: $\Delta = Ab \cap Cn_{\vdash}(\Gamma \cup \Delta)$. The motivation for this is that a closed set satisfies the closure rationality postulate [6]. However, as shown in [15], for simple contrapositive ABFs (with collective attacks) this additional requirement can be avoided.

The set of the complete (respectively, the grounded, well-founded, preferred, stable) extensions of ABF is denoted $\mathsf{Com}(\mathsf{ABF})$ (respectively, $\mathsf{Grd}(\mathsf{ABF})$, $\mathsf{WF}(\mathsf{ABF})$, $\mathsf{Prf}(\mathsf{ABF})$, $\mathsf{Stb}(\mathsf{ABF})$). In what follows we shall denote by $\mathsf{Sem}(\mathsf{ABF})$ any of the above-mentioned sets. The entailment relations that are induced from an ABF (with respect to a certain semantics) are defined as follows:

Definition 2.4. Given an assumption-based framework $\mathsf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ and $\mathsf{Sem} \in \{\mathsf{Com}, \mathsf{WF}, \mathsf{Grd}, \mathsf{Prf}, \mathsf{Stb}\}$, we denote:

- $\mathsf{ABF} \mid\!\sim^{\cap}_{\mathsf{Sem}} \psi$ iff $\Gamma, \Delta \vdash \psi$ for *every* $\Delta \in \mathsf{Sem}(\mathsf{ABF})$.
- $\mathsf{ABF} \mid\!\sim^{\cup}_{\mathsf{Sem}} \psi$ iff $\Gamma, \Delta \vdash \psi$ for *some* $\Delta \in \mathsf{Sem}(\mathsf{ABF})$.

Example 2.5. Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \emptyset$, $Ab = \{p, \neg p, q\}$, and $\sim\psi = \{\neg\psi\}$ for every formula ψ. A corresponding attack diagram is shown in Figure 1.[3]

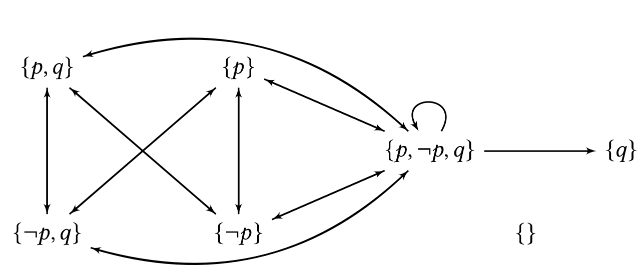

Figure 1: Attack diagram for Example 2.5

Note that since in classical logic inconsistent sets of premises imply *any* conclusion, the classically inconsistent set $\{p, \neg p, q\}$ attacks all the other sets in the diagram (For instance, $\{p, \neg p, q\}$ attacks $\{q\}$, since $p, \neg p, q \vdash \neg q$).

Thus, $\mathsf{Grd}(\mathsf{ABF}) = \mathsf{WFF}(\mathsf{ABF}) = \{\}$, while $\mathsf{Prf}(\mathsf{ABF}) = \mathsf{Stb}(\mathsf{ABF}) = \{\{p, q\}, \{\neg p, q\}\}$. It follows that for $* \in \{\cup, \cap\}$, $\mathsf{Sem} \in \{\mathsf{Grd}, \mathsf{WFF}\}$ we have that $\mathsf{ABF} \mid\!\sim^{*}_{\mathsf{Sem}} \psi$ iff ψ is a $\vdash$-tautology, while when $* \in \{\cup, \cap\}$ and $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}\}$ it holds that, e.g., $\mathsf{ABF} \mid\!\sim^{*}_{\mathsf{Sem}} q$.

3 COLLECTIVE ATTACKS

Let's reconsider Example 2.5. The grounded and the well-founded semantics in this case look overly cautious, since they allow only tautological inferences, while the inference of q (or any other formula in $Cn_{\vdash}(\{q\})$) is forbidden, although there is no indication whatsoever that q is related to the inconsistency in Ab. One way to overcome this weakness is to extend the notion of attacks, and apply it to *sets* of formulas. The extended notion, called *collective attacks*, is defined next (cf. Definition 2.2).

Definition 3.1. Let $\mathsf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \neg \rangle$ be a simple-contrapositive ABF. Then Δ *collectively attacks* $\{\psi_1, \ldots, \psi_n\}$ if $\Gamma, \Delta \vdash \neg \bigwedge_{i=1}^{n} \psi_i$. This notion is carried on to supersets: Δ collectively attacks a set $\Theta \subseteq Ab$, if Δ collectively attacks some $\{\psi_1, \ldots, \psi_n\} \subseteq \Theta$.

Example 3.2. Consider again the assumption-based framework of Example 2.5, but now attacks are replaced by collective attacks. The attack diagram that is obtained is similar to that in Figure 1, with two additional collective attacks: One, from $\{\}$ on $\{p, \neg p, q\}$

[3] To simplify the diagram, the set $\{p, \neg p\}$ is omitted (it behaves just like $\{p, \neg p, q\}$).

(since $\vdash \neg(p \wedge \neg p)$) and the other one from $\{q\}$ on $\{p, \neg p, q\}$ (since $q \vdash \neg(p \wedge \neg p)$). As a consequence, $\{q\}$ defends itself from any (collective) attacker, which implies that this set is the grounded as well as the well-founded extension of the assumption-based framework with collective attacks. In particular, any formula in $Cn_{\vdash}(\{q\})$ is now inferred also by the grounded and the well-founded semantics in this case.

The fact that the transition in the last example to collective attacks provides more intuitive conclusions is not a coincidence. Indeed, in [15] the following interesting facts about simple contrapositive frameworks with collective attacks are shown:

(1) Preferential and stable semantics coincide: $\mathsf{Prf}(\mathsf{ABF}) = \mathsf{Stb}(\mathsf{ABF})$
(2) Grounded and well-founded semantics equals to the intersection of the maximally consistent sets: $\mathsf{Grd}(\mathsf{ABF}) = \mathsf{WF}(\mathsf{ABF}) = \{\cap \mathsf{MCS}(\mathsf{ABF})\}$, where

$$\mathsf{MCS}(\mathsf{ABF}) = \{\Delta \subseteq Ab \mid \Gamma, \Delta \nvdash \mathsf{F} \text{ and } \Gamma, \Delta' \vdash \mathsf{F} \text{ for all } \Delta \subsetneq \Delta' \subseteq Ab\}.$$

Our goal in this paper is to examine to what extent these and other properties carry on to more expressive ABFs, in particular those in which preferences among the defeasible assumptions are introduced. In the next section we define such extended settings, and in Section 5 we study the properties of the induced entailment relations. We show, in particular, that the two facts above carry on to the prioritized case (with some obvious adjustments). In particular, the extension of the second fact to preferential argumentation frameworks provides, to the best of our knowledge, a novel characterization of the grounded semantics in such frameworks.

4 ADDING PREFERENCES

We now extend simple contrapositive ABFs with priorities, expressing the relative strengths (or reliability) of assumptions. Adding quantitative measurements to qualitative information is a common approach in argumentation-based reasoning in general (see e.g. [17, 18]) and in assumption-based argumentation frameworks in particular (ABA$^+$ [8, 10]) is a notable example). In our case, the extension is made by allocating to every defeasible assumption a preference value (where, intuitively, lower values indicate higher preferences), and then making sure that assumptions with higher preferences cannot be attacked by assumptions with strictly lower preferences. This is formalized next.

Definition 4.1. [2] Let Ab be a set of formulas (which, in our case, will be the defeasible assumptions of an ABF).

- An *allocation function* on Ab is a total function $g : Ab \longrightarrow \mathbb{N}$.
- A *numeric aggregation function* f is a total function that maps multisets of natural numbers into non-negative real numbers, such that $\forall x \in \mathbb{N}\, f(\{x\}) = x$.[4] We also assume that an aggregation function is $\subseteq$*-coherent* in its values, namely, it is either non-decreasing with respect to the subset relation ($f(X') \leq f(X)$ whenever $X' \subseteq X$) or non-increasing with respect to the subset relation ($f(X') \geq f(X)$ whenever $X' \subseteq X$).
- A *preference setting* on Ab is a pair $\mathsf{P} = \langle g, f \rangle$, where g is an allocation function on Ab and f is a numeric aggregation function.

[4] In what follows, the set signs of a singleton will sometimes be omitted.

An allocation function makes preferences among the defeasible information. The sets $Ab_i = \{\psi \in Ab \mid g(\psi) = i\}$ form a partition of Ab, which in turn may be viewed as a stratified set. This is sometimes denoted by $Ab = Ab_1 \oplus \ldots \oplus Ab_n$. Aggregation functions are then used for aggregating the preferences. The maximum, minimum, and the summation functions are common aggregation functions. We shall write $f(g(\Delta))$ instead of $f(\{g(\delta) \mid \delta \in \Delta\})$.

Some useful properties of preference settings are defined next.

Definition 4.2. [2] Let $\mathsf{P} = \langle g, f \rangle$ be a preference setting for Ab and $\Delta \subseteq Ab$ an arbitrary non-empty set of assumptions.

- We say that P is *selecting*, if $f(g(\Delta)) \in \bigcup_{\delta \in \Delta} f(g(\delta))$.
- We say that P is *max-lower-bounded*, if $\max\{f(g(\delta)) \mid \delta \in \Delta\} \leq f(g(\Delta))$.
- We say that P is *conservative under union*, if $f(g(\Delta)) \leq f(g(\phi))$ and $f(g(\psi)) \leq f(g(\phi))$ imply that $f(g(\Delta \cup \{\psi\})) \leq f(g(\phi))$.

The selection property assures that $f(g(\Delta))$ is a selection of values in $\{f(g(\delta)) \mid \delta \in \Delta\}$, i.e., $f(g(\Delta))$ does not introduce 'new' values other than those that are assigned to the elements in Δ. Preference settings where $f = \min$ or $f = \max$ are clearly selecting. In the latter case, the setting is also max-lower-bounded and conservative under union.

Prioritized ABFs are defined now as follows:

Definition 4.3. A *prioritized assumption-based framework* (prioritized ABF, or pABF, for short) is a pair $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P} \rangle$, where ABF is a (simple contrapositive) assumption-based argumentation framework and P is a preference setting.

A prioritized ABF is called selecting, max-lower-bounded, or conservative under union, if so is its preference setting.

As noted before, for defining collective attacks in pABFs one has to take care that assumptions with higher priorities will not be attacked by assumptions with lower priorities. This is assured by the following definition.

Definition 4.4. Let $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P} \rangle$ be a prioritized ABF with a preference setting $\mathsf{P} = \langle g, f \rangle$. Let also $\Delta, \Theta \subseteq Ab$ and $\psi_1, \ldots \psi_n \in Ab$. Suppose further that Δ collectively attacks $\{\psi_1, \ldots \psi_n\}$.

- The P*-attacking value* of Δ on $\{\psi_1, \ldots, \psi_n\}$ is: $\mathsf{val}_{f,g}(\Delta, \psi) = \min\{f(g(\Delta')) \mid \Delta'$ is a $\subseteq$-minimal subset of Δ that collectively attacks $\{\psi_1, \ldots \psi_n\}\}$.
- Δ *collectively p-attacks* $\{\psi_1, \ldots, \psi_n\}$ iff Δ collectively attacks $\{\psi_1, \ldots, \psi_n\}$, and $\mathsf{val}_{f,g}(\Delta, \psi) \leq f(g(\psi_i))$ for every $1 \leq i \leq n$. As in the non-prioritized case, we say that Δ collectively p-attacks Θ if Δ collectively p-attacks some $\{\psi_1, \ldots, \psi_n\} \subseteq \Theta$.

The semantics of the structures in Definition 4.4 are defined just as before (Definition 2.3), where 'attacks' are replaced by 'collective-p-attacks'. The corresponding entailment relations are defined just as in Definition 2.4.

Example 4.5. Consider the pABF that is obtained from the ABF of Examples 2.5 and 3.2, together with the allocation function $g(p) = 1$, $g(\neg p) = 2$, $g(q) = 3$, and the aggregation function $f = \max$. The diagram for this case is shown in Figure 2 (cf. Figure 1). In the figure, dashed lines denote collective attacks that are not standard (pointed) attacks.[5]

[5] Note that although q collectively attacks $\{p, \neg p, q\}$, it does not collectively p-attack the latter, since q is less preferred than both p and $\neg p$.

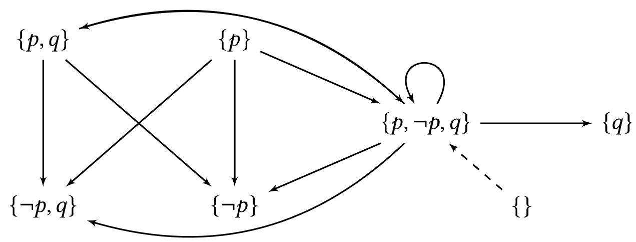

Figure 2: Attack diagram for Example 4.5

Note that this time, since p is strictly preferred over $\neg p$, the grounded extension is $\{p, q\}$ and not just $\{q\}$ as in the non-prioritized case. Thus, intuitively, p and q belong to the grounded extension in this case for two different reasons: p due to its high priority, and q since it is not related to the inconsistency in Ab. In the next section we shall prove that this is not a coincidence.

5 REASONING WITH COLLECTIVE ATTACKS

In this section we consider some basic properties of the semantics and the entailment relations that are induced by pABFs with collective attacks.

5.1 Preferred and Stable Semantics

First, we check the correspondence between preferred and stable extensions in such frameworks. This is shown in Proposition 5.2. For this proposition, we need the following lemma.

Lemma 5.1. *Let* pABF $= \langle$ABF, P$\rangle$, *be a selecting prioritized ABF with collective attacks that is conservative under union, and let* Δ *be a conflict-free set in* Ab. *Then* Δ *is maximally admissible iff it collectively attacks any* $\psi \in Ab \setminus \Delta$.

Proof. One direction is clear: as already shown in [13] (for regular attacks), if a conflict-free set Δ p-attacks any $\psi \in Ab \setminus \Delta$ it must be maximally admissible. Since Δ p-attacks a formula ψ iff it collectively p-attacks $\{\psi\}$, we are done.

Let now Δ be a maximally admissible set and suppose towards a contradiction that there is some $\psi \in Ab \setminus \Delta$ s.t. Δ does not collectively p-attack ψ. Let $\{\psi_1, \ldots, \psi_n\} = Ab \setminus \Delta$ s.t. $g(\psi_i) \geq g(\psi_j)$ when $i < j$ (that is, the ψ_i's are all the assumptions not in Δ, arranged according to theirs strengths). We now construct an admissible set $\Delta^\star$ s.t. $\Delta \subsetneq \Delta^\star$, which contradicts the maximal admissibility of Δ. We define: $\Delta^\star = \bigcup_{i \geq 0} \Delta_i$, where: $\Delta_0 = \Delta$ and for every $0 \leq i \leq n-1$,

$$\Delta_{i+1} = \begin{cases} \Delta_i \cup \{\psi_{i+1}\} & \text{if } \Gamma, \Delta_i \nvdash \neg\psi_{i+1}, \\ \Delta_i & \text{otherwise.} \end{cases}$$

We first show that [C1]: for no $i \geq 0$, if $\psi_i \in \Delta_i$ then $\Gamma, \Delta_i \nvdash \neg\psi_i$. The case where $i = 0$ is clear, since Δ is conflict-free. Now, given any $i \geq 0$, suppose towards a contradiction that $(*)$ $\psi_{i+1} \in \Delta_{i+1}$, yet $(**)$ $\Gamma, \Delta_{i+1} \vdash \neg\psi_{i+1}$. By the construction of Δ_{i+1}, $(*)$ means that $\Gamma, \Delta_i \nvdash \neg\psi_{i+1}$. Thus $\Delta_{i+1} \neq \Delta_i$ (otherwise we get a contradiction to $(**)$), i.e., $\Delta_{i+1} = \Delta_i \cup \{\psi_{i+1}\}$, and so $(**)$ means that $\Gamma, \Delta_i, \psi_{i+1} \vdash \neg\psi_{i+1}$. By contraposition, $\Gamma, \Delta_i \setminus \delta, \psi_{i+1} \vdash \neg\delta$ for any $\delta \in \Delta_i$, and by contraposition again $\Gamma, \Delta_i, \vdash \neg\psi_{i+1}$, a contradiction to the assumption that $\Gamma, \Delta_i \nvdash \neg\psi_{i+1}$.

We now show that [C2]: for every $i \geq 0$, Δ_i is conflict-free. We show this by an induction on i. The inductive base is clear since Δ is conflict-free. Suppose now that [C2] holds for Δ_i and suppose towards a contradiction that Δ_{i+1} collectively p-attacks some $\Theta \subseteq \Delta_{i+1}$. This means, in particular, that $\Gamma, \Delta_{i+1} \vdash \neg\bigwedge\Theta$. If $\psi_{i+1} \notin \Delta_{i+1}$, then $\Delta_i = \Delta_{i+1}$ and by the induction hypothesis $\Delta_i = \Delta_{i+1}$ is conflict-free, so we are done. If $\psi_{i+1} \in \Delta_{i+1}$, then by contraposition, $\Gamma, \Delta_{i+1} \cup (\Theta \setminus \{\psi_{i+1}\}) \vdash \neg\psi_{i+1}$. As $\Theta \subseteq \Delta_i \cup \{\psi_{i+1}\}$, this means that $\Gamma, \Delta_i \vdash \neg\psi_{i+1}$. This is a contradiction to C1.

We now show that [C3]: $\Delta^\star$ is admissible. Suppose towards a contradiction that some $\Theta \subseteq Ab$ collectively p-attacks $\Delta^\star$ and $\Delta^\star$ does not collectively p-attack Θ. Since $\Delta^\star$ does not collectively p-attack Θ, and $\Delta \subseteq \Delta^\star$, Δ does not collectively p-attack Θ (as p-attacks are closed under supersets). Since $\{\psi_1, \ldots, \psi_n\}$ contains all the assumptions not collectively p-attacked by Δ, we have that $(\Theta \setminus \Delta^\star) \subseteq \{\psi_1, \ldots, \psi_n\}$. Let $\phi \in \Theta \setminus \Delta^\star$ (Note that since by C2, $\Delta^\star$ is conflict-free, $\Theta \not\subseteq \Delta^\star$ and so such ϕ exists). Since $\phi \notin \Delta^\star$ yet $\phi = \psi_k$ for some $1 \leqslant k \leqslant n$, necessarily $\Gamma, \Delta_{k-1} \vdash \neg\phi$. Since $\Delta^\star$ does not collectively p-attack ϕ, again by the closure of p-attacks to supersets, also Δ_{k-1} does not collectively p-attack ϕ, and thus $\phi <_{\mathsf{P}} \Delta_{k-1}$, i.e., $f(g(\phi) < f(g(\Delta_{k-1}))$. By the selecting property, there is some $\delta \in \Delta_{k-1}$ s.t. $x = f(g(\delta))$. Suppose first that $\delta \notin \Delta$, i.e., for some $1 \leq i < k$, $f(g(\phi)) = f(g(\psi_k)) < f(g(\psi_i))$. This contradicts the construction of $\{\psi_1, \ldots, \psi_n\}$. Thus, $\delta \in \Delta$. Take $\delta^\star \in \Delta$ s.t. $f(g(\delta)) \leq f(g(\delta^\star))$ and for no $\delta' \in \Delta$, $f(g(\delta^\star)) < f(g(\delta'))$. We show that $\delta^\star \not<_{\mathsf{P}} \Delta_{k-1} \cup \phi \setminus \delta^\star$:

Indeed, suppose for a contradiction that $f(g(\delta^\star)) < f(g(\Delta_{k-1} \cup \{\phi\} \setminus \{\delta^\star))$. Again, by the selecting property, there is some $\gamma \in \Delta_{k-1} \cup \{\phi\}$ such that $x = f(g(\gamma))$. Suppose first that $\gamma \notin \Delta$. Then since $f(g(\delta)) \leq f(g(\delta^\star))$ and $f(g(\phi)) < f(g(\delta))$, we have that $f(g(\phi)) < f(g(\gamma))$, a contradiction to the construction of $\{\psi_1, \ldots, \psi_n\}$ (which are arranged according to their strengths). Thus, $\gamma \in \Delta$, but this contradicts the way $\delta^\star$ was selected.

We have shown that $\delta^\star \not<_{\mathsf{P}} \Delta_{k-1} \cup \phi \setminus \delta^\star$. This means that $\Delta_{k-1} \cup \{\phi\} \setminus \{\delta^\star\}$ collectively p-attacks $\delta^\star \in \Delta$, and thus, by the admissibility of Δ, Δ collectively p-attacks some $\Theta \subseteq \Delta_{k-1} \cup \{\phi\} \setminus \{\delta^\star\}$ (which implies that there is some $\Delta' \subseteq \Delta$ s.t. $\Gamma, \Delta' \vdash \neg\bigwedge\Theta$ and $f(g(\Delta)) \leq f(g(\gamma))$ for every $\gamma \in \Theta$). Since Δ is conflict-free, $\Theta \setminus \Delta \neq \emptyset$. Notice that $\Theta \setminus \Delta = \{\phi, \psi_{l_1}, \ldots, \psi_{l_k}\}$ for some $\{l_1, \ldots, l_k\} \subseteq \{1, \ldots, i+1\}$. As $g(\psi_j) \leq g(\phi)$ for every $j = l_1, \ldots, l_k$ (in view of how the indices were chosen), also $f(g(\psi_j)) \leq f(g(\phi))$ for every $j = l_1, \ldots, l_k$, and so, by conservativeness under union,$f(g(\Delta' \cup \{\psi_{l_1}, \ldots, \psi_{l_k}\})) \leq f(g(\phi))$. Thus, $\Delta' \cup \{\psi_{l_1}, \ldots, \psi_{l_k}\}) \subseteq \Delta_{i+1}$ attacks ϕ.

Finally, we show that [C4]: $\Delta \subsetneq \Delta^\star$. Suppose towards a contradiction that $\Delta = \Delta_1$. This means that $\Gamma, \Delta \vdash \neg\psi_1$. Since Δ is conflict-free and it does not p-attack ψ_1, by [2, lemma4] there is a $\phi \in \Delta$ s.t. $\Delta \cup \psi_1 \setminus \phi$ p-attacks ϕ (and $\phi \neq \psi_1$). Since Δ is admissible, Δ collectively p-attacks some $\sigma \in (\Delta \setminus \phi) \cup \psi_1$. Since Δ is conflict-free, $\sigma = \psi_1$, which contradicts the assumption that Δ does not collectively p-attack ψ_1, and so Δ does not p-attack ψ_1. We thus conclude that $\Delta \subsetneq \Delta_1 \subseteq \Delta^\star$.

By [C3] and [C4] we get a contradiction to the maximal admissibility of Δ. □

By the last lemma, we conclude the following result:

Proposition 5.2. *Let* pABF *be a prioritized ABF with collective attacks, which is both selecting and conservative under union. Then the stable extensions and the preferred extensions of* pABF *coincide.*

In [2] it is shown that when $f = \max$ (i.e., when the comparison criterion is with respect to the least preferred formulas, also known as the *'weakest link principle'*), the stable extensions of a prioritized ABF (with regular attacks) are the same as the $\sqsubset_g$-preferred maximally consistent subsets of Ab, where $\sqsubset_g$ is Brewka's preference order [5]. Formally:

- Let $Ab_i = \{\psi \in Ab \mid g(\psi) = i\}$ for $i = 1, \dots, n$ be a partition of Ab, and let $\Delta, \Theta \subseteq Ab$. We denote $\Delta \sqsubset_g \Theta$ (intuitively read as: 'Δ is *preferred* over Θ'), if there is an $1 \leq i \leq n$ such that $Ab_j \cap \Delta = Ab_j \cap \Theta$ for every $1 \leq j < i$, and $Ab_i \cap \Delta \supsetneq Ab_i \cap \Theta$.
- Δ is a *maximally consistent set* (MCS) in ABF, if (a) $\Gamma, \Delta \nvdash \mathsf{F}$ and (b) $\Gamma, \Delta' \vdash \mathsf{F}$ for every $\Delta \subsetneq \Delta' \subseteq Ab$. The set of the maximally consistent sets in ABF is denoted MCS(ABF).
- Δ is a *preferred maximally consistent set* (pMCS) in pABF, if $\Delta \in$ MCS(ABF) and there is no $\Theta \in$ MCS(ABF) that is $\sqsubset_g$-preferred over Δ. The set of the preferred maximally consistent sets in pABF is denoted $\mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF})$.

Proposition 5.3. *Let* $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P} \rangle$ *be a prioritized ABF in which* $\mathsf{P} = \langle g, \max \rangle$ *for some allocation function* g. *Then* $\mathsf{Prf}(\mathsf{pABF}) = \mathsf{Stb}(\mathsf{pABF}) = \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF})$.

Proposition 5.2 allows us to extend Proposition 5.3 to prioritized ABFS with collective attacks:

Corollary 5.4. *Let* $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P} \rangle$ *be a prioritized ABF with collative attacks, in which* $\mathsf{P} = \langle g, \max \rangle$ *for some allocation function* g. *Then* $\mathsf{Prf}(\mathsf{pABF}) = \mathsf{Stb}(\mathsf{pABF}) = \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF})$.

Proof. By Proposition 5.2, since settings with $f = \max$ are both selecting and conservative under union. □

The last corollary allows us to easily prove some properties of the induced entailment relations. The following properties were introduced by Kraus, Lehmann and Magidor in [19] and [20], and their formulations are adjusted to our setting. Some of the properties (CM, CC, and LLE) take into account also the priority setting. In such cases, the original formulation in [19] is obtained just by ignoring the conditions about the allocation function.

Below, instead of writing $\mathsf{ABF} \mid\!\sim \psi$, where $\mid\!\sim$ is as defined in Definition 2.4 and $\mathsf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \neg \rangle$, we just write $\Gamma, Ab \mid\!\sim \psi$.

Definition 5.5. A relation $\mid\!\sim$ between pABFs and formulas is called *cumulative*, if the following conditions are satisfied:

- *Cautious Reflexivity* (CR): For every $\vdash$-consistent formula $\psi \in \Gamma$ it holds that $\Gamma, Ab \mid\!\sim \psi$.
- *Cautious Monotonicity* (CM): If $\Gamma, Ab \mid\!\sim \phi$ for ϕ s.t. $g(\phi) \leq \min\{g(\varphi) \mid \varphi \in Ab\}$, and $\Gamma, Ab \mid\!\sim \psi$, then $\Gamma \cup \{\phi\}, Ab \mid\!\sim \psi$.
- *Cautious Cut* (CC): If $\Gamma, Ab \mid\!\sim \phi$ where $g(\phi) \leq \min\{g(\varphi) \mid \varphi \in Ab\}$, and $\Gamma \cup \{\phi\}, Ab \mid\!\sim \psi$, then $\Gamma, Ab \mid\!\sim \psi$.
- *Left Logical Equivalence* (LLE): If $\phi \vdash \psi$ and $\psi \vdash \phi$ and $g(\phi) = g(\psi)$, then $\Gamma \cup \{\phi\}, Ab \mid\!\sim \sigma$ iff $\Gamma \cup \{\psi\}, Ab \mid\!\sim \sigma$.
- *Right Weakening* (RW): If $\phi \vdash \psi$ and $\Gamma, Ab \mid\!\sim \phi$ then $\Gamma, Ab \mid\!\sim \psi$.

A cumulative relation is called *preferential*, if it satisfies the following condition:

- *Distribution* (OR): If $\Gamma \cup \{\phi\}, Ab \mid\!\sim \sigma$ and $\Gamma \cup \{\psi\}, Ab \mid\!\sim \sigma$ then also $\Gamma \cup \{\phi \vee \psi\}, Ab \mid\!\sim \sigma$.

Another property that is sometimes useful for reasoning with conflicts is the following:

Definition 5.6. Given a logic $\mathfrak{L} = \langle \mathsf{L}, \vdash \rangle$, let $\mathsf{ABF}_i = \langle \mathfrak{L}, \Gamma_i, Ab_i, \sim_i \rangle$ $(i = 1, 2)$ be two ABFs based on $\mathfrak{L}$.

- We denote by Atoms(Γ) the set of all atoms occurring in Γ.
- We say that Γ_1 and Γ_2 are *syntactically disjoint* if $\mathsf{Atoms}(\Gamma_1) \cap \mathsf{Atoms}(\Gamma_2) = \emptyset$.
- We say that ABF_1 and ABF_2 are *syntactically disjoint* if so are $\Gamma_1 \cup Ab_1$ and $\Gamma_2 \cup Ab_2$.
- We denote: $\mathsf{ABF}_1 \cup \mathsf{ABF}_2 = \langle \mathfrak{L}, \Gamma_1 \cup \Gamma_2, Ab_1 \cup Ab_2, \sim_1 \cup \sim_2 \rangle$.

We say that the entailment $\mid\!\sim$ satisfies *non-interference* [7], if for every two syntactically disjoint assumption-based frameworks $\mathsf{ABF}_1 = \langle \mathfrak{L}, \Gamma_1, Ab_1, \sim_1 \rangle$ and $\mathsf{ABF}_2 = \langle \mathfrak{L}, \Gamma_2, Ab_2, \sim_2 \rangle$ where $\Gamma_1 \cup \Gamma_2$ is consistent, it holds that $\mathsf{ABF}_1 \mid\!\sim \psi$ iff $\mathsf{ABF}_1 \cup \mathsf{ABF}_2 \mid\!\sim \psi$ for every formula ψ such that $\mathsf{Atoms}(\psi) \subseteq \mathsf{Atoms}(\Gamma_1 \cup Ab_1)$.

For extending non-interference to the prioritized case, we further suppose that there are priority settings $\mathsf{P}_i = \langle g_i, f \rangle$ over Ab_i $(i = 1, 2)$. When ABF_1 and ABF_2 are syntactically disjoint, we can define a priority setting $\mathsf{P} = \langle g, f \rangle$ over $Ab_1 \cup Ab_2$, where g coincides with g_i on Ab_i. In such a case, non-interference is defined as in the non-prioritized case, except that now we require that $\mathsf{pABF}_1 \mid\!\sim \psi$ iff $\mathsf{p}(\mathsf{ABF}_1 \cup \mathsf{ABF}_2) \mid\!\sim \psi$, where $\mathsf{p}(\mathsf{ABF}_1 \cup \mathsf{ABF}_2) = \langle \mathsf{ABF}_1 \cup \mathsf{ABF}_2, \mathsf{P} \rangle$.

Proposition 5.7. *Let* $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P} \rangle$ *be a prioritized simple contrapositive ABF with collective attacks, in which* $\mathsf{P} = \langle g, \max \rangle$ *for some allocation function* g. *Then both* $\mid\!\sim^{\cup}_{\mathsf{Sem}}$ *and* $\mid\!\sim^{\cap}_{\mathsf{Sem}}$ *are cumulative for* $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}\}$.

Proposition 5.8. *Let* $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P} \rangle$ *be a prioritized simple contrapositive ABF with collective attacks, in which* $\mathsf{P} = \langle g, \max \rangle$ *for some allocation function* g. *Then both* $\mid\!\sim^{\cup}_{\mathsf{Sem}}$ *and* $\mid\!\sim^{\cap}_{\mathsf{Sem}}$ *satisfy non-interference for* $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}\}$.

Propositions similar to the the last two results, but for standard (pointed) attacks, are shown in [2], based on Proposition 5.3. By Corollary 5.4, their proofs are carried on to Propositions 5.7 and 5.8, incorporating collective attacks.

What about preferentiality? It turns out that skeptical reasoning ($\mid\!\sim^{\cap}_{\mathsf{Sem}}$) is preferential, while credulous reasoning ($\mid\!\sim^{\cup}_{\mathsf{Sem}}$) is not.

Proposition 5.9. *Let* $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P} \rangle$ *be a prioritized simple contrapositive ABF with collective attacks, in which* $\mathsf{P} = \langle g, \max \rangle$ *for some allocation function* g. *Then, for every* $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}\}$ *it holds that* $\mid\!\sim^{\cap}_{\mathsf{Sem}}$ *is a preferential logic, while* $\mid\!\sim^{\cup}_{\mathsf{Sem}}$ *is not a preferential logic.*

Proof. By Proposition 5.7 it suffices to check the distribution property. Below, for some $\mathsf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \neg \rangle$ and a formula ϕ, we denote $\mathsf{ABF}^{\phi} = \langle \mathfrak{L}, \Gamma \cup \{\phi\}, Ab, \neg \rangle$.

- Let $\mid\!\sim$ abbreviate $\mid\!\sim^{\cap}_{\mathsf{Sem}}$ for some $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}\}$. To see that $\mid\!\sim$ satisfies the distribution property, suppose towards a contradiction that $\Gamma \cup \{\phi\}, Ab \mid\!\sim \sigma$ and $\Gamma \cup \{\psi\}, Ab \mid\!\sim \sigma$, but $\Gamma \cup \{\phi \vee \psi\}, Ab \not\mid\!\sim \sigma$. Then there is a set $\Delta \in \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF}^{\phi \vee \psi})$ such that $\Gamma \cup \{\phi \vee \psi\}, \Delta \nvdash \sigma$, and so $\Gamma \cup \{\phi\}, \Delta \nvdash \sigma$ or $\Gamma \cup \{\psi\}, \Delta \nvdash \sigma$. We show that this implies that $\Delta \in \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF}^{\phi})$ or $\Delta \in \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF}^{\psi})$, contradicting the assumption that $\Gamma \cup \{\phi\}, Ab \mathrel{|\!\sim} \sigma$ and $\Gamma \cup \{\psi\}, AB \mathrel{|\!\sim} \sigma$. Note, first, that it is not possible that both $\Gamma \cup \{\psi\}, \Delta \vdash \mathsf{F}$ and $\Gamma \cup \{\phi\}, \Delta \vdash \mathsf{F}$, otherwise $\Gamma \cup \{\psi \vee \phi\}, \Delta \vdash \mathsf{F}$, and so also $\Gamma \cup \{\phi \vee \psi\}, \Delta \vdash \sigma$. Without loss of generality, we assume that $\Gamma \cup \{\psi\}, \Delta \nvdash \mathsf{F}$. If there were some $\Delta' \supsetneq \Delta$ s.t. $\Gamma \cup \{\psi\}, \Delta' \nvdash \mathsf{F}$, then also $\Gamma \cup \{\phi \vee \psi\}, \Delta' \nvdash \mathsf{F}$, contradicting the assumption that $\Delta \in \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF}^{\phi \vee \psi})$. Thus, $\Delta \in \mathsf{MCS}(\mathsf{ABF}^{\psi})$. To see that $\Delta \in \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF}^{\psi})$, suppose that there is some $\Theta \in \mathsf{MCS}(\mathsf{ABF}^{\psi})$ s.t. $\Theta \sqsubset_g \Delta$. It can be shown that in this case there is some $\Theta' \in \mathsf{MCS}(\mathsf{ABF}^{\psi \vee \phi})$ with $\Theta' \supseteq \Theta$ (since $\Gamma \cup \{\psi\}, \Theta \nvdash \mathsf{F}$ implies that $\Gamma \cup \{\psi \vee \phi\}, \Theta \nvdash \mathsf{F}$). By the definition of $\sqsubset_g$, it holds that $\Theta' \sqsubset_g \Lambda$. But this is a contradiction to $\Delta \in \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF}^{\phi \vee \psi})$. Thus, $\Delta \in \mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF}^{\psi})$, but as noted above, this contradicts the assumption that $\Gamma \cup \{\psi\}, AB \mathrel{|\!\sim} \sigma$.

- Let $\mathrel{|\!\sim}$ abbreviate $\mathrel{|\!\sim}^{\cup}_{\mathsf{Sem}}$ for some $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}\}$. To see that $\mathrel{|\!\sim}$ *does not* satisfy the distribution property, consider the following prioritized ABF: $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \emptyset$, $Ab = \{r \wedge (q \supset p), \neg r \wedge (\neg q \supset p)\}$, and $g(\psi) = 1$ for every $\psi \in Ab$. Then $\{q\}, Ab \mathrel{|\!\sim} p$ and $\{\neg q\}, Ab \mathrel{|\!\sim} p$ but $\{q \vee \neg q\}, Ab \not\mathrel{|\!\sim} p$. □

Note 2. Another property that is considered in [20], called *rational monotonicity* (RM), requires that if $\Gamma, Ab \mathrel{|\!\sim} \sigma$ and $\Gamma, Ab \not\mathrel{|\!\sim} \neg\psi$ then $\Gamma \cup \{\psi\}, Ab \mathrel{|\!\sim} \sigma$. In [15, Example 11] it is shown that RM fails for $\mathrel{|\!\sim}^{\cap}_{\mathsf{Prf}}$ and $\mathrel{|\!\sim}^{\cap}_{\mathsf{Stb}}$ already when the framework is not prioritized.

5.2 Grounded and Well-Founded Semantics

Let us turn now to the grounded and the well-founded semantics. As Examples 2.5 and 3.2 show, a transition from pointed attacks to collective attacks may affect the grounded extension. Yet, the following proposition carries on to frameworks with collectives attacks:

Proposition 5.10. *Let* pABF *be a prioritized ABF with collective attacks. Then* $\mathsf{WF}(\mathsf{pABF}) = \bigcap \mathsf{Grd}(\mathsf{pABF})$.

Proof. Since $\mathsf{Grd}(\mathsf{pABF}) \subseteq \mathsf{Cmp}(\mathsf{pABF})$, we have: $\mathsf{WF}(\mathsf{pABF}) = \bigcap \mathsf{Cmp}(\mathsf{pABF}) \subseteq \bigcap \mathsf{Grd}(\mathsf{pABF})$. For the converse, note that every element in $\bigcap \mathsf{Grd}(\mathsf{pABF})$ belongs to every $\subseteq$-minimal complete extension of pABF, and so it belongs to every complete extension (not necessarily minimal) of pABF, thus $\bigcap \mathsf{Grd}(\mathsf{pABF}) \subseteq \bigcap \mathsf{Cmp}(\mathsf{pABF}) = \mathsf{WF}(\mathsf{pABF})$. □

Hence, the grounded and the well-founded semantics of a pABF with collective attacks coincide iff the pABF has a unique grounded extension. Such a case is assured by the characterization of the grounded extensions in terms of minimally inconsistent subsets that we provide below (Proposition 5.16). To the best of our knowledge, this is the first time that such a characterization has been provided for logic-based argumentation with prioritized knowledge-bases.

For the characterization of the grounded extensions, we need the following definition.

Definition 5.11. Let $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P} \rangle$ be a prioritized argumentation framework. For every $i \geq 1$, we define:

- $\mathsf{Free}_0(\mathsf{pABF}) = \emptyset$,
- $\mathsf{MIC}_i(\mathsf{pABF}) = \{\Lambda \subseteq \bigcup_{j \leq i} Ab_j \mid \Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}), \Lambda \vdash \mathsf{F}$ and there is no $\Delta' \subsetneq \Delta$ such that $\Gamma, \Delta' \vdash \mathsf{F}\}$,
- $\mathsf{Free}_i(\mathsf{pABF}) = \bigcup_{j<i} \mathsf{Free}_j(\mathsf{pABF}) \cup (Ab_i \setminus \bigcup \mathsf{MIC}_i(\mathsf{pABF}))$.

The idea behind this construction is the following: we proceed iteratively, starting from the assumptions with the best (lowest) priority, and select all the free formulas there (those that do not belong to any minimally inconsistent set). Then, we use these free formulas as strict premises in the next step, where we construct MIC_{i+1} as the sets that are minimally conflicting in view of the strict premises Γ and the free formulas $\mathsf{Free}_i(\mathsf{pABF})$ obtained in the previous step. All the formulas not involved in any such conflict are then designated as free on the $(i+1)$th level.

Note 3. An equivalent way of defining $\mathsf{Free}_i(\mathsf{pABF})$ is by the (union of the intersections of the) maximally consistent subset of Ab_j $(j \leq i)$ w.r.t. Γ and $\mathsf{Free}_{i-1}(\mathsf{pABF})$, namely:

- $\mathsf{Free}_0(\mathsf{pABF}) = \emptyset$,
- $\mathsf{MCS}_i(\mathsf{pABF}) = \{\Delta \subseteq \bigcup_{j \leq i} Ab_j \mid \Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}), \Delta \nvdash \mathsf{F}$ and there is no $\Delta \subsetneq \Delta'$ such that $\Gamma, \Delta' \nvdash \mathsf{F}\}$.

Note that the definition above is different than the notion of prioritized MCS w.r.t. preferred subtheories ($\mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF})$), introduced in Section 5.1. This difference is illustrated in Example 5.15 bellow.

The following result shows the validity of the alternative definition $\mathsf{Free}_i(\mathsf{pABF})$:

Lemma 5.12. $\mathsf{Free}_i(\mathsf{pABF}) = \bigcup_{j \leq i} \bigcap \mathsf{MCS}_j(\mathsf{pABF})$ *for all* $i \geq 0$.

Proof. By induction on i. The base case is trivial. For the inductive step, suppose that $\mathsf{Free}_i(\mathsf{pABF}) = \bigcup_{j \leq i} \bigcap \mathsf{MCS}_j(\mathsf{pABF})$. We first show that $\mathsf{Free}_{i+1}(\mathsf{pABF}) \subseteq \bigcup_{j \leq i+1} \bigcap \mathsf{MCS}_j(\mathsf{pABF})$. Indeed, consider some $\phi \in \mathsf{Free}_{i+1}(\mathsf{pABF})$ and suppose there is some $\Delta \in \mathsf{MCS}_j(\mathsf{pABF})$ s.t. $\phi \notin \Delta$. Then $\Gamma, \mathsf{Free}_i(\mathsf{pABF}), \Delta, \phi \vdash \mathsf{F}$ and thus there is some $\phi \in \Delta' \subseteq \Delta \cup \{\phi\}$ that is minimal in this regard, a contradiction to $\phi \in \mathsf{Free}_{i+1}(\mathsf{pABF})$.

To see that $\bigcup_{j \leq i+1} \bigcap \mathsf{MCS}_j(\mathsf{pABF}) \subseteq \mathsf{Free}_{i+1}(\mathsf{pABF})$, suppose now that $\phi \in \bigcup_{j \leq i+1} \bigcap \mathsf{MCS}_j(\mathsf{pABF})$ and suppose towards a contradiction that for some $\Delta \in \mathsf{MIC}_{i+1}(\mathsf{pABF})$ it holds that, $\phi \in \Delta$. Then $\Gamma, \mathsf{Free}_i(\mathsf{pABF}), \Delta \setminus \{\phi\} \nvdash \mathsf{F}$, which implies there is some maximally consistent $\Delta' \supset \Delta$ s.t. $\phi \notin \Delta'$. This is contradiction to the assumption that $\phi \in \bigcup_{j \leq i+1} \bigcap \mathsf{MCS}_j(\mathsf{pABF})$. □

Example 5.13. Let $\mathsf{ABF} = \langle \mathsf{CL}, \emptyset, \{s, \neg s, p, \neg p, r\}, \neg \rangle$ with $Ab_1 = \{s\}$, $Ab_2 = \{p, \neg p, r\}$ and $Ab_3 = \{\neg s\}$. Then:

- $\mathsf{MIC}_1(\mathsf{pABF}) = \{\emptyset\}$, $\mathsf{MCS}_1(\mathsf{pABF}) = \{\{s\}\}$, $\mathsf{Free}_1(\mathsf{pABF}) = \{s\}$.
- $\mathsf{MIC}_2(\mathsf{pABF}) = \{\{p, \neg p\}\}$, $\mathsf{MCS}_2(\mathsf{pABF}) = \{\{s, p, r\}, \{s, \neg p, r\}\}$, $\mathsf{Free}_2(\mathsf{pABF}) = \{s, r\}$.
- $\mathsf{MIC}_3(\mathsf{pABF}) = \{\{p, \neg p\}, \{s, \neg s\}\}$, $\mathsf{MCS}_3(\mathsf{pABF}) = \{\{s, p, r\}, \{s, \neg p, r\}\}$, $\mathsf{Free}_3(\mathsf{pABF}) = \{s, r\}$.

We further illustrate this definition with another example:

Example 5.14. Let $\mathsf{ABF} = \langle \mathsf{CL}, \emptyset, \{s, \neg s, s \wedge \neg r\}, \neg \rangle$ with $Ab_1 = \{s\}$, $Ab_2 = \{\neg s, s \wedge \neg r\}$. Then:

- $\mathsf{MIC}_1(\mathsf{pABF}) = \{\emptyset\}$ $\mathsf{Free}_1(\mathsf{pABF}) = \{s\}$.
- $\mathsf{MIC}_2(\mathsf{pABF}) = \{\{\neg s\}\}$ (as $\mathsf{Free}_1(\mathsf{pABF}), \{\neg s\} \vdash \mathsf{F}$), $\mathsf{Free}_2(\mathsf{pABF}) = \{s, s \wedge \neg r\}$.

Thus, since $\mathsf{Free}_1(\mathsf{pABF})$ is assumed to be a strict set of premises in the computation of MIC_2, $s \wedge \neg r$ does not get "drowned" by $\neg s$.

Suppose now that we add an assumption r to the second level (i.e., to Ab_2). Then $s \wedge \neg r$ is no longer considered a free formula, which is to be expected, as it is involved in a conflict independent of s (i.e.,

$\{s \wedge \neg r, r\}$). In more detail, let $\mathsf{ABF} = \langle \mathsf{CL}, \emptyset, \{s, \neg s, s \wedge \neg r, r\}, \neg\rangle$ with $Ab_1 = \{s\}$, $Ab_2 = \{\neg s, s \wedge \neg r, r\}$. Then:

- $\mathsf{MIC}_1(\mathsf{pABF}) = \{\emptyset\}$, $\mathsf{Free}_1(\mathsf{pABF}) = \{s\}$.
- $\mathsf{MIC}_2(\mathsf{pABF}) = \{\{\neg s\}, \{s \wedge \neg r, r\}\}$, $\mathsf{Free}_2(\mathsf{pABF}) = \{s\}$.

So in this case the only free formula of the prioritized framework is s, as expected.

Note 4. One might wonder whether the intersection of preferred subtheories coincides with the free formulas, since in the non-prioritized case it holds that $\mathsf{Free}(\mathsf{ABF}) = \bigcap \mathsf{MCS}(\mathsf{ABF})$. As is observed in [11], this is not the case when taking into account priorities. Here is a counter-example:

Example 5.15. Let $\mathsf{ABF} = \langle \mathsf{CL}, \emptyset, Ab_1 \cup Ab_2, \neg\rangle$ with $Ab_1 = \{(\neg a \vee \neg b) \wedge d, (\neg a \vee \neg b) \wedge \neg d\}$, $Ab_2 = \{a, b, (a \wedge b) \rightarrow \neg c, c\}$. Then:

- $\mathsf{MIC}_1(\mathsf{pABF}) = \{Ab_1\}$ and $\mathsf{MCS}_1(\mathsf{pABF}) = \mathsf{MCS}(Ab_1)$, thus $\mathsf{Free}_1(\mathsf{pABF}) = \emptyset$.
- $\mathsf{MIC}_2(\mathsf{pABF}) = \mathsf{MIC}(Ab_1 \cup Ab_2)$, and also $\mathsf{MCS}_2(\mathsf{pABF}) = \mathsf{MCS}(Ab_1 \cup Ab_2)$, thus $\mathsf{Free}_2(\mathsf{pABF}) = \emptyset$.

On the other hand, the preferred subtheories are $\mathsf{MCS}_{\sqsubset_g}(\mathsf{ABF}) = \{\{(\neg a \vee \neg b) \wedge d, a, (a \wedge b) \rightarrow \neg c, c\}, \{(\neg a \vee \neg b) \wedge d, b, (a \wedge b) \rightarrow \neg c, c\}, \{(\neg a \vee \neg b) \wedge \neg d, a, (a \wedge b) \rightarrow \neg c, c\}, \{(\neg a \vee \neg b) \wedge \neg d, b, (a \wedge b) \rightarrow \neg c, c\}\}$.

Thus, $\bigcap \mathsf{MCS}_{\sqsubset_g}(\mathsf{pABF}) = \{c\} \neq \emptyset = \mathsf{Free}(\mathsf{pABF})$.

We can now show the following characterization of the grounded extension for prioritized argumentation frameworks with collective attacks and the weakest-link principle (*max*-based aggregations).

Proposition 5.16. *Let* $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P}\rangle$ *be a prioritized ABF with collective attacks, in which* $\mathsf{P} = \langle g, \max\rangle$ *and* n *is the maximal number in the image of* g. *Then:* $\mathsf{Grd}(\mathsf{pABF}) = \mathsf{Free}_n(\mathsf{pABF})$.

Proof. We first show that $\mathsf{Free}_n(\mathsf{pABF})$ is contained in every complete extension. We do this by induction on the maximal value of g, showing that $\mathsf{Free}_i(\mathsf{pABF}))$ is contained in every complete extension for every $i \geq 1$. For the base case, suppose that $\Theta \subseteq \mathsf{Free}_1(\mathsf{pABF})$ and some Δ collectively p-attacks Θ. Without loss of generality, let Δ be a $\subseteq$-minimal set attacking Θ. Then $f(g(\Theta))$ is not strictly $\mathbb{P}$-stronger than $\mathsf{val}_{f,g}(\Delta, \Theta)$. As $f = \max$, and $g(\psi) = 1$ for every $\phi \in \Theta$ (since $\Theta \subseteq \mathsf{Free}_1(\mathsf{pABF})$), this means that $g(\delta) = g(\phi) = 1$ for every $\delta \in \Delta$ and $\phi \in \Theta$. But then there is $\Delta' \subseteq \Delta \cup \Theta \in \mathsf{MIC}_1(\mathsf{pABF})$ with $\Delta' \cap \Theta \neq \emptyset$ or $\Gamma, \Delta \vdash \mathsf{F}$. The first case constitutes a contradiction against $\Theta \subseteq \mathsf{Free}_1(\mathsf{pABF})$. Suppose that $\Gamma, \Delta \vdash \mathsf{F}$. Then $\emptyset$ collectively attacks Δ (since $\Gamma \vdash \neg\bigwedge\Delta$) and thus Θ is defended by $\emptyset$. Hence, Θ is included in any complete extension.

For the inductive step, suppose that $\mathsf{Free}_i(\mathsf{pABF})$ is contained in a complete extension Φ of pABF and let $\Theta \subseteq \mathsf{Free}_{i+1}(\mathsf{pABF})) \setminus \mathsf{Free}_i(\mathsf{pABF}))$. Suppose that some Δ collectively p-attacks Θ. Again, without loss of generality, let Δ be a $\subseteq$-minimal set attacking Θ. Then $f(g(\Theta))$ is not strictly $\mathbb{P}$-stronger than $\mathsf{val}_{f,g}(\Delta, \Theta)$, which means, in our case, that $\mathsf{val}_{f,g}(\Delta, \Theta) \leq f(g(\Theta)) = \max(g(\Theta)) \leq i+1$ (since $\Theta \subseteq Ab_{i+1}$). Thus, either $\Gamma, \Delta \vdash \mathsf{F}$ (in which case we have already shown in the base case that Θ is defended by $\emptyset$, and so it is in Φ as the latter is complete), or there is $\Delta' \subseteq \Delta \cup \Theta \in \mathsf{MIC}_{i+1}(\mathsf{pABF})$ with $\Delta' \cap \Theta \neq \emptyset$, a contradiction to the assumption that $\Theta \subseteq \mathsf{Free}_{i+1}(\mathsf{pABF})$. We have thus shown that $\Phi \supseteq \mathsf{Free}_n(\mathsf{pABF})$.

We now show that $\mathsf{Free}_n(\mathsf{pABF})$ is complete. We first make the following observation:

$$(\dagger)\text{: if } \Theta \text{ collectively p-attacks } \Delta, \text{ then } g(\theta) \leq g(\delta) \text{ for every } \theta \in \Theta \text{ and } \delta \in \Delta.$$

This follows immediately from the fact that $f = \max$. (Indeed, for that attack to take place, for every $\delta \in \Delta$ we require that $f(g(\delta))$ should not be preferred over $f(g(\Theta))$. In our case, and since the preference order is linear, this means that $\max(g(\Theta))$ should be less than or equal to $g(\delta)$ for every $\delta \in \Delta$. Thus $\forall \delta \in \Delta, \forall \theta \in \Theta$, $g(\theta) \leq g(\delta)$.)

We now show that $\mathsf{Free}_n(\mathsf{pABF})$ *is conflic-free.* Suppose towards a contradiction that there are some $\Delta_1, \Delta_2 \subseteq \mathsf{Free}_n(\mathsf{pABF})$ s.t. Δ_1 collective p-attacks Δ_2. Then $(\Delta_1 \cup \Delta_2) \cap \mathsf{MIC}_i(\mathsf{pABF}) \neq \emptyset$ for $i = \max_{\delta \in \Delta_1 \cup \Delta_2} g(\delta)$, thus $\mathsf{Free}_n(\mathsf{pABF}) \cap \mathsf{MIC}_i(\mathsf{pABF}) \neq \emptyset$, a contradiction to the definition of $\mathsf{Free}_n(\mathsf{pABF})$.

We now show that $\mathsf{Free}_n(\mathsf{pABF})$ *defends all of its elements.* We show by induction on i that $\mathsf{Free}_i(\mathsf{pABF})$ defends all of its elements. For the base case, suppose that $\Delta_1 \subseteq Ab$ attacks some $\Delta_2 \subseteq \mathsf{Free}_1(\mathsf{pABF})$. Then with ($\dagger$), $\Delta_1 \subseteq Ab_1$ and thus $\Gamma, \Delta_1 \cup \Delta_2 \vdash \mathsf{F}$. As $\Delta_2 \subseteq \mathsf{Free}_1(\mathsf{pABF})$, it holds that $\Gamma, \Delta_1 \vdash \mathsf{F}$, which implies that $\emptyset$ attacks Δ_1. For the inductive case, suppose that $\mathsf{Free}_i(\mathsf{pABF})$ defends all of its elements, and suppose that some $\Delta_1 \subseteq Ab$ attacks some $\Delta_2 \subseteq \mathsf{Free}_{i+1}(\mathsf{pABF})$. Let $j = \max_{\delta \in \Delta_2} g(\delta)$. By ($\dagger$), we know that $\max_{\delta \in \Delta_1} g(\delta) \leq \max_{\delta \in \Delta_2} g(\delta)$. With the inductive hypothesis, $j = i + 1$. Then $\Gamma, \mathsf{Free}_i(\mathsf{pABF}), \Delta_1, \Delta_2 \vdash \mathsf{F}$. Thus, there is a minimal $\Theta \subseteq \Delta_1 \cup \Delta_2$ s.t. $\Gamma, \mathsf{Free}_i, \Theta \vdash \mathsf{F}$. As $\Delta_2 \subseteq \mathsf{Free}_{i+1}(\mathsf{pABF})$, necessarily $\Theta \cap \Delta_2 = \emptyset$. Thus, $\Gamma, \mathsf{Free}_i(\mathsf{pABF}) \vdash \neg\bigwedge\Theta$. Suppose now that $\mathsf{Free}_i(\mathsf{pABF})$ does not collectively p-attack Θ. This means that there is some $\theta \in \Theta$ s.t. $f(g(\theta)) < i$. But this contradicts the definition of $\mathsf{Free}_i(\mathsf{pABF})$. Thus, we have established that $\mathsf{Free}_i(\mathsf{pABF})$ defends Δ_2 from the attack of Δ_1.

Next, we show that $\mathsf{Free}_n(\mathsf{pABF})$ *contains every set of assumptions that it defends.* For this, suppose that $\mathsf{Free}_n(\mathsf{pABF})$ collectively p-attacks every Δ that collectively p-attacks Θ. We have to show that $\Theta \subseteq \mathsf{Free}_n(\mathsf{pABF})$. Our assumption means that $\Gamma, \mathsf{Free}_n(\mathsf{pABF}) \vdash \neg\bigwedge\Delta$. With ($\dagger$), where $i = \max_{\delta \in \Delta} g(\delta)$, we get $\Gamma, \mathsf{Free}_i(\mathsf{pABF}) \vdash \neg\bigwedge\Delta$. Suppose now that $\Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}) \nvdash \neg\bigwedge\Delta$. Then it holds that $\Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}), \Delta \nvdash \mathsf{F}$ whereas $\Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}), \Delta, \Delta' \vdash \mathsf{F}$ for some $\Delta' \subseteq \mathsf{Free}_i(\mathsf{pABF}) \setminus \mathsf{Free}_{i-1}(\mathsf{pABF})$, a contradiction to the definition of $\mathsf{Free}_i(\mathsf{pABF})$ (since $\Delta \subset \bigcup_{j=1}^{i} Ab_j$). Thus, we have established that

$$(\ddagger)\text{: for every } \Delta \subseteq \textstyle\bigcup_i Ab_i \text{ that collectively attacks } \Theta, \; \Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}), \Delta \vdash \mathsf{F}.$$

Suppose now towards a contradiction that $\Theta \cap \mathsf{MIC}_i(\mathsf{pABF}) \neq \emptyset$, i.e., there is some $\Delta \subseteq \bigcup_{j=1}^{i} Ab_i$ and some $\emptyset \neq \Theta' \subseteq \Theta$ s.t. $\Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}), \Delta, \Theta' \vdash \mathsf{F}$ and $\Delta \cup \Theta'$ is minimal in this regard. Then $\mathsf{Free}_{i-1}(\mathsf{pABF}) \cup \Delta$ collectively p-attacks Θ' (thus also Θ). But then with ($\ddagger$), $\Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}), \Delta \vdash \mathsf{F}$, which is a contradiction to the assumed minimality of $\Delta \cup \Theta'$. We have shown that $\Theta \cap \mathsf{MIC}_i(\mathsf{pABF}) = \emptyset$, thus $\Theta \subseteq \mathsf{Free}_i(\mathsf{pABF})$, and so $\Theta \subseteq \mathsf{Free}_n(\mathsf{pABF})$, as required.

We finally show that $\mathsf{Free}_n(\mathsf{pABF})$ is *closed.* For this, suppose that $\Gamma, \mathsf{Free}_n(\mathsf{pABF}) \vdash \phi$ and $\phi \in Ab$. Suppose that some Δ collectively p-attacks ϕ (if there is no such Δ then $\phi \in \mathsf{Free}_n(\mathsf{pABF})$ and we are done). Then with ($\dagger$), where $\phi \in Ab_i$, it holds that $\Delta \subseteq \bigcup_{j=0}^{i} Ab_j$. As $\Gamma, \Delta \vdash \neg\phi$, it holds that $\Gamma, \phi \vdash \neg\bigwedge\Delta$. Thus, with transitivity, $\Gamma, \mathsf{Free}_n(\mathsf{pABF}) \vdash \neg\bigwedge\Delta$. Suppose now that $\Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}) \nvdash$

$\neg\bigwedge\Delta$. Then $\Gamma, \mathsf{Free}_{i-1}(\mathsf{pABF}), \Delta, \Psi \vdash \mathsf{F}$ for some $\Psi \subseteq \mathsf{Free}_n(\mathsf{pABF})$, which contradicts the definition of $\mathsf{Free}_n(\mathsf{pABF})$. It followed, then, that $\mathsf{Free}_{i-1}(\mathsf{pABF})$ collectively p-attacks Δ, which means that ϕ is defended by $\mathsf{Free}_n(\mathsf{pABF})$. As $\mathsf{Free}_n(\mathsf{pABF})$ contains every set of assumptions that it defends, $\phi \in \mathsf{Free}_n(\mathsf{pABF})$.

Altogether, we have shown that $\mathsf{Free}_n(\mathsf{pABF}) \subseteq \mathsf{WF}(\mathsf{pABF})$ and that $\mathsf{Free}_n(\mathsf{pABF})$ is complete, which means that $\mathsf{Free}_n(\mathsf{pABF})$ coincides with the unique minimal complete extension, i.e., the grounded extension of pABF. □

Example 5.17. In Example 4.5, the grounded extension of pABF is $\{p, q\}$. This is also $\mathsf{Free}_3(\mathsf{pABF})$, as Proposition 5.16 indeed assures.

Now we can show that non-interference is satisfied also for grounded and well-founded semantics.

PROPOSITION 5.18. *Let* $\mathsf{pABF} = \langle \mathsf{ABF}, \mathsf{P}\rangle$ *be a prioritized simple contrapositive ABF with collective attacks, in which* $\mathsf{P} = \langle g, \max\rangle$ *for some allocation function* g. *Then* $\mathrel{|\!\sim}^{\cup}_{\mathsf{Grd}} = \mathrel{|\!\sim}^{\cap}_{\mathsf{Grd}} = \mathrel{|\!\sim}^{\cup}_{\mathsf{WF}} = \mathrel{|\!\sim}^{\cap}_{\mathsf{WF}}$ *satisfy non-interference.*

PROOF SKETCH. The fact that the four entailment relations coincide follows from Propositions 5.10 and 5.16. By Proposition 5.16, it suffices to show that for two syntactically disjoint pABF_1 and pABF_2, $\mathsf{Free}(\mathsf{pABF}_1 \cup \mathsf{pABF}_2) = \mathsf{Free}(\mathsf{pABF}_1) \cup \mathsf{Free}(\mathsf{pABF}_2)$. This can be shown by a simple induction on the construction of Free, where the crucial observation is the fact that $\mathsf{MIC}_i(\mathsf{pABF}_1 \cup \mathsf{pABF}_2) = \mathsf{MIC}_i(\mathsf{pABF}_1) \cup \mathsf{MIC}_i(\mathsf{pABF}_2)$ for any $i \geq 0$. □

The last result provides another justification for switching to collective attacks, since as shown in [2, Example 18] non-interference is not satisfied by entailment relations that are induced by the grounded semantics of pABFs with standard attacks.

6 CONCLUSION AND RELATED WORKS

The primary goal of this paper is to demonstrate the usefulness of incorporating collective attacks in (prioritized) assumption-based frameworks and to investigate some of the properties of the resulting argumentation frameworks. In passing, we have also obtained some further interesting results, such as the characterization of grounded extensions in prioritized ABFs. To the best of our knowledge, this is the first such characterization. Indeed, it has been observed before that in prioritized logic-based argumentation, the grounded extension does not always coincide with the intersection of preferred subtheories [11]. We now give a precise characterization of what *is* included in the grounded extension. This also allows us to derive further properties of the grounded extension, such as non-interference, which is not guaranteed for the grounded extension in logic-based argumentation [1] and prioritized ABFs with standard attacks [2, Example 18].

As noted previously, some preliminary results concerning collective attacks in ABFs have already been introduced in [15]. Those results are carried on in this paper to the prioritized case. Using uniform allocation functions (namely, those that assign the same preference value to all the defeasible assumptions) brings us back to the results in [15], thus some results in this paper are conservative extensions of those in [15], and some others are new.

Attacks of sets of arguments on other sets of arguments have recently been considered also for other frameworks for argumentative reasoning. For such a work in the context of abstract argumentation frameworks, we refer to [12]. In sequent-based argumentation [3] collective attacks are enabled by attack rules on subsets of the arguments' supports.

In future work, it would be interesting to extend the notion of collective attacks to other structured argumentation settings, such as rule-based assumption-based argumentation [4] and systems allowing for structured argumentation with defeasible rules [16, 21].

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