UNIVERSITÀ DEGLI STUDI DI GENOVA

DIBRIS

DEPARTMENT OF COMPUTER SCIENCE AND TECHNOLOGY,
BIOENGINEERING, ROBOTICS AND SYSTEM ENGINEERING

MODELLING AND CONTROL OF MANIPULATORS

Second Assignment

Manipulator Geometry and Direct Kinematics

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Mathematical expression	Definition	MATLAB expression
$< w >$	World Coordinate Frame	w
${}^a_bR$	Rotation matrix of frame $< b >$ with respect to frame $< a >$	aRb
${}^a_bT$	Transformation matrix of frame $< b >$ with respect to frame $< a >$	aTb

Table 1: Nomenclature Table

1 Introduction

The second assignment of Modelling and Control of Manipulators consists of one exercise and it focuses on the manipulators geometry and direct kinematics.

2 Exercise 1

Given the CAD model of an industrial 7 dof manipulator composed of rotational joints only, it is necessary to define the z-axis coinciding with the joint rotation axis and then set the remaining axes of each reference frames according to the right hand's rule for each link; the resulting scheme, with labels for each frames, is shown in the Appendix.

2.1 Q1.1 - Geometric model

In order to study the possible movements of the manipulator, it is necessary to compute its initial configuration or also called the *geometric model*, i.e. the case in which each joint is at rest, and to express with this each transformation matrix between a link *i* and the subsequent *i+1*, so as to be able to obtain the tree of frames.
This task is carried out by the custom function *BuildTree()*, which sets the square transformation matrix of dimension four ${}^{i}_{i+1}T$, that has the following structure:

$$
{}^{i}_{i+1}T = \begin{bmatrix} {}^{i}_{i+1}R & {}^{i}r_{i+1/i} \\ 0_{1x3} & 1 \end{bmatrix}
$$

The relevant components are the entry 1,1 which is the rotation matrix between the link *i+1* w.r.t the link *i* and the entry 1,2, that sets the distance between the link *i+1* observed by the link *i*.
The resulting *geometric model* obtained is the following multidimensional array:

$$
{}^{0}_{1}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 175 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{1}_{2}T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 98 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{2}_{3}T = \begin{bmatrix} 0 & 0 & 1 & 93.5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{4}T = \begin{bmatrix} 0 & -1 & 0 & 145.5 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 326.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{4}_{5}T = \begin{bmatrix} 0 & 0 & 1 & 35 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{5}_{6}T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 385 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{6}_{7}T = \begin{bmatrix} 0 & 0 & 1 & 153 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

We can make some observations on the geometric model obtained; for example, in the transformation matrix ${}^{0}_{1}T$ the rotation part is equal to the identity matrix so the two frames are equal, whereas the other rotation matrices were computed by inspection by observing how the axes in the frames were positioned.

2.2 Q1.2 - Motion of the joints

The case of study of this section is to calculate how the matrix attached to a joint will rotate if the joint rotates, i.e. it is required to obtain the transformation matrix between a link and the subsequent when moving from the inital configuration.
This is done by developing a custom function called *GetDirectGeometry()*, that works with the following parameters: the geometric model, the new joint configuration *q* and the joint type.
Starting from these, it exploits a function named *DirectGeometry()* that computes the correct transformation matrix ${}^{i}_{i+1}T$ based on the joint type.
This is because if the joint is *rotational* then the matrix ${}^{i}_{i+1}R$ is multiplied for a rotation around the z-axis; whereas if the joint is *prismatic* then a displacement, equal to the product between the z-axis of the *i+1* joint and the corresponding q, is added to the distance between the two close joints.

In this case, the joint configuration given was the following: **q** $= [1.3, 1.3, 1.3, 1.3, 1.3, 1.3, 1.3]$; whereas the resulting transformation matrices between each link and the subsequent one:

$$
{}^{0}_{1}T = \begin{bmatrix} 0.2675 & -0.9636 & 0 & 0 \\ 0.9636 & 0.2675 & 0 & 0 \\ 0 & 0 & 1 & 175 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{1}_{2}T = \begin{bmatrix} -0.2675 & 0.9636 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.9636 & 0.2675 & 0 & 98 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{2}_{3}T = \begin{bmatrix} 0 & 0 & 1 & 93.5 \\ 0.2675 & -0.9636 & 0 & 0 \\ 0.9636 & 0.2675 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$ {}^3_4T = \begin{bmatrix} -0.9636 & -0.2675 & 0 & 145.5 \\ 0 & 0 & 1 & 0 \\ -0.2675 & 0.9636 & 0 & 326.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^4_5T = \begin{bmatrix} 0 & 0 & 1 & 35 \\ -0.2675 & 0.9636 & 0 & 0 \\ -0.9636 & -0.2675 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^5_6T = \begin{bmatrix} 0.9636 & 0.2675 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.2675 & -0.9636 & 0 & 385 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^6_7T = \begin{bmatrix} 0 & 0 & 1 & 153 \\ 0.2675 & -0.9636 & 0 & 0 \\ 0.9636 & 0.2675 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

It is relevant to observe that since the manipulator is composed of rotational joints only, the component related to the distance and the z-axis, which is the axis of rotation, between two close joints remained unchanged.

2.3 Q1.3 - Quantities concerning the orientation of the joints

To further develop the study of the motion of the joints, it is possible to analyze the transformation matrices between any two links, between a link and the base and the corresponding distance vectors, with the related custom function *GetFrameWrtFrame()*, *GetTransformationWrtBase()*, *GetBasicVectorWrtBase()*.

In order to compute the transformation matrix between two different frames, it is necessary to multiply the matrices involved in between those frames, so as to move from one link to the other and maintain a correspondence among the different projections that are related to distinct reference systems.
In this case, it was chosen to compute ${}^2_6T$ so the needed computation for what said above is the following: ${}^2_6T =^2_3 T^3_4T^5_6T$, which resulted in the matrix:

$$ {}^2_6T = \begin{bmatrix} 0.8989 & 0.4330 & -0.0664 & -149.1037 \\ -0.4303 & 0.8441 & -0.3199 & -249.7489 \\ -0.0825 & 0.3162 & 0.9451 & 375.8923 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The transformation matrices between the links and the base were computed with the same reasoning and the results are:

$$ {}^0_1T = \begin{bmatrix} 0.2675 & -0.9636 & 0 & 0 \\ 0.9636 & 0.2675 & 0 & 0 \\ 0 & 0 & 1 & 175 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^0_2T = \begin{bmatrix} -0.0716 & 0.2578 & -0.9636 & 0 \\ -0.2578 & 0.9284 & 0.2675 & 0 \\ 0.9636 & 0.2675 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_3T = \begin{bmatrix} -0.8595 & -0.5061 & -0.0716 & -6.6905 \\ 0.5061 & -0.8231 & -0.2578 & -24.0997 \\ 0.0716 & -0.2578 & 0.9636 & 363.0927 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^0_4T = \begin{bmatrix} 0.8473 & 0.1610 & -0.5061 & -155.1101 \\ -0.4187 & -0.3837 & -0.8231 & -34.6165 \\ -0.3267 & 0.9093 & -0.2578 & 688.1058 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} 0.4446 & 0.2905 & 0.8473 & -125.4540 \\ 0.8957 & -0.1496 & -0.4187 & -49.2716 \\ 0.0051 & 0.9451 & -0.3267 & 676.6713 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^0_6T = \begin{bmatrix} 0.6551 & -0.6975 & 0.2905 & 200.7626 \\ 0.7511 & 0.6431 & -0.1496 & -210.4776 \\ -0.0825 & 0.3162 & 0.9451 & 550.8923 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_7T = \begin{bmatrix} 0.0933 & 0.7498 & 0.6551 & 300.9868 \\ 0.0279 & -0.6596 & 0.7511 & -95.5649 \\ 0.9952 & -0.0518 & -0.0825 & 538.2763 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The distances w.r.t the base were computed and resulted to be:

$$ bri = \begin{bmatrix} 0 & 0 & -6.6905 & -155.1101 & -125.4540 & 200.7626 & 300.9868 \\ 0 & 0 & -24.0997 & -34.6165 & -49.2716 & -210.4776 & -95.5649 \\ 175.0000 & 273.0000 & 363.0927 & 688.1058 & 676.6713 & 550.8923 & 538.2763 \end{bmatrix} $$

2.4 Q1.4 - Spatial configuration of the manipulator

Given a starting and ending configuration, it is required to plot the intermediate link positions in between the two configurations.
The initial configuration of the manipulator with all the joints at rest can be observed in Figure 1.

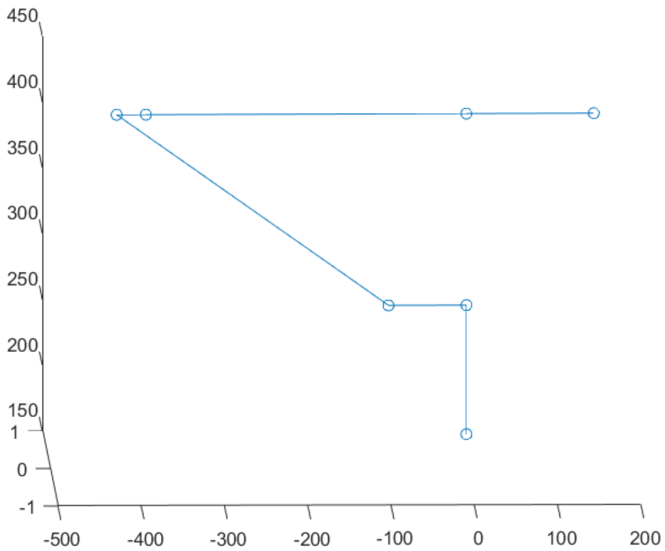

Figure 1: Manipulator initial configuration at rest

Considering a case where all the joints have the same initial and final configuration: $\mathbf{q}_i = [1.3, 1.3, 1.3, 1.3, 1.3, 1.3, 1.3]$ and $\mathbf{q}_f = [2, 2, 2, 2, 2, 2, 2]$, we can observe in Figure 2 how each link affects the others when turning.

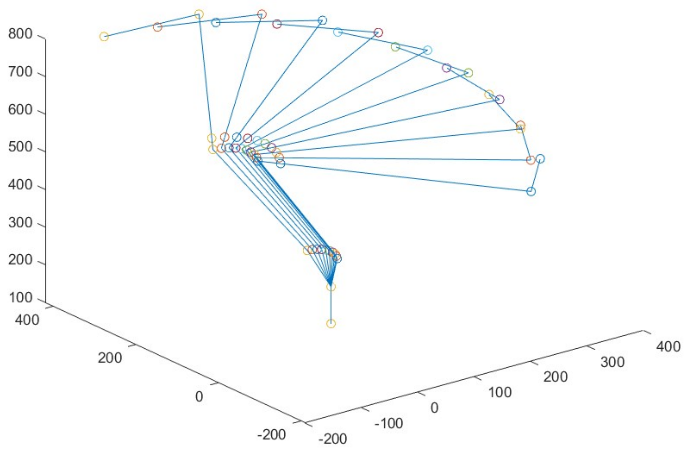

Figure 2: Rotation of all the joints with the same initial configuration

Considering a case where only the first joint actually changes its configuration: $\mathbf{q}_i = [1.3, 0, 1.3, 1.7, 1.3, 0.8, 1.3]$ and $\mathbf{q}_f = [2, 0, 1.3, 1.7, 1.3, 0.8, 1.3]$, we can observe in Figure 3 that the points in space reached by the manipulator are not the same w.r.t before, therefore different tasks could be implemented.

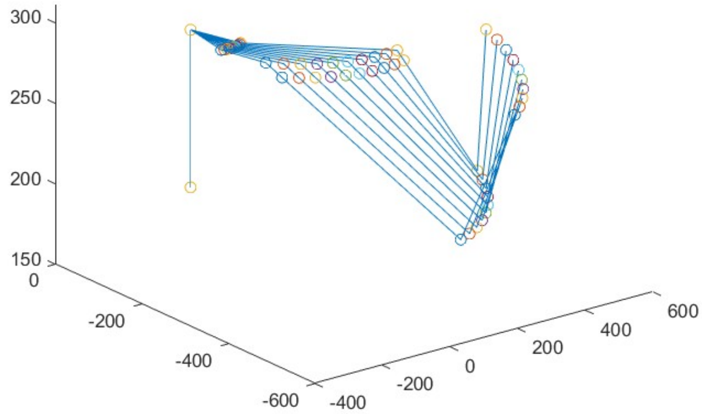

Figure 3: Rotation of the first joint only

Considering a case where all the joints rotate to end up in the same final configuration: $\mathbf{q}_i = [1.3, 0.1, 0.1, 1, 0.2, 0.3, 1.3]$ and $\mathbf{q}_f = [2, 2, 2, 2, 2, 2, 2]$, we can observe in Figure 4 how the links moved in a counterclockwise direction and the different points reached in space.

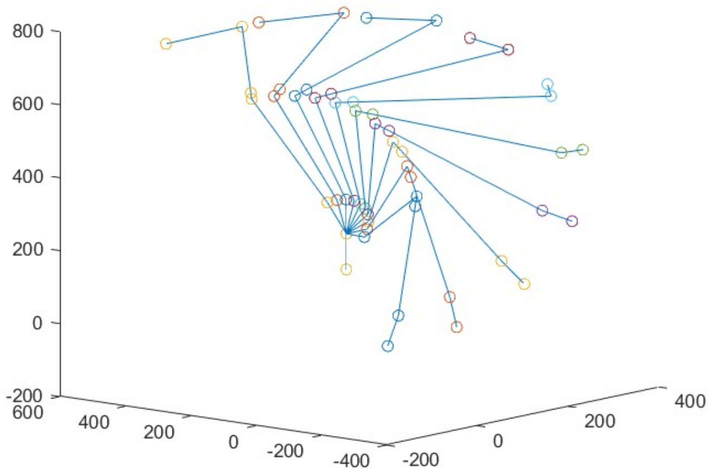

Figure 4: Rotation of all the joints with a different initial configuration

2.5 Q1.5 - Configuration of the manipulator with one joint changing at the time

To further develop the observations made in the previous sections, one joint position is changed at time to analyze the behaviour for three different configurations, except the last joint which was not considered meaningful since no end-effector is attached to it.
In the first case, the configuration chosen was $\mathbf{q}_1 = [1.3, 0, 0, 0, 0, 0, 0]$ and the plot obtained can be observed in Figure 5:

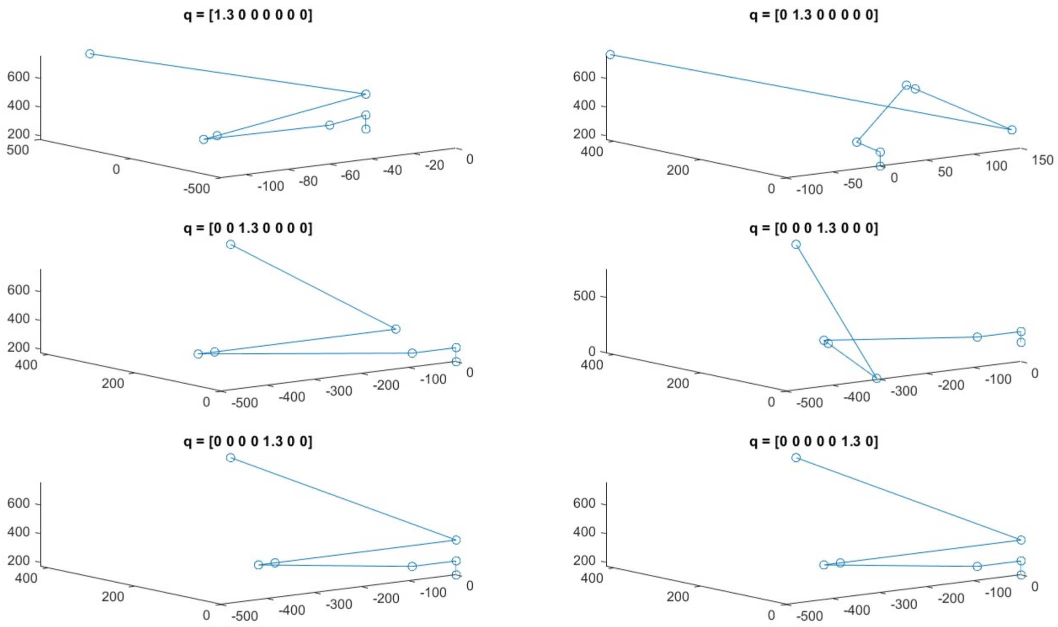

Figure 5: Configuration $\mathbf{q}_1 = [1.3, 0, 0, 0, 0, 0, 0]$

In the second case, the configuration chosen was $\mathbf{q}_2 = [2, 0, 0, 0, 0, 0, 0]$ and the plot obtained can be observed in Figure 6:

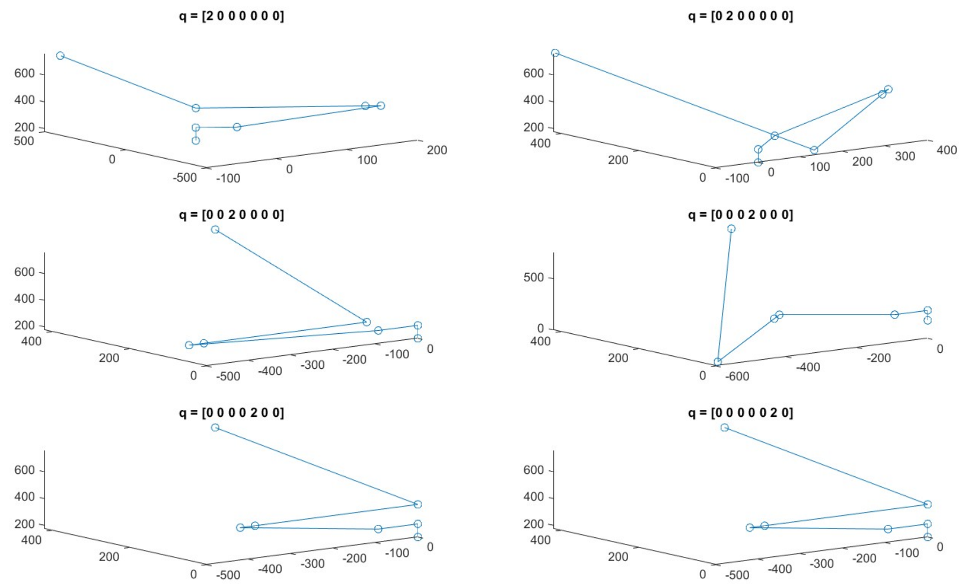

Figure 6: Configuration $\mathbf{q}_2 = [2, 0, 0, 0, 0, 0, 0]$

In the last case, the configuration chosen was $\mathbf{q}_3 = [2, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]$ and the plot obtained can be observed in Figure 7:

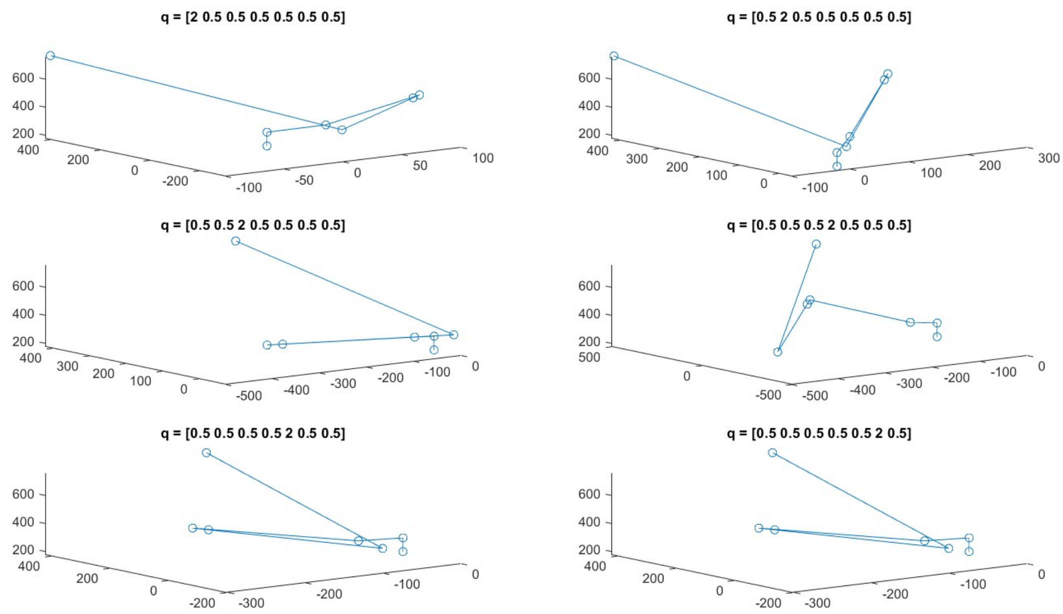

Figure 7: Configuration $\mathbf{q}_3 = [2, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]$

It is important to notice how the joints moved based on the rotation of the others, to further develope the kinematics of manipulators in understanding the independence that exists between joints in the serial chain and how this affects the achievement of the task by the end-effector.

3 Appendix

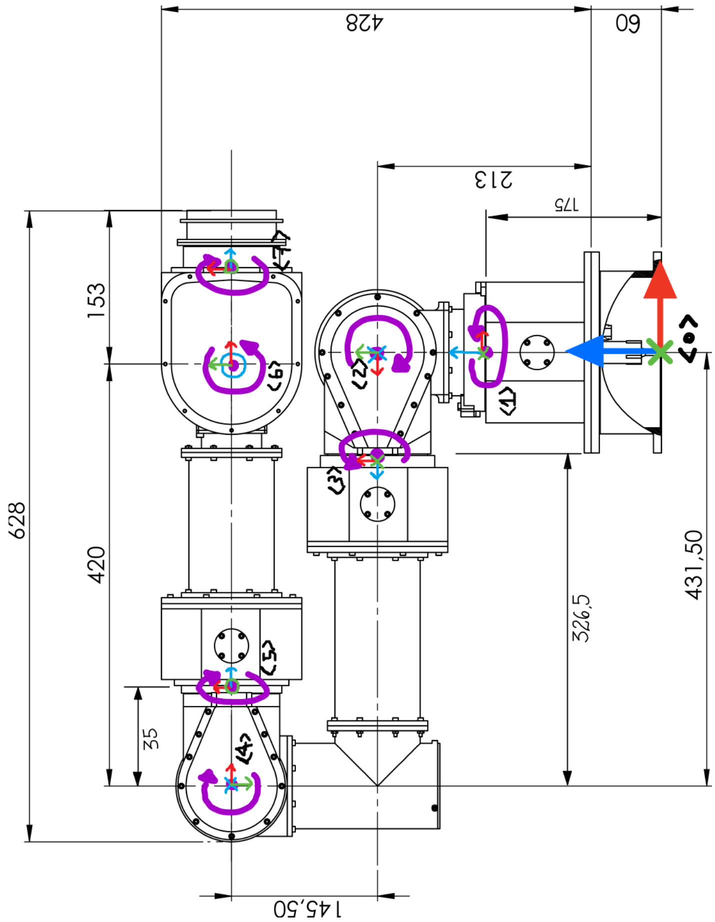

Figure 8: CAD model of the robot