ACOUSTICAL QUANTA AND THE THEORY OF HEARING

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In popular expositions of wave mechanics, acoustical illustrations have been used by several authors, with particular success by Landé[1]. In a recent paper on the "Theory of Communication"[2] I have taken the opposite course. Acoustical phenomena are discussed by mathematical methods closely related to those of quantum theory. While in physical acoustics a new formal approach to old problems cannot be expected to reveal much that is not already known, the position in subjective acoustics is rather different. In fact, the new methods have already proved their heuristic value, and can be expected to throw more light on the theory of hearing. In my original paper the point of view was mainly that of communication engineering; in the following survey I have emphasized those features which may be of interest to physicists and to physiologists.

What do we hear? The answer of the standard text-books is one which few students, if any, can ever have accepted without a grain of salt. According to the theory chiefly connected with the names of Ohm and Helmholtz, the ear analyses the sound into its spectral components, and our sensations are made up of the Fourier components, or rather of their absolute values. But Fourier analysis is a timeless description, in terms of exactly periodic waves of infinite duration. On the other hand, it is our most elementary experience that sound has a time pattern as well as a frequency pattern. This duality of our sensations finds no expression either in the description of sound as a signal $s(t)$ in function of time, or in its representation by Fourier components $S(f)$. A mathematical description is wanted which *ab ovo* takes account of this duality. Let us therefore consider both time and frequency as co-ordinates of sound, and see what meaning can be given to such a representation.

If t and f are laid down as orthogonal co-ordinates, a diagram is obtained which may be called an 'information diagram' (Fig. 1). A simple harmonic oscillation is represented by a vertical line with abscissa f, a sharp pulse (delta function) by a horizontal line at the 'epoch' t. These are extreme cases. In general, signals cannot be represented by lines; but it is possible to associate with them a certain characteristic rectangle or 'cell' by the following process, which at first sight might perhaps appear somewhat complicated.

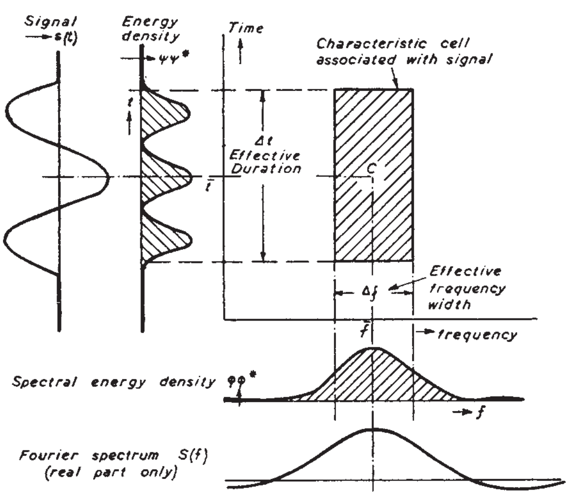

Fig. 1. CHARACTERISTIC RECTANGLE OF A SIGNAL IN THE INFORMATION DIAGRAM

Consider a given signal described as $s(t)$ in 'time language' and by its Fourier transform $S(f)$ in 'frequency language'. If $s(f)$ is real, $S(f)$ will be in general complex, and the spectrum will extend over both positive and negative frequencies. This creates an unwelcome asymmetry between the two representations, which can be eliminated by operating with a complex signal $\psi(t) = s(t) + i\sigma(t)$, where $\sigma(t)$ is the Hilbert transform of $s(t)$, instead of with the real signal $s(t)$. This choice makes the Fourier transform $\varphi(f)$ of $\psi(t)$ zero for all negative frequencies. Next we define the 'energy density' of the signal as $\psi\psi^*$, where the asterisk denotes the conjugate complex value, and similarly $\varphi\varphi^*$ as the spectral energy density. In Fig. 1 the two energy distributions are shown as shaded areas. The two are of equal size; that is, the total energy of the signal is the same by both definitions. We can now define a 'mean epoch' $\bar{t}$ of the signal, and similarly a 'mean frequency' $\bar{f}$, as the co-ordinates of the centres of gravity of the two distributions. This gives a point C in the information diagram as the centre of the signal. Going a step further, we define the 'effective duration' Δt and the 'effective frequency width' Δf of the signal by means of the mean square deviations of the two energy distributions from the mean values $\bar{t}$ and $\bar{f}$. In the figure these appear as the inertial radii of the two shaded areas. For reasons which will become evident later, it is convenient to define Δt and Δf as $2\sqrt{\pi}$ times the respective inertial radii†. With these definitions a mathematical relation, the Schwarz inequality, can be applied, which states that

$$\Delta t \, \Delta f \geqslant 1 ; \qquad (1)$$

that is to say, *the area of the characteristic rectangle or cell of a signal is at least unity.* This is the exact formulation of the uncertainty relation between time and frequency. It is clear now that what we have obtained is a classical model of one-dimensional static wave mechanics, in which unity replaces Planck's constant h. Formally this 'quantum' is represented by a cell of unit (dimensionless) area in the information diagram. Its physical significance becomes evident from the following consideration.

If in a time interval T a signal is expanded in a Fourier series, there will be T complex Fourier components per unit frequency band-width. Thus we can say that the information diagram contains one complex datum per unit area. (This, of course, is to be considered as an asymptotic theorem for large areas, as by equation (1) we cannot analyse the information area into cells of less than unit order.) We see now that the quanta in this model of wave mechanics are *quanta of information*. Each quantum represents one complex numerical datum or two real data.

This result, in combination with the uncertainty relation (1), suggests the question whether there are signals for which the inequality turns into an equality, and which could be used for the representation of quanta. It can be shown that such simplest or 'elementary' signals exist, and their form is

$$s(t) = \exp - \alpha^2(t - t_0)^2 \, . \exp i2\pi f_0 t, \qquad (2)$$

† In the original paper the factor was $\sqrt{2\pi}$ instead of $2\sqrt{\pi}$, so that $\frac{1}{2}$ appears in equation 1 instead of unity. The present choice is more advantageous for a direct comparison with quantum theory.

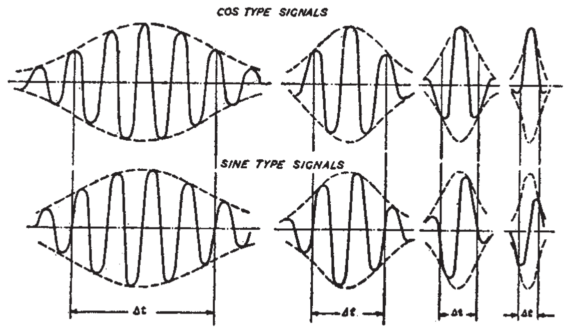

Fig. 2. ELEMENTARY SIGNALS

where α is a real constant. Thus the elementary signals are harmonic oscillations of any frequency f_0, modulated by a probability pulse. Their Fourier transforms are of the same mathematical form

$$S(f) = \exp - (\pi/\alpha)^2 (f - f_0)^2 \,.\, \exp i 2\pi t_0 f. \qquad (3)$$

The real components of the elementary signals are conveniently specified as cosine-type and sine-type signals, and are illustrated in Fig. 2 for several values of the constant α, which is connected with the effective duration and effective band-width by the equations $\Delta t = \sqrt{\pi}/\alpha$ and $\Delta f = \alpha/\sqrt{\pi}$. If $\alpha = 0$, the elementary signals become simple harmonic oscillations; if $\alpha = \infty$, they turn into the delta function and its derivative.

These elementary signals can be used for the representation of quanta of information, as any arbitrary signal can be expanded in terms of them, with any choice of the constant α. That is to say, we can expand a signal by dividing up the information area into unit rectangular cells of any aspect $\Delta t/\Delta f = \pi/\alpha^2$, and by associating with every cell an elementary signal with a complex amplitude factor c_{ik}. We thus obtain a sort of matrix, as illustrated in Fig. 3. This method of analysis contains 'time language' and 'frequency language' as special extreme cases. If the cells are infinite in the time direction we obtain Fourier analysis; if they are infinite in the frequency direction we obtain an expansion into delta functions, that is to say, the function $s(t)$ itself.

Expansion into elementary functions is in general a rather inconvenient mathematical process, as they do not form an orthogonal set. There exists, however, an experimental method, which, though not equivalent, gives somewhat similar and very valuable results. This is the method of 'sound spectrography'

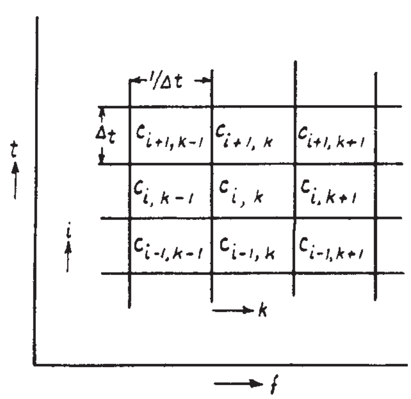

Fig. 3. MATRIX OF ELEMENTARY SIGNALS REPRESENTING AN ARBITRARY SIGNAL

developed by the Bell Telephone Laboratories during the War, and published after the original paper by the author was written and accepted[3]. There is no space here to describe this method and the difference between 'sound portraits' and matrices of elementary signals, but it may be noted that sound portraits are not complete representations of the original sound, though they contain most of its subjectively important features.

It is to be expected that the analysis of hearing sensations will prove a fruitful field for the new methods of representation, and the results so far obtained seem to indicate very clearly that the 'quantum of sound' is a concept of considerable physiological significance. Acoustical workers have collected a large amount of material on the threshold of ear discrimination for epoch and frequency differences. It appears that at least a good part of these results can be summed up in the statement that *the ear possesses a threshold area of discrimination of the order unity*.

One series of experiments which were analysed were those of Bürck, Kotowski and Lichte[4]. Their object was the threshold of time discrimination in different frequency-ranges. Mach showed that if a sinoidal oscillation is sounded for a few cycles only, the ear will hear it as a noise, while after a minimum duration of about 10 milliseconds it will be heard as a short musical note, with ascertainable frequency. The authors carried out a series of tests on this phenomenon, and in a second series went a step further. In these tests the intensity was doubled after a certain time from the start. If this was less than a certain minimum (21 msec. at 500 cycles), the sound could not be distinguished from a note which was sounded from the start with double intensity; beyond this the step could be heard as such. One could say that in the interval between 10 and 21 msec. the ear has just got ready to register a second, distinct sensation.

Another series of experiments is due to Shower and Biddulph[5] and relates to the threshold of differential frequency discrimination of the ear. Their technique was quite different. The frequency of a note was modulated between narrow limits, and the minimum frequency swing was noted which gave the impression of a 'trill'. At 500 cycles this minimum swing was 2·3 cycles. Below this the trill could not be distinguished from a pure note. The cycle of modulation was so chosen that it gave the finest frequency discrimination (0·5 sec. for the full cycle). In these experiments, as in those of Bürck, Kotowski and Lichte, intensity played no essential part, provided that the tests were carried out at a level of good audibility.

Though it might appear that in these experiments the frequency discrimination of the ear was tested, and in the previously mentioned experiments the time response of the ear, in reality, of course, both time and frequency are involved in both cases. In the experiments of Bürck, Kotowski and Lichte, the effective width of a sinoidal oscillation which has lasted only for a finite time has to be considered. In the tests by Shower and Biddulph the time of 0·25 sec. is necessary to notice one 'up' or 'down' of the frequency. Thus from the time and frequency intervals involved in each case it is possible to form a product, the 'threshold area', in which the ear was capable of registering one sensation only, and which must be exceeded if it is to register a second. The numerical value of this area is to some extent a matter of definition. With the conventions adopted in the original paper, the threshold area calculated from the data of Bürck, Kotowski and Lichte was 1·05 at 500 cycles and 1·5 at 1,000 cycles, while the figures of Shower and Biddulph gave 1·17 in both cases. No such accuracy can of course be claimed, and the absolute values depend on the exact definition, but by any reasonable convention the threshold area is found to be of the order unity, that is to say, near the 'acoustical quantum limit'. It must be understood, of course, that there is an important difference between an acoustical quantum as registered by a physical measuring instrument, and as registered by the ear. In the experiments considered the ear was called upon only to answer 'yes' or 'no' to a simple question. To a measuring instrument, on the other hand, a quantum of information conveys a complex numerical datum (two real data), and every exact datum carries in itself an infinite number of 'yes's' and 'no's'. The difference is expressed by talking of 'discrimination' in the case of the ear and 'registering' in the case of a measuring instrument. In this sense we can say that, at least in simple experiments, the best ears in the optimum frequency-range can just about discriminate one acoustical quantum.

The importance of this result lies in the remarkably wide range of aspect ratios in which it seems to apply. As shown in Fig. 4 for the case of signals near 500 cycles, the threshold area does not vary appreciably while the time interval varies from about 20 to 250 msec.; that is to say, by more than a factor of ten. Within these wide limits the ear has the power of searching out by an automatic adjustment the finest details in the sound pattern offered to it. If a short sound strikes the ear, with poorly defined frequency, the time discrimination is of the order of 20 msec. If the sound is prolonged, so that its effective band-width shrinks to a few cycles per second only, the frequency discrimination of the ear keeps in step with this process for at least a quarter of a second. An old problem of physiological acoustics appears here in particularly sharp relief. No simple physical system of resonators could account for this degree of perfection. Resonators can be tuned to

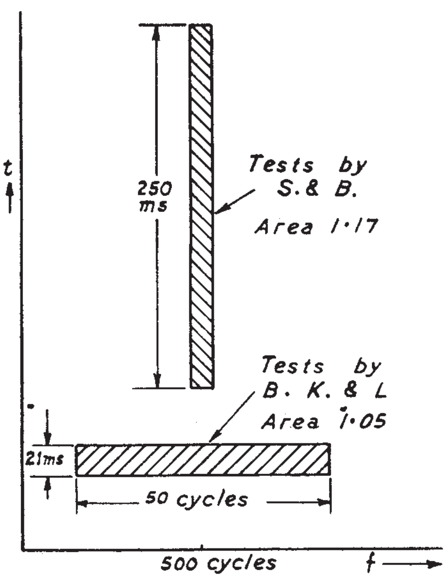

Fig. 4. THRESHOLD AREAS OF THE EAR AT 500 CYCLES/SEC.

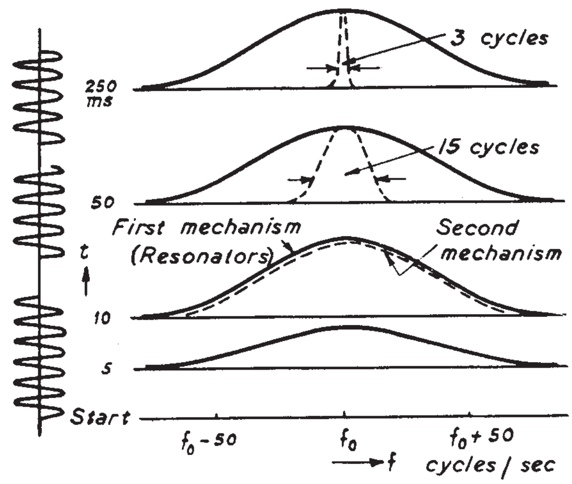

Fig. 5. THE TWO MECHANISMS OF HEARING. IF A NOTE IS SOUNDED FOR MORE THAN ABOUT 10 MSEC. THE RESONANCE PATTERN OF THE INNER EAR REMAINS UNCHANGED; BUT A SECOND MECHANISM STARTS SEARCHING FOR THE MAXIMUM OF EXCITATION AND LOCATES IT WITH AN ACCURACY WHICH INCREASES UP TO ABOUT 250 MSEC.

wide bands or to narrow ones, but not to both simultaneously. From physiological data, and in particular from the experiments of Wegel and Lane[6] on 'masking', it is practically certain that the ear resonators are very strongly damped, with decay times of the order of 10 msec. or less. They are tuned to bands approximately as wide as would account for the results of Bürck, Kotowski and Lichte, but not for those of Shower and Biddulph, which would require for their explanation decay times at least ten times longer. Thus the postulation of a second mechanism in the ear, which accounts for its apparent progressively refined tuning in the case of prolonged sounds, appears to be inescapable.

The role of the two 'mechanisms' is roughly illustrated in Fig. 5. The first is probably the real mechanism of the ear resonators; the second is almost certainly non-mechanical. One might be tempted to locate it in the brain, but this assumption is not unavoidable. It might well be a new phenomenon in nervous conduction, and could be explained by the hypothesis that conduction in adjacent nerve fibres is to some extent unstable, so that the more strongly excited fibre gradually suppresses the excitation of its less strongly excited neighbour. Thus if a pure note is sounded for a sufficient time, less and less fibres will be excited, until finally only the fibre corresponding to the position of maximum amplitude in the basilar membrane remains in action. This makes it possible to ascertain the frequency with far greater accuracy than one would expect from the broad tuning of the mechanical resonators. It would be most interesting, though probably very difficult, to prove or to disprove this effect, perhaps by a further refinement of the techniques of Galambos and Davis[7].

It is interesting to note that we begin to perceive a sound as 'musical' just at the point where the second mechanism takes over. Speech would be perfectly intelligible by the first mechanism alone; but the second is necessary to enable us to appreciate music.

Finally, it may be asked whether quantum theory has anything to learn from the acoustical model. In a formal sense the answer must be, of course, in the negative. From the fact, however, that two different fields admit the same formal treatment it follows that, so far as the mathematics goes, there can be nothing in one which is not implicit in the other. But familiar models of less familiar phenomena have at least one advantage. They make it easier to distinguish between intrinsic features of the phenomenon, and others which are introduced by the method of analysis. One might be inclined to think that sharply defined states, characterized by integral numbers, are peculiar to quantum phenomena, or at least that they require special mechanisms to imitate them classically, such as strings or membranes. But we have seen that in the acoustical model the integers emerge as a part of the mathematical background before any physical phenomenon has appeared on the stage. The ultimate reason for the emergence of 'acoustical quanta' is that we have viewed the same phenomenon simultaneously from two different aspects, and described it by two 'quantities of interest', time and frequency. Any number of other classical models may be found, if only we ask simultaneously two kinds of questions about the same thing, or perform two lines of experiments on it, provided that the questions are neither identical nor independent.

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A CHEMICAL BICENTENARY AT THE UNIVERSITY OF GLASGOW

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ARISTOTELIAN physics and astronomy had had place in the arts curriculum at the College of Glasgow ever since King Jamie wi' the Fiery Face welcomed the advent of Scotland's second University (1451). In the eighteenth century a more modern science was recognized by the institution of chairs in natural philosophy (1717) and practical astronomy (1760).

Between these events in the Faculty of Arts, William Cullen completed the first course of lectures in chemistry (1747). This lectureship in the Faculty of Medicine was part of a renewed and finally successful effort to provide his *Alma Mater* with the prestige of a comprehensive medical school, the steady expansions of which included the later regius chair of chemistry (1818).

Cullen was perhaps the greatest physician to teach chemistry in this or any other British university. Although there is some evidence that he would have preferred to place the subject in the wider field of arts, there was, of course, nothing unusual in its association with eighteenth-century medicine. The notable factors are the broad and enlightened views which this polymathic personality adopted in the scope and emphasis of his teaching, and his lasting influence on the new department.

During 1747–1874 the Department was headed, for all but three years, by professional physicians. Although the degree of B.Sc. was first conferred in 1873, and the belated Faculty of Science (1893) included lectureships in organic chemistry (1898), in metallurgical chemistry (1899) and in "Physical Chemistry including Radio-activity" (1904), the chair of chemistry was first detailed in the calendar under 'Faculty of Science' only in 1920. Although even the nineteenth-century class of 'Practical Chemistry' pursued a course of "bases and acids, poisons and the pharmaceutical compounds", there was always a group of 'laymen' in the principal class of systematic chemistry and in the laboratory. For these freedoms of outlook and emphasis a lasting gratitude is due to William Cullen.

So the portentous association of Joseph Black (1756) with James Watt has its place in world history. John Robison (1766) was not a physician. William Irvine (1769) and T. C. Hope (1787) were physicists of note. Robert Cleghorn (1791) may have concentrated unduly on his duties to the students of medicine; but with Thomas Thomson (1817), the first regius professor, the lectures certainly reverted to a wider scope, which was undiminished during the tenure of his successor, Thomas Anderson (1852). With the advent of John Ferguson (1874) the medical dynasty is at an end: the now legendary 'Soda' took an active part in framing the new Faculty of Science, and in completing a near century of service by the first three regius professors. The fourth, G. G. Henderson (1919), guided the Department to its present Faculty. The 'auld alliance' has since been represented by a lecturer in 'Chemistry for students of Medicine' and by the research interests of George Barger (1937) and of James W. Cook (1939). In the Gardiner chair T. S. Patterson (1919) and J. Monteath Robertson (1942) have emphasized the modern significance of organic and of physical chemistry.

These heads of department, six of whom were products of the chemistry class, have seen their opportunities grow fiftyfold since some twenty students first gathered around Cullen. Since 1861 they have enjoyed the support of official colleagues such as, among others, William Ramsay, J. J. Dobbie, Frederick Soddy and C. H. Desch. The present teaching and research staff, some thirty strong, installed in a new and well-equipped building, have also behind them two hundred years of provocative tradition.

The Department has not only carried out its natal responsibilities in the teaching of medicine. It has helped to fashion in earlier days physician-scientists like Joseph Black, John MacLean of Princeton, and Thomas Graham, professional scientific workers like John Robison, William Thomson (Lord Kelvin) and (Sir) William Ramsay, *entrepreneurs* like Charles Macintosh and William Couper.

The Duke of Hamilton, descendant of Cullen's friend and patron, will preside over the first of a bicentenary series of lectures to be given this month by Dr. Douglas Guthrie, Prof. John Read, Dr. Alexander Fleck and Prof. A. R. Todd. Later, it is intended to publish a commemorative volume, edited by the present writer, in which scientific men of diverse interests and of many colleges will combine in tribute to those early pioneers whose far-sighted ambitions have eventuated in the multifold activities and the wide influence of the school of chemistry at the University of Glasgow.

The careless dust of time has overlaid some points of interest. It is not easy now to discover the type of chemistry taught by Cleghorn during a tenure of