Exercícios de Eq. Linear ñ Homogênea

(a) $y'' + 3y' + 2y = 6$.

eq. auxiliar $\rightarrow r^2 + 3r + 2 = 0$

$\Delta = 3^2 - 4 \cdot 1 \cdot 2 \rightarrow \Delta = 1$

$r = \dfrac{-3 \pm \sqrt{1}}{2} \rightarrow r_1 = -2$

$\qquad \rightarrow r_2 = -1$

$y_p = \begin{cases} y_p = A \\ y_p' = 0 \\ y_p'' = 0 \end{cases}$

Logo: $y_c = c_1 e^{-2x} + c_2 e^{-1x}$

Substi[redacted]: $0^2 + 0 \cdot 3 + 2A = 6 \rightarrow 2A = 6 \rightarrow A = 3$

$y_p = 3$

Solução geral: $y(x) = c_1 e^{-2x} + c_2 e^{-x} + 3$

(b) $y'' - 10y + 25y = 30x + 3$.

eq. auxiliar : $r^2 - 10 + 25 = 0$

$\Delta = (-10)^2 - 4 \cdot 1 \cdot 25 \rightarrow \Delta = 0 \qquad r = \dfrac{10}{2} = 5$

$y_c = c_1 e^{5x} + c_2 x e^{5x}$

$y_p \begin{cases} y_p = ax + b \\ y_p' = a \\ y_p'' = 0 \end{cases}$

Substituindo:

$0 - 10(ax + b) + \cancel{25}\,a = 30x + 3$

$25(ax+b)$

$-10ax - 10b + 25a = 30x + 3$

$+ 25ax + 25b$

$(-10a + 25a)x + (-10b + 25$

$\begin{cases} -10a + 25a = 30 \\ -10b + 25b = 3 \end{cases}$

$a = 2 \quad b = 10$

$\begin{cases} -10a = 30 \rightsquigarrow a = -3 \\ -10b + 25a = 3 \rightsquigarrow -10b + 25 \cdot (-3) = 3 \end{cases}$

$b = -7{,}8$

$y_p = -3x - 7{,}8$

Solução geral : $y(x) = c_1 e^{5x} + c_2 x e^{5x} - 3x - 7{,}8$

(c) $\frac{1}{4}y'' + y' + y = x^2 - 2x$.

eq. auxiliar: $\frac{1}{4}r^2 + r + 1 = 0$

$\Delta = 1^2 - 4 \cdot \frac{1}{4} \cdot 1 \rightsquigarrow \Delta = 0$

$r = -\frac{1}{\frac{2}{4}} \rightsquigarrow r = -2$ $\qquad y_c = c_1 e^{-2x} + c_2 x e^{-2x}$

$$\begin{cases} y_p = ax^2 + bx + c \\ y_p' = 2ax + b \\ y_p'' = 2a \end{cases}$$

Substituindo: $\frac{1}{4}(2a) + 2ax + b + ax^2 + bx + c = x^2 - 2x$

$ax^2 + (2a + b)x + (\frac{2}{4}a + b + c) = x^2 - 2x$

Comparando: $\begin{cases} a = 1 \\ 2a + b = -2 \rightsquigarrow b = -4 \\ \frac{2}{4}a + b + c = 0 \rightsquigarrow \frac{2}{4} - 4 + c = 0 \rightsquigarrow c = \frac{7}{4} \end{cases}$

$y_p = x^2 - 4x + \frac{7}{4}$

Solução geral: $y(x) = c_1 e^{-2x} + c_2 x e^{-2x} + x^2 - 4x + \frac{7}{4}$

(d) $y'' + 3y' = -48x^2 e^{3x}$.

eq. auxiliar: $r^2 + 3r = 0$

$r(r+3) = 0 \rightsquigarrow r = 0$

$r + 3 = 0 \rightsquigarrow r = -3$

$y_c = c_1 + c_2 e^{-3x}$

$$\begin{cases} y_p = e^{3x}(Ax^2 + b + c) \\ y_p' = e^{3x}[(2ax + b) + 3(ax^2 + b + c)] \\ y_p'' = e^{3x}[3ax^2 + (2A + 3b)x + b + 3c)] \end{cases}$$

(e) $y'' - y' = -3$.

eq. auxiliar: $r^2 - r = 0$

$r(r-1) = 0 \rightarrow r = 0$

$\rightarrow r - 1 = 0 \rightarrow r = 1$

$y_c = c_1 + c_2 e^x$

$$\begin{cases} y_p = A x \\ y_p' = 0\ A \\ y_p'' = 0 \end{cases}$$

substituindo: $0 - 0 = -3$

$0 - A = -3$

$A = 3$

$y_p = 3x$

Solução geral: $y(x) = c_1 + c_2 e^x + 3x$

(f) $y'' - y' + \frac{1}{4} y = 3 + e^{x/2}$.

eq. auxiliar: $r^2 - r + 1/4 = 0$

Método variação dos Parâmetros

exercício) $4y'' + 36y = \text{cossec}\,x \quad \div 4$

$y'' + 9y = \frac{1}{4}\text{cossec}\,x$

eq. auxiliar: $r^2 + 9 = 0 \rightsquigarrow r = \sqrt{9} = \pm 3$

$y_c = c_1 e^{3x} + c_2 e^{-3x}$

$$W(e^{3x}, e^{-3x}) \begin{vmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{vmatrix} = -3 + (-3) = -9$$

$-3 \quad -3$

$$u_1' = \frac{-e^{-3x} \cdot \text{cossec}\,x}{-9}$$

$$u_1 = \int -\frac{1}{9} \cdot (-e^{-3x}\,\text{cossec}\,x)\,dx$$

$u_1 = \frac{1}{9} \int e^{-3x}\,\text{cossec}\,x\,dx$

$\int u\,dv = uv - \int v\,du$

$u = \text{cossec}\,x \qquad e^{-3x}dx = dv$

$du = -\text{cossec}\,x \cdot \text{cotg}\,x \qquad -\frac{1}{3}e^{-3x} = v$

$\int \text{cossec}\,x\, e^{-3x}\,dv = \text{cossec}\,x \cdot (-\frac{1}{3} e^{-3x})$

$- \int -\frac{1}{3} e^{-3x} \cdot \text{cossec}\,x \cdot \text{cotg}\,x\,dx$

$= -\frac{1}{3}\text{cossec}\,x \cdot e^{-3x} + \frac{1}{3} \int e^{-3x} \cdot \text{cossec}\,x \cdot \text{cotg}\,x\,dx$

$u = e^{-3x} \qquad \text{cossec}\,x \cdot \text{cotg}\,x\,dx = dv$

$du = -3e^{-3x}dx \qquad -\text{cossec}\,x = v$

$\int e^{-3x} \cdot \text{cossec}\,x \cdot \text{cotg}\,x\,dx = e^{-3x} \cdot (-\text{cossec}\,x) -$

$\int -\text{cossec}\,x \cdot (-3e^{-3x})\,dx$

$= -e^{-3}\,\text{cossec}\,x - 3\int \text{cossec} \cdot e^{-3x}\,dx$

$2\int \text{cossec}\,x\, e^{-3}\,dx = \text{cossec}\,x \cdot (-\frac{1}{3} e^{-3x}) + e^{-3x}\,\text{cossec}\,x\,-3$

$$2\int \text{cossec}\,x\, e^{-3}\,dx = \frac{-10\,\text{cossec}\,x\, e^{-3x}}{3}$$

$$\int \text{cossec}\,x\, e^{-3}\,dx = \frac{-5e^{-3}\,\text{cossec}\,x}{3} = u_1$$

$$u_2' = \frac{e^{3x} \cdot \operatorname{cossec} x}{-9}$$

$$u_2 = -\frac{1}{9} \int e^{3x} \operatorname{cossec} x$$