Applied Data Mining Homework 04

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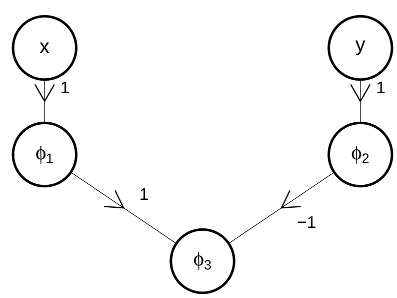
Problem 1:

The function f can be interpreted as:

$$f(x,\, y) = I\{g(x,\, y) > 0\}$$
$$g(x,\, y) = \sigma(x) - \sigma(y)$$

So the network is:



Here, the three ϕ nodes are:

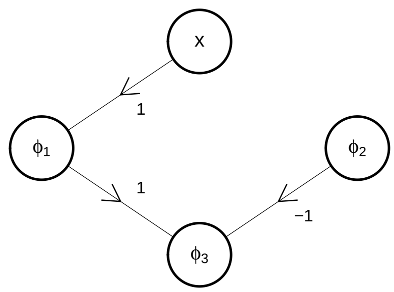
$$\phi_1(\cdot) = \sigma(\cdot)$$
$$\phi_2(\cdot) = \sigma(\cdot)$$
$$\phi_3(\cdot) = I\{\cdot > 0\}$$

Problem 2:

According to the definition of sigmoid function, $\sigma(x) = \frac{1}{1+e^{-x}}$, we can get:

$$
\begin{aligned}
-\sigma(-x) &= -\frac{1}{1+e^x} \\
&= \frac{e^x}{1+e^x} - 1 \\
&= \frac{1}{e^{-x}+1} - 1 \\
&= \sigma(x) - 1
\end{aligned}
$$

So, the network is:



Here, the three ϕ nodes are:

$$
\begin{aligned}
\phi_1(\cdot) &= \sigma(\cdot) \\
\phi_2(\cdot) &= c = 1 \\
\phi_3(\cdot) &= \cdot
\end{aligned}
$$

Problem 3:

According basic logical relationship:

$$
\begin{aligned}
x \oplus y &= (x = y = 1) \vee (x = y = 0) \\
&= (x \wedge y) \vee (\neg x \wedge \neg y)
\end{aligned}
$$

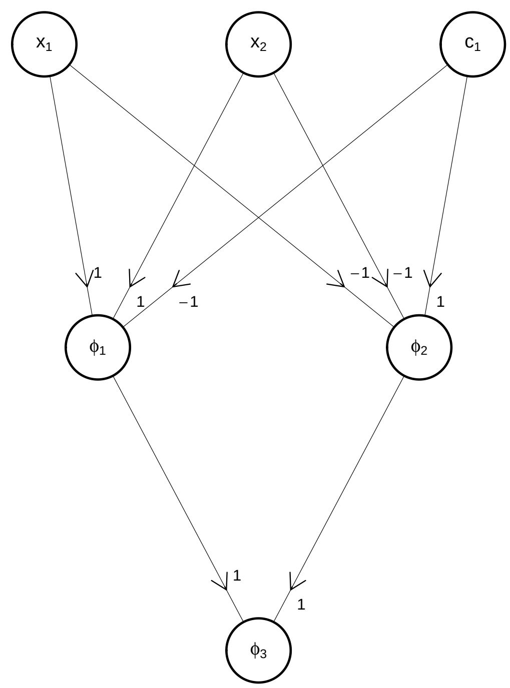
It indicates that "`XOR`" is the "`or`" of two "`and`".

The single unit of "`and`" can be presented as $f(x,\, y) = I\{x + y > 1\}$. The single unit of "`or`" can be presented as $f(x,\, y) = I\{x + y > 0\}$. The single unit of "`not`" can be presented as $f(x) = 1 - x$.

Therefore,

$$\begin{aligned} x \oplus y &= \text{or}(\text{and}(x,\, y),\, \text{and}(\text{not}(x),\, \text{not}(y))) \\ \text{and}(x,\, y) &= I\{x + y - 1 > 0\} \\ \text{or}(x,\, y) &= I\{x + y > 0\} \\ \text{not}(x) &= 1 - x \\ \text{and}(\text{not}(x),\, \text{not}(y)) &= I\{-x - y + 1 > 0\} \end{aligned}$$

So the network is:



Here, the bias nodes are:

$$c_1 = 1$$

And other three ϕ nodes are:

$$\phi_1(\cdot) = I\{\cdot > 0\}$$
$$\phi_2(\cdot) = I\{\cdot > 0\}$$
$$\phi_3(\cdot) = I\{\cdot > 0\}$$

Problem 4:

Classification function is:

$$\begin{aligned} f(\vec{x}) &= \phi(\vec{x} \cdot \vec{w} + w_c \cdot c) \\ &= I\{\vec{x} \cdot \vec{w} \geq c\} \end{aligned}$$

So, the classification result of $\vec{x}$ is:

$$\begin{aligned} f(\vec{x}) &= I\{\begin{pmatrix} -3 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} - \frac{1}{2\sqrt{2}} > 0\} \\ &= I\{-\frac{3}{\sqrt{2}} - \frac{1}{2\sqrt{2}} > 0\} \\ &= I\{-\frac{7}{2\sqrt{2}} > 0\} \\ &= 0 \end{aligned}$$

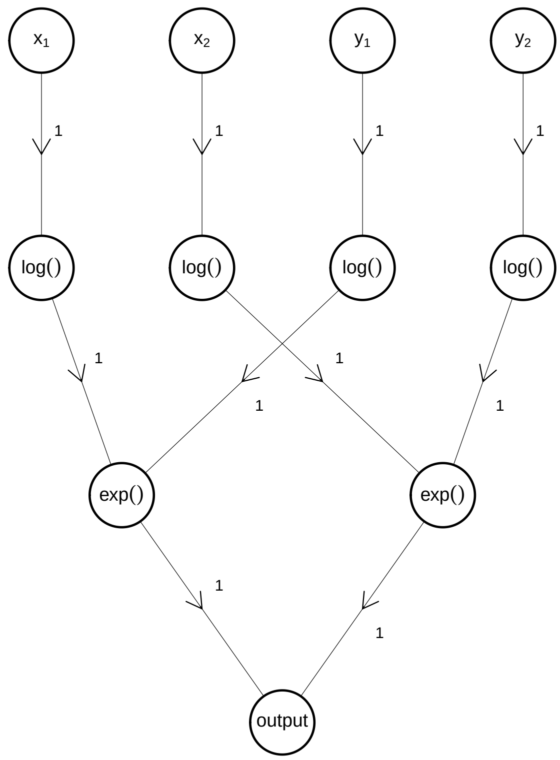
And the classification reuslt of $\vec{x'}$ is:

$$\begin{aligned} f(\vec{x'}) &= I\{\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} - \frac{1}{2\sqrt{2}} > 0\} \\ &= I\{\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} > 0\} \\ &= I\{\frac{1}{2\sqrt{2}} > 0\} \\ &= 1 \end{aligned}$$

Problem 5:

$$\vec{x} \cdot \vec{y} = e^{\log(x_1)+\log(y_1)} + e^{\log(x_2)+\log(y_2)}$$

The network is:


x1
x2
y1
y2
1
1
1
1
log()
log()
log()
log()
1
1
1
1
exp()
exp()
1
1
output