EEEB UN3005/GR5005
Lab - Week 04 - 18 and 20 February 2019

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Bayesian Basics

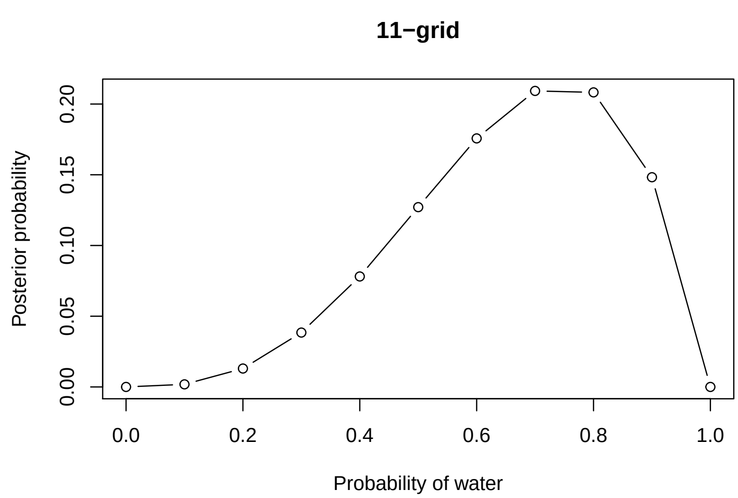
Exercise 1: Applying Bayes' Theorem using Grid Approximation

Imagine the series of observations in the globe tossing example from the *Statistical Rethinking* text and class were: W L W W, where "W" corresponds to water and "L" corresponds to land.

With this set of observations, use grid approximation (with 11 grid points) to construct the posterior for p (the probability of water). Assume a flat prior.

Plot the posterior distribution.

```
grid.length = 11
prob = seq(0, 1, length.out = grid.length)
prior = rep(1, grid.length)
likelihood = dbinom(3, size = 4, prob = prob)
unstd.post = likelihood * prior
std.post = unstd.post / sum(unstd.post)
plot(prob, std.post, type  ='b',
    xlab = 'Probability of water',
    ylab = 'Posterior probability',
    main = paste(c(as.character(grid.length), '-grid'), collapse = ''))
```



Exercise 2: Thinking Deeper with Bayes' Theorem

Suppose in the globe tossing scenario there are two globes, one for Earth and one for Mars. The Earth globe is 70% covered in water. The Mars globe is 100% land. Further suppose that one of these globes-you don't know which-was tossed in the air and produced a land observation. Assume that each globe was equally likely to be tossed. Show that the posterior probability that the globe was the Earth, conditional on seeing land (Pr(Earth|land)), is 0.23.

Note, this problem might seem like it has a lot of information to consider, but it is actually a direct application of Bayes' Theorem. If you're having problems getting started, write out Bayes' Theorem. Also, R is not strictly necessary for this problem. You could do the math by hand, so R is just a glorified calculator here.

Prove:

From the information given above, directly we can get:

$$Pr(Land|Earth) = 0.3,\ Pr(Water|Earth) = 0.7$$
$$Pr(Land|Mars) = 1,\ Pr(Water|Mars) = 0$$
$$Pr(Mars) = Pr(Earth) = 0.5$$

Accoding to the **Bayes' Theorem**,

$$Pr(A|B) = \frac{Pr(B|A) \times Pr(A)}{Pr(B)}$$

To calculate $Pr(Earth|Land)$, we assume that $A = Earth$, $B = Land$.

Thus, the equation is,

$$Pr(Earth|Land) = \frac{Pr(Land|Earth) \times Pr(Earth)}{Pr(Land)}$$

For all the probabilities in this equation, the only one we do not know is $Pr(Land)$, but we can calculate as follows according to **Total Probability Theorem**,

$$\begin{aligned} Pr(Land) &= Pr(Land|Mars) \times Pr(Mars) + Pr(Land|Earth) \times Pr(Earth) \\ &= 1 \times 0.5 + 0.3 \times 0.5 \\ &= 0.65 \end{aligned}$$

Put all known values into the equation,

$$
\begin{aligned}
Pr(Earth|Land) &= \frac{Pr(Land|Earth) \times Pr(Earth)}{Pr(Land)} \\
&= \frac{0.3 \times 0.5}{0.65} \\
&= 0.23
\end{aligned}
$$

q.e.d.