Kr revision notes and mock exam

Knowledge Representation (Vrije Universiteit Amsterdam)

Vrije Universiteit Amsterdam

School of Computer Science
Department of Artificial Intelligence
Knowledge Representation and Reasoning Group

Revision Notes

Knowledge Representation

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Propositional Logic and SAT Solvers

1.1 Definitions

1.2 Exercise

Exercise 1

How would you define the difference between “Soundness” and “Completeness”?

soundness every derived conclusion is correct according to the semantics of the logic

completeness every conclusion that should be derived according to the semantics of the logic is indeed correct.

Exercise 2

What exactly is the Conflict driven learning & non-Chronological backtracking Algorithm?

Non-chronological backtracking jumps not simply to most recent place where there is still an unexplored choice (= chronological backtracking), but jumps over all such places where the choice would not affect the current failure anyway. Conflict-driven learning remembers the reasons for the current failure, so that it will also be of benefit in other places in the search tree.

Exercise 3

What is the exact difference between consistency and satisfiability?

It is consistent when there is no contradiction. It is satisfiable when there is a possible satisfying interpretation to the problem. Every satisfiable problem is consistent (since the semantic interpretation of consistency is satisfiability). Soundness: every satisfiable set is consistent. Completeness: every consistent set is satisfiable.

Exercise 4

$(A \vee B) \wedge (\neg A \vee \neg B)$ is satisfiable, but not valid (slides 1c, sheet 8). Why is it not valid?

Validity means that it is true in all models/interpretations. Something is satisfiable if you have at least one model/interpretation where the sentence is true.

To prove that is satisfiable it suffices to show a true model for our formula. Thus, if A=T and B=F our formula is true which means that our formula is satisfiable too.

To show that is not valid we have to present a false model. A false model is A=T, B=T. That leads to the conclusion that our formula is not valid.

Exercise 5

Why is a 2-SAT solvable in linear time?

2SAT is a logical expression that is the conjunction of a set of clauses, where each clause is the disjunction of two literals.

In 2-SAT, every clause $p_1 \vee p_2$ can be represented as implications: $\neg p_1 \Rrightarrow p_2$ and $\neg p_2 \Rrightarrow p_1$ which means that if we set p1 to false, p2 must be true as well. Thus, those implications are straightforward because there is only one possibility(in contrast with 3-SAT) and if we set a value to a variable, we follow a possible implication chain. So, if we set a value to a variable p_1, through unit propagation we set values for every variable that interacts with p_1.

Exercise 6

Why are horn clause SAT solvable in linear time?

A clause is a Horn clause if at most one literal is positive. These problems can be solved in linear time. For instance, if we have a set of horn clauses we can solve the formula using a greedy algorithm in linear time by seting all the variables to False and then we change only the variable we are forced to true. If we can't change a value from false to true when we are forced to and there is no other option then the formula cannot be satisfied

Exercise 7

How does GSAT (Greedy SAT) work?

Exercise 8

For what kind of problems is GSAT (almost) guaranteed to be faster than the DPLL algorithm?

Answer For a problem that has many possible solution GSAT has a better chance of performing better, since it is has a higher chance of randomly starting close to or on a solution.

Exercise 9

Use Truth table and find if the following formula satisfiable or not. It is valid?

1. $(A \vee B) \wedge (A \wedge \neg B)$.
2. $((A \wedge \neg C) \vee (A \wedge B)) \wedge (C \wedge \neg B)$.
3. $(A \vee \neg B) \vee (B \vee \neg A)$.

Exercise 10

Convert the following to CNF:

1. $\neg(A \vee B) \wedge \neg(A \wedge \neg B)$

2. $(\neg(A \wedge \neg C) \vee (A \wedge B)) \wedge (C \wedge \neg B)$

Exercise 11

When is a CNF formula a Horn formula? Are Horn formula harder to solve or easier?

Every clause has at most one positive literal. It is possible to find satisfiable interpretation in polynomial time.

Exercise 12

Explain Threshold phenomena.

Exercise 13

List 4 applications of SAT solvers to real world problems and give examples.

Exercise 14

- Describe MAXSAT problem.
  the maximum satisfiability problem (MAX-SAT) is the problem of determining the maximum number of clauses, of a given Boolean formula in conjunctive normal form, that can be made true by an assignment of truth values to the variables of the formula. It is a generalization of the Boolean satisfiability problem, which asks whether there exists a truth assignment that makes all clauses true.

- Let $X = (\neg x) \wedge (x \vee y) \wedge (z \vee \neg y) \wedge \neg z$. Is the formulae satisfiable?
  No it is not satisfiable

- If it is not satisfiable, can you find an assignment that satisfies the most clauses?
  Truth assignment $\pi = x, -y, z$

- What are hard/soft clauses?
  Some clauses may be more important than others to solve . If not there is a cost incurred C. For hard clauses the cost is ∞ , otherwise it is a soft clause

- What is Partial Weighted MaxSAT problem (a.k.a Weighted Partial MaxSAT problem)? Give an example.
  They are the ones which have hard clauses and soft clauses but the cost associated with soft clauses is 1 . Let $X = (\neg x, \infty) \wedge (x \vee y, 1) \wedge (z \vee \neg y, 1) \wedge (\neg z, 1)$

Exercise 15

Given the individuals v_1, v_2 and v_3, rewrite the following sentences into propositional form.

1. $\forall x.(p(x) \rightarrow q(x))$
2. $\forall x.(p(x) \wedge q(x))$
3. $\exists x.(p(x) \vee q(x))$

1. $(-p \vee q)$
2. $(p \wedge q)$
3. $(p \wedge q)$

Exercise 16

Explain how you can reduce the amount of clauses you have by introducing dummy variables to the constraint $at\_most\_one(v_1, v_2, v_3, v_4)$. How many clauses have you reduced?

Exercise 17

1. Describe the Davis-Putnam procedure. Use the terms “empty clause”, “tautology”, “unit clause”, “pure literal” and “split”.

Answer: TODO

2. Discuss how the GSAT procedure differs from the Davis-Putnam procedure. Compare the completeness of the GSAT and Davis-Putnam procedures

   Answer: DP gradually builds up a truth-assignment while GSAT starts with a full truth assignment and modifies it DP is a deterministic procedure, while GSAT depends on the randomly chosen initial configuration DP systematically backtracks through all possibilities, while GSAT heuristically explores neighbouring solutions Because of the previous point, DP knows when it has tried all possibilities, so it will end after finite time with either success or failure. GSAT may continue to restart and keep trying, never noticing that the formula is unsatisfiable, or always continuing to miss a solution. Because of the previous point, DP is a complete procedure, GSAT isn't.

3. An often used heuristic in the split step of the Davis Putnam procedure is to pick the literal that occurs most often in the shortest clauses. Explain why this is a good heuristic.

Exercise 18

In clause learning

1. Explain the what triggers the learning of a particular clause in the Davis Putnam procedure.

2. Explain the advantages and disadvantages of learning many or learning few clauses.

Answer:

1. Clause learning is triggered by a conflict (= a set of literals that cannot all be false at the same time)

2. Learning too few clauses = individual steps are efficient, but redoing too many previous failures Learning too many clauses = avoiding many previous failures, but individual steps become very expensive (because set of clauses becomes very large)

Exercise 19

Prove exercise 10 using DPLL and GSAT algorithm.

2 Description logics and Tableaux Method

2.0.1 Exercise

Exercise 1

What is meant by the expressiveness and decidability of a language? Why do we use DL instead of FOL or PL?

First order logic is expressive but undecidable, Propositional logic is inexpressive and decidable. DL is a good balance.

Exercise 2

in a TBox what is the exact difference between concept inclusion and concept equivalence? i.e. what is the exact difference between necessary and necessary and sufficient?

2.1 Tableaux Method

See the Tableaux Tutorial: `https://docs.google.com/document/d/1G4UGcUntWNkuCh0Tn0e37Z8m8` `edit?usp=sharing`.

This Youtube video is also helpful: `https://www.youtube.com/watch?v=18o0VwcowYE`.

2.1.1 Answer to the Tutorial

Negation Normal Form

$NNF(\neg(A \sqcup \forall r.B)) = \neg A \sqcap \exists r.\neg B.$

$NNF(\neg(\exists r.(A \sqcap \neg C))) = \forall r.(\neg A \sqcup C)$.

Tableaux Algorithm

1. Question: What is the proof like if it is consistent?

   Answer: If it is consistent, all branches should be closed.

2. Question: Why do we need to transform the formula to NNF?

   Answer: It allows systematic procedural for consistency checking.

3. Question: True or false? The Tableaux Algorithm decides the consistency of ABox w.r.t. TBox.

   Answer: True.

4. Question: Does the order of rules matter ?

   Answer: No. It does not matter. However, applying rules in order makes the reasoner procedural clearer.

5. Question: I see no rule corresponding to subsumption and equivalence, why is that?

   Answer: Subsumption is not defined primitively. We will have to rewrite it using $\sqcap$ and $\neg$ (see the tutorial for details).

6. Question: Can you close a branch if you see obvious contradictions during the reasoning process?

   Answer: No. You will have to perform all the steps until reaching atoms.

7. Question: How should I define fresh individual names (in $\exists$ rule)?

   Answer: The name of individuals must be different from existing names.

8. Question: What if I have multiple entries meeting the precondition of the $\forall$ rule? For example, we have $(a, b1) : r$ and $(a, b2) : r$.

Answer: You will have to apply the $\forall$ rule on all of them.

9. Question: What if I have multiple parts connected by $\sqcap$, can I use the $\sqcap$ rule and add new elements to the set all together? Similarly, what if I have multiple parts connected by $\sqcup$, can I branch out all at once?

   Answer: Yes you can. Please comment where each term comes from.

10. Question: What can I conclude if I cannot apply rules any more?

    Answer: You have reached an open branch. Your formula is therefore satisfiable. The subsumption does not hold.

11. Question: If I spot a:T somewhere along a branch, can I conclude that it is an open branch?

    Answer: No. You will have to keep going until you finish all branches.

Exercise

A. Describe in words the meaning of

1. $\forall$ created.Painting: those things that creates only paintings (if he/it/she has any).

2. $\forall$ created.Painting $\sqcup \exists$ created.T : those things that creates only paintings and he/it/she has at least one.

3. $\exists$ created.Painting :

B. Is the first formulae subsumed by the third, why?

No. Prove using Tableaux Method. TODO.

C. Use a tableau algorithm to prove whether the second formulae is subsumed by the third one.

All branches are closed. TODO.

2.1.2 More Exercises

Exercise 1

Describe the following using ALC:

- A planet is a celestial body that orbits around some star.
- Moons orbit only around planets.
- Planets and stars are disjoint classes of objects.
- Earth is a planet.
- The Moon orbits around the Earth.

Given the above axioms, give tableau proofs of the following statements:

- The Moon is a moon.
- The Moon cannot orbit around any star.
- Moons and planets are disjoint classes of objects

3 Constraint Satisfaction Problem

3.1 Time Points and Time Intervals

3.1.1 Definitions

time point actions and events that are instantaneous.

time interval actions and events that have duration.

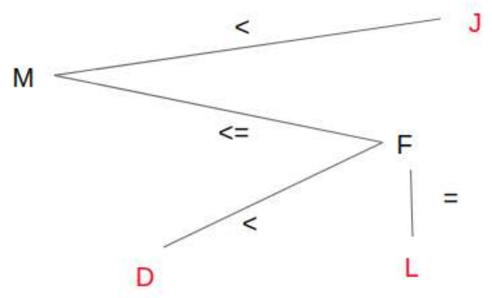
3.1.2 Modelling Exercise

Exercise 1

- Jack is taller than Mike
- Mike is no taller than Finn
- Finn is the same height as Luuk
- Duke is smaller than Finn

Is it possible that:

1. Jack has the same height as Luuk
2. Jack is the smaller than Duke



Answer: It is possible that Jack is as tall as Luuk. It is also possible that Jack is smaller than Duke.

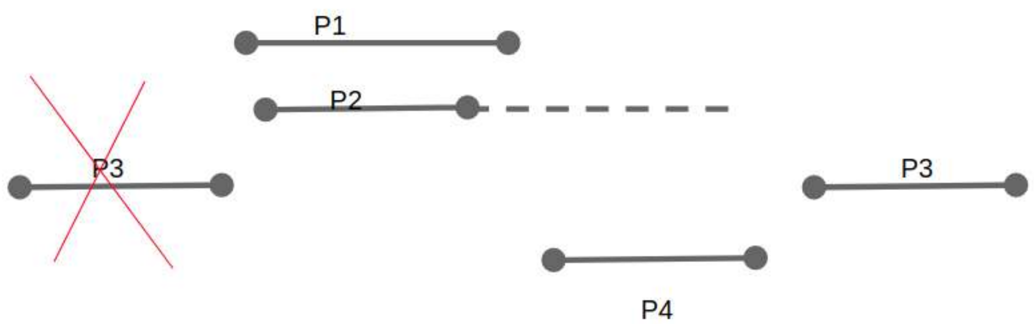
Exercise 2

- P1: Finn marks the reports
- P2: Finn goes to OKEANOS for a competition
- P3: It rains
- P4: Frank gives the lecture

The constraints are as follows:

- P2 starts not before than P1
- P2 should not happen during P3
- P4 should happen before P3 starts and after P1 finishes

Question: is it possible that P2 and P4 overlaps?



Answer: It is possible that event P2 and P4 overlaps.

3.1.3 Allen's Interval Calculus

The Base Relations

There are 13 binary relations and 8 basic binary relations when considering symmetricity. Fill in Table 3.1.

3.2 AC-1 Algorithm

3.2.1 Algorithm

3.2.2 Exercise

3.3 Local Consistency

3.3.1 Definition

Definition 3.3.1. Local consistency A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X.

Definition 3.3.2. Arc consistency A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X.

Definition 3.3.3. Path consistency A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X.

Examples:

Relations:

Definition 3.3.4. k-consistency A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X.

Definition 3.3.5. Strong k-consistency A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X.

Tab. 3.1.: The Base Relations

Set Representation	Name	Symbol	Graphical Representation	Comment
$\{(X, Y) \mid X- < X+ < Y- < Y+\}$	before	$<$		
	after			
	meets			
	meet-by			
	overlaps			
	overlaps-by			
	starts			
	started-by			
	finishes			
	finished-by			
	during			
	contains			
	equals			

Exercise

Represent the following:

1. time point x is before y
2. time point x is not after y
3. time point x is not at the same time as y
4. event X happens before Y
5. event X happens after the Y
6. event X finishes event Y
7. event X is overlapped by event Y

3.3.2 Exercise

Exercise 1

Consider the CSP problem $(x \wedge y = z; x = true, y \in true, false, z \in true, false)$.

Is this problem arc consistent? hyper-arc consistent? How can a SAT problem be seen as a constraint problem?

Exercise 2

Solve the CSP problem $x < y, y < z; x \in 1, 2, 3, y \in 2, 3, z \in 1, 2, 3$ using AC-1 (aka draw the boxes and arrows). Is it arc-consistent?

Exercise 3

Consider the problem $x < y, y < z, x < z; x \in [0..4], y \in [1..5], z \in [5..10])$. Is this problem arc consistent? Is it path consistent?

The problem is arc-consistent: every value of x is covered by some value of y (and vice versa), and similar for (y,z)-pairs and (x,z)-pairs. It is not path consistent, since the consistent pair x=4 en z=5 cannot be extended to a triple of consistent values.

3.4 Search Tree

3.4.1 Exercise

Exercise 1

Consider the CSP problem $x < y, y < z; x \in 1, 2, 3, y \in 2, 3, z \in 1, 2, 3$. Draw the search tree for solving this problem.

Exercise 2

4 Qualitative Reasoning

4.1 Influence vs. Proportional

- Proportional:
  The *derivative* of Q1 affects the *derivative* of Q2.
  E.g. if Q1 increases ($\delta Q1 > 0$), then Q2 increases ($\delta Q2 > 0$)

- Influence:
  The *magnitude* of Q1 affects the *derivative* of Q2.
  E.g. if Q1 is active ($Q1 > 0$), then Q2 increases ($\delta Q2 > 0$)

4.2 Sign algebra

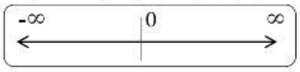
Addition

Q1 + Q2	-	0	+
-	-	-	?
0	-	0	+
+	?	+	+

Multiplication

Q1 * Q2	-	0	+
-	+	0	-
0	0	0	0
+	-	0	+

Quantity space: {-,0,+}



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4.3 Ontology

4.3.1 Process ontology

In a given world with objects, processes arise from the relationships and properties of those objects. These processes are treated as distinct categories of entities. Process ontologies are a natural fit to most everyday physical phenomena. However, it requires reasoning about the relationships and creates additional complexities in the reseasoning.

4.3.2 Field ontology

In field ontologies, the spatial structure is represented by dividing the space into regions where some parameters of interest takes on qualitatively equivalent values. Qualitative reasoning in this ontology typically uses representations and algorithms drawn from computer vision and computational geometry to construct symbolic representations of numerical data.

5 Mini-Mock Exam

Propositional Logic [20 points]

A [10 points]

Consider the sentence $(A \vee \neg B) \wedge (\neg A \wedge \neg B)$.

1. Is this sentence satisfiable?

2. Is this sentence valid?

Draw the Truth table and explain your answers.

B [5 points]

Given the individuals i_1 and i_2, rewrite the sentences $\forall x.(p(x) \wedge r(x) \rightarrow q(x))$ and $\exists x.(q(x) \rightarrow p(x))$ into propositional form.

C [5 points]

What is a Partial Weighted MaxSAT problem? How does that differ from SAT?

SAT Solvers [15 points]

A [10pt]

Use DPLL and show that $(\neg A \vee B) \wedge (A \vee \neg B) \wedge (C \wedge \neg A)$ is satisfiable.

B [5pt]

What heuristic would you use and which literal did you choose at the first decision level and why?

Description Logic [25 points]

A [10 points]

Define concepts and role names. Write down the description logic axioms in TBox and ABox.

Students are those who study in a university. Researchers and professors are those who work in a university. Some students are PhD students and they also work for a university. VU is a university. Frank is a professor at VU. Finn is a PhD student at VU. Wouter and Albert are researchers at VU. Luuk and Michel are (non-PhD) students at UvA. Sara is a researcher at UvA.

B [10 points]

The Chinese have no siblings (due to the One-Child policy). Finn has siblings. Can you prove that he is not a Chinese using the Tableaux method?

C [5 points]

Why are we not using First Order Logic here? What advantage does Description Logics have? How can we express "Frank is supervising 3 students"?

CSP [20 points]

A [10 points]

Consider the CSP problem $(x \wedge y = z; x = true, y \in \{true, false\}, z \in \{true, false\})$. Is this problem arc consistent? hyper-arc consistent?

B [10 points]

How can a SAT problem be seen as a constraint problem? Give an example.

A

Additional Material and Tools

A.1 SMT solver

SMT [1] stands for Satisfiability Modulo Theory. The corresponding solvers are playing a more and more important role today in research. SMT solvers usually have one or more SAT solver(s) integrated. SMT solvers do not increase the problems solvable by SAT solver (a.k.a. NP-complete) but they are much better in dealing with constraints in different format. Z3 and Yices are common solvers.

A.2 CSP

WolframAlpha is a handy tool to do some primitive analysis, especially when there are actual values involved. The following is an example of qualitative reasoning.

[1] https://en.wikipedia.org/wiki/Satisfiability_modulo_theories

WolframAlpha computational knowledge engine.

$0<x<5,\ 1<y<7,\ 3<z<5,\ x<y,\ y<z,\ x<z$

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Input:

$$\{0 < x < 5, 1 < y < 7, 3 < z < 5, x < y, y < z, x < z\}$$

Open code

Alternate form:

$$\{x < 5 \land x > 0, y < 7 \land y > 1, z < 5 \land z > 3, x < y, y < z, x < z\}$$

$e_1 \land e_2 \land \ldots$ is the logical AND function

Solutions:

$$0 < x \le 1, \quad 1 < y \le 3, \quad 3 < z < 5$$

$$0 < x \le 1, \quad 3 < y < 5, \quad y < z < 5$$

$$1 < x \le 3, \quad 3 < y < 5, \quad y < z < 5$$

$$1 < x \le 3, \quad x < y \le 3, \quad 3 < z < 5$$

$$3 < x < 5, \quad x < y < 5, \quad y < z < 5$$

Integer solutions:

$$x = 1, \quad y = 2, \quad z = 4$$

$$x = 1, \quad y = 3, \quad z = 4$$

$$x = 2, \quad y = 3, \quad z = 4$$

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Special thanks to the students in Group B for editing and proof-reading this revision note. Good luck with your exam!

Colophon

This thesis was typeset with $\LaTeX\,2_\varepsilon$. It uses the *Clean Thesis* style developed by Ricardo Langner. The design of the *Clean Thesis* style is inspired by user guide documents from Apple Inc.

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