Sea $\{(x_i, y_i) : i = 1, \dots, n\}$ → muestra aleatoria tamaño n

Para todo i:

idea de una recta ← $y_i = \beta_0 + \beta_1 x_i + u_i$ (1)

→ forma parte de la pendiente

↓ intercepto

↳ error

$$\boxed{E(u) = 0}$$

$$\boxed{Cov(x, u) = E(xu) = 0}$$

a nivel población aun sin estimar

$$\begin{cases} E(y - \beta_0 - \beta_1 x) = 0 & (2) \\ E[x(y - \beta_0 - \beta_1 x)] = 0, & (3) \end{cases}$$

La parte muestral:

$$\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0 \quad (2.1)$$

$$\frac{1}{n} \sum_{i=1}^{n} x_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0 \quad (2.2)$$

De (2.1):

✓ aplicamos propiedad de sumatorias

$$\bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{X} \quad (3)$$

donde $\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} y_i$

✓ eliminaremos $1/n$ porque no ayuda

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \quad (3.1)$$

La parte muestral:

$$\frac{1}{n}\sum_{i=1}^{n}(y_i-\hat{\beta}_0-\hat{\beta}_1 x_i)=0 \quad (2.1)$$

$$\frac{1}{n}\sum_{i=1}^{n}x_i(y_i-\hat{\beta}_0-\hat{\beta}_1 x_i)=0 \quad (2.2)$$

aplicamos propiedad de sumatorias

De (2.1)

$$\bar{y}=\hat{\beta}_0+\hat{\beta}_1\bar{x} \quad (3)$$

donde $\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i$

eliminaremos 1/n porque no ayuda

$$\hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x} \quad (3.1)$$

(3.1) se reemplaza en (2.2)

$$\sum_{i=1}^{n}x_i\left[y_i-(\bar{y}-\hat{\beta}_1\bar{x})-\hat{\beta}_1 x_i\right]=0$$

Reordenando:

$$\sum_{i=1}^{n}x_i(y_i-\bar{y})=\hat{\beta}_1\sum_{i=1}^{n}x_i(x_i-\bar{x})$$

aplicando propiedades de sumatoria

$$\sum_{i=1}^{n} x_i(x_i - \bar{x}) = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\sum_{i=1}^{n} x_i(y_i - \bar{y}) = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

Por tanto, siempre que:

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 > 0 \qquad (4)$$

La pendiente estimada es:

$$\hat{\beta}_1 = \frac{\text{cov}(x,y)}{\sqrt{\text{var}(x)}} \rightarrow$$

$$\boxed{\hat{\beta}_1 = \frac{\sum_{i=0}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=0}^{n} (x_i - \bar{x})^2}} \qquad (5)$$