# Homework 1

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In [1]:
```julia
# Load libraries first of all
using LinearAlgebra # load LinearAlgebra standard library
using Plots # loads the Plots module
```

In [2]:
```julia
gr()
```

Out[2]: Plots.GRBackend()

In [3]:
```julia
xrange = -3.0 : 0.1 : 3.0
```

Out[3]: -3.0:0.1:3.0

## Question 1

1. a) $H^a = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

1. b) $|H^a| = 4$, so the matrix is negative definite, this means it is a concave function and it has a global maximum at x = 0 and y = 0.

In [4]:
```julia
Hª = [-2 0; 0 -2]
```

Out[4]: 2×2 Array{Int64,2}:
 -2   0
  0  -2

1. c)

In [5]:
```julia
@assert det(Hª) == 4
```

In [6]:
```julia
@assert det(Hª) == 3
```

AssertionError: det(Hª) == 3

Stacktrace:
 [1] top-level scope at In[6]:1

1. d) Eigenvalues

In [7]: ▶ `eigvals(Hᵃ)`

Out[7]: 2-element Array{Float64,1}:
  -2.0
  -2.0

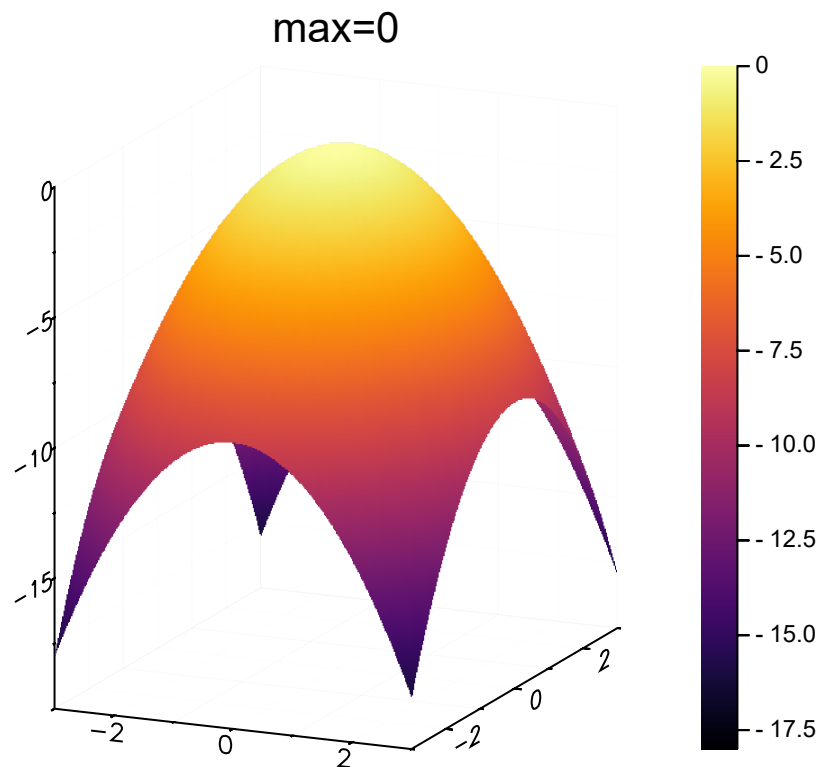1. e) Function to plot

In [8]: ▶
```julia
# functions to plot
fa(x,y) = -x^2 - y^2
```

Out[8]: fa (generic function with 1 method)

In [9]: ▶ `plot(xrange, xrange, fa, st = :surface, title = "max=0")`

Out[9]:



## Question 2

2. a) $H^b = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$

2. b) $|H^b| = 3$, so the matrix is negative definite, this means it is a concave function and it has a global maximum at x = 0 and y = 0.

```
In [10]:  ▶  Hᵇ = [-2 1; 1 -2]
```

```
Out[10]:  2×2 Array{Int64,2}:
           -2   1
            1  -2
```

2. c)

```
In [11]:  ▶  @assert det(Hᵇ) == 3
```

```
In [12]:  ▶  @assert det(Hᵇ) == 2
```

```
AssertionError: det(Hᵇ) == 2

Stacktrace:
 [1] top-level scope at In[12]:1
```

2. d)

```
In [13]:  ▶  eigvals(Hᵇ)
```

```
Out[13]:  2-element Array{Float64,1}:
           -3.0
           -1.0
```
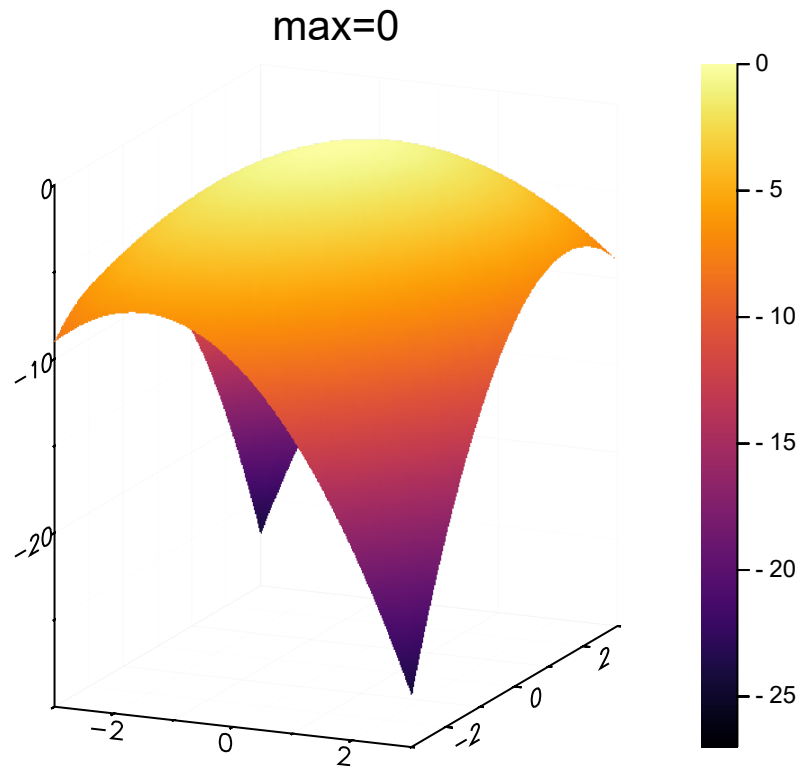
2. e) Function to plot

```
In [14]:  ▶  fb(x,y) = -x^2 +x*y - y^2
```

```
Out[14]:  fb (generic function with 1 method)
```

```
In [15]:  ▶ plot(xrange, xrange, fb, st = :surface, title = "max=0")
```

Out[15]:



max=0

## Question 3

3. a) $H^c = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

3. b) $|H^c| = 0$, so the matrix is negative semi-definite, this means it is a concave function and it
has a global maximum at the line where x = y.

```
In [16]:  ▶ Hᶜ = [-2 2; 2 -2]
```

Out[16]:  2×2 Array{Int64,2}:
          -2   2
           2  -2

3. c)

```
In [17]:  ▶ @assert det(Hᶜ) == 0
```

```
In [18]:   ▶  @assert det(Hᶜ) == 1
```

```
AssertionError: det(Hᶜ) == 1

Stacktrace:
 [1] top-level scope at In[18]:1
```

3. d)

```
In [19]:   ▶  eigvals(Hᶜ)
```

```
Out[19]:  2-element Array{Float64,1}:
           -4.0
            0.0
```
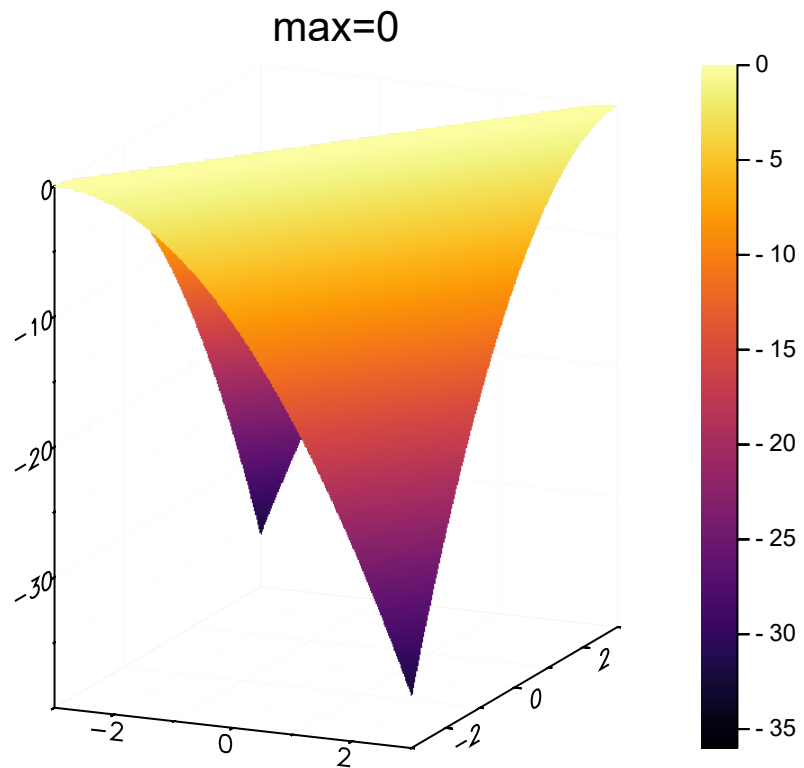
3. e) Function to plot

```
In [20]:   ▶  fc(x,y) = -x^2 + 2*x*y - y^2
```

```
Out[20]:  fc (generic function with 1 method)
```

```
In [21]:   ▶  plot(xrange, xrange, fc, st = :surface, title = "max=0")
```

Out[21]:

# Question 4

4. a) $H^d = \begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix}$

4. b) $|H^d| = -5$, so the matrix is indefinite, this means it is neither a concave nor a convex function. It does not have a global maximum nor a global minimum.

In [22]: ▶ | `Hᵈ = [-2 3; 3 -2]`

Out[22]: 2×2 Array{Int64,2}:
 -2   3
  3  -2

4. c)

In [23]: ▶ | `@assert det(Hᵈ) == -5`

In [24]: ▶ | `@assert det(Hᵈ) == 0`

AssertionError: det(Hᵈ) == 0

Stacktrace:
 [1] top-level scope at In[24]:1

4. d)

In [25]: ▶ | `eigvals(Hᵈ)`

Out[25]: 2-element Array{Float64,1}:
 -5.0
  0.9999999999999998

4. e) Function to plot

In [26]: ▶ | `fd(x,y) = -x^2 + 3*x*y - y^2`

Out[26]: fd (generic function with 1 method)

In [27]: ▶| `plot(xrange, xrange, fd, st = :surface)`