UNet-HCRF Integration for EEG Pattern Recognition

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Overview

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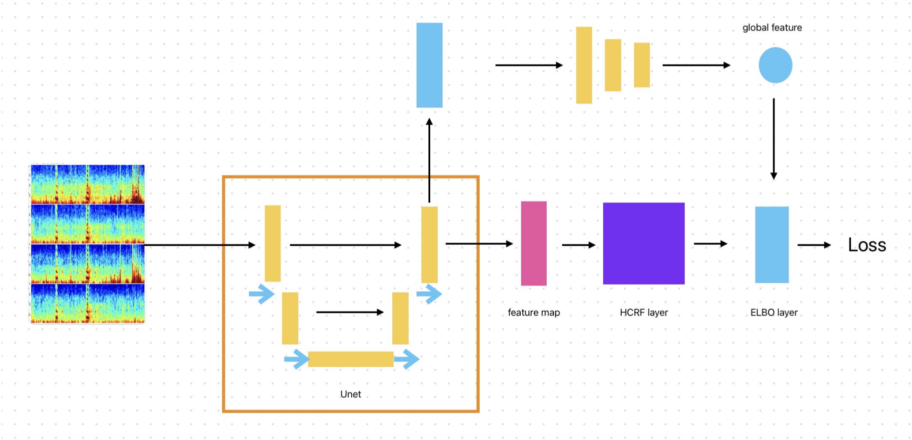
Introduction

- Electroencephalography (EEG) pattern recognition tasks are crucial for healthcare, where accurate and timely data classification can significantly impact patient outcomes.

- Deep learning has recently emerged as a powerful tool for sequence labeling.

- Conditional random fields (CRFs) have been employed for modeling label dependencies, demonstrating promising results.

Framework



Overview

CNN-HCRF Model Structure

- Given the feature of the input data $\mathbf{I}$ learned by CNN, i.e. $\mathbf{x} = \mathbf{F}(\mathbf{I})$, its corresponding hidden part states $\mathbf{h}$, and the class label y, a hidden conditional random field (HCRF) has the exponential form:

$$\mathbb{P}(y, \mathbf{h}|\mathbf{x}; \theta) = \frac{\exp\left(\Phi(y, \mathbf{h}, \mathbf{x}; \theta)\right)}{\sum_{y' \in Y} \sum_{\mathbf{h} \in H^N} \exp\left(\Phi(y', \mathbf{h}, \mathbf{x}; \theta)\right)},$$

  where θ is the model parameter, H^N denotes the set of all possible hidden part states of N hidden parts, and $\Phi(y, \mathbf{h}, \mathbf{x}; \theta)$ refers to potential function depending on the feature $\mathbf{x}$.

- The probability of class label y for the given feature $\mathbf{x}$ is:

$$\mathbb{P}(y|\mathbf{x}; \theta) = \sum_{\mathbf{h} \in H^N} \mathbb{P}(y, \mathbf{h} \mid \mathbf{x}; \theta) = \frac{\sum_{\mathbf{h} \in H^N} \exp\left(\Phi(y, \mathbf{h}, \mathbf{x}; \theta)\right)}{\sum_{y' \in Y} \sum_{\mathbf{h} \in H^N} \exp\left(\Phi(y', \mathbf{h}, \mathbf{x}; \theta)\right)}.$$

CNN-HCRF Model Structure (cont.)

The jointly probability distribution of the HCRF model is:

$$\Phi(y, \mathbf{h}, \mathbf{x}; \theta) = \underbrace{\sum_{j \in \nu} \phi(h_j, x_j; \omega) + \sum_{i \neq j} \psi(h_i, h_j, x_i, x_j; \eta)}_{\text{Measures log-likelihood } \log \mathbb{P}(\mathbf{h}|\mathbf{x}; \theta)}$$

$$+ \underbrace{\sum_{j \in \nu} \varphi(y, h_j, x_j; \delta) + \vartheta(y, x_0; \varpi)}_{\text{Measures log-likelihood } \log \mathbb{P}(y|\mathbf{h}, \mathbf{x}; \theta)},$$

where:

- Unary potential $\phi(h_j, x_j; \omega)$: Measures the likelihood of the local feature x_j is assigned as the hidden part state h_j, with parameter ω is learned by end-to-end CNN-UNet structure.

CNN-HCRF Model Structure (cont..)

- Binary potential $\psi(h_i, h_j, x_i, x_j; \eta)$: Balances the hidden label compatibility of the neighboring pixels p on the feature $\mathbf{x}$:

$$\psi(h_i, h_j, x_i, x_j; \eta) = \mu(h_i, h_j)\left[\omega_1 \exp\left(-\frac{|p_i - p_j|^2}{2\eta_\alpha^2} - \frac{|x_i - x_j|^2}{2\eta_\beta^2}\right) + \omega_2 \exp\left(-\frac{|p_i - p_j|^2}{2\eta_\gamma^2}\right)\right],$$

where $\mu(h_i, h_j)$ is a label compatibility function, ω_1 and ω_2 are linear combination weights of different kernels, and the parameters η_α, η_β and η_γ control the influence of the corresponding feature spaces.

CNN-HCRF Model Structure (cont...)

- Unary potential $\varphi(y, h_j; \delta)$: Measures the compatibility between global class label y and the hidden local state h_j:

$$\varphi(y, h_j; \delta) = \sum_{a \in Y} \sum_{b \in H} \delta_{a,b} \cdot \mathbb{1}(y = a) \cdot \mathbb{1}(h_j = b),$$

  where $\mathbb{1}(\cdot)$ is a indicator function, and $\delta_{a,b}$ denotes the likelihood of class label $y = a$ containing a joint with hidden state $h_j = b$, learned during training.

- Global potential $\vartheta(y, x_0; \varpi)$: Measures the likelihood of the global feature x_0 is assigned as the action label y, with parameter ϖ learned during training process.

Variational Inference of HCRF

- The mean-field varepsilon family approximation of $\log P(\mathbf{h} \mid y, \mathbf{x}, \theta)$ is expressed as $\mathbb{Q}(h) = \prod_{i=1}^{N} q_i(h_i)$.
- The evidence lower bound (ELBO) under the mean-field family is:

$$
\begin{aligned}
\mathrm{ELBO}(\mathbb{Q}) &= \mathbb{E}_{\mathbb{Q}(\mathbf{h})} \log \mathbb{P}(y, \mathbf{h}|\mathbf{x}; \theta) - \mathbb{E}_{\mathbb{Q}(\mathbf{h})} \log q(\mathbf{h}) \\
&= \sum_{i=1}^{N} \mathbb{E}_{q_i(h_i)} \left[\phi(h_j, x_j; \omega) + \varphi(y, h_j, x_j; \delta)\right] \\
&+ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l,l'} \mathbb{1}_{\{i \neq j\}} \mathbb{Q}(h_i = l)\mathbb{Q}(h_j = l')\mu(l, l') \\
&\left[\omega_1 \exp\left(-\frac{|p_i - p_j|^2}{2\eta_\alpha^2} - \frac{|\mathbf{F}_i(\mathbf{I}) - \mathbf{F}_j(\mathbf{I})|^2}{2\eta_\beta^2}\right) + \omega_2 \exp\left(-\frac{|p_i - p_j|^2}{2\eta_\gamma^2}\right)\right] \\
&- \sum_{i=1}^{N} \mathbb{E}_{q_i} \log q_i(h_i) + \vartheta(y, \mathbf{x}; \varpi).
\end{aligned}
$$

Variational Inference of HCRF (cont.)

- By the coordinate ascent variational inference (CAVI) algorithm, fixing $q_j(h_j), \forall j \neq i$, the optimal $q_i(h_i)$ that maximizes $\mathrm{ELBO}(q_i(h_i))$ is:

$$\mathbb{Q}(h_i) \propto \exp\{\mathbb{E}_{\mathbb{Q}(\mathbf{h}_{-i})} \log \mathbb{P}(y, h_i, \mathbf{h}_{-i} \mid \mathbf{x}; \theta)\}.$$

- The explicit form of $q_i(l)$ is derived as follows:

$$q_i(l) = \frac{1}{Z_i} \exp\left\{ \phi(h_j, x_j; \omega) + \varphi(y, h_j, x_j; \delta) + \sum_{j \neq i} \sum_{l'} q(l')\mu(l, l') \right.$$
$$\left. \left[\omega_1 \exp\left(-\frac{|p_i - p_j|^2}{2\eta_\alpha^2} - \frac{|\mathbf{F}_i(\mathbf{I}) - \mathbf{F}_j(\mathbf{I})|^2}{2\eta_\beta^2} \right) + \omega_2 \exp\left(-\frac{|p_i - p_j|^2}{2\eta_\gamma^2} \right) \right] \right\},$$

where Z_i is the normalization constant.

Mean Field Inference in HCRF

Algorithm 1: Mean-field inference in fully-connected CRF

Input: $Q_i(h_i) \leftarrow U_i(h_i),\ i = 1, 2, \ldots, N$ `// Initialize` $Q(h)$

while *not reach max iteration number* **do**

- $\hat{Q}_i^{(m)}(h_i) \leftarrow \sum_{j \neq i} k^{(m)}(f_i, f_j) Q_j(h_i)$ for all m `// Message Passing`;
- $\tilde{Q}_i^{(m)}(h_i) \leftarrow \sum_{l \in \mathcal{L}} \mu^{(m)}(h_i, l) \sum_m \omega^{(m)} \hat{Q}_i^{(m)}(l)$ `// Compatibility Transform`;
- $Q_i(h_i) \leftarrow \exp\{\phi(h_j, x_j; \omega) + \varphi(y, h_j, x_j; \delta) - \tilde{Q}_i(x_i)\}$ `// Local Update`;
- Normalize $Q_i(h_i)$;

end

Overview

Harmful Brain Activity Classification Dataset

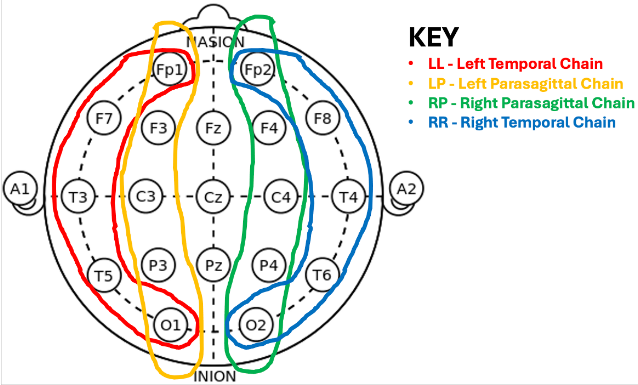
- The dataset contains 50-second long EEG samples covering a 10-minute window as well as spectrograms matching each sample.
- The EEG samples were labeled by expert annotators to determine which of the six features each spectrogram belonged to.
- The longer 10-minute window allows for the model to pick up on further insights regarding the nature of the brain activity.

Data Preparation

- Extract 10-minute long spectrograms centered at the midpoint time, where the expert ratings (Global EEGs) were provided.
- Convert the 50-second long EEG waveforms into spectrograms.

Make Spectrogram from EEG

- Bipolar Montage combines multiple EEG electrode signals by calculating the difference between adjacent electrodes, yielding signals representing specific brain regions.



Make Spectrogram from EEG (cont.)

- A single time series signal is required to generate a spectrogram. To create four spectrograms from the dataset's 19 EEG time series signals, we employ Bipolar Montage to combine them into four representative signals.

$$\text{LL Spec} = \frac{1}{4}\left[spec(Fp1 - F7) + spec(F7 - T3) + spec(T3 - T5) + spec(T5 - O1)\right],$$

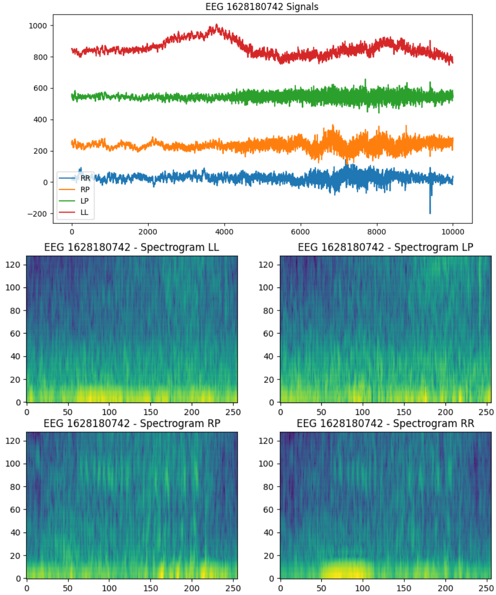
$$\text{LP Spec} = \frac{1}{4}\left[spec(Fp1 - F3) + spec(F3 - C3) + spec(C3 - P3) + spec(P3 - O1)\right],$$

$$\text{RP Spec} = \frac{1}{4}\left[spec(Fp2 - F4) + spec(F4 - C4) + spec(C4 - P4) + spec(P4 - O2)\right],$$

$$\text{RR Spec} = \frac{1}{4}\left[spec(Fp2 - F8) + spec(F8 - T4) + spec(T4 - T6) + spec(T6 - O2)\right],$$

where $spec(\cdot)$ is a custom function implemented using the `pywt` and `librosa` packages for signal transformation.

Make Spectrogram from EEG (cont..)



Training Schedule

The loss function was designed as Kullback-Leibler (KL) divergence.

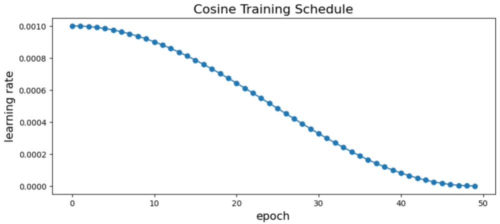


Figure 1: Pre-train UNet learning rate schedule.

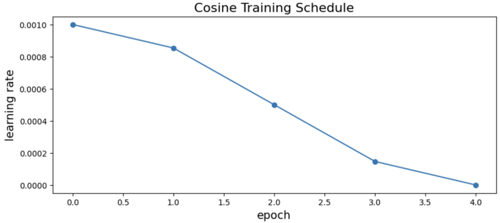


Figure 2: Fine tuning HCRF learning rate schedule.

Overview

Results

We evaluate our model by 5 fold cross-validation.

- Our model achieves a **0.4992** KL-divergence.
- The baseline model from Kaggle using EfficientNet obtains a 0.59 cross-validation KL-divergence.

However, there still are many teams on Kaggle have achieved even lower KL-divergence scores (0.2-0.3) by leveraging the power of large models like Vision Transformers (ViT).

Conclusion

- We build a Hidden Conditional Random Fields for EEG based Spectrograms image classification.
- The latent CRF layer is desgined to capture the latent segmentation information of the output of Neural Networks.
- A variational mean-field inference scheme is used to address the posterior distribution.
- We integrate the Unet and HiddenCRF into an end-to-end training structure. The Neural Networks feature learning and variational inference can be accelerated jointly by GPUs.