

## Exame Recurso 2021 - 9/07/2021

1.  $k = 28,3 \text{ N/m}$

$$m = 520 \text{ g} \quad r = 4,8 \text{ cm}$$

$$r_g = 4,12 \text{ cm}$$

a)  $I = 0,520 \times 0,0412^2 = 8,827 \times 10^{-4} \text{ kg} \cdot \text{m}^2$

↳ Inércia da roda em relação ao eixo

As variáveis de estado  $\rightarrow$  posição  $x$   
 $\rightarrow$  velocidade  $\dot{x}$

$$\omega = \frac{\dot{x}}{0,048}$$

$$U_e = \frac{1}{2} 28,3 x^2 = 14,15 x^2$$

$$E_c = \frac{1}{2} 0,520 \dot{x}^2 + \frac{1}{2} 8,827 \times 10^{-4} \left( \frac{\dot{x}}{0,048} \right)^2 = 0,4516 \dot{x}^2$$

$$\frac{d}{dt} \left( \frac{\partial (E_c)}{\partial \dot{x}} \right) - 0 + \frac{\partial (U_e)}{\partial x} = 0 \Leftrightarrow \ddot{x} = -31,34 x$$

b)  $\begin{cases} \dot{x} = v \\ \dot{v} = -31,34 x \end{cases} \Rightarrow \begin{bmatrix} 0 & 1 \\ -31,34 & 0 \end{bmatrix} \Rightarrow \begin{matrix} \lambda_1 = -5,598 i \\ \lambda_2 = 5,598 i \end{matrix}$

$$\Omega = 5,598 \text{ s}^{-1}$$

$$T = \frac{2\pi}{\Omega} = 1,122 \text{ s}$$

2.  $a_t = 4\sqrt{v^2 + 5}$   $v_f = 65 \text{ m/s}$   
 $v_i = 0 \text{ m/s}$

$$a = \frac{dv}{ds} \cdot \frac{ds}{dt} \Leftrightarrow a = v \frac{dv}{ds} \Leftrightarrow a ds = v dv \Leftrightarrow ds = \frac{v}{a} dv \Leftrightarrow \int_0^s ds = \int_0^{65} \frac{v}{4\sqrt{v^2 + 5}} dv \Leftrightarrow$$

$$\Leftrightarrow \int_0^s ds = \int_0^{65} \frac{v}{4\sqrt{v^2 + 5}} dv \Leftrightarrow s = 15,1 \text{ m} \quad \mathcal{D}_{//}$$

3. C



4.  $R = 1 \text{ m}$        $v = 3t^2$        $a^2 = a_n^2 + a_t^2$

$$a_n = \frac{v^2}{R} = \frac{(3t^2)^2}{1} = 9t^4$$

$$a_t = R \alpha \quad \alpha = \frac{d\omega}{dt} \quad \omega = \frac{v}{R} = \frac{3t^2}{1} = 3t^2$$

$$\alpha = \frac{d(3t^2)}{dt} = 6t \quad a_t = 1 \times 6t = 6t$$

$$a = \sqrt{(9t^4)^2 + (6t)^2} = \sqrt{81t^8 + 36t^2} \quad A //$$

5.  $M = \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \rightarrow \lambda_1 = 1 - i \quad \text{foco repulsivo} \quad A //$   
 $\lambda_2 = 1 + i$


6.  $x = \frac{a}{4} (2\theta + \sin(2\theta)) \quad y = \frac{a}{4} (1 - \cos(2\theta))$

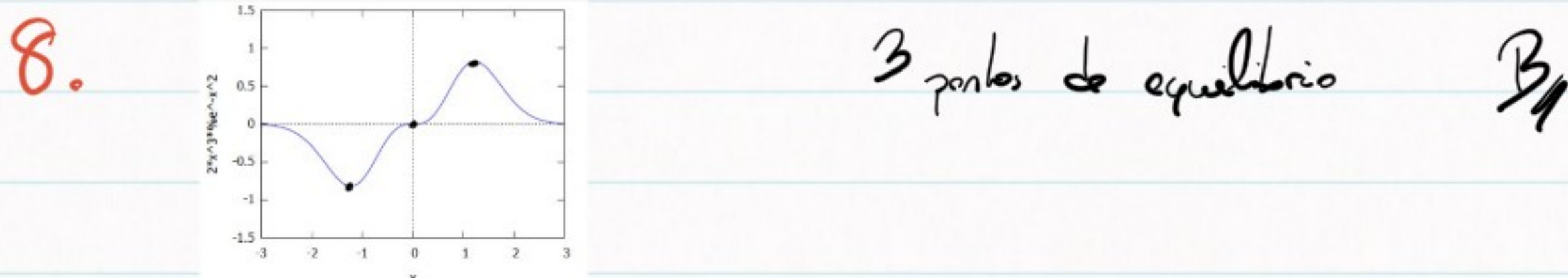
$$v_x = \frac{dx}{dt} = \frac{a}{4} (2\cos(2\theta)\dot{\theta} + 2\dot{\theta})$$

$$v_y = \frac{dy}{dt} = \frac{a}{2} \sin(2\theta)\dot{\theta}$$

$$v^2 = v_x^2 + v_y^2 \Leftrightarrow v^2 = \frac{a^2}{16} (2\cos(2\theta)\dot{\theta} + 2\dot{\theta})^2 + \frac{a^2}{4} \sin^2(2\theta)\dot{\theta}^2$$

$$E_c = \frac{1}{2} m v^2 = \frac{a^2 m \dot{\theta}^2}{4} (\cos(2\theta) + 1) \quad C //$$

7.   $\vec{a} \cdot \vec{b} = 0 \quad C //$



9.  $\lim_{y \rightarrow 0} x\sqrt{y-x} + xy^2 = x\sqrt{-x}$

$$\lim_{y \rightarrow 0} 2xy^2 - x \cos y = -x$$

$$\lim_{y \rightarrow 0} x\sqrt{y+1} - 5yx^2 = x$$

$$\lim_{y \rightarrow 0} y^3 - 3x \sin(x) = -3x \sin x$$

$$\lim_{y \rightarrow 0} 2xy^2 + y \cos x = 0 \quad C //$$



10. Há conservação de  $E_m$  logo,

$$\Delta E_c = -\Delta E_p \Leftrightarrow \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = -m g h_f + m g h_0 \Leftrightarrow \frac{1}{2} v_f^2 - \frac{1}{2} v_0^2 = 9,8 \times h_0 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2} v_f^2 = 9,8 h_0 + \frac{1}{2} v_0^2 \Leftrightarrow v_f^2 = 2 g h_0 + v_0^2 \Leftrightarrow v_f = \sqrt{v_0^2 + 2 g h} \quad A //$$

11. D

12.  $\vec{F} = 2\hat{i} + 4t\hat{j}$        $m = 2 \text{ kg}$        $F = m \times a$

$$a = \hat{i} + 2t\hat{j} \Rightarrow (1, 2t)$$

$$\Delta t = 6 \text{ s}$$

$$v(0) = 8\hat{i} \text{ m/s}$$

$$\hat{i}: v(6) = v(0) + \int_0^6 1 dt = 8 + \int_0^6 1 dt = 14,0 \hat{i}$$

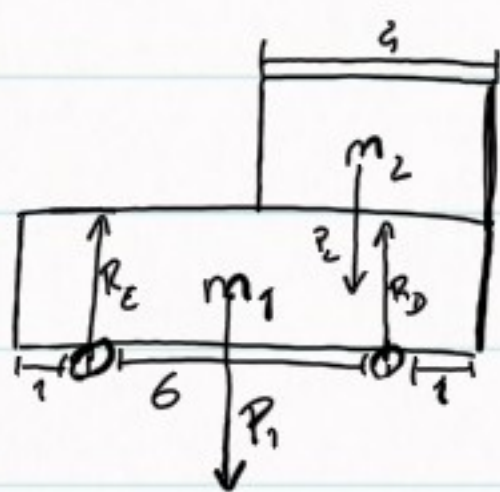
$$\hat{j}: v(6) = v(0) + \int_0^6 2t dt = \int_0^6 2t dt = 36 \hat{j} \quad C //$$

Como  $F$  é nula quando  $t > 6$  a velocidade é constante, logo  $v(6) = v(9)$

13.  $W = 0$  pois o  $cr$  é desprezível      A

14. A  $2x - x^2$   $\cap$   $\leftarrow$   
B  $-2x + x^2$   $\cup$       A //

15.



$$m_1 = 100 \text{ g}$$

$$m_2 = 280 \text{ g}$$

$$P_1 = 0,1 \times 9,8 = 0,98 \text{ N}$$

$$P_2 = 0,28 \times 9,8 = 2,744 \text{ N}$$

$$3 \times P_1 + 1 \times P_2 - 6 R_E = 0 \Leftrightarrow R_E = 0,947 \text{ N} \quad B //$$