Homework 3!

March 7, 2016

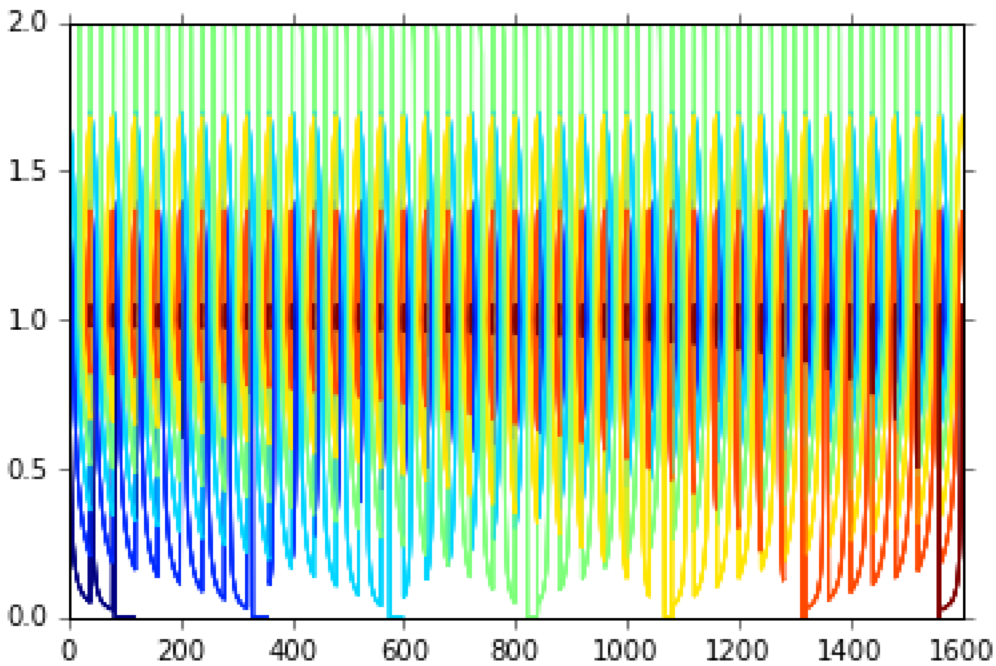
1 Welcome to homework 3 by WIlly G

Let's do some imports.

```
In [1]: %matplotlib inline

        import scipy.io
        from sklearn import svm
        from sklearn.preprocessing import normalize
        import numpy as np
        from sklearn.metrics import confusion_matrix
        import matplotlib.pylab as plt
        from scipy.stats import multivariate_normal


        from skimage.feature import hog
        from skimage import data, color, exposure
```

```
In [26]: delta = 0.25
         x = np.arange(-5.0, 5.0, delta)
         y = np.arange(-5.0, 5.0, delta)
         X, Y = np.meshgrid(x, y)

         prob2mu = [np.array([1,1]),
                    np.array([-1,2]),
                    np.array([0,2]),
                    np.array([0,2]),
                    np.array([0,2])]

         prob2cov = [np.array([[2, 0]
                           ,[0, 1]]),
                    np.array([[3,1 ]
                          ,[1,2 ]]),
                    np.array([[1,1 ]
                          ,[1,2 ]]),
                    np.array([[2,1 ]
                          ,[1,2 ]]),
                    np.array([[1,1 ]
                          ,[1,2 ]])]

         for mu,cov in zip(prob2mu, prob2cov):
             plt.rcParams['xtick.direction'] = 'out'
             plt.rcParams['ytick.direction'] = 'out'
```

```
gaussian = multivariate_normal(np.array(mu), np.array(cov), True)
z = [np.array([p1[idx],p2[idx], gaussian.pdf(np.array([p1[idx],p2[idx]]))]) for idx, p1 in

plt.contour(np.transpose(z))            # line colours
```

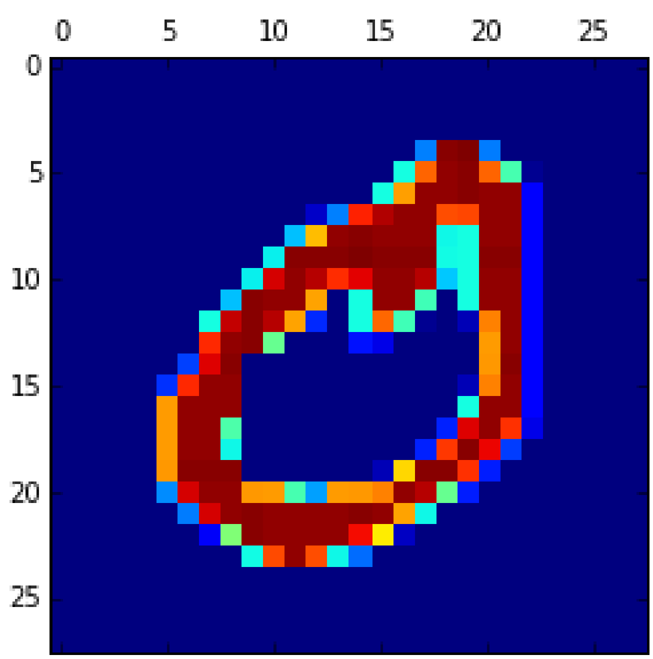


Time to get some awesome data for MNIST and do some dundiddly preprocessing.

```
In [12]: def featurize(image, i ):
             lol = image.reshape(28,28)
             extra1 = hog(lol, orientations=8, pixels_per_cell=(14, 14), cells_per_block=(1, 1), visuali
             extra2 = hog(lol, orientations=8, pixels_per_cell=(7, 7), cells_per_block=(1, 1), visualise
             pixelvector = list(image* 1/255.0*2 - 1)
             if (i-1) % 10000 == 0:
                 print("Featurization progress: %s" % i)
             pixelvector.extend(list(extra1[1].flatten()*1/255.0*4))
             pixelvector.extend(list(extra2[1].flatten()*1/255.0*4))
             return np.array(pixelvector)

         print("Starting!")

         image_train = scipy.io.loadmat("../data/digit_dataset/train.mat")
         image_test = scipy.io.loadmat("../data/digit_dataset/test.mat")['test_images']

         image_train_data_raw =  image_train['train_images']
         image_train_label_raw =  image_train['train_labels']

         print("Data loaded!")
         image_tdata = np.array([featurize(image_train_data_raw[:,:,i].flatten(), i) for i in range(len
         image_tlabel = image_train_label_raw.ravel()
```

```
image_test_data = np.array([featurize(test.reshape(28,28).T.flatten(), i) for i, test in enume

#Shuffle that image data good.
shuffle = np.random.permutation(np.arange(image_tdata.shape[0]))
image_tdata, image_tlabel = image_tdata[shuffle], image_tlabel[shuffle]

#VALIDATION
image_valid_data  =  image_tdata[0:10000]
image_valid_label = image_tlabel[0:10000]

#TRAINING
image_train_data  =image_tdata[10000:]
image_train_label =image_tlabel[10000:]

plt.matshow(image_train_data_raw[:,:,1])
print(image_train_label[0]) #unrelated to the plot!
```

```
Starting!
Data loaded!
Featurization progress: 1
Featurization progress: 10001
Featurization progress: 20001
Featurization progress: 30001
Featurization progress: 40001
Featurization progress: 50001
Featurization progress: 1
6
```



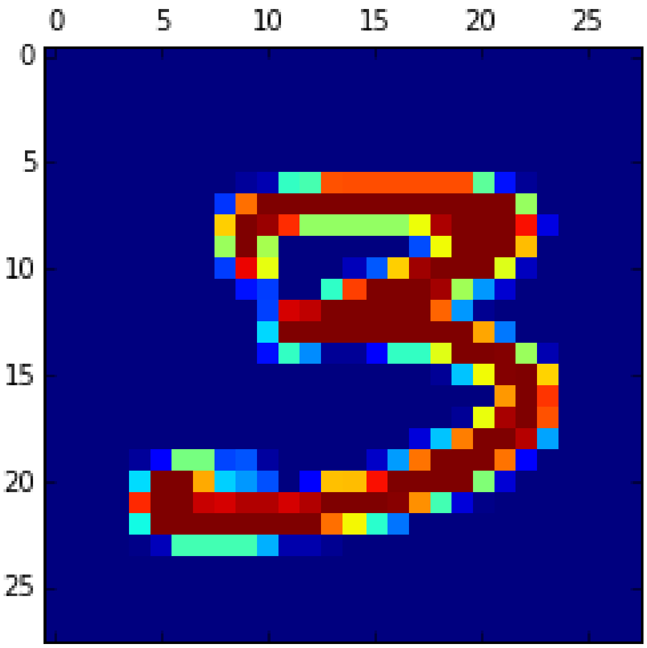
Did you like that. I did. Okay so, let's make a class which model data classes!

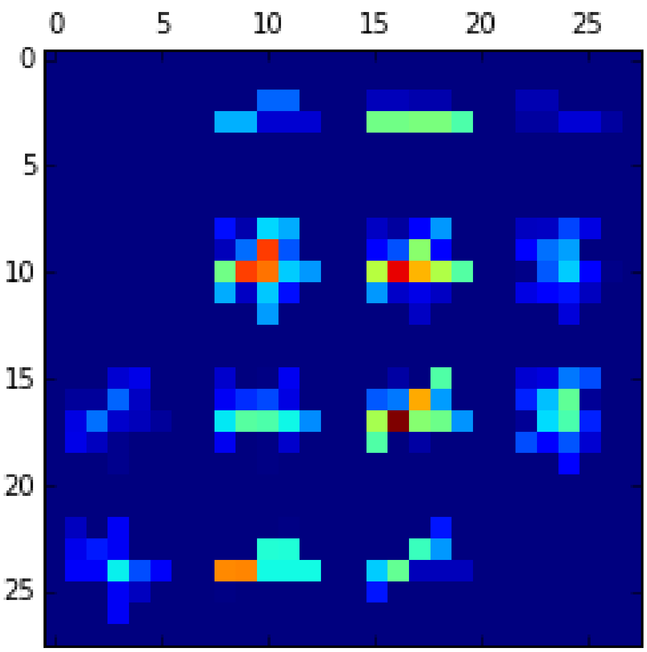
```
In [13]: def perturb_eig(mat, alpha):
             return mat + alpha*np.eye(mat.shape[0])



         class dataclass:
             def __init__(self, datum, labels, class_label, alpha):
                 # Find all of the datapoints in datum with the labels class_lab.
                 # add them to this dataclass
                 ids_classc = [idx for idx,label in enumerate(labels) if label == class_label ]
                 self.data = datum[ids_classc]

                 #calculate some things!
                 singcov = np.cov(self.data.T)
                 self.mean = np.mean(self.data, axis = 0)
                 self.prior = len(ids_classc)/len(labels)

                 #Let's make sure we can invert the covariance matrix using a bincary search.
                 self.cov = singcov

                 self.gaussian = multivariate_normal(self.mean, self.cov, True)
                 self.covall = None

             def lda(self, covall, x):
                 if self.covall is covall:
                     return self.ldagaussian.logpdf(x) + self.prior
                 else:
                     self.covall =covall
                     self.ldagaussian = multivariate_normal(self.mean, covall, True)
                     return self.ldagaussian.logpdf(x) + self.prior

             def qda(self, x):
                 return self.gaussian.pdf(x)*self.prior
```

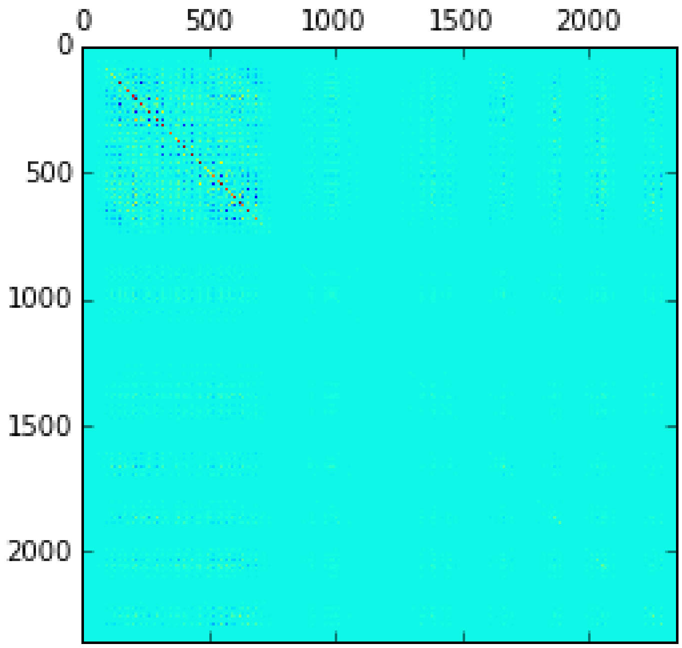
We gotta make sure this class makes sense

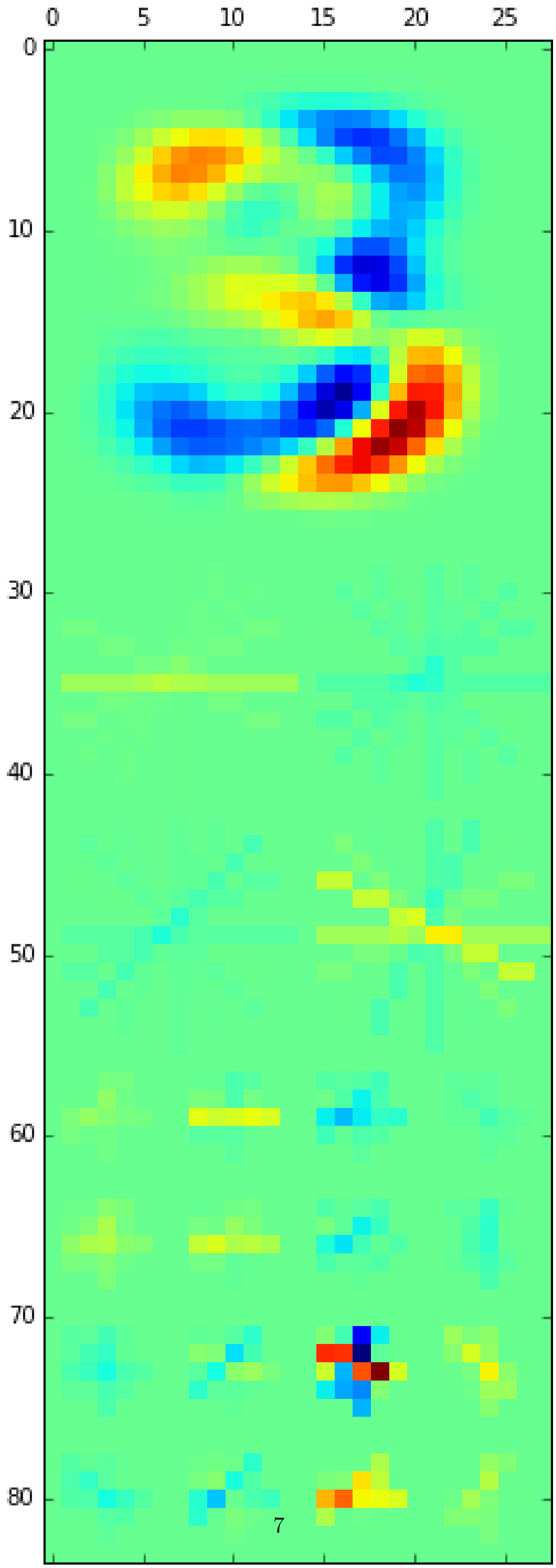
```
In [17]: a = dataclass(image_train_data, image_train_label, 3.0, 0.1)

         plt.matshow(a.data[0][0:28*28].reshape(28,28))
         plt.matshow(a.data[0][28*28*2:].reshape(28,28))
         plt.matshow(a.cov)

         plt.matshow(a.cov[1333,:].reshape(56+28,28))
```

```
Out[17]: <matplotlib.image.AxesImage at 0x7f72d5d8a4a8>
```


0 5 10 15 20 25
0 5 10 15 20 25


0 5 10 15 20 25
0 5 10 15 20 25


0
500
1000
1500
2000
0
500
1000
1500
2000

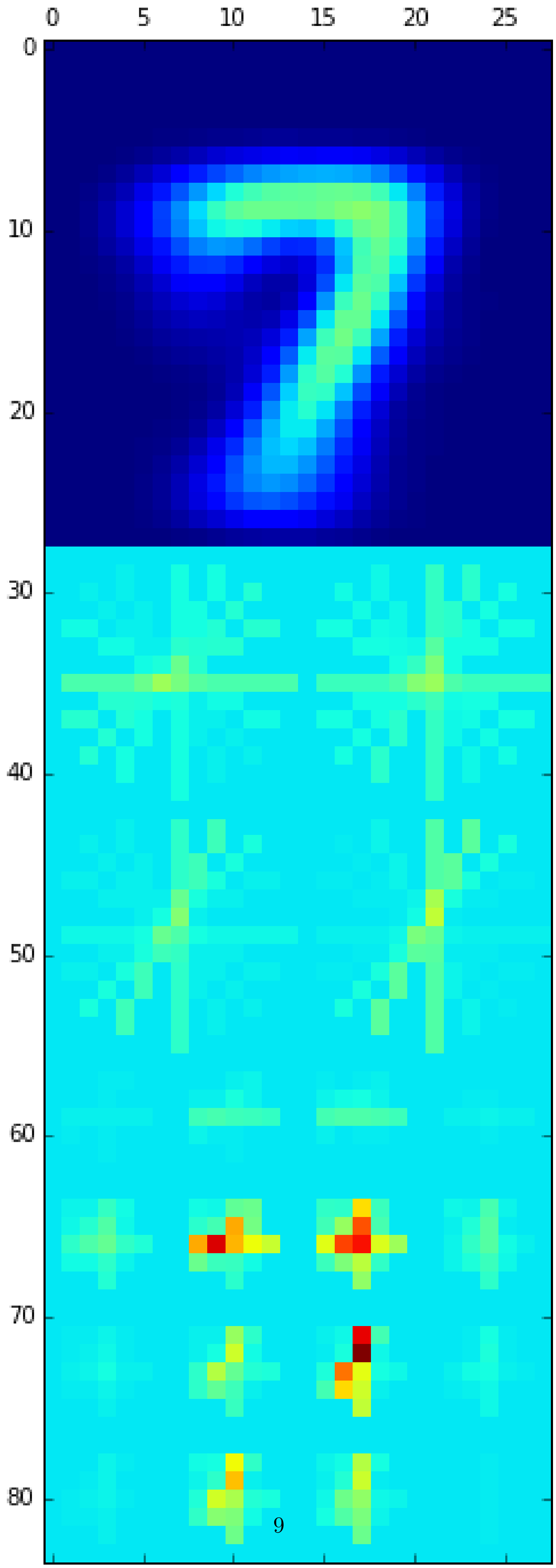

0
5
10
15
20
25
0
10
20
30
40
50
60
70
80

Let's actually make all of the image classes!

```
In [18]: image_train_class = [dataclass(image_train_data, image_train_label, float(x), 0.1) for x in ran
```

```
In [19]: plt.matshow(image_train_class[7].mean.reshape((56+28,28)))
```

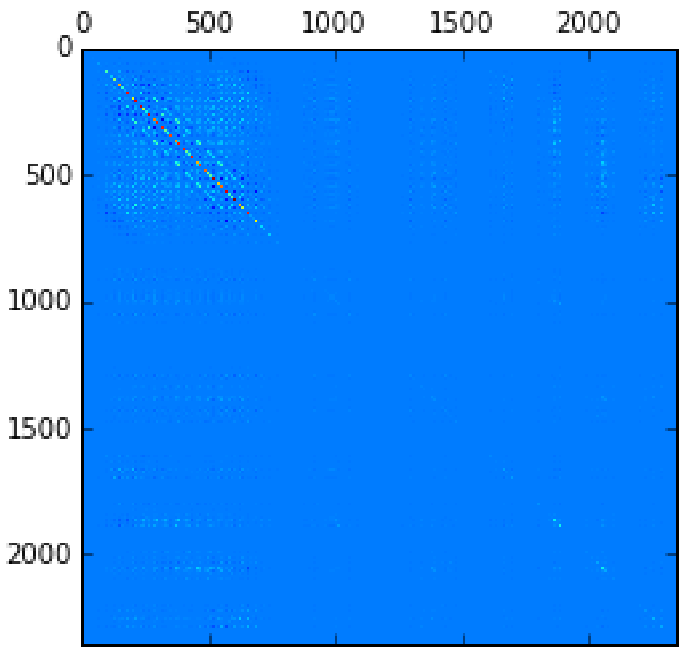
```
Out[19]: <matplotlib.image.AxesImage at 0x7f72d88f9048>
```

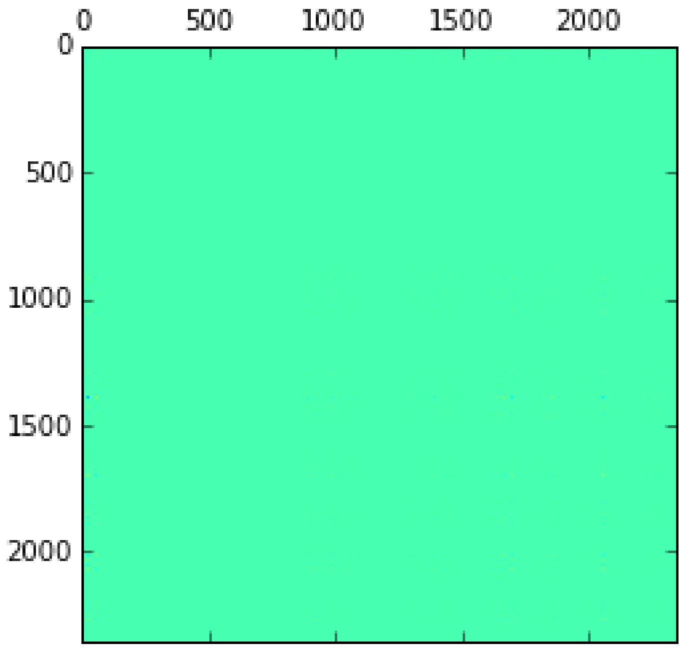

0
5
10
15
20
25
0
10
20
30
40
50
60
70
80

Well that worked thank god! Now we need to add all of those peski general properties

```
In [20]: covall = 0.1*np.sum([image_train_class[i].cov for i in range(len(image_train_class))], axis=0)
         precall = np.linalg.pinv(covall)
         plt.matshow(covall)
         plt.matshow(precall)
```

```
Out[20]: <matplotlib.image.AxesImage at 0x7f72d8c2f5f8>
```





2 Question 5 part(d)

Now we make some helper methods.

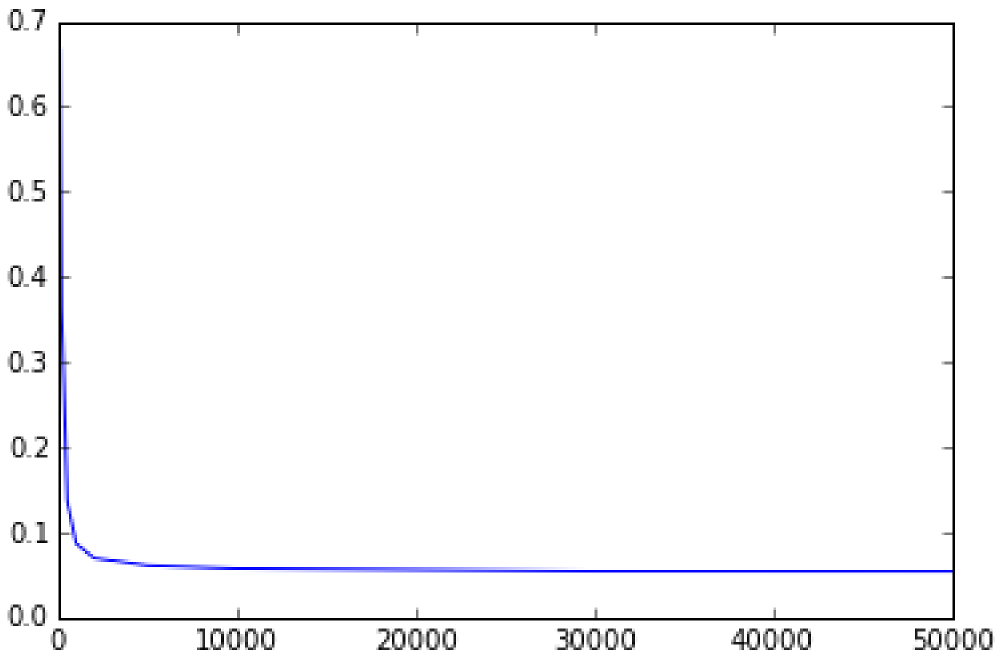
```
In [21]: def best_prediction(classes,method,  x):
             return np.argmax(list(map(lambda cls: method(cls, x), classes)))

         def validate(classes, method,  x, y):
             if int(best_prediction(classes, method, x)) == int(y):
                 return 1.0
             else:
                 return 0

         def lda(cls, x):
             return cls.lda(covall, x)

         def qda(cls, x):
             return cls.qda(x)

         def experiment(method, num_class, train_data, train_labels, validation, labels, samples=1000000
             classes = [dataclass(train_data[0:samples], train_labels[0:samples], float(x), 0.1) for x i
             print("classes created for experiment!")
             net = 0
             for idx, sample in enumerate(validation):
                 score = validate(classes, method, sample, labels[idx])
                 net += score

             return 1.0 - net/len(validation)
```

```
def do_test(method, num_class, train_data, train_labels, test_data):
    classes = [dataclass(train_data, train_labels, float(x), 0.1) for x in range(num_class)]
    print("classes classes created for test!")
    pred = [int(best_prediction(classes, method, test)) for test in test_data]
    pairs = list(enumerate(pred))
    for idx, pair in enumerate(pairs):
        pairs[idx] =(pair[0]+1, pair[1])
    return pairs

experiment_count = [100,200, 500, 1000, 2000, 5000, 10000, 30000, 50000]
```

Now we can actually run some experiments

```
In [22]: #LDA
         print("LDA")
         errors = []
         for count in experiment_count:
             print("Running at %s," %  count)
             error = experiment(lda, 10, image_train_data,image_train_label , image_valid_data, image_va
             print("\tError: %s" % error)
             errors.append(error)

         plt.plot(experiment_count, errors)
         plt.show()
```

```
LDA
Running at 100,
classes created for experiment!
        Error: 0.6674
Running at 200,
classes created for experiment!
        Error: 0.36329999999999996
Running at 500,
classes created for experiment!
        Error: 0.13729999999999998
Running at 1000,
classes created for experiment!
        Error: 0.08689999999999998
Running at 2000,
classes created for experiment!
        Error: 0.06969999999999998
Running at 5000,
classes created for experiment!
        Error: 0.061000000000000054
Running at 10000,
classes created for experiment!
        Error: 0.05740000000000001
Running at 30000,
classes created for experiment!
        Error: 0.054200000000000026
Running at 50000,
classes created for experiment!
        Error: 0.05410000000000004
```

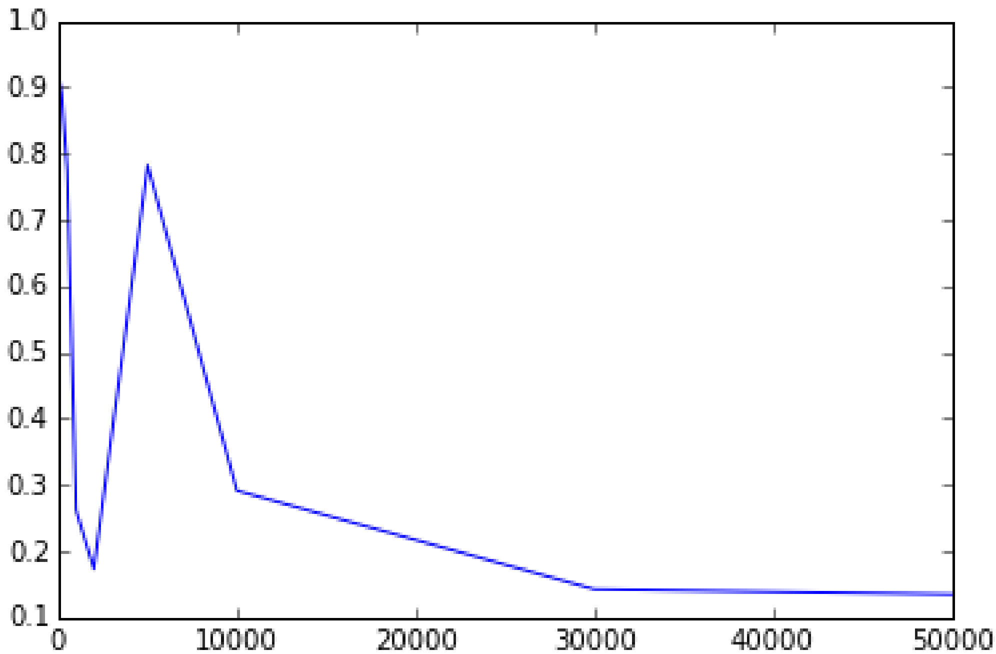



```
In [23]: #QDA
         print("QDA")
         errors = []
         for count in experiment_count:
             print("Running at %s," %  count)
             error = experiment(qda, 10, image_train_data, image_train_label, image_valid_data, image_va
             print("\tError: %s" % error)
             errors.append(error)

         plt.plot(experiment_count, errors)
         plt.show()
```

```
QDA
Running at 100,
classes created for experiment!
        Error: 0.9071
Running at 200,
classes created for experiment!
        Error: 0.8905
Running at 500,
classes created for experiment!
        Error: 0.7746
Running at 1000,
classes created for experiment!
        Error: 0.2619
Running at 2000,
classes created for experiment!
        Error: 0.17300000000000004
Running at 5000,
```

```
classes created for experiment!
        Error: 0.7823
Running at 10000,
classes created for experiment!
        Error: 0.2914
Running at 30000,
classes created for experiment!
        Error: 0.14239999999999997
Running at 50000,
classes created for experiment!
        Error: 0.13470000000000004
```

```
/home/william/anaconda3/lib/python3.4/site-packages/scipy/stats/_multivariate.py:603: RuntimeWarning: ov
  return np.exp(self.logpdf(x))
```




```
In [27]: thedata = list(do_test(lda, 10, image_tdata, image_tlabel, image_test_data))
         np.savetxt(
             'kaggleimage.csv',           # file name
             thedata,                  # array to save
             fmt='%i',               # formatting, 2 digits in this case
             delimiter=',',            # column delimiter
             newline='\n')
```

```
classes classes created for test!
```

Now let's do the spam thing and work it all out!

```
In [25]: spam_data = scipy.io.loadmat("../data/spam_dataset/spam_data.mat")
         spam_train = spam_data['training_data']
         spam_test = spam_data['test_data']
```

```
spam_train_data_raw =  spam_train
spam_train_label_raw =  spam_data['training_labels']

spam_tlabel = spam_train_label_raw.ravel()
div_train = [1.0/max(arr) for arr in spam_train_data_raw.T]
spam_tdata = np.array([np.multiply(dp, div_train) for dp in spam_train_data_raw ])
spam_test_data = np.array([np.multiply(dp, div_train) for dp in spam_test ])

#Shuffle that spam data good.
shuffle = np.random.permutation(np.arange(spam_tdata.shape[0]))
spam_tdata, spam_tlabel = spam_tdata[shuffle], spam_tlabel[shuffle]

#VALIDATION
spam_valid_data  =  spam_tdata[0:750]
spam_valid_label = spam_tlabel[0:750]

#TRAINING
spam_train_data  =spam_tdata[750:]
spam_train_label =spam_tlabel[750:]
```

Looks good! Let's train on everything and build a Kaggle.

```
In [26]: thedata = list(do_test(qda, 2, spam_tdata, spam_tlabel, spam_test_data))
         np.savetxt(
             'kagglespam.csv',           # file name
             thedata,                # array to save
             fmt='%i',             # formatting, 2 digits in this case
             delimiter=',',          # column delimiter
             newline='\n')
```

```
classes classes created for test!
```

3 Linear Regressive

Let's do it! We know that $\langle w, x \rangle = \hat{y}$. Therefore, we can consider the following line of reasoning.

Let X be the design matrix containing all of the data points. Since for each data point there is weight whose dot product yeilds an entry of the final output y, we let w be some weight vector, along which we will multiply each data point. This gives

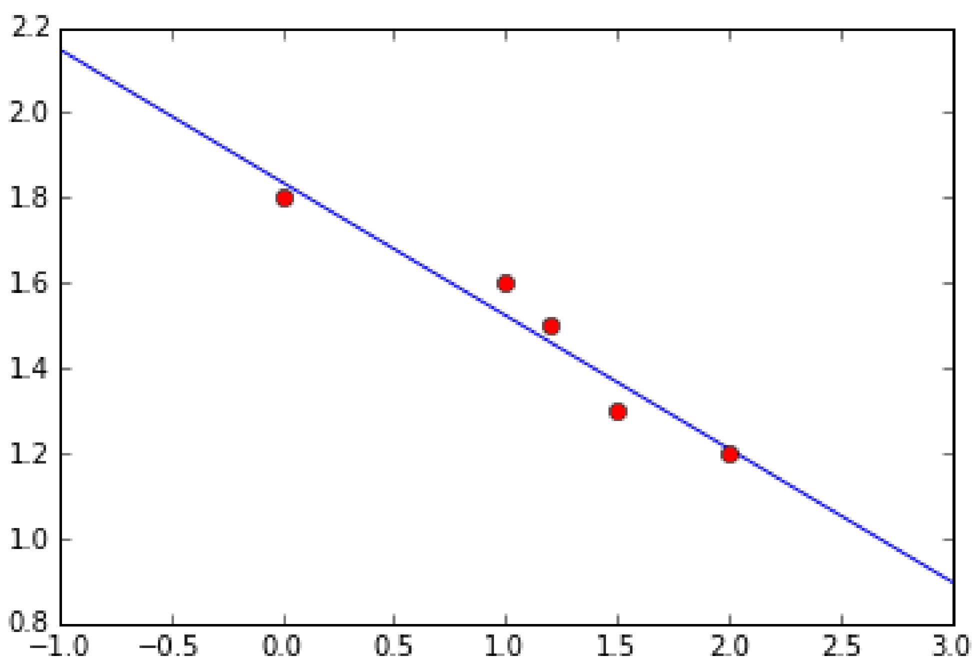
$$Xw = y.$$

Performing a pseudo inverse will give us the least squares solution $w = (X^T X)^{-1} X^T y$.

```
In [142]: def linreg(data, y, alpha=0):
              #Add one to all the data points.
              extended = np.array([np.append(sample, [1]) for sample in data])
              print(extended)
              w = np.dot(np.dot(np.linalg.inv(np.dot(extended.T, extended)), extended.T), y)
              return w

          def linrss(data, y, w):
              extended = np.array([np.append(sample, [1]) for sample in data])
              return np.linalg.norm(np.dot(extended,w) - y,2)**2

          def plot_weights(w):
              plt.bar(range(len(w)-1),w[:-1])
```

```
In [143]: test = np.array([[0],[1],[1.2],[1.5],[2]])
          test_y = np.array([1.8,1.6,1.5,1.3,1.2])

          w = linreg(test,test_y, 0)
          #plot_weights(w)

          X = np.array([[-1,1],
                       [-0.5,1],
                       [0,1],
                       [0.5,1],
                       [1,1],
                        [1.5,1],
                        [2,1],
                        [2.5,1],
                        [3,1]])
          x = np.array([-1,-0.5,0,0.5,1,1.5,2,2.5,3])

          plt.plot(x, np.dot(X,w))
          plt.plot(test, test_y, 'ro')

          print(linrss(test, test_y, w))
```

```
[[ 0.   1. ]
 [ 1.   1. ]
 [ 1.2  1. ]
 [ 1.5  1. ]
 [ 2.   1. ]]
0.0133120437956
```

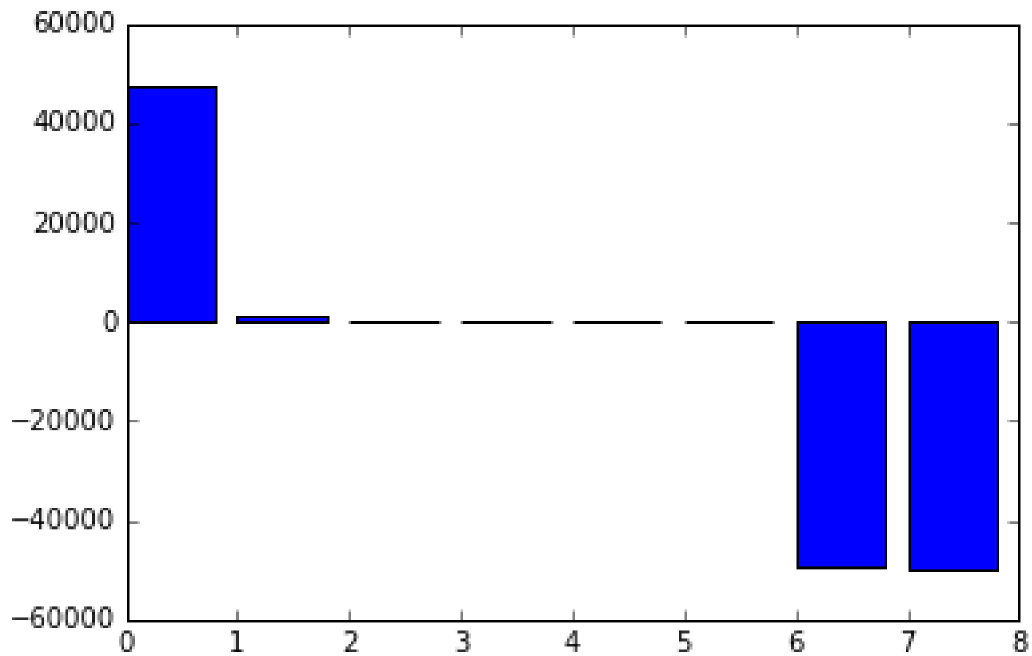



```
In [149]: real_data =  scipy.io.loadmat("../data/housing_dataset/housing_data.mat")
          real_Xtrain = (real_data['Xtrain'])/ np.linalg.norm((real_data['Xtrain']),2)
```

```
real_Ytrain = real_data['Ytrain'].ravel()/np.linalg.norm(real_data['Ytrain'].ravel(),2)
real_Xvalidate = real_data['Xvalidate'] /np.linalg.norm(((real_data['Xvalidate'])),2)
real_Yvalidate = real_data['Yvalidate'].ravel()/np.linalg.norm(real_data['Yvalidate'].ravel()
```

In [151]:
```
w = linreg(real_Xtrain, real_Ytrain)

plot_weights(w)
print(linrss(real_Xvalidate, real_Yvalidate, w))
```

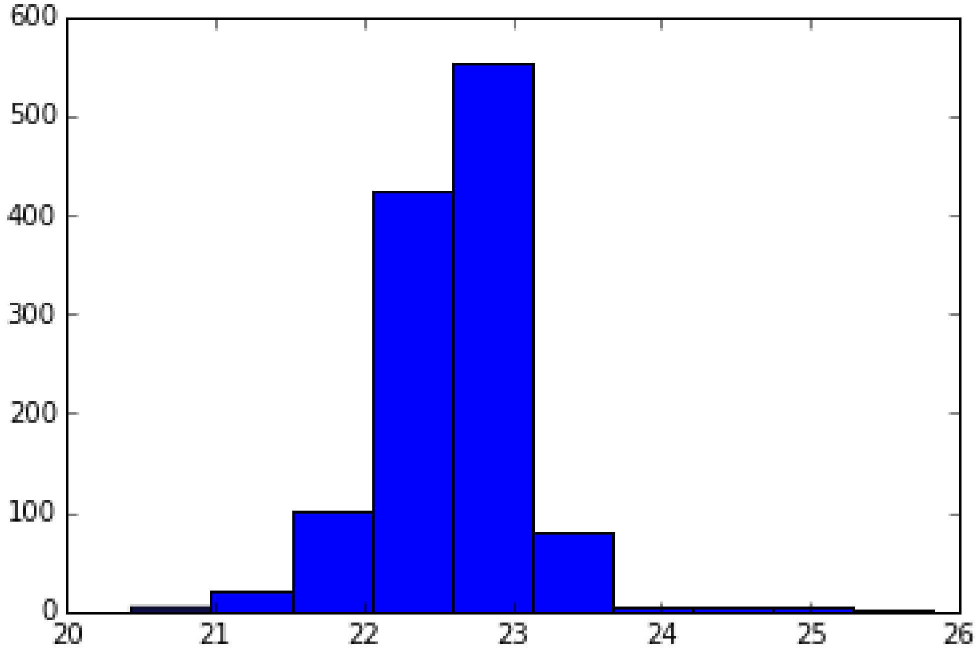
```
[[  5.32672469e-06   5.82932150e-05   1.97286100e-03 ...,   6.92960594e-05
   -2.20621602e-04   1.00000000e+00]
 [  8.92487339e-06   6.19365410e-05   6.02241778e-03 ...,   6.92049762e-05
   -2.23244797e-04   1.00000000e+00]
 [  9.20449865e-06   4.73632372e-05   4.10967166e-03 ...,   6.94600090e-05
   -2.20967718e-04   1.00000000e+00]
 ...,
 [  2.73249445e-06   7.10448558e-05   1.84898791e-03 ...,   6.79480288e-05
   -2.19473955e-04   1.00000000e+00]
 [  1.69068540e-05   5.64715521e-05   4.94945829e-03 ...,   6.80573285e-05
   -2.22388615e-04   1.00000000e+00]
 [  5.94208244e-06   9.47264744e-05   6.59077662e-03 ...,   6.89499434e-05
   -2.22716515e-04   1.00000000e+00]]
613157.990429
```



In [154]:
```
extended = np.array([np.append(sample, [1]) for sample in real_Xvalidate])
plt.hist(np.dot(extended, w) - real_Yvalidate)
```

Out[154]:
```
(array([   4.,   21.,  102.,  423.,  553.,   81.,    6.,    4.,    5.,    1.]),
 array([ 20.43552208,  20.97386487,  21.51220766,  22.05055045,
         22.58889324,  23.12723604,  23.66557883,  24.20392162,
```

```
       24.74226441,  25.28060721,  25.81895   ]),
<a list of 10 Patch objects>)
```




```
In [ ]:
```