

FRE7251 Algo Trading & High-frequency Finance HW4

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1. Explain the notion of risk aversion. When relative risk aversion may be more preferable than the absolute one?

Risk aversion denotes an individual's reluctance to choose a bargain with uncertain payoff over a bargain with certain but possibly lower payoff. While it is relatively easy to use CARA in theoretical analysis, absolute aversion may not always be an accurate assumption. Indeed, the loss may be perceived quite differently by a millionaire and a college student. In this case, relative risk aversion function (CRRA) may be more preferable than the absolute one.

2. Explain the differences between risk-neutral and risk-averse inventory market microstructure models.
 1. Risk-neutral models (Garman (1976), Amihud & Mendelson (1980)) are based on the Walrasian framework according to which lower (higher) price drives (depresses) demand. These models demonstrate that rational dealers attempting to maximize their profits must establish certain bid / ask spread and manipulate its size for maintaining preferred inventory.
 2. Inventory models with risk aversion are based on optimizing the utility function. Inventory models are used to address the market maker's risk of maintaining inventory as well as show how the size of the bid / ask spread can compensate this risk. Risk aversion is a concept that refers to individual's reluctance to choose a bargain with uncertain payoff rather than a bargain with certain but possibly a lower payoff. In mathematical terms, risk aversion is usually described with some utility function that is non-linear upon wealth. Inventory models with risk aversion are based on optimizing this utility function.
 3. Different inventory models differ in whether they account for risk aversion explicitly. The Stoll's model considers a single dealer who optimizes the CARA utility function and yields the bid / ask spread that depends linearly on the dealer's risk aversion and the asset volatility. In the multi-period model, the optimal spread narrows since the risk for maintaining inventory diminishes as trading comes to the end. The spread also decreases with growing number of competing dealers as the risk is spread among them.
3. Describe the risks that market dealers face. How to manage these risks?
 1. Dealers must maintain inventory on both sides of the market. There is an **inventory risk**: buy order flows and sell order flows are not balanced. Bid / ask spread compensates inventory risk.
 2. Inventory models address the dealer's problem of maintaining inventory on both sides of the market. Since order flows are not synchronized, dealers face possibility of running out of cash (bankruptcy) or out of inventory (failure).
 3. In the multi-period model, the optimal spread narrows since the risk for maintaining inventory diminishes as trading comes to the end. The spread also decreases with growing number of competing dealers as the risk is spread among them.
 4. Adverse selection: Informed traders and market makers make conjectures on rational behavior of their counterparts, and they do behave rationally in the sense that all their actions are focused on maximizing their wealth. Market in these models reaches an equilibrium state that satisfies

participants' expectations. Obviously, informed investors trade only on one side of the market at any given time. Hence market makers face adverse selection problem.

5. In the batch auctions, the Kyle's (1985) model, dealers compensate potential losses from adverse selection by setting asset price as increasing with demand. In the sequential trade models, Glosten & Milgrom (1985), dealers compensate potential losses from adverse selection by setting the bid/ask spread that grows with the number of informed traders and with order size.

4. Explain Walrasian equilibrium and why information market microstructure models contradict this concept.

1. In Walrasian equilibrium, price is determined by equilibrium between supply and demand. Meanwhile, lower prices drive demand and suppress supply while higher prices decrease demand and increase supply in it.
2. Price is a source of information that investors can use for their trading decisions. For example, if security price falls, investors may suggest that price will further deteriorate and refrain from buying this security. Hence, it contradicts the Walrasian framework.

5. Consider the following situation: you intend to sell your stocks today. You observe the market and notice that the order size on the bid side of the market is growing. What order would you submit?

According to Parlour model, seller is more likely to submit a market order in case of a thick LOB on the ask. On the other hand, a thick LOB on the bid side motivates a seller to submit a limit order. Hence, I would **submit a limit order** considering that the order size on the bid side of the market is growing. Also, market orders are more likely submitted after market orders on the same side of the market. Indeed, one may expect that a buy order diminishes liquidity on the ask side, which motivates a seller to submit a limit ask order. The order size on the bid side of the market is growing, which will diminish liquidity on the ask side.

6. Your bid order is on top of the book @12.55. There is an order @12.54 and several orders @12.52 behind you. Suddenly you notice that the order @12.54 gets cancelled. What are your actions (if any)?

According to the Foucault model, Winner's curse that the highest bid and lowest offer "picked off" by market orders may be mispriced occurs when a limit order filled due to price volatility turns out to be mispriced. Moreover, buyer (seller) is more likely to submit market order in case of thick LOB on the bid (ask) side. Buyer (seller) is more likely to submit limit order in case of thick LOB on the ask (bid) side. Therefore, I will consider actions both bid and ask sides. To be more specific,

1. From the perspective of bid side, if there are new orders with higher prices than 12.55, I may keep my order or bid higher. The thick bid LOB may lead me to cancel the limit order and submit a market order. On the contrary, if there are new orders with lower prices than 12.55 and no new orders, I may keep my order or bid lower.
2. From the perspective of offer side, if the Limit Order Book (LOB) is thick, I will modify to a lower price. If it's thin, I will keep my order, or cancel order and change to a market order.