Problem 1

When $x_1 > x_2 > x_3 > x_4$,

$$\Rightarrow \left(W^{(1)} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + b^{(1)} \right)^T W^{(2)} + b^{(2)} \geq 0$$

else < 0

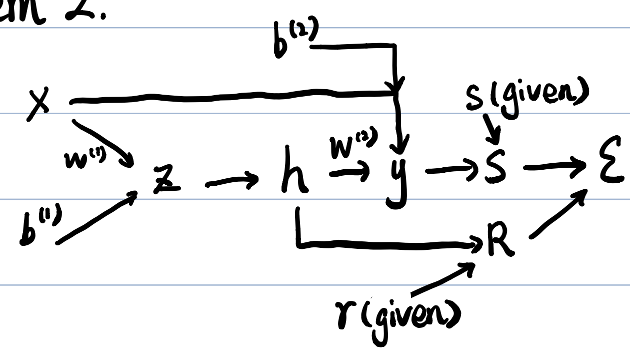
$\therefore$

$$W^{(1)} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad b^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad W^{(2)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad b^{(2)} = -3$$

can correctly implement this function.

Problem 2.

(1)



(2) $\bar{\mathcal{E}} = 1$

$$\bar{R} = \bar{\mathcal{E}} \frac{d\mathcal{E}}{dR} = \bar{\mathcal{E}} = 1$$

$$\bar{S} = \bar{\mathcal{E}} \frac{d\mathcal{E}}{dS} = \bar{\mathcal{E}} = 1$$

$$\bar{y} = \bar{S} \frac{dS}{dy} = \bar{S}(y-s) = y-s$$

$$\bar{h} = \bar{y} \cdot \frac{dy}{dh} + \bar{R} \cdot \frac{dR}{dh} = (y-s) \cdot W^{(2)} + r$$

$$\bar{z} = \bar{h} \cdot \frac{dh}{dz} = \left((y-s) W^{(2)} + r \right) \sigma'(z)$$

$$\bar{x} = \bar{z} \cdot \frac{dz}{dx} + \bar{y} \cdot \frac{dy}{dx} = \left((y-s) W^{(2)} + r \right) \sigma'(z) \cdot W^{(1)} + (y-s)$$

(3) $$W^{(2)} = \bar{y} \cdot \frac{dy}{dW^{(2)}} = (y-s) \cdot h$$

$$b^{(2)} = \bar{y} \cdot \frac{dy}{db^{(2)}} = (y-s)$$

$$W^{(1)} = \bar{z} \cdot \frac{dz}{dW^{(1)}} = \left((y-s) W^{(2)} + r \right) \sigma'(z)\, x$$

$$b^{(1)} = \bar{z} \cdot \frac{dz}{db^{(1)}} = \left((y-s) W^{(2)} + r \right) \sigma'(z)$$

Problem 3.

$$\bar{\mathcal{L}} = 1$$

$$\bar{y} = \bar{\mathcal{L}} \cdot \frac{\partial \mathcal{L}}{\partial y} = (\log y - \log t) \cdot \frac{1}{y}$$

$$\bar{z} = \bar{y} \cdot \frac{\partial y}{\partial z} = \bar{y}\, e^{z}$$

$$\bar{w_2} = \bar{z} \cdot \frac{\partial z}{\partial w_2} = \bar{z} x^2 = \bar{\mathcal{L}}(\log y - \log t) \frac{1}{y} e^{z} x^2$$

$$\frac{\partial L}{\partial W_2} = (\log y - \log t) \frac{1}{y} e^{z} x^2$$

Problem 4

(1)

$$\overline{o^{(t)}} = \overline{h^{(t)}} \tanh(c^{(t)})$$

$$\overline{c^{(t)}} = \overline{h^{(t)}}\, o^{(t)} [1 - \tanh^2(c^{(t)})] + \overline{c^{(t+1)}} f^{(t+1)}$$

$$\overline{g^{(t)}} = \overline{c^{(t)}}\, i^{(t)}$$

$$\overline{f^{(t)}} = \overline{c^{(t)}}\, c^{(t-1)}$$

$$\overline{i^{(t)}} = \overline{c^{(t)}}\, g^{(t)}$$

$$\overline{h^{(t-1)}} = \overline{i^{(t)}} \sigma'(W_{ix} X^{(t)} + W_{ih} h^{(t-1)}) W_{ih} + \overline{f^{(t)}} \sigma'(W_{fx} X^{(t)} + W_{fh} h^{(t-1)}) W_{fh}$$
$$+ \overline{o^{(t)}} \sigma'(W_{ox} X^{(t)} + W_{oh} h^{(t-1)}) W_{oh} + \overline{g^{(t)}} (1 - \tanh^2(W_{fg} X^{(t)} + W_{gh} h^{(t-1)})) W_{gh}$$

$$\therefore \overline{h^{(t)}} = \overline{i^{(t+1)}} \sigma'(W_{ix} X^{(t+1)} + W_{ih} h^{(t)}) W_{ih}$$
$$+ \overline{f^{(t+1)}} \sigma'(W_{fx} X^{(t+1)} + W_{fh} h^{(t)}) W_{fh}$$
$$+ \overline{o^{(t+1)}} \sigma'(W_{ox} X^{(t+1)} + W_{oh} h^{(t)}) W_{oh}$$
$$+ \overline{g^{(t+1)}} (1 - \tanh^2(W_{fg} X^{(t+1)} + W_{gh} h^{(t)})) \cdot W_{gh}$$

(2) $$\overline{W_{ix}} = \sum_t \overline{i^{(t)}} \sigma'(W_{ix} X^{(t)} + W_{ih} h^{(t-1)}) x^{(t)}$$

(3) $$\overline{X^{(t)}} = \overline{i^{(t)}} \frac{\partial i^{(t)}}{\partial x^{(t)}} + \overline{f^{(t)}} \frac{\partial f^{(t)}}{\partial x^{(t)}} + \overline{g^{(t)}} \frac{\partial g^{(t)}}{\partial x^{(t)}} + \overline{o^{(t)}} \frac{\partial O^{(t)}}{\partial x^{(t)}}$$

$$= \overline{i^{(t)}}\sigma'(W_{ix}x^{(t)}+W_{ih}h^{(t-1)})W_{ix}+\overline{g^{(t)}}\sigma'(W_{fx}x^{(t)}+W_{fh}h^{(t-1)})W_{fx}$$
$$+\overline{g^{(t)}}(1-\tanh^2(W_{gx}x^{(t)}+W_{gh}h^{(t-1)}))W_{gx}$$
$$+\overline{o^{(t)}}\sigma'(W_{ox}x^{(t)}+W_{oh}h^{(t-1)})W_{ox}$$

when $i^{(t)}\to 0$, $o^{(t)}\to 0$, $f^{(t)}\to 1$

by changing value of o.f.i. one can control the amount of information the network retains or discards over the entire input series as well as control depending on individual inputs.

$$c(t)\approx c(t-1),\ h(t)=o^{(t)}\tanh(c^{(t)})\approx o^{(t)}\tanh(c^{(t-1)})$$

Both $c^{(t)}$ and $h^{(t)}$ stay almost the same as before

Hence, the gradient won't explode.

Problem 5.

suppose $\mathcal{L}=\frac{1}{2}(y-t)^2$

$\overline{\mathcal{L}}=1$

$\bar{y}=\bar{\mathcal{L}}\cdot(y-t)=y-t$

$\overline{w_1}=\bar{y}\cdot h_1=h_1(y-t)=0$ Yes

$\overline{w_2}=\overline{h_1}\cdot\frac{\partial h_1}{\partial z_1}\cdot\frac{\partial z}{\partial w_2}$ where $\frac{\partial h_1}{\partial z_1}=0$ since $z_1=-1$ ($\mathrm{ReLU}'(-1)=0$)

$=(y-t)\cdot w_1\cdot 0\cdot h_3=0$ Yes

$\overline{W_3}=\overline{h_3}\times\frac{\partial h_3}{\partial W_3}=(\overline{h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial h_3}+\overline{h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial h_3})x_1$

$=(\overline{h_1}\cdot 0\cdot W_2+\overline{h_2}\cdot\frac{\partial h_2}{\partial z_2}\cdot\frac{\partial z_3}{\partial h_3})x_1$

$=\overline{h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_3}{\partial h_3}x_1$ No