OPTIMIZATION METHODS AND ALGORITHMS

PROBLEM FORMALIZATION

	NOTATION:	Domain:	
1	s	$\epsilon\{1, \|S\|\}$	Student index
2	e	$\epsilon\{1, \|E\|\}$	Exam index
3	t	$\epsilon\{1, t_{max}\}$	Timeslot index
4	t_1	$\epsilon\{1, t_{max}\}$	Auxiliary timeslot index
5	i	$\epsilon\{1, t_{max}-1\}$	Difference between 2 timeslots indices

DATA:	
$\|S\| \epsilon$ N	Total number of students enrolled in at least 1 exam
$\|E\| \epsilon$ N	Total number of exams
$a_{s,e}\ \epsilon\{0,1\}$	1 if student s is enrolled in exam e, 0 o/w
$t_{max}\ \epsilon$ N	Number of available timeslots

	VARIABLES:		
1	$x_{e,t}\ \epsilon\{0,1\}$	$\forall\, e, t$	1 if exam e is scheduled on timeslot t, 0 o/w
2	$z_{s,t}\ \epsilon$ N	$\forall\, s, t$	Number of exams that student s has in a certain timeslot t. **NB: due to the constraint #3, the real domain of $z_{s,t}$ is {0,1}**
3	$u_{s,t,t_1}\ \epsilon\ \{0,1\}$	$\forall\, s, t, t_1$	1 if student s is occupied in both timeslots t and t_1, 0 o/w

	CONSTRAINTS:		
1	$\sum_{t=1}^{t_{max}} x_{e,t} = 1$	$\forall\, e$	Each exam is scheduled in one and only one timeslot.
2	$z_{s,t} = \sum_{e=1}^{\|E\|} a_{s,e} x_{e,t}$	$\forall\, s, t$	$z_{s,t}$ is the number of exams that student s has in a certain timeslot t.
3	$z_{s,t} \leq 1$	$\forall\, s, t$	Student s cannot be enrolled in more exams which are scheduled in the same timeslot, hence the sum of all exams in which student s is enrolled and which take place in timeslot t is 1 or 0.
4	$u_{s,t,t_1} \geq z_{s,t} + z_{s,t_1} - 1$	$\forall\, s, t, t_1$	$u_{s,t,t1}$ represents a conflict: it is 1 if student s is occupied in both timeslots t and t_1, 0 o/w.

COST FUNCTION:	
$c(i,t) = \begin{cases} 0, & i > 5 \\ 2^{5-i} * \frac{\sum_{s=1}^{\|S\|} u_{s,t,t+i}}{\|S\|}, & i \leq 5 \end{cases}$	For a given distance i and a given timeslot t, we have a penalty that depends on the number of people occupied in both timeslots t and $t+i$. The penalty is 0 if i>5.

OBJECTIVE FUNCTION:

$$\sum_{i=1}^{5} \sum_{t=1}^{t_{max}-i} c(i,t)$$

For each penalizing distance $i \in \{1,5\}$ between timeslots we sum the penalty generated by each pair of timeslots which distance is i.

$$= \sum_{i=1}^{5} \sum_{t=1}^{t_{max}-i} 2^{5-i} * \frac{\sum_{s=1}^{|S|} u_{s,t,t+i}}{|S|}$$

$$= \sum_{i=1}^{5} \sum_{t=1}^{t_{max}-i} \sum_{s=1}^{|S|} 2^{5-i} * \frac{u_{s,t,t+i}}{|S|}$$

i.e. for each considered distance, we multiply the respective penalty $\frac{2^{5-i}}{|S|}$ by the number of students which are occupied both in timeslot t and $t+i$*. This number of students is calculated by summing over all students the boolean variable $u_{s,t,t+i}$ which is 1 if student s is occupied in timeslots t and $t+i$ and 0 o/w.

* We only consider the **subsequent** timeslots to avoid counting penalties twice.