OPTIMIZATION METHODS AND ALGORITHMS

PROBLEM FORMALIZATION

	NOTATION:	Domain:	
1	q	$\epsilon\{1, \|S\|\}$	Student index
2	j	$\epsilon\{1, \|E\|\}$	Exam index
3	h	$\epsilon\{1, t_{max}\}$	Timeslot index
4	k	$\epsilon\{1, t_{max}\}$	Auxiliary timeslot index
5	i	$\epsilon\{1, t_{max}\}$	Difference between timeslots index

	DATA:	
	$\|S\|$	Total number of student enrolled in at least 1 exam
	$\|E\|$	Total number of exams
	$a_{q,j}$	1 if student q is enrolled in exam j, 0 o/w
	t_{max}	Number of available timeslots

	VARIABLES:		
1	$x_{j,h}\epsilon\{0,1\}$	$\forall j,h$	1 if exam j is scheduled on timeslot h, 0 o/w
2	$z_{q,h}\epsilon\{0,1\}$	$\forall q,h$	1 if student q is occupied during timeslot h, 0 o/w
3	$u_{q,h,k}\epsilon\{0,1\}$	$\forall q,h,k$	1 if student q is occupied in both timeslots h and k, 0 o/w

	CONSTRAINTS:		
1	$\sum_{h=1}^{t_{max}} x_{j,h} = 1$	$\forall j$	Each exam will have one and only one time slot
2	$\sum_{j=1}^{\|E\|} a_{q,j}x_{j,k} \leq 1$	$\forall q,k$	Student q cannot be enrolled in more exams which are in the same timeslot, hence the sum of all exams in which student q is enrolled and which takes place in timeslot k is 1 or 0.
3	$z_{q,k} = \sum_{j=1}^{\|E\|} a_{q,j}x_{j,k}$	$\forall q,k$	$z_{q,k}$ is 1 if student q is occupied in timeslot k, 0 o/w.
4	$u_{q,h,k} \geq z_{q,h} + z_{q,k} - 1$	$\forall q,h,k$	$u_{q,h,k}$ is 1 if student q is occupied in both timeslots h and k, 0 o/w.

COST FUNCTION:

$c(i,h)$

$$c(i,h) = \begin{cases} 0, & i > 5 \\ 2^{5-i} * \frac{\sum_{q=1}^{\|S\|} u_{q,h,h+i}}{\|S\|}, & i \leq 5 \end{cases}$$

OBJECTIVE FUNCTION:

$$\sum_{i=1}^{5} \sum_{h=1}^{t_{max}-i} c(i,h)$$

For each distance i between timeslots that generates a penalty (1,2,3,4,5) we sum the cost function generated by each timeslot configuration.

$$= \sum_{i=1}^{5} \sum_{h=1}^{t_{max}-i} 2^{5-i} * \frac{\sum_{q=1}^{|S|} u_{q,h,h+i}}{|S|}$$

$$= \sum_{i=1}^{5} \sum_{h=1}^{t_{max}-i} \sum_{q=1}^{|S|} 2^{5-i} * \frac{u_{q,h,h+i}}{|S|}$$

i.e. for each distance between timeslots that generates a penalty, we multiply the corrispective cost (2^{5-i}) by the number of students which are occupied both in timeslot h and $h+i$. This number of students is calculated summing over all students the boolean variable $u_{q,h,h+i}$ which is 1 if student q is occupied in timeslots h and $h+i$ and 0 o/w.