OPTIMIZATION METHODS AND ALGORITHMS

PROBLEM FORMALIZATION

	NOTATION:	Domain:	
1	s	$\epsilon\{1, \lvert S\rvert\}$	Student index
2	e	$\epsilon\{1, \lvert E\rvert\}$	Exam index
3	t	$\epsilon\{1, t_{max}\}$	Timeslot index
4	t_1	$\epsilon\{1, t_{max}\}$	Auxiliary timeslot index
5	i	$\epsilon\{1, t_{max} - 1\}$	Difference between timeslots index

	DATA:	
	$\lvert S\rvert$	Total number of student enrolled in at least 1 exam
	$\lvert E\rvert$	Total number of exams
	$a_{s,e}\ \epsilon\{0,1\}$	1 if student s is enrolled in exam e, 0 o/w
	$t_{max}\ \epsilon$ N	Number of available timeslots

	VARIABLES:		
1	$x_{e,t}\ \epsilon\{0,1\}$	$\forall\ e, t$	1 if exam e is scheduled on timeslot t, 0 o/w
2	$z_{s,t}\ \epsilon\ \{0,1\}$	$\forall\ s, t$	1 if student s is occupied during timeslot t, 0 o/w
3	$u_{s,t,t_1}\ \epsilon\ \{0,1\}$	$\forall\ s, t, t_1$	1 if student s is occupied in both timeslots t and t_1, 0 o/w

	CONSTRAINTS:		
1	$\sum_{t=1}^{t_{max}} x_{e,t} = 1$	$\forall\ e$	Each exam will have one and only one time slot
2	$z_{s,t} = \sum_{e=1}^{\lvert E\rvert} a_{s,e} x_{e,t}$	$\forall\ s, t$	$z_{s,t}$ is 1 if student s is occupied in timeslot t, 0 o/w.
3	$z_{s,t} \leq 1$	$\forall\ s, t$	Student s cannot be enrolled in more exams which are in the same timeslot, hence the sum of all exams in which student s is enrolled and which takes place in timeslot t is 1 or 0.
4	$u_{s,t,t_1} \geq z_{s,t} + z_{s,t_1} - 1$	$\forall\ s, t, t_1$	$u_{s,t,t1}$ is 1 if student s is occupied in both timeslots t and t_1, 0 o/w.

COST FUNCTION:

$c(i,t)$

$$c(i,t) = \begin{cases} 0, & i > 5 \\ 2^{5-i} * \frac{\sum_{s=1}^{\lvert S\rvert} u_{s,t,t+i}}{\lvert S\rvert}, & i \leq 5 \end{cases}$$

OBJECTIVE FUNCTION:

$$\sum_{i=1}^{5} \sum_{t=1}^{t_{max}-i} c\,(i,t)$$

For each distance i between timeslots that generates a penalty (1,2,3,4,5) we sum the cost function generated by each timeslot configuration.

$$= \sum_{i=1}^{5} \sum_{t=1}^{t_{max}-i} 2^{5-i} * \frac{\sum_{s=1}^{\lvert S\rvert} u_{s,t,t+i}}{\lvert S\rvert}$$

$$= \sum_{\boldsymbol{i=1}}^{\boldsymbol{5}} \sum_{\boldsymbol{t=1}}^{\boldsymbol{t_{max}-i}} \sum_{\boldsymbol{s=1}}^{|S|} 2^{5-i} * \frac{u_{s,t,t+i}}{|S|}$$

i.e. for each distance between timeslots that generates a penalty, we multiply the corrispective cost (2^{5-i}) by the number of students which are occupied both in timeslot t and $t+i$. This number of students is calculated summing over all students the boolean variable $u_{s,t,t+i}$ which is 1 if student s is occupied in timeslots t and $t+i$ and 0 o/w.