University of Zurich[UZH]

Some Statistics

Master Rasch Seminar 2 – 23.09.2020

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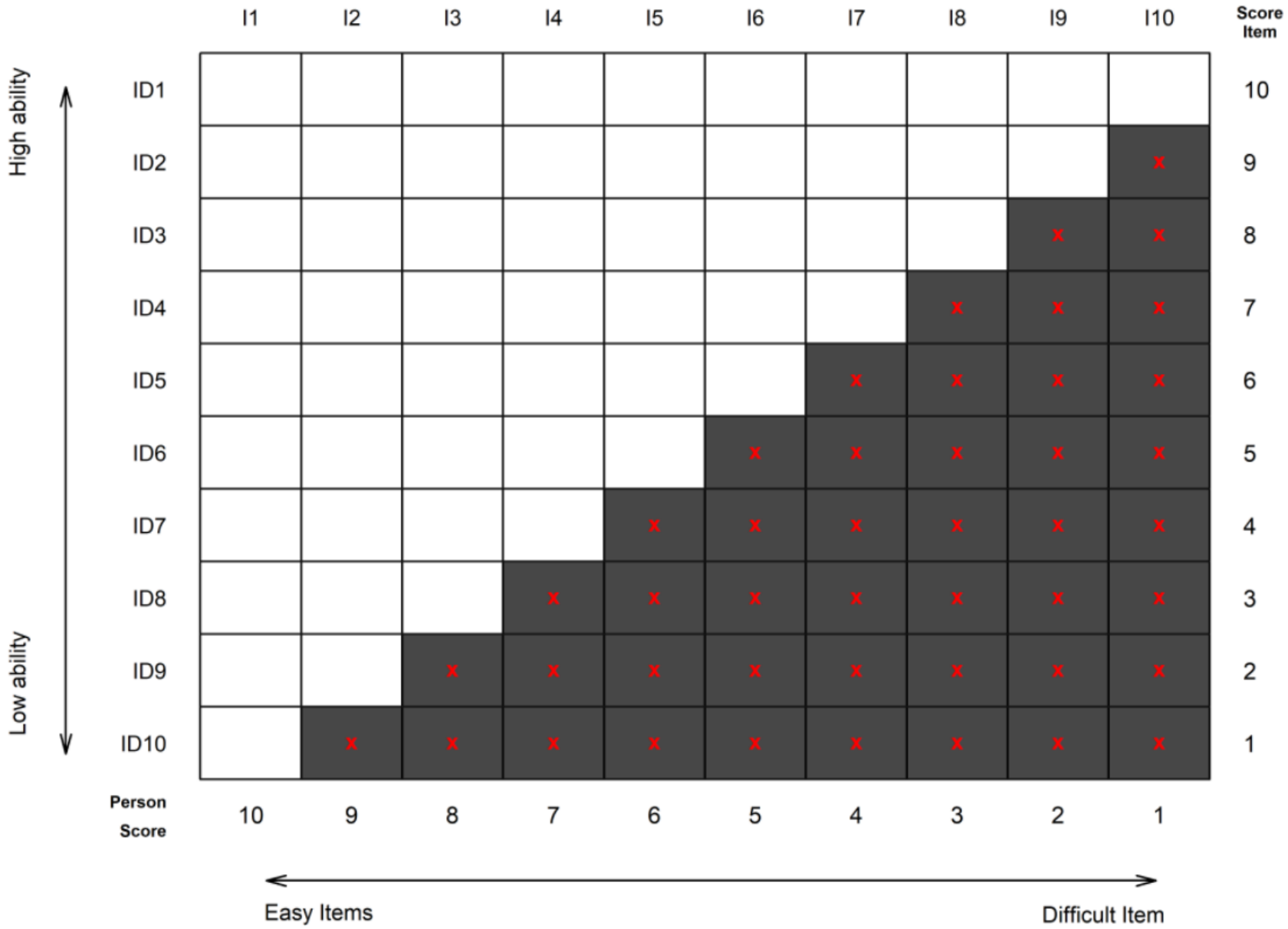
Rasch Analysis

The probability of a response is
a function of the ability of a respondent and
of the difficulty of an item.

Guttman Pattern



The Rasch Assumptions

The Rasch analysis tests if data follow a certain pattern by testing a serie of assumptions

- Fit, Reliability, Targeting
- Monotonicity
- Local Item Dependency
- Unidimensionality
- Absence of Differential Item Functioning

The seminar lectures will focus on how to test these assumptions and what to do if assumptions are not fulfilled.

Different Rasch Models

Rasch Model for dichotomous responses (Rasch 1960).

The probability of a person *j* with an ability θ to respond correctely (Xij = 1) to the item *i*, having difficulty δ is formalized:

$$P(X_{ij} = 1|\theta_j) = \frac{exp(\theta_j - \delta_i)}{1 + exp(\theta_j - \delta_i)}$$

Rasch Equations

Rasch Model for dichotomous responses (Rasch 1960)

Example: The probability that a person with an ability of 5 responds correctly to an item with difficulty 6?

$$P(X_{ij} = 1) = \frac{e^{5-6}}{1 + e^{5-6}} = \frac{e^{-1}}{1 + e^{-1}} = 0.27$$

Short Exercise

What is the Probability that the person responds wrongly, i.e. $P(X_{ij} = 0 \mid \theta_j)$?

What happens if the person ability equals the item difficulty, i.e. $\theta_j = \delta_i$?

Short Exercise

What is the Probability that the person responds X = 0?

```
# a) The probability of not-responding correctly is
theta <- 5
delta <- 6

1 - exp(theta - delta)/(1 + exp(theta - delta))
```

```
## [1] 0.7310586
```

```
# or simply 1 - 0.27
```

Short Exercise

What happens if the person ability equals the item difficulty?

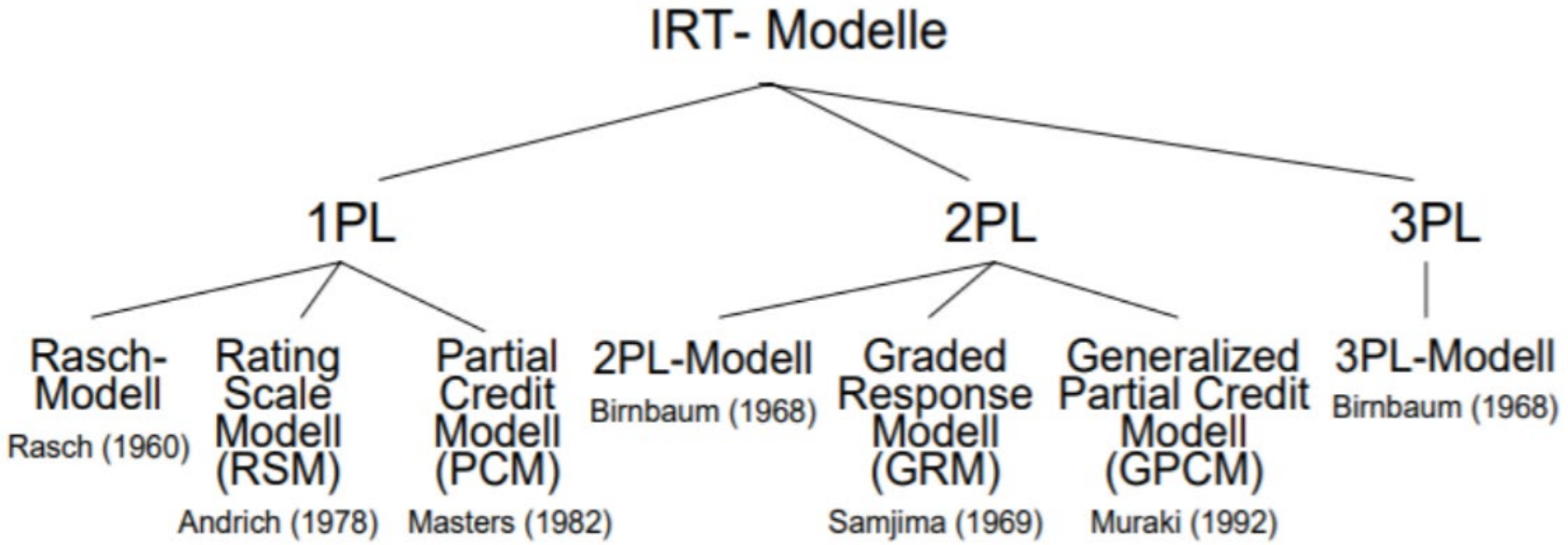
```
# b) When ability equals the item difficulty

theta <- 6
delta <- 6

1 - exp(theta - delta)/(1 + exp(theta - delta))
```

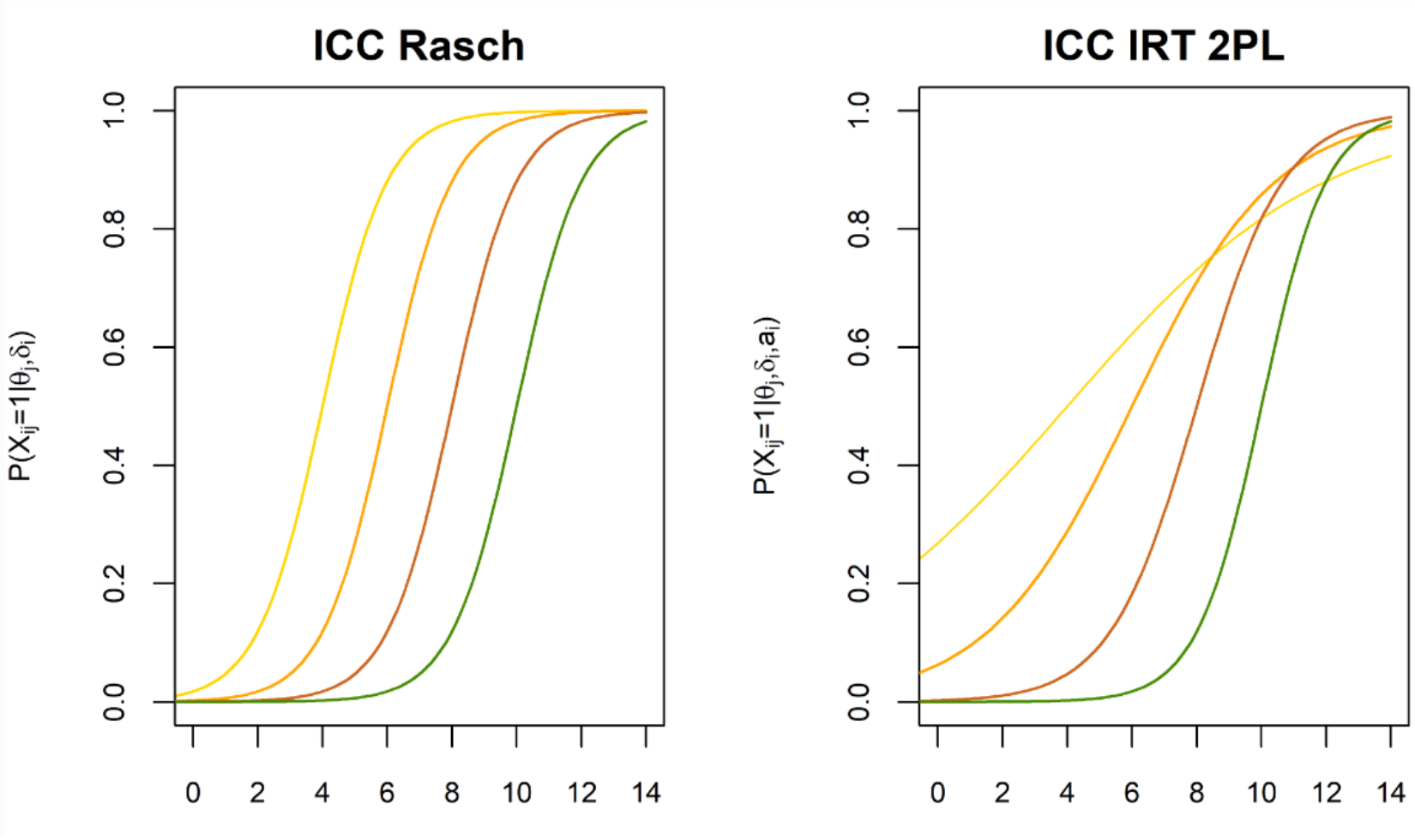
```
## [1] 0.5
```

Family of Rasch and IRT Models[1]



[1] A simplified view

Family of Rasch and IRT Models



Rasch vs. IRT Equations

Rasch Equation

$$P(X_{ij} = 1|\theta_j) = \frac{exp(\theta_j - \delta_i)}{1 + exp(\theta_j - \delta_i)}$$

2-Parameter Logistic

$$P(X_{ij} = 1|\theta_j) = \frac{exp[a_i(\theta_j - \delta_i)]}{1 + exp[a_i(\theta_j - \delta_i)]}$$

Rasch vs. IRT Equations

Rasch Equation

$$P(X_{ij} = 1|\theta_j) = \frac{exp(\theta_j - \delta_i)}{1 + exp(\theta_j - \delta_i)}$$

2-Parameter Logistic

$$P(X_{ij} = 1|\theta_j) = \frac{exp[a_i(\theta_j - \delta_i)]}{1 + exp[a_i(\theta_j - \delta_i)]}$$

a_i : the parameter for the item discrimination

Probabilistic Models of Measurement

Free parameter in the various models

Rasch Models : item difficulty

1-Parameter Logistic (1-PL) : item difficulty (very similar to the Rasch model)

2-Parameter Logistic (2-PL): item difficulty, item discrimination

3 Parameter Logistic (3-PL): item difficulty, item discrimination, guessing parameter

4 Parameter Logistic (4-PL): item difficulty, item discrimination, guessing parameter, ‘slipping parameter’.

Rasch Analysis in Practice

Rasch analysis is applied to test if an instrument possess psychometric properties.

The assumptions of the Rasch model are tested iteratively.

The data is «adjusted» until all assumptions are met.

The assumptions of the Rasch model are:

1. Stochastic Ordering (fit of data to the model)
2. Monotonicity (ordering of response options)
3. No local response dependencies or LID (no correlations between items)
4. Unidimensionality (1 construct being measured)
5. No differential item functioning (no sample subgroup effects)

Let’s go to R

Open the R-Script MS2_Rscript.r that you can find, in the OLAT or the MS-Teams Course Materials.

Exercise

Create a random sample of polytomous data with item discrimination constraint, for N = 500 persons and 15 items with 4 response categories, with difficulties ranging from -6 to 6 and with the spread of the latent variable set to 2.5 using `sim.poly.npl`.

Test which Rasch model fits the data better.

Please use `set.seed(2020)` for the random sampling and make sure that the results are invariant.