



University of  
Zurich<sup>UZH</sup>

# Some Psychometric Equations

Master Rasch Seminar 2 – 23.09.2020

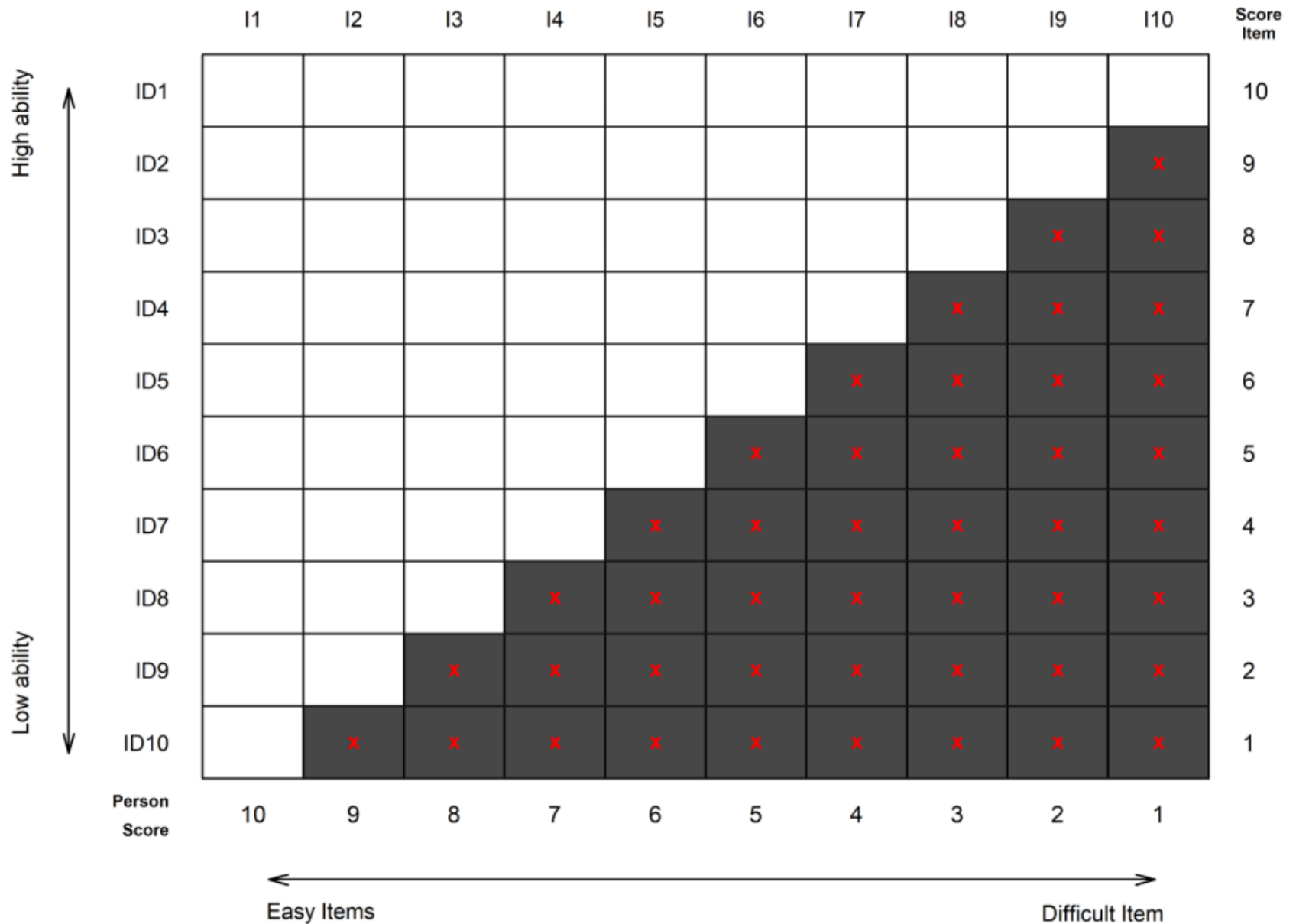
Carolina Fellinghauer : [c.fellinghauer@psychologie.uzh.ch](mailto:c.fellinghauer@psychologie.uzh.ch)

# Rasch Analysis

The probability of a response is  
a function of the ability of a respondent and  
of the difficulty of an item.



# Guttman Pattern



# Different Rasch Models

Rasch Model for dichotomous responses (Rasch 1960).

The probability of a person  $j$  with an ability  $\theta$  to respond correctly ( $X_{ij} = 1$ ) to the item  $i$ , having difficulty  $\beta$  is formalized:

$$P(X_{ij} = 1|\theta_j) = \frac{\exp(\theta_j - \beta_i)}{1 + \exp(\theta_j - \beta_i)}$$

# Rasch Equations

Rasch Model for dichotomous responses (Rasch 1960)

Example: The probability that a person with an ability of 5 responds correctly to an item with difficulty 6?

$$P(X_{ij} = 1) = \frac{e^{5-6}}{1 + e^{5-6}} = \frac{e^{-1}}{1 + e^{-1}} = 0.27$$

# Short Exercise

What is the Probability that the person responds wrongly, i.e.  $P(X_{ij} = 0 \mid \theta_j)$  ?

What happens if the person ability equals the item difficulty, i.e.  $\theta_j = \beta_i$  ?

# Short Exercise

What is the Probability that the person responds  $X = 0$ ?

```
# a) The probability of not responding correctly is  
theta <- 5  
beta <- 6  
  
1 - exp(theta - beta)/(1 + exp(theta - beta))
```

```
## [1] 0.7310586
```

```
# or simply 1 - 0.27
```

# Short Exercise

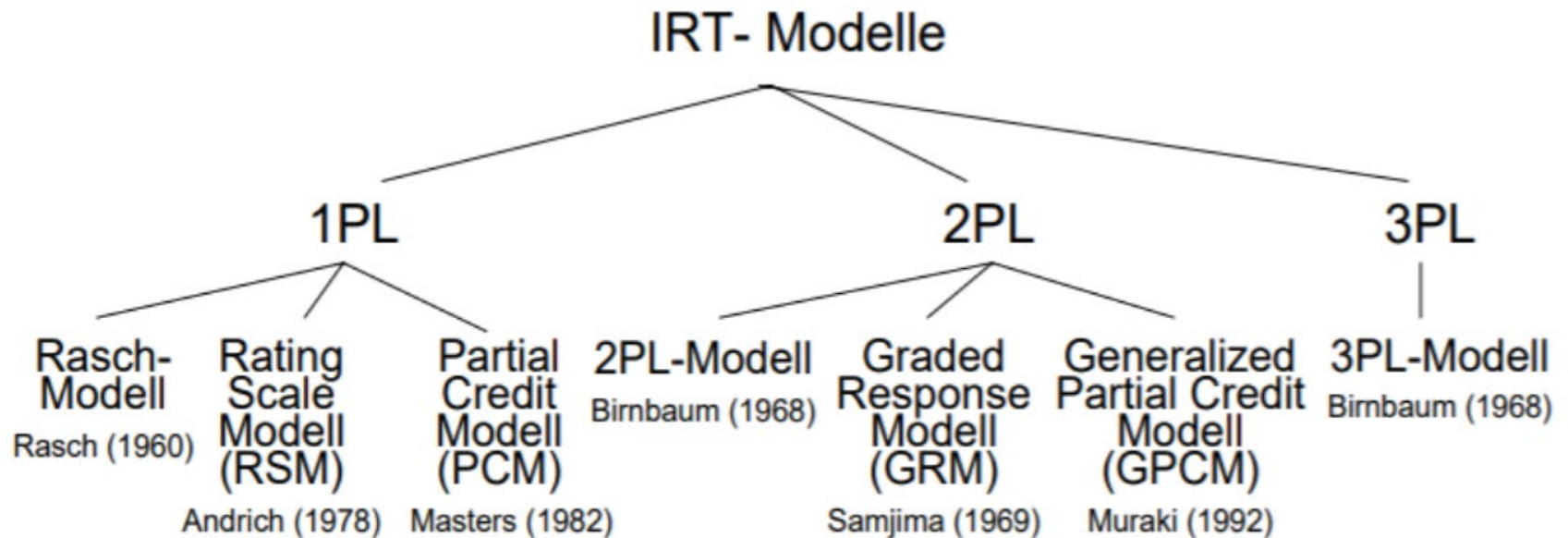
What happens if the person ability equals the item difficulty?

```
# b) When ability equals the item difficulty  
  
theta <- 6  
beta <- 6  
  
1 - exp(theta - beta)/(1 + exp(theta - beta))
```

```
## [1] 0.5
```



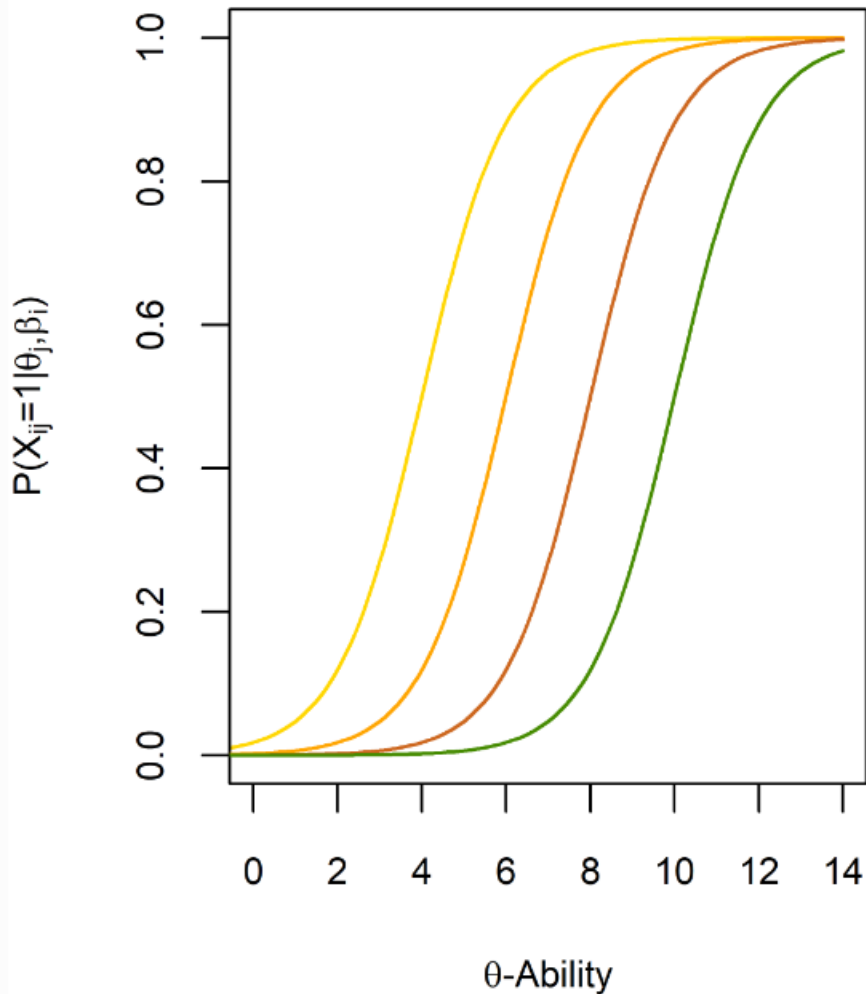
# Family of Rasch and IRT Models<sup>1</sup>



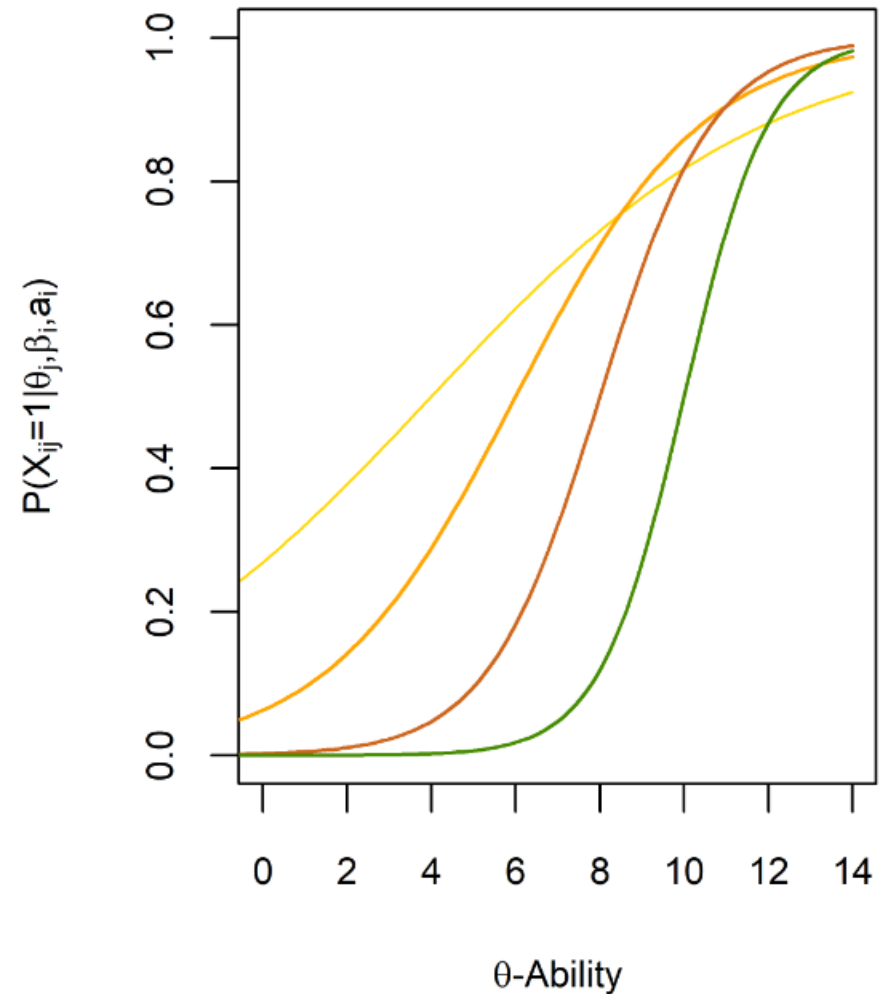
<sup>1</sup> A simplified view

# Family of Rasch and IRT Models

ICC Rasch



ICC IRT 2PL



# Rasch vs. IRT Equations

Rasch Equation

$$P(X_{ij} = 1|\theta_j) = \frac{\exp(\theta_j - \beta_i)}{1 + \exp(\theta_j - \beta_i)}$$

2-Parameter Logistic

$$P(X_{ij} = 1|\theta_j) = \frac{\exp[a_i(\theta_j - \beta_i)]}{1 + \exp[a_i(\theta_j - \beta_i)]}$$

# Rasch vs. IRT Equations

Rasch Equation

$$P(X_{ij} = 1|\theta_j) = \frac{\exp(\theta_j - \beta_i)}{1 + \exp(\theta_j - \beta_i)}$$

2-Parameter Logistic



$$P(X_{ij} = 1|\theta_j) = \frac{\exp[a_i(\theta_j - \beta_i)]}{1 + \exp[a_i(\theta_j - \beta_i)]}$$

# Score Sufficiency

Rasch perspective: The raw score has all the information about the «ability» of the respondent. = Score Sufficiency

IRT perspective: The pattern of responses has all the information about the «ability» of the respondent.

# Probabilistic Models of Measurement

**Free parameter in the various models**

**Rasch Models** : item difficulty

**1-Parameter Logistic (1-PL)** : item difficulty (very similar to the Rasch model)

**2-Parameter Logistic (2-PL)**: item difficulty, item discrimination

**3 Parameter Logistic (3-PL)**: item difficulty, item discrimination, guessing parameter

**4 Parameter Logistic (4-PL)**: item difficulty, item discrimination, guessing parameter, 'slipping parameter'.

# Rasch Analysis in Practice

Rasch analysis is applied to test if an instrument possess psychometric properties.

The assumptions of the Rasch model are tested iteratively.

The data is «adjusted» until all assumptions are met.

The assumptions of the Rasch model are:

1. Stochastic Ordering (fit of data to the model)
2. Monotonicity (ordering of response options)
3. No local response dependencies or LID (no correlations between items)
4. Unidimensionality (1 construct being measured)
5. No differential item functioning (no sample subgroup effects)

# Let's go to R

Open the R-Script MS2\_Rscript.r that you can find, in the OLAT or the MS-Teams Course Materials.



# Exercise

Create a random sample of polytomous data with item discrimination constraint, for  $N = 500$  persons and 15 items with 4 response categories, with difficulties ranging from -6 to 6 and with the spread of the latent variable set to 2.5 using

```
sim.poly.npl.
```

Test which Rasch model fits the data better.

Please use `set.seed(2020)` for the random sampling and make sure that the results are invariant.