World Health Organization

Some Psychometric Equations

Rasch Technical Training 2

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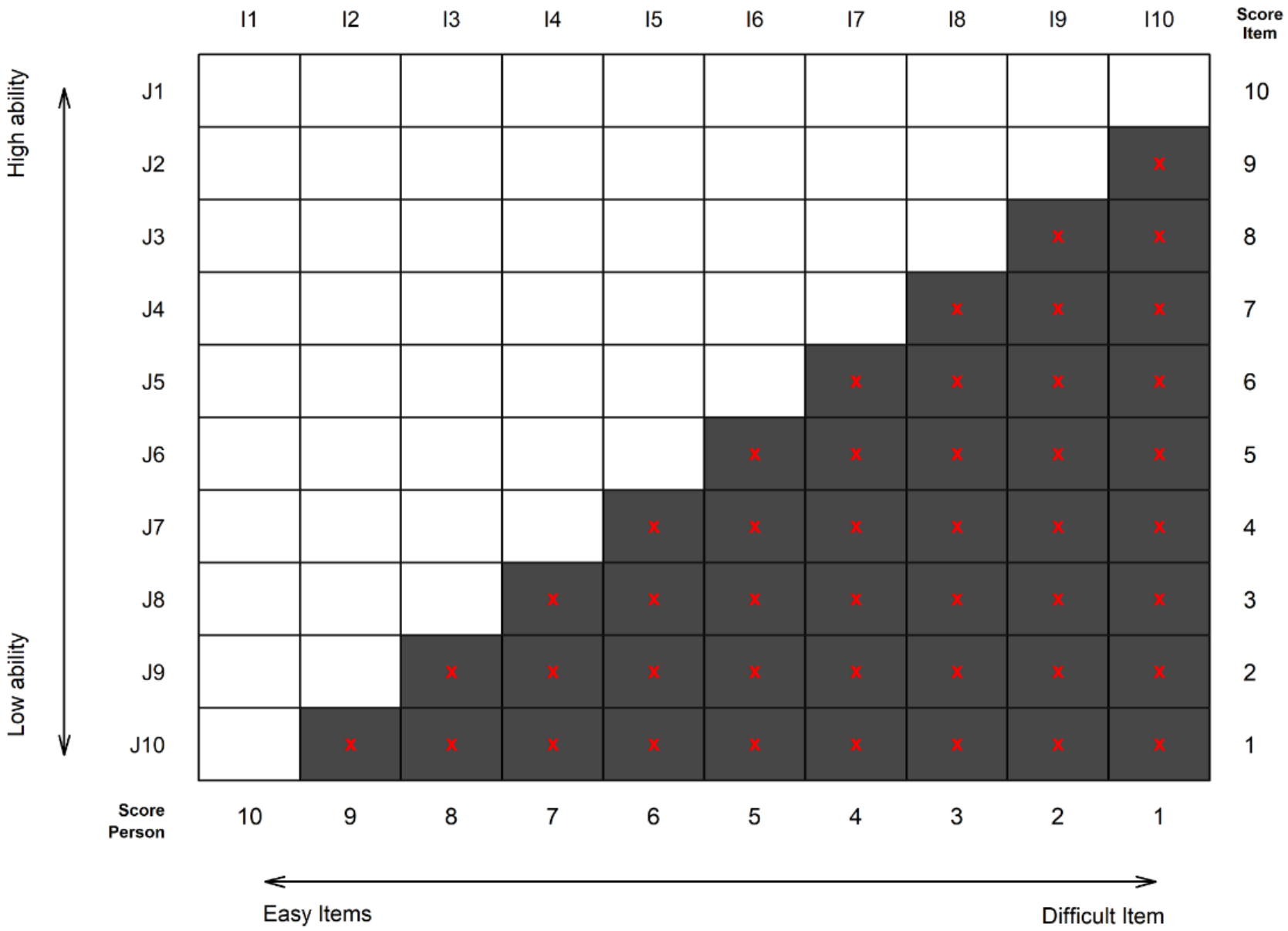
Rasch Analysis

The probability of a response is
a function of the ability of a respondent and
of the difficulty of an item.

Guttman Pattern



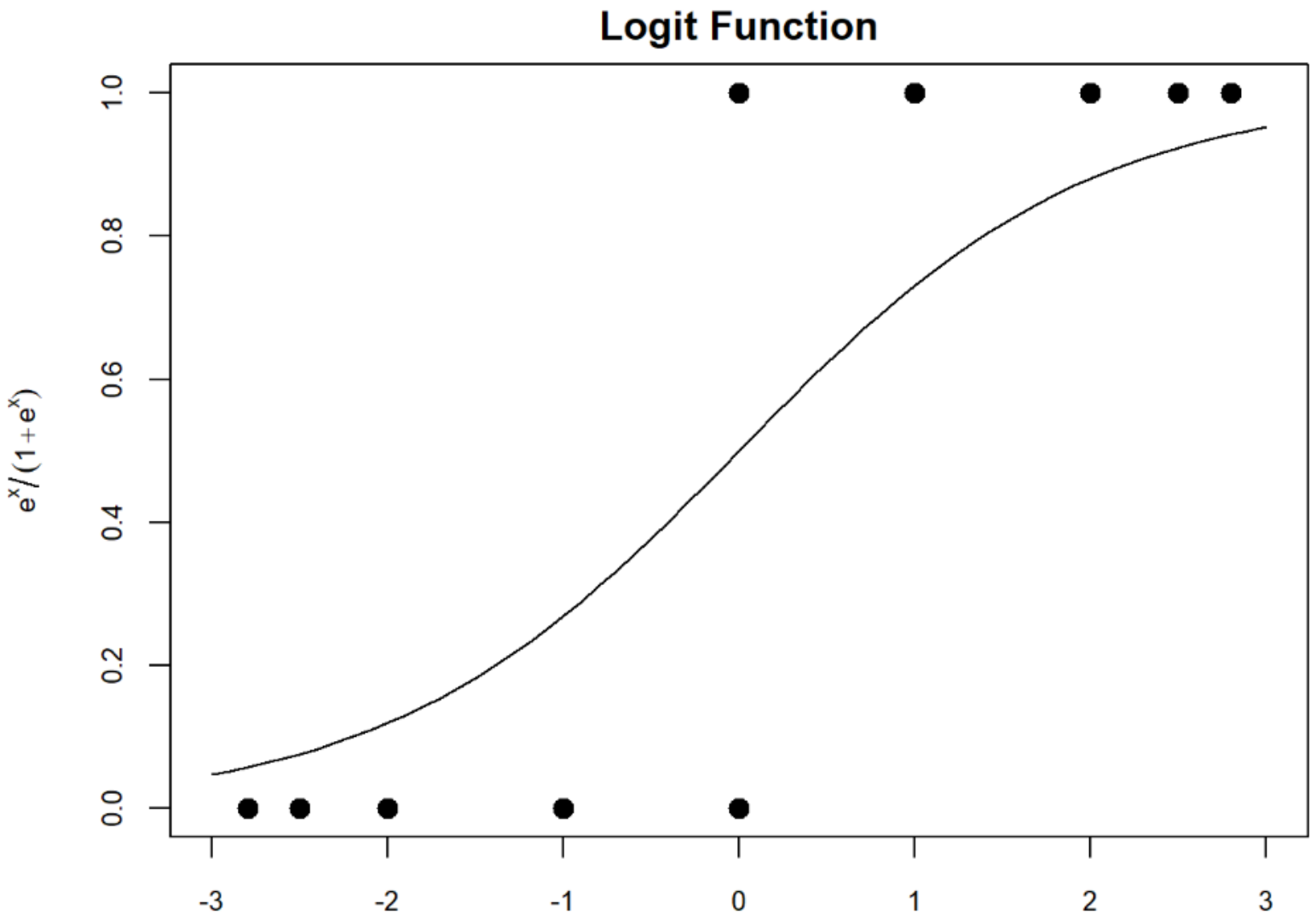
Different Rasch Models

Rasch Model for dichotomous responses (Rasch 1960).

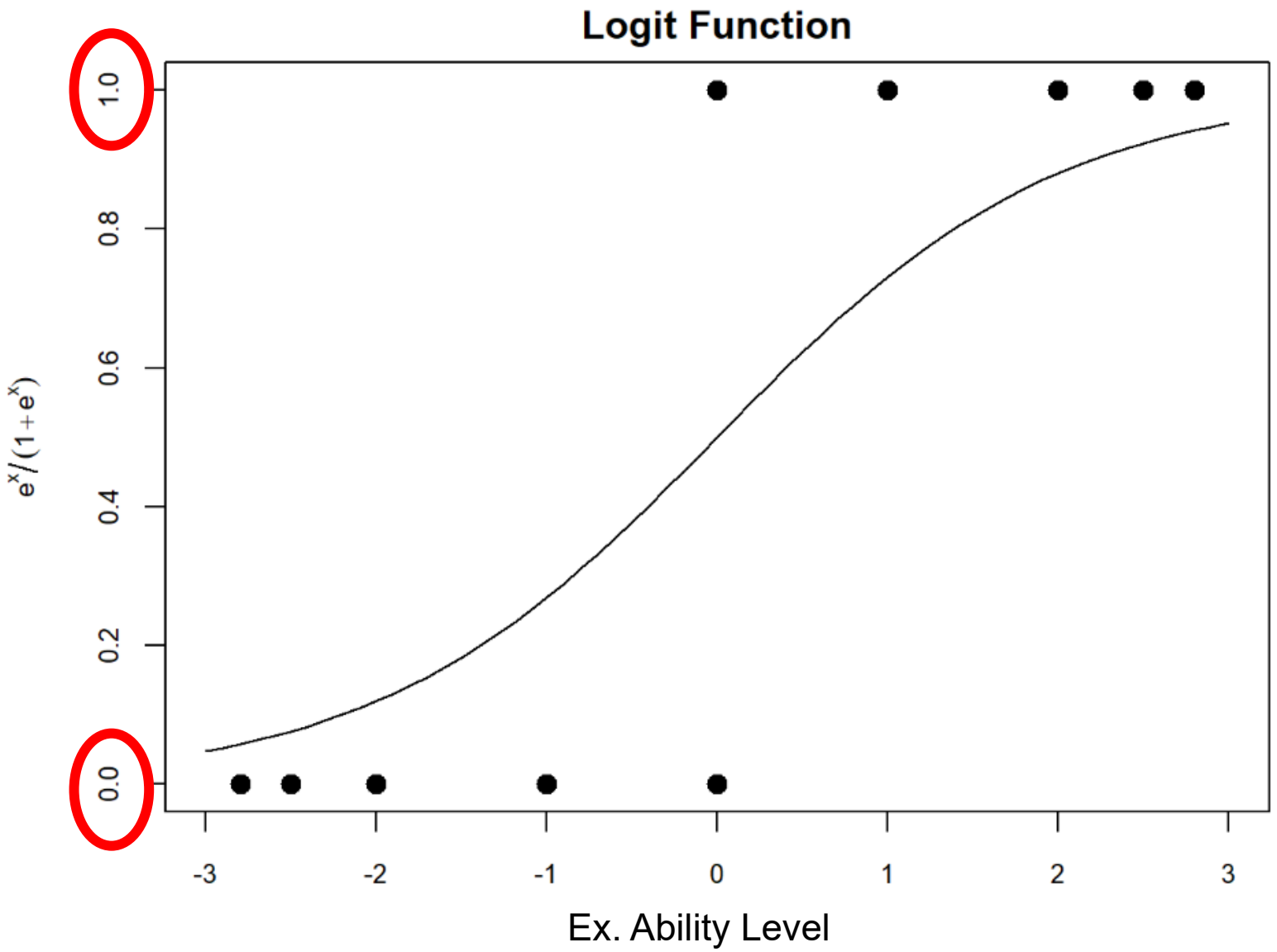
The probability of a person *j* with an ability θ to respond correctely (Xij = 1) to the item *i*, having difficulty β is formalized:

$$P(X_{ij} = 1|\theta_j) = \frac{exp(\theta_j - \beta_i)}{1 + exp(\theta_j - \beta_i)}$$

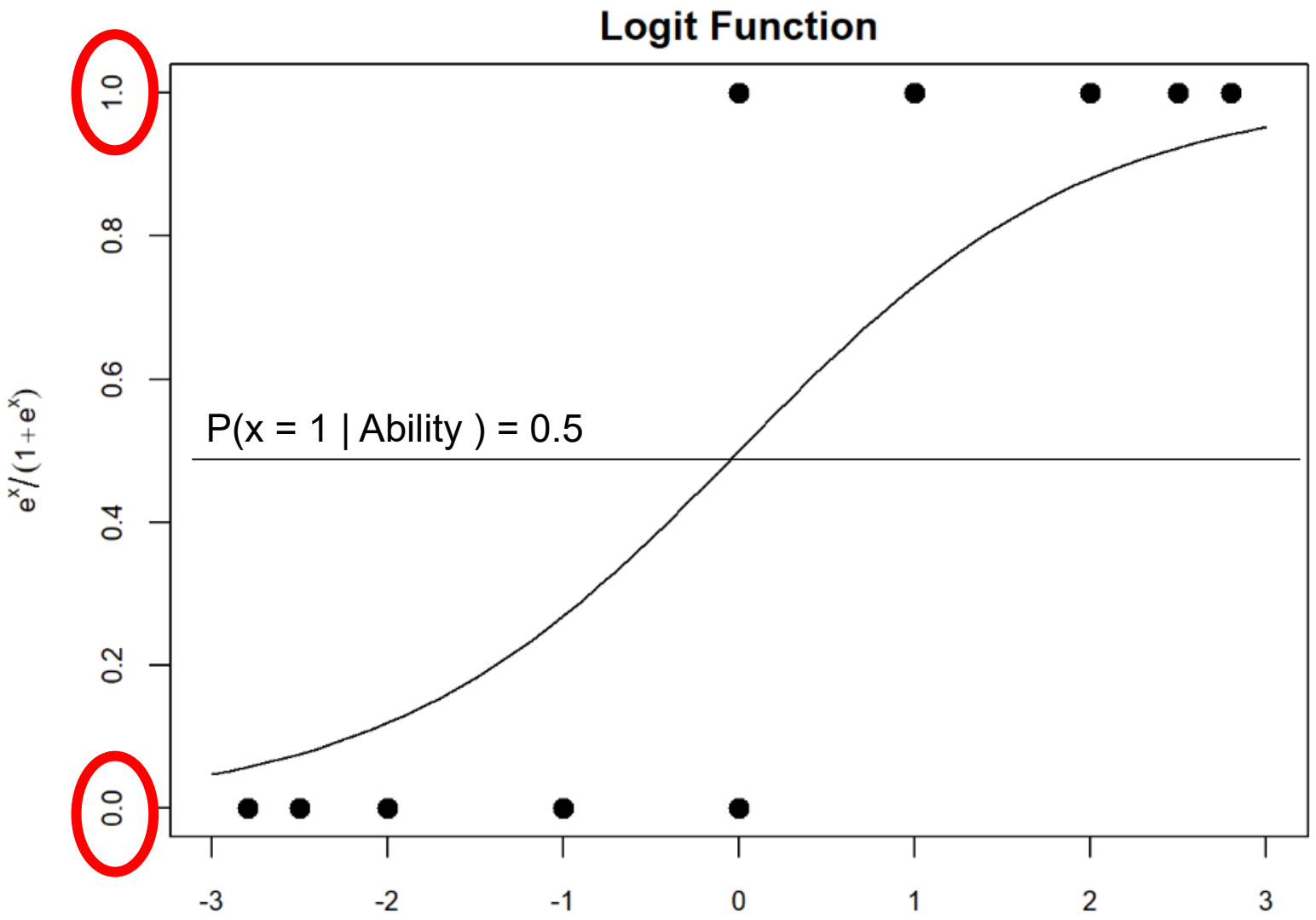
Logit Function



Logit Function



Logit Function



Ex. Probability to respond per ability level when the item difficulty = 0

Rasch Equations

Rasch Model for dichotomous responses (Rasch 1960)

Example: The probability that a person with an ability of 5 responds correctly to an item with difficulty 6?

$$P(X_{ij} = 1) = \frac{e^{5-6}}{1 + e^{5-6}} = \frac{e^{-1}}{1 + e^{-1}} = 0.27$$

Short Exercise

What is the probability that the person gives a wrong response, i.e. $P(X_{ij} = 0 \mid \theta_j)$?

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```
# a) The probability of not responding correctly is
theta <- 5
beta <- 6

1 - exp(theta - beta)/(1 + exp(theta - beta))
```

```
## [1] 0.7310586
```

```
# or simply 1 - 0.27
```

Short Exercise

What happens if the person ability equals the item difficulty, i.e. $\theta_j = \beta_i$?

Short Exercise

What happens if the person ability equals the item difficulty, i.e. $\theta_j = \beta_i$?

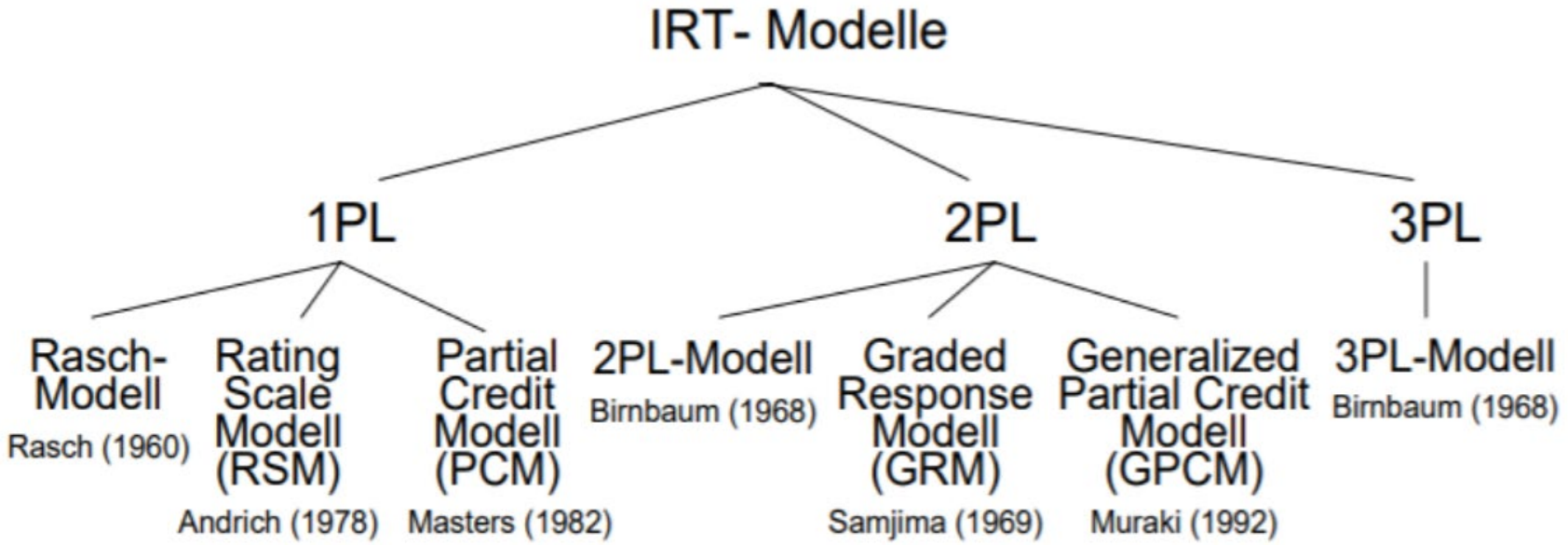
```
# b) When ability equals the item difficulty

theta <- 6
beta <- 6

1 - exp(theta - beta)/(1 + exp(theta - beta))
```

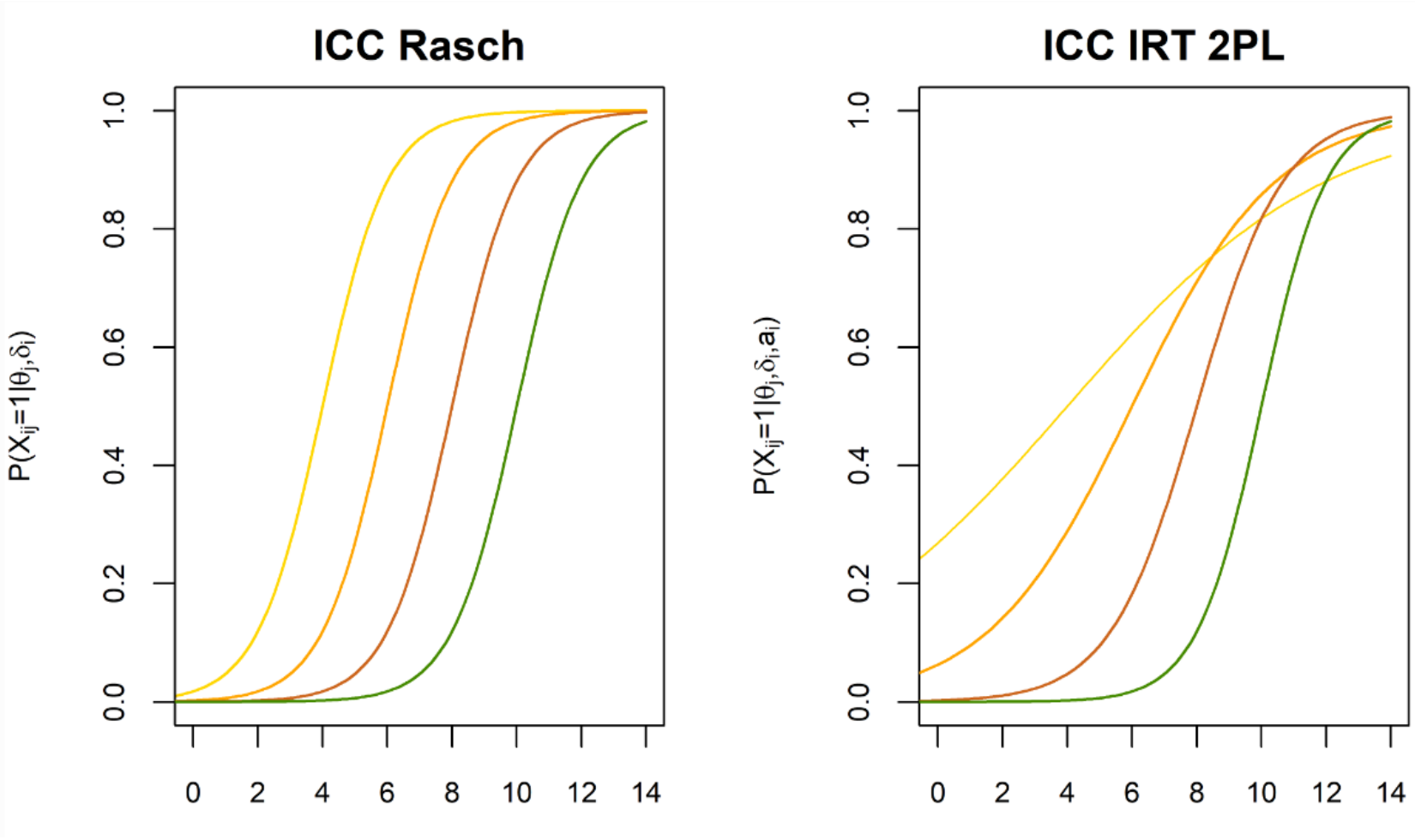
```
## [1] 0.5
```

Family of Rasch and IRT Models[1]



[1] A simplified view

Family of Rasch and IRT Models



Score Sufficiency

Rasch perspective: The raw score has all the information about the «ability» of the respondent. = **Score Sufficiency**

IRT perspective: The pattern of responses has all the information about the «ability» of the respondent.

Rasch vs. IRT Equations

Rasch Equation

$$P(X_{ij} = 1|\theta_j) = \frac{exp(\theta_j - \beta_i)}{1 + exp(\theta_j - \beta_i)}$$

2-Parameter Logistic

$$P(X_{ij} = 1|\theta_j) = \frac{exp[a_i(\theta_j - \beta_i)]}{1 + exp[a_i(\theta_j - \beta_i)]}$$

Rasch vs. IRT Equations

Rasch Equation

$$P(X_{ij} = 1|\theta_j) = \frac{exp(\theta_j - \beta_i)}{1 + exp(\theta_j - \beta_i)}$$

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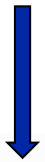

$$P(X_{ij} = 1|\theta_j) = \frac{exp[a_i(\theta_j - \beta_i)]}{1 + exp[a_i(\theta_j - \beta_i)]}$$

Probabilistic Models of Measurement

Free parameter in the various models

Rasch Models : item difficulty

1-Parameter Logistic (1-PL) : item difficulty (very similar to the Rasch model)

2-Parameter Logistic (2-PL): item difficulty, item discrimination

3 Parameter Logistic (3-PL): item difficulty, item discrimination, guessing parameter

4 Parameter Logistic (4-PL): item difficulty, item discrimination, guessing parameter, ‘slipping parameter’.

Further IRT-models

3-Parameter Logistic

Adjustment for guessing

$$P(X_{ij}=1|\theta_j) = g_i + (1-g_i)\frac{exp[a_i(\theta_j-\beta_i)]}{1+exp[a_i(\theta_j-\beta_i)]}$$

4-Parameter Logistic

Adjustment for carelessness

$$P(X_{ij}=1|\theta_j) = g_i + (u_i - g_i)\frac{exp[a_i(\theta_j-\beta_i)]}{1+exp[a_i(\theta_j-\beta_i)]}$$

Let’s go to R-Studio

Open the R-Script TT2_Rscript.r that you can find Github.

Exercise

Create a random sample of polytomous data with item discrimination constraint, for N = 500 persons and 15 items with 4 response categories, with difficulties ranging from -6 to 6 and with the spread of the latent variable set to 2.5 using `sim.poly.npl`.

Test which Rasch model fits the data better.

Please use `set.seed(2020)` for the random sampling and make sure that the results are invariant.