APPENDIX E

APPENDIX E: EXPLICIT RESULTS OF INTEGRALS

Here we present explicit results for the integrals used in this work. They were obtained combining the IREG procedure in Section. 3.1.1 with 1-loop evaluation of some integrals performed with *Package X* (26).

E.1 General IREG integrals

Where k is the internal momentum in the loop.

E.1.1 1-loop: two point functions

$$\int_k \frac{1}{k^2(k+p)^2} = I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + 2b, \tag{E.1}$$

$$\int_k \frac{k_\mu}{k^2(k+p)^2} = -\frac{p_\mu}{2}\left[I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + 2b\right], \tag{E.2}$$

$$\begin{aligned}\int_k \frac{k_\mu k_\nu}{k^2(k+p)^2} &= -\frac{g_{\mu\nu}p^2}{12}\left[I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + \frac{8b}{3}\right] \\ &+ \frac{p_\mu p_\nu}{3}\left[I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + \frac{13b}{6}\right]\end{aligned} \tag{E.3}$$

E.1.2 1-loop: three point functions

$$\int_k \frac{k_\mu k_\nu}{k^4(k+p)^2} = \frac{g_{\mu\nu}}{4}\left[I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + 2b\right] + \frac{p_\mu p_\nu}{p^2}\frac{b}{2} \tag{E.4}$$

E.1.3 2-loop: explicit results involving logarithms

The general structure is:

$$I^{\nu_1\ldots\nu_m} = \int\limits_{k_l} \frac{A^{\nu_1\ldots\nu_m}(k_l,q_i)}{\prod_i[(k_l-q_i)^2-\mu^2]} \ln^{l-1}\left(-\frac{k_l^2-\mu^2}{\lambda^2}\right), \tag{E.5}$$

Considering the 2-loop case ($l=2$) and relabeling $k_2=k$, it simplifies to:

$$I^{\nu_1\ldots\nu_m} = \int\limits_{k} \frac{A^{\nu_1\ldots\nu_m}(k,q_i)}{\prod_i[(k-q_i)^2-\mu^2]} \ln\left(-\frac{k^2-\mu^2}{\lambda^2}\right), \tag{E.6}$$

E.1.3.1 One point functions

$$\int_k \frac{1}{(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right) = \frac{p^2}{2} I_{log}(\lambda^2) + \text{finite}, \tag{E.7}$$

E.1.3.2 Two point functions

$$\int_k \frac{1}{k^2(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right) = I_{log}^{(2)}(\lambda^2) + \text{finite}, \tag{E.8}$$

$$\int_k \frac{k_\mu}{k^2(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right) = -\frac{p_\mu}{2}\left[I_{log}^{(2)}(\lambda^2) + \frac{I_{log}(\lambda^2)}{2} + \text{finite}\right], \tag{E.9}$$

E.1.3.3 Three point functions

$$\int_k \frac{k_\mu k_\nu}{k^4(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right) = \frac{g_{\mu\nu}}{4}\left[I_{log}^{(2)}(\lambda^2) + \frac{I_{log}(\lambda^2)}{2} + \text{finite}\right] + \frac{p_\mu p_\nu}{p^2}\text{finite}, \tag{E.10}$$

E.1.4 2-loop: overlapped integrals

We will have the general structure:

$$I[f(k,q)] = \int_k \int_q \frac{f(k,q)}{k^2(k-p)^2(k-q)^2 q^2 (q-p)^2} \tag{E.11}$$

$$I[k_\mu q_\nu] = \frac{g_{\mu\nu}}{4}\left[I_{log}(\lambda^2) + \text{finite}\right] + \frac{p_\mu p_\nu}{p^2}\text{finite} \tag{E.12}$$

$$\begin{aligned} I[k_\mu k_\nu] = \frac{g_{\mu\nu}}{4}&\left\{I_{log}^2(\lambda^2) - bI_{log}^{(2)}(\lambda^2) + bI_{log}(\lambda^2)\left[\frac{9}{2} - \ln\left(-\frac{p^2}{\lambda^2}\right)\right] + \text{finite}\right\} \\ &+ \frac{p_\mu p_\nu}{p^2}\text{finite} \end{aligned} \tag{E.13}$$

$$I[k_\mu k.q] = \frac{p_\mu}{8}\left\{I_{log}^2(\lambda^2) - bI_{log}^{(2)}(\lambda^2) + bI_{log}(\lambda^2)\left[\frac{19}{2} - \ln\left(-\frac{p^2}{\lambda^2}\right)\right] + \text{finite}\right\}. \tag{E.14}$$

E.2 Integrals for QED

E.2.1 Massless case

$$\int_k\int_l \frac{\not{k}(k\cdot p)}{(l+p)^2k^2(l^2)^2(k-l)^2} = \frac{1}{12}bI_{log}(\lambda^2)\not{p} + \text{finite}, \tag{E.15}$$

$$\begin{aligned} \int_k\int_l \frac{l^2\not{k}}{(l+p)^2k^2(l^2)^2(k-l)^2} = &\frac{1}{4}bI_{log}^{(2)}(\lambda^2)\not{p} + \frac{1}{4}bI_{log}(\lambda^2)\ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} \\ &- \frac{7}{8}bI_{log}(\lambda^2)\not{p} - \frac{1}{4}I_{log}^2(\lambda^2)\not{p} + \text{finite}, \end{aligned} \tag{E.16}$$

$$\begin{aligned} \int_k\int_l \frac{(k\cdot l)\not{l}}{(l+p)^2k^2(l^2)^2(k-l)^2} = &\frac{1}{4}bI_{log}^{(2)}(\lambda^2)\not{p} + \frac{1}{4}bI_{log}(\lambda^2)\ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} \\ &- \frac{7}{8}bI_{log}(\lambda^2)\not{p} - \frac{1}{4}I_{log}^2(\lambda^2)\not{p} + \text{finite}, \end{aligned} \tag{E.17}$$

$$\begin{aligned} \int_k\int_l \frac{\not{k}(k\cdot l)}{(l+p)^2k^2(l^2)^2(k-l)^2} = &\frac{1}{8}bI_{log}^{(2)}(\lambda^2)\not{p} + \frac{1}{8}bI_{log}(\lambda^2)\ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} \\ &- \frac{7}{16}bI_{log}(\lambda^2)\not{p} - \frac{1}{8}I_{log}^2(\lambda^2)\not{p} + \text{finite}, \end{aligned} \tag{E.18}$$

$$\begin{aligned} \int_k\int_l \frac{(k\cdot p)\not{l}}{(l+p)^2k^2(l^2)^2(k-l)^2} = &-\frac{1}{8}bI_{log}^{(2)}(\lambda^2)\not{p} - \frac{1}{8}bI_{log}(\lambda^2)\ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} \\ &+ \frac{11}{16}bI_{log}(\lambda^2)\not{p} + \frac{1}{8}I_{log}^2(\lambda^2)\not{p} + \text{finite}, \end{aligned} \tag{E.19}$$

$$\int_k \int_l \frac{\not{k}(l \cdot p)}{(l+p)^2 k^2 (l^2)^2 (k-l)^2} = -\frac{1}{8} b I_{log}^{(2)}(\lambda^2)\not{p} - \frac{1}{8} b I_{log}(\lambda^2) \ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} + \frac{11}{16} b I_{log}(\lambda^2)\not{p} + \frac{1}{8} I_{log}^2(\lambda^2)\not{p} + \text{finite}, \tag{E.20}$$

$$\int_k \int_l \frac{(k \cdot l)\not{l}}{(l+p)^2 k^2 (l^2)^2 (k-l)^2} = \frac{1}{4} b I_{log}^{(2)}(\lambda^2)\not{p} + \frac{1}{4} b I_{log}(\lambda^2) \ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} - \frac{7}{8} b I_{log}(\lambda^2)\not{p} - \frac{1}{4} I_{log}^2(\lambda^2)\not{p} + \text{finite}, \tag{E.21}$$

$$\int_k \int_l \frac{\not{k}(k \cdot l)}{l^2 k^2 (k-l)^2 (k-p)^2 (l-p)^2} = -\frac{1}{8} b I_{log}^{(2)}(\lambda^2)\not{p} - \frac{1}{8} b I_{log}(\lambda^2) \ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} + \frac{19}{16} b I_{log}(\lambda^2)\not{p} + \frac{1}{8} I_{log}^2(\lambda^2)\not{p} + \text{finite}, \tag{E.22}$$

$$\int_k \int_l \frac{(k \cdot l)\not{l}}{l^2 k^2 (k-l)^2 (k-p)^2 (l-p)^2} = -\frac{1}{8} b I_{log}^{(2)}(\lambda^2)\not{p} - \frac{1}{8} b I_{log}(\lambda^2) \ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} + \frac{19}{16} b I_{log}(\lambda^2)\not{p} + \frac{1}{8} I_{log}^2(\lambda^2)\not{p} + \text{finite}, \tag{E.23}$$

$$\int_k \int_l \frac{\not{k}(l \cdot p)}{l^2 k^2 (k-l)^2 (k-p)^2 (l-p)^2} = \frac{1}{4} b I_{log}(\lambda^2)\not{p} + \text{finite}, \tag{E.24}$$

$$\int_k \int_l \frac{\not{l}(l \cdot p)}{l^2 k^2 (k-l)^2 (k-p)^2 (l-p)^2} = -\frac{1}{4} b I_{log}^{(2)}(\lambda^2)\not{p} - \frac{1}{4} b I_{log}(\lambda^2) \ln\left(-\frac{p^2}{\lambda^2}\right)\not{p} + \frac{9}{8} b I_{log}(\lambda^2)\not{p} + \frac{1}{4} I_{log}^2(\lambda^2)\not{p} + \text{finite}. \tag{E.25}$$

Notice all the integrals are proportional to $\not{p}$ as expected for the massless case.