IREG-Finite terms: vectorial part (p_slashed)

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Notation: $\vec{A} = \underline{A}$; $\int \frac{d^4k}{(2\pi)^4} = \int_k$; $\not{A} = \gamma^\mu A_\mu$

Color code: Equations/Drawings, Titles, Extras, Pencil, Normal text, Important concepts

$$\widetilde{\Sigma}_{ij}(p)\Big|_{\text{finite terms}} = -i g_s^2 C_2(r)\Big\{ 2\not{p} \lim_{\mu^2\to 0}\int_l \frac{p^2+2p\cdot l - m^2}{(l^2-\mu^2)^2[(l+p)^2-m^2-\mu]}$$

$$+ 2\gamma^\mu \Big[p^2 \lim_{\mu^2\to 0}\int_l \frac{l_\mu}{(l^2-\mu^2)^3} - m^2 \lim_{\mu^2\to 0}\int_l \frac{l_\mu}{(l^2-\mu^2)^3}$$

$$- \lim_{\mu^2\to 0}\int_l \frac{l_\mu[(p^2+2p\cdot l)-m^2]^2}{(l^2-\mu^2)^3[(l+p)^2-m^2-\mu^2]}\Big]$$

$$- 4m \lim_{\mu^2\to 0}\int_l \frac{p^2+2p\cdot l-m^2}{(l^2-\mu^2)^2[(l+p)^2-m^2-\mu^2]}\Big\}$$

We are interested on this terms

$-ig_s^2C_2(r)\, 2\not{p}\,\{$

Something $\cdot\, p_\mu$

(*) I think there's a little mistake in Marcos paper

$$\widetilde{\Sigma}_{ij}(p)\Big|_{\text{divergent part}} = +i g_s^2 C_2(r)(\not{p} - 4m)\lim_{\mu^2\to 0} I_{log}(\mu^2)$$

$$\not{p} \lim_{\mu^2\to 0} I_{log}(\mu^2)$$

where

$$\lim_{\mu^2\to 0} I_{log}(\mu^2) = I_{log}(\lambda^2) - \lim_{\mu^2\to 0}\frac{i}{(4\pi)^2}\ln\left(\frac{\mu^2}{\lambda^2}\right)$$

Let's check the integrals of the finite part that goes with $\not{p}$:

$$\lim_{\mu^2\to 0}\int_l \frac{p^2+2p\cdot l-m}{(l^2-\mu^2)^2[(l+p)^2-m^2-\mu]} - \lim_{\mu^2\to 0}\int_l \frac{l_\mu[(p^2+2p\cdot l)-m^2]^2}{(l^2-\mu^2)^3[(l+p)^2-m^2-\mu^2]}$$

$$= \lim_{\mu^2\to 0}\left(-\frac{3}{2}\right)\left(-\text{Log}\left[\frac{\mu^2}{m^2+\mu^2}\right]\right) + \lim_{\mu^2\to 0}\left(\frac{(\cancel{2m^4} + \cancel{2m^2\mu^2}^{\,0})\,\text{Log}\left[\frac{\mu^2}{m^2+\mu^2}\right]}{\cancel{4m^4}_{\,2}}\right) + \lim_{\mu^2\to 0}\left(\cancel{\frac{2\mu^2\,\text{Log}\left[\frac{2\mu^2+2\sqrt{-m^2\mu^2}}{2\mu\sqrt{m^2+\mu^2}}\right]}{\sqrt{-m^2\mu^2}}}^{\,0}\right)$$

$$= \lim_{\mu^2\to 0}\left\{\frac{1}{2}\left(-3 - \text{Log}\left[\frac{\mu^2}{m^2+\mu^2}\right]\right)\right\}$$

This integrals are also all multiply by a global factor $b = \frac{i}{(4\pi)^2}$ in the Package-X.

$\longrightarrow$ integrals that goes with $2\not{p}$ $= -\frac{3}{2}b - \frac{1}{2}b \lim_{\mu^2\to 0}\ln\left(\frac{\mu^2}{m^2+\mu^2}\right)$

Serie expansion

$$\lim_{\mu^2\to 0}\left[\ln\frac{\mu^2}{m^2} - \frac{\mu^2}{m^2} + \frac{\mu^4}{2m^4} - O(\mu^6)\right]$$

$$= -\frac{3}{2}b - \frac{1}{2}b\lim_{\mu^2\to 0}\ln\left(\frac{\mu^2}{m^2}\right)$$

Putting the divergent and finite parts together:

$$-ig_s^2\, C_2(r)\left\{\not p\left[-I\log\Lambda^2 + b\lim_{\mu^2\to 0}\ln\left(\frac{\mu^2}{\Lambda^2}\right) + 2\cdot\left(-\frac{3}{\cancel{2}}b - \frac{1}{\cancel{2}}b\lim_{\mu^2\to 0}\ln\left(\frac{\mu^2}{m^2}\right)\right)\right]\right\}$$

$$= -ig_s^2 C_2(r)\left\{\not p\left[-I\log\Lambda^2 + b\lim_{\mu^2\to 0}\ln\left(\frac{\mu^2}{\Lambda^2}\right) - 3b - b\lim_{\mu^2\to 0}\ln\left(\frac{\mu^2}{m^2}\right)\right]\right\}$$

$$b\lim_{\mu^2\to 0}\left[\ln\frac{\mu^2}{\Lambda^2} - \ln\frac{\mu^2}{m^2}\right] = \lim_{\mu^2\to 0}\ln\left(\frac{\cancel{\mu^2}/\Lambda^2}{\cancel{\mu^2}/m^2}\right)$$

$$= -ig_s^2 C_2(r)\left\{\not p\left[-I\log\Lambda^2 - 3b + b\ln\left(\frac{m^2}{\Lambda^2}\right)\right]\right\}$$