## IREG-Finite terms: scalar part

martes, 30 de marzo de 2021 15:45

Notation $\vec{A} = \not{A}$

$\int \frac{d^4k}{(2\pi)^4} = \int_k$

$\not{A} = \gamma^\mu A_\mu$

Color code: Equations/Drawings, Titles, Extras, Pencil, Normal text, Important concepts

$$\tilde{\sigma}_{ij}(p)\Big|_{\text{finite terms}} = -i g_s^2 C_2(r)\Big\{ 2\not{p} \lim_{\mu^2\to 0}\int_l \frac{p^2+2p\cdot l - m^2}{(l^2-\mu^2)^2[(l+p)^2-m^2-\mu^2]}$$

$$+ 2\gamma^\mu\Big[ p^2 \lim_{\mu^2\to 0}\int_l \frac{l_\mu}{(l^2-\mu^2)^3} - m^2 \lim_{\mu^2\to 0}\int_l \frac{l_\mu}{(l^2-\mu^2)^3}$$

$$- \lim_{\mu^2\to 0}\int_l \frac{l_\mu[(p^2+2p\cdot l)-m^2]^2}{(l^2-\mu^2)^3[(l+p)^2-m^2-\mu^2]}\Big]$$

$$-4m \lim_{\mu^2\to 0}\int_l \frac{p^2+2p\cdot l - m^2}{(l^2-\mu^2)^2[(l+p)^2-m^2-\mu^2]}\Big\}$$

We are interested on this

(*) I think there's a little mistake in Marcos paper

$$\bar{\sigma}_{ij}(p)\Big|_{\text{divergent part}} = +i g_s^2 C_2(r)(\not{p} - 4m)\lim_{\mu^2\to 0} I_{log}(\mu^2)$$

$$-4m \lim_{\mu^2\to 0} I_{log}(\mu^2)$$

where

$$\lim_{\mu^2\to 0} I_{log}(\mu^2) = I_{log}(\lambda^2) - \lim_{\mu^2\to 0}\frac{i}{(4\pi)^2}\ln\left(\frac{\mu^2}{\lambda^2}\right)$$

Let's check the integral of the finite part that goes with $m^2$

$$\lim_{\mu^2\to 0}\int_l \frac{p^2+2p\cdot l - m^2}{(l^2-\mu^2)^2[(l+p)^2-m^2-\mu^2]} = \lim_{\mu^2\to 0}\left(-2 - \text{Log}\left[\frac{\mu^2}{m^2+\mu^2}\right] + \frac{2\mu^2\,\text{Log}\left[\frac{2\mu^2+2\sqrt{-m^2\mu^2}}{2\mu\sqrt{m^2+\mu^2}}\right]}{\sqrt{-m^2\mu^2}}\right) =$$

$$\lim_{\mu^2\to 0}\left(-2\right)\left(-\text{Log}\left[\frac{\mu^2}{m^2+\mu^2}\right]\right)\lim_{\mu^2\to 0}\left(\cancel{\frac{2\mu^2\,\text{Log}\left[\frac{2\mu^2+2\sqrt{-m^2\mu^2}}{2\mu\sqrt{m^2+\mu^2}}\right]}{\sqrt{-m^2\mu^2}}}\right)$$

$-2$ $\quad 0$

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<< X`
Package-X v2.1.1, by Hiren H. Patel
For more information, see the guide

Clear[d]
borra

HoldForm[LoopIntegrate[1, l, {l, μ}, {l, μ}]]
forma sin evaluación

∫ d^d l/(2π)^d  1/((l^2 - μ^2) (l^2 - μ^2))

LoopIntegrate[1, l, {l, μ}, {l, μ}]
PVA[0, μ, Weights → {2}]

LoopRefine[%]
1/ε + Log[μ^2/μ^2]
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And Ilog always have that b in DReg

$$I_{log}(m^2) = \frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{m^2}\right)\right]$$

Package-X doesn't put the b

$$= -2 - \lim_{\mu^2\to 0}\ln\left(\frac{\mu^2}{m^2+\mu^2}\right)$$

But these integrals as they were solve with the package-X in Mathematica, we must multiply then by $b = i/(4\pi)^2$. (*)

Serie expansion

$$\lim_{\mu^2\to 0}\int_l \frac{p^2+2p\cdot l - m^2}{(l^2-\mu^2)^2[(l+p)^2-m^2-\mu^2]} = -b2 - b\cdot\Big\{\lim_{\mu^2\to 0}\Big[\ln\frac{\mu^2}{m^2} - \frac{\mu^2}{m^2} + \frac{\mu^4}{2m^4} - O(\mu^6)\Big]\Big\}$$

$$= -b2 - b\cdot\lim_{\mu^2\to 0}\ln\frac{\mu^2}{m^2} = -\frac{i2}{(4\pi)^2} - \frac{i}{(4\pi)^2}\lim_{\mu^2\to 0}\ln\frac{\mu^2}{m^2}$$

So, putting both scalar part Together: (divergent and finite)

$$-i g_s^2 C_2(r)\Big\{-4m\Big[I_{log}(\lambda^2) + \lim_{\mu^2\to 0}\frac{i}{(4\pi)^2}\ln\left(\frac{\mu^2}{\lambda^2}\right) - \frac{i2}{(4\pi)^2} - \frac{i}{(4\pi)^2}\lim_{\mu^2\to 0}\ln\frac{\mu^2}{m^2}\Big]\Big\}$$

$$= -i g_s C_2(r)\Big\{-4m\Big[I_{log}(\lambda^2) - 2b - b\lim_{\mu^2\to 0}\Big(\ln\left(\frac{\mu^2}{\lambda^2}\right) + \ln\left(\frac{\mu^2}{m^2}\right)\Big)\Big]\Big\}$$

$$= -i g_s C_2(r) \left\{ -4m \left[ -I \log(\Lambda^2) - 2b - b \lim_{\mu^2 \to 0} \left( -\ln\left(\frac{\mu^2}{\Lambda^2}\right) + \ln\left(\frac{\mu^2}{m^2}\right) \right) \right] \right\}$$

$$= -i g_s C_2(r) \left\{ -4m \left[ -I \log \Lambda^2 - 2b - b \lim_{\mu^2 \to 0} \ln \frac{(\cancel{\mu^2}/m^2)}{(\cancel{\mu^2}/\Lambda^2)} \right] \right\}$$

$$= -i g_s C_2(r) \left\{ -4m \left[ - \underbrace{I \log \Lambda^2}_{\text{divergent}} \underbrace{- 2b - b \ln \frac{\Lambda^2}{m^2}}_{\text{finite}} \right] \right\}$$