MIS 381N

Stochastic Control and Optimization: Homework 1

Non-graded problems

This assignment is graded on Credit/No-Credit.

That is, if you complete the homework and it is acceptable, you will get credit. If you don't submit or if the submitted work is not acceptable, you will not get credit. Getting a credit is required to obtain a grade for the group project that follows.

Problem 1

Use definitions to prove

$$(A^{-1})^T = (A^T)^{-1},$$

Where A is an invertible square matrix and A^T means the transpose of matrix A.

Problem 2

Given the square matrices

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 3 \\ 3 & -2 & -5 \end{bmatrix}, \ B = \begin{bmatrix} 3 & -6 & -3 \\ 7 & -14 & -7 \\ -1 & 2 & 1 \end{bmatrix}$$

Verify that AB = 0 although neither A nor B is. Also calculate BA. Is AB = BA?

Problem 3

A bank makes four kinds of loans to its personal customers and these loans yield the following annual interest rates to the bank:

- First mortgage 14%

- Second mortgage 20%
- Home improvement 20%
- Personal overdraft 10%

We are interested in the bank's lending strategy. The information we know is as following:

1. In total $250 million is lent out.
2. First mortgages are 55% of all mortgages (i.e. first and second mortgage) issued.
3. Second mortgages are 25% of all loans issued.
4. The average interest rate on all loans is 15%.

Calculate the lending strategy using matrix inversion.

Problem 4

A company manufactures four variants of the same product and in the final part of the manufacturing process there are assembly, polishing and packing operations. For each variant the time required for these operations is shown below (in minutes) as is the profit per unit sold.

	Assembly	Polish	Pack	Profit($)
Variant 1	2	3	2	1.50
2	4	2	3	2.50
3	3	3	2	3.00
4	7	4	5	4.50

Given the current state of the labor force the company estimate that, each year, they have 100000 minutes of assembly time, 50000 minutes of polishing time and 60000 minutes of packing time available.

Write these constrains in the form of $Ax \leq b$. Non-negative constrains are excluded.

Problem 5

Add the nonnegative constraints in form of $Ax \leq b$ to problem 4.

Deliverables

Scan your homework and name it as **hw1_x.pdf** (where x is your eid) and submit it online.