Homework 1

① $(A^{-1})^T = (A^T)^{-1}$

Let $B = A^{-1}$ such that $AB = BA = I$,

By definition: $AA^{-1} = I$, $(AB)^T = B^T A^T$

$\Rightarrow B^T = (A^T)^{-1}$

$B^T A^T = (A^T)^{-1}(A^T)$

↳ $(AB)^T$

↳ $(AA^{-1})^T$

↳ $I^T = I$ $\quad I = I$ □

↘ Let $C = A^T$.

$(A^T)^{-1}A^T = C^{-1}C$

$= I$

② $A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 3 \\ 3 & -2 & -5 \end{bmatrix}$ $\quad B = \begin{bmatrix} 3 & -6 & -3 \\ 7 & -14 & -7 \\ -1 & 2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 3\cdot 3 - 7 - 2 & -18+14+4 & -9+7+2 \\ 3-3 & -6+6 & -3+3 \\ 9-14+5 & -18+28-10 & -9+14-5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -6 & 3 & 3 \\ -14 & 7 & 7 \\ 2 & -1 & -1 \end{bmatrix}$$

$AB \neq BA$

(3) x_1 = # mortgages (first)

x_2 = # second mortgages

x_3 = # home improvements

x_4 = # personal overdrafts

$$0.14x_1 + 0.2x_2 + 0.2x_3 + 0.1x_4 = 0.15(x_1 + x_2 + x_3 + x_4)$$

$$x_1 + x_2 + x_3 + x_4 = 250M$$

$$x_1 = 0.55(x_1 + x_2) \Rightarrow 0 = -0.45x_1 + 0.55x_2$$

$$x_2 = 0.25(x_1 + x_2 + x_3 + x_4) \Rightarrow 0 = 0.25x_1 - 0.75x_2 + 0.25x_3 + 0.25x_4$$

Solve: $\underset{A}{\begin{bmatrix} -0.01 & 0.05 & 0.05 & -0.05 \\ 1 & 1 & 1 & 1 \\ -0.45 & 0.55 & 0 & 0 \\ 0.25 & -0.75 & 0.25 & 0.25 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 250M \\ 0 \\ 0 \end{bmatrix}}$

$x_1 = \$76,388,888.89$, $x_2 = \$62,500,000$, $x_3 = \$31,944,444.44$, $x_4 = \$79,166,666.67$

↳ 30.56% ↳ 25% ↳ 12.78% ↳ 31.67%

(4) $2v_1 + 4v_2 + 3v_3 + 7v_4 \leq 100,000$

$3v_1 + 2v_2 + 3v_3 + 4v_4 \leq 50,000$

$2v_1 + 3v_2 + 2v_3 + 5v_4 \leq 60,000$

(5) $v_1 \geq 0$, $v_2 \geq 0$, $v_3 \geq 0$, $v_4 \geq 0$

(v_1, v_2, v_3, v_4 are all integers)