① Choose x_1, x_2

max $4x_1 + 5x_2$

s.t $2x_1 + 3x_2 \leq 60$

$x_1 \geq 0$

$x_2 \geq 0$

$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$

$\hookrightarrow x_1 = 30, x_2 = 0 \Rightarrow 120$ points

b) new restriction:

$x_2 \geq x_1 \Rightarrow 0 \geq x_1 - x_2$

$A \qquad x$

$\begin{bmatrix} 2 & 3 \\ 0 & -1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 60 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\hookrightarrow x_1 = 12, x_2 = 12 \Rightarrow 108$ points

$120 - 108 = \boxed{12 \text{ fewer points}}$

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② $x_1 + x_2 \leq 450$

$3x_1 + 2x_2 \leq 1000$

$2x_1 + 4x_2 \leq 1200$

Choose x_1, x_2

max $2000x_1 + 3000x_2$

$x_1, x_2 \geq 0$

$\dfrac{1000 - 3x_1}{2} = \dfrac{1200 - 2x_1}{4}$

$500 - \dfrac{3}{2}x_1 = 300 - \dfrac{1}{2}x_1$

$200 - x_1 = 0$

$\boxed{200 = x_1}$

$\dfrac{1000 - 3(200)}{2} = \dfrac{1000 - 600}{2} = 200$

$\boxed{x_2 = 200}$

b) Solved with R ✓

x_2 Corn

2000

1000

355   1000   2000   3000 x_1

wheat

c) duals. from   to   (from R)

	from	to	
0	$-\infty$	∞	
250	600	1200	
625	667	1600	

→ He should discontinue wheat when there are less than 667 tons of fertilizer available. (700)

→ He should discontinue corn when there are more than 1600 tons available.

→ Profit increases by $6250 for every 10-unit increment.
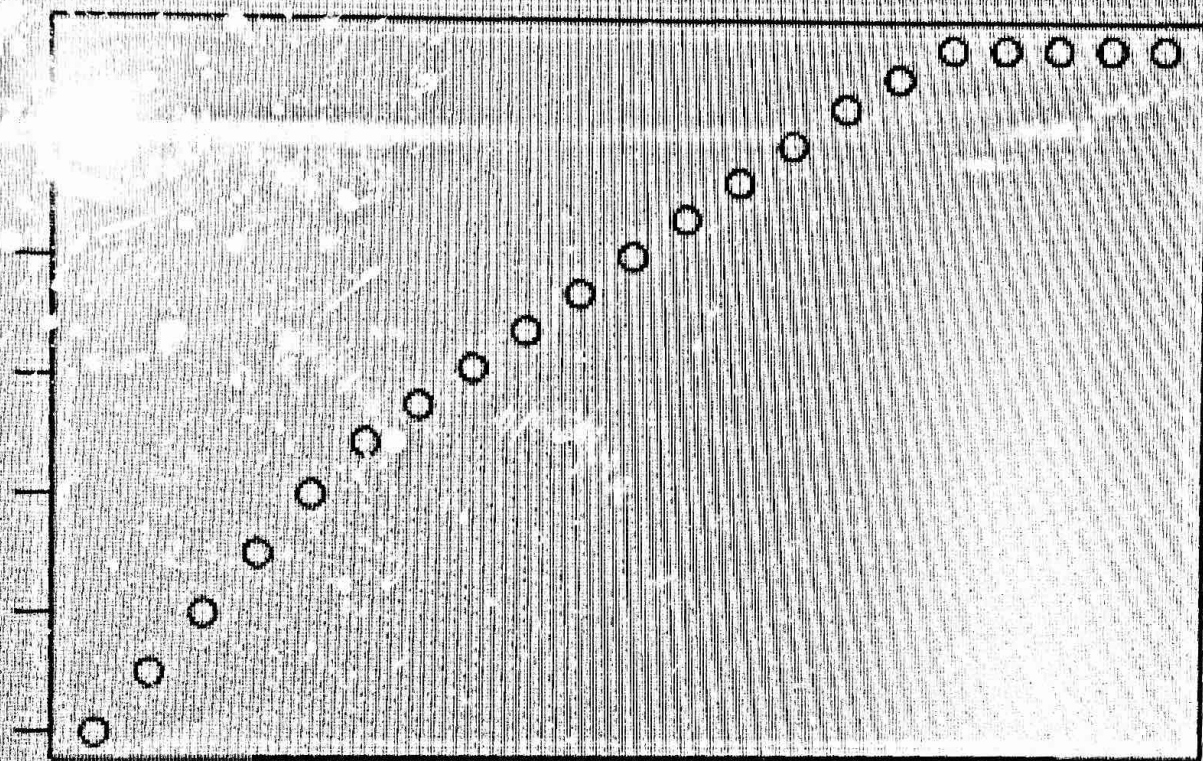
③ Choose x_1, x_2, x_3, x_4, x_5

max $13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$

st $0 \leq x_i \leq 1$, x_i represents fractions of investments

$11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 40$

$3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 \leq 20$

$\boxed{x_1 = 1, x_2 = 0.2, x_3 = 1, x_4 = 1, x_5 = 0.288}$

$\Rightarrow \$57.432M$

④ choose C, M, B

obj min $0.18C + 0.23M + 0.05B$

Constraints:

$C, M, B \leq 10$

$2000 \leq 72C + 121M + 65B \leq 2250$

$3000 \leq 107C + 500M \leq 50000$

$\boxed{C = 1.94, M = 10, B = 10}$

Cost $= \$3.15$

⑤ Choose $x_1, x_2, x_3, y_1, y_2, y_3$

max $\Sigma x_i + \Sigma y_i$

st $x_1, y_1 \leq 1$

$x_2 \leq 1.3, y_2 \leq 1.2$

$x_3 \leq 1.4, y_3 \leq 1.6$

$1.2 \leq x_1 + y_1 \leq 2$

$1.5 \leq x_2 + y_2 \leq 2$

$2 \leq x_3 + y_3 \leq 3$

$\boxed{x_1 = 1, x_2 = 1.3, x_3 = 1.4, y_1 = 1, y_2 = 0.7, y_3 = 1.6}$

$\Rightarrow 7$ tons