① Max $-x_1 + 4x_2$

s.t $-10x_1 + 20x_2 \leq 22$

$5x_1 + 10x_2 \leq 49$

$x_1 \leq 5$

$x_i \geq 0$, x_i are integers

$LP_1 \Rightarrow x_1 = 3.8, x_2 = 3 \Rightarrow 8.2$

$x_1 \geq 4$
$\rightarrow x_1 = 4, x_2 = 2.9$
$\Rightarrow 7.6$

$x_1 \leq 3$
$\rightarrow x_1 = 3, x_2 = 2.6$
$\Rightarrow 7.4$

$+$
$x_2 \leq 2$
$\rightarrow x_1 = 4,$
$x_2 = 2$
$\rightarrow 4$

$x_2 \geq 3$
$\rightarrow$ Infeasible

$x_2 \leq 2$
$\rightarrow x_1 = 1.8$
$x_2 = 2$
$\Rightarrow 6.2$

$x_2 \geq 3$
$\rightarrow$ Infeasible

$x_1 \leq 1$
$x_1 = 1.0$
$x_2 = 1.6$
$\Rightarrow 5.4$

$x_1 \geq 2$
$\rightarrow x_1 = 2, \Rightarrow 6$
$x_2 = 2$

② There are only 2 feasible solutions $(4,2) + (2,2)$

③ There is one feasible solution per branch.

② Choose x_1, x_2, x_3, x_4 (binary)

max $9x_1 + 5x_2 + 6x_3 + 4x_4$

s.t
$$\sum x_i \leq 11 \Rightarrow 6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 11$$

$$x_3 + x_4 \leq 1$$

$$x_1 + x_2 \geq 1$$

Build a factory in Austin and Dallas, and a warehouse in Dallas.

Profit = $18M

③ Choose $x_1, \dots, x_{12}$ (binary — hub/not)

min $\sum x_i$

s.t

$$x_1 + x_3 + x_5 + x_7 + x_8 + x_9 \geq 1$$
$$x_2 + x_8 + x_9 \geq 1$$
$$x_1 + x_3 + x_7 + x_8 + x_9 \geq 1$$
$$x_4 + x_{10} \geq 1$$
$$x_1 + x_4 + x_6 \geq 1$$
$$x_6 + x_{10} + x_{11} \geq 1$$
$$x_1 + x_3 + x_5 + x_7 \geq 1$$
$$x_1 + x_2 + x_3 + x_8 + x_9 \geq 1$$
$$x_4 + x_6 + x_{10} + x_{11} + x_{12} \geq 1$$
$$x_6 + x_{10} + x_{11} + x_{12} \geq 1$$
$$x_{10} + x_{11} + x_{12} \geq 1$$

$\Rightarrow$ ATL, SLC, NY

④ Choose x_1, x_2, x_3, y_1

    $\underbrace{}_{\text{number of each width}}$ $\underbrace{}_{\text{number of 120 inch rolls}}$

 min $120y_1 - (25x_1 + 37x_2 + 54x_3)$

 s.t

   $25x_1 + 37x_2 + 54x_3 \leq 120y_1$

   $x_1 \geq 233$

   $x_2 \geq 148$

   $x_3 \geq 106$

$\Rightarrow x_1 = 242, \; x_2 = 148, \; x_3 = 211, \; x_4 = 191$

   0 waste!

⑤ Choose $x_1, x_2, \ldots x_7$

    $\underbrace{}_{\text{number of people who start on each day}}$

 min $330x_1 + 300x_2 + 330x_3 + 360x_4 + 360x_5 + 360x_6 + 360x_7$

 s.t

   $x_1 + x_4 + x_5 + x_6 + x_7 \geq 5$

   $x_2 + x_5 + x_6 + x_7 + x_1 \geq 13$

   $x_3 + x_6 + x_7 + x_1 + x_2 \geq 12$

   $x_4 + x_7 + x_1 + x_2 + x_3 \geq 10$

   $x_5 + x_1 + x_2 + x_3 + x_4 \geq 14$

   $x_6 + x_2 + x_3 + x_4 + x_5 \geq 8$

   $x_7 + x_3 + x_4 + x_5 + x_6 \geq 6$

$\Rightarrow \boxed{x_1 = 1, \; x_2 = 8, \; x_3 = 2, \; x_4 = 0, \; x_5 = 3, \; x_6 = 0, \; x_7 = 1}$

It is less expensive for someone to start on Monday (M-F).