Thomas H. Jørgensen

2023

Introduction

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• Dynamic Labor supply w. HC and **children**Adda, Dustmann and Stevens (2017): "The Career Costs of Children"

# Plan for today

Introduction •000

- Dynamic Labor supply w. HC and children Adda, Dustmann and Stevens (2017): "The Career Costs of Children"
- Reading guide:
  - What are the main research questions?
  - What is the (empirical) motivation?

3 What are the central mechanisms in the model?

What is the simplest model in which we could capture these?

Introduction

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- Dynamic Labor supply w. HC and children
   Adda, Dustmann and Stevens (2017): "The Career Costs of Children"
- Reading guide:
  - 1 What are the main research questions?
    - How **costly** are children for careers over the life cycle?
    - How does pro-fertility reforms affect completed fertility?
  - What is the (empirical) motivation?

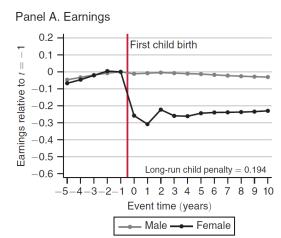
3 What are the central mechanisms in the model?

4 What is the simplest model in which we could capture these?

Introduction

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• "Child penalty" (Kleven, Landais and Søgaard, 2019)



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### Empirical Motivation: II

Introduction

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TABLE 1 DESCRIPTIVE STATISTICS, BY OCCUPATION

	Routine	Abstract	Manual	Whole Sample
Initial occupation	25.0%	44.8%	30.3%	100%
Occupation of work	25.4%	52.7%	21.9%	
A				
Annual occupational transition rates:				
If in routine last year	97.9%	1.5%	.5%	
If in abstract last year	.7%	99.0%	.2%	
If in manual last year	.9%	.8%	98.3%	
B				
Log wage at age 20	3.598	3.742	3.470	3.634
	(.297)	(.301)	(.386)	(.337)
Log wage growth, at potential	,	,	,,	,
experience = 5 years	.0485	.0551	.0450	.051
	(.187)	(.156)	(.196)	(.175)
Log wage growth, at potential	(,	( /	,,	, ,
experience = 10 years	.0181	.0240	.0152	.020
	(.187)	(.206)	(.223)	(.206
Log wage growth, at potential	()	(	(/	(
experience = 15 years	.00995	.0147	.0127	.013
	(.206)	(.195)	(.211)	(.200
C	()	()	()	(
Total work experience after 15 years	11.55	12.81	12.14	12.34
·· <del> </del>	(3.273)	(2.624)	(2.880)	(2.909)
Full-time work experience after 15 years	10.32	11.92	10.86	11.29
	(3.907)	(3.348)	(3.570)	(3.617
Part-time work experience after 15 years	1.229	.889	1.274	1.056
	(2.187)	(1.828)	(2.125)	(1.997
D	(2.201)	(1.020)	(=:==0)	(2.00)
Total log wage loss, after interruption = 1 year	0968	147	105	121
	(.560)	(.636)	(.633)	(.613)
Total log wage loss, after interruption = 3 years	152	253	223	216
zour rog mgc ross, accer interruption — 5 years	(.604)	(.639)	(.619)	(.625)
E	(.501)	(1355)	(.515)	(1040)
Age at first birth	27.27	28.39	25.94	27.56
- Be at mot onen	(4.138)	(3.783)	(3.517)	(3.943)

Introduction

- Selection into family friendly occupations
  - $\rightarrow$  correlation  $\neq$  causation!
  - $\rightarrow$  we need a model!

• **Short run** effects of pro-fertility reforms on labor supply are substantial Reduced form evidence Long run effects: "need" a model!

### Outline

Model and Mechanisms

Simulation Results

Simple Mode

#### Model Overview

#### Choices:

```
b_t \in \{0,1\}: fertility, try to conceive a child c_t \in (0,\overline{M}]: consumption (household) o_t \in \{1,2,3\}: occupation of women (effect end of period) l_t \in \{\mathsf{OLF},\mathsf{U},\mathsf{PT},\mathsf{FT}\}: labor supply of women (effect end of period) (and o_0 initial occupation/education)
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```

### • States, $\Omega_t$ :

```
A_{t-1}: wealth, no borrowing h_{t-1} \in \{0,1\}: presence of husband last period age_t^M: age of woman x_t: human capital of women n_t: number of children age_t^K: age of youngest child
```

 $f = (f^P, f^F, f^L, f^C)$ : heterogeneity

 $Y_t$ : preference and income shocks (see e.g. p.331)

 $l_{t-1}$  and  $o_{t-1}$ : Past labor market

(productivity, fertility, taste for leisure, taste for children)

#### Model Overview

### Key mechanisms:

- **Early life occupational choices** (age 15) locks in on fertility effects
  - Family friendliness differs across occupations

Wage level/growth

Costs of temporary leave (atrophy)

Offer probabilities

Amenities (utility value)

- Human capital leads to persistent effects of early life behavior
- Endogenous fertility trades of timing of children and career

# State Transitions, $\Omega_{t+1} \sim \Gamma(\Omega_t, b_t, c_t, o_t, l_t)$

•  $I_t$  and  $o_t$  are choices.

$$egin{aligned} A_t &= (1+r)A_{t-1} + \textit{net}(\textit{GI}_t; \textit{h}_t, \textit{n}_t) \ &- c_t - \kappa(\textit{age}_t^M, \textit{n}_t) \mathbf{1}(\textit{n}_t > 0, \textit{I}_t \in \{\mathsf{PT}, \mathsf{FT}\}) \end{aligned}$$

where  $GI_t$  is gross household income (next slide)

- $h_t$  is random and function of  $age_t^M$ ,  $x_t$ ,  $f^C$ . Husbands earnings are exogenous function of women's characteristic
- $age_{t+1}^{M} = \frac{1}{2} + age_{t}^{M}$

$$x_{t+1} = \left\{ egin{array}{ll} x_t + 1 & ext{if } I_t = \mathsf{FT} \ x_t + rac{1}{2} & ext{if } I_t = \mathsf{PT} \ x_t 
ho(x_t, o_t) & ext{else} \end{array} 
ight.$$

where  $\rho(x_t, o_t)$  is deprecation rate.

- $n_{t+1} = n_t + 1$  with prob.  $\pi(age_t^M, f^C)\mathbf{1}(b_t = 1)$ . Else  $n_{t+1} = n_t$ .
- $age_{t+1}^{K} = \frac{1}{2} + age_{t}^{K}$  if  $n_{t+1} = n_{t}$  else  $age_{t+1}^{K} = 0$

### State Transitions, Gross Income

- Never write up the gross income. This is my take.
- Gross household income is (note timing)

$$GI_t = w_t I_{t-1} + \mathbf{1}(h_t = 1) earn_t^h + \text{benefits...}$$

Husbands earnings are exogenous

$$earn_t^h = \alpha_0^h + \alpha_1^h age_t^M + \alpha_2^h (age_t^M)^2 + \alpha_P^h f^P + \eta_t^h$$

Wages of women are Mincer-type

$$\log w_t = f^P + a_0(o_t) + a_X(o_t)x_t + a_{XX}(o_t)x_t^2 + \eta_t$$

#### • Bellman equation is

$$\begin{split} V_t(\Omega_t) &= \max_{b_t, c_t, o_t, l_t} u(\bullet) + \beta \mathbb{E}_t[V_{t+1}(\Omega_{t+1})] \\ \text{s.t.} \\ \Omega_{t+1} &\sim \Gamma(\Omega_t, b_t, c_t, o_t, l_t) \end{split}$$

#### Recursive Formulation

#### Bellman equation is

$$egin{aligned} V_t(\Omega_t) &= \max_{b_t, c_t, o_t, l_t} u(ullet) + eta \mathbb{E}_t[V_{t+1}(\Omega_{t+1})] \ & ext{s.t.} \ &\Omega_{t+1} \sim \Gamma(\Omega_t, b_t, c_t, o_t, l_t) \end{aligned}$$

### Implemented "sequentially"

Split up the different discrete choices. See their appendix. I will illustrate how the fertility choice is made (conditional on  $o_t$ ,  $I_t$ )

#### Recursive Formulation

• Bellman equation is

$$\begin{split} V_t(\Omega_t) &= \max_{b_t, c_t, o_t, l_t} u(\bullet) + \beta \mathbb{E}_t \big[ V_{t+1}(\Omega_{t+1}) \big] \\ \text{s.t.} \\ \Omega_{t+1} &\sim \Gamma(\Omega_t, b_t, c_t, o_t, l_t) \end{split}$$

- Implemented "sequentially"
   Split up the different discrete choices. See their appendix.
   I will illustrate how the fertility choice is made (conditional on o<sub>t</sub>, I<sub>t</sub>)
- Discuss: What is good/bad about this model for the research purpose?

# Recursive Formulation, working + conceiving

ullet Value of **W**orking ( $I_t \in \{\mathsf{PT},\mathsf{FT}\}$ ) and **C**onceiving ( $b_t = 1$ )

$$\begin{split} V^{W,C}(\Omega_t) &= \max_{c_t} u(c_t, \bullet) + \pi(age_t^M, f^C) \beta \mathbb{E}_t[V^{L_w}(\Omega_{t+1}^P)] \\ &+ \delta(1 - \pi(age_t^M, f^C)) \beta \mathbb{E}_t[V^U(\Omega_{t+1})] \\ &+ (1 - \delta)(1 - \pi(age_t^M, f^C))(1 - \phi_0(o_t, I_t)) \beta \textit{Emax}_t \\ &+ (1 - \delta)(1 - \pi(age_t^M, f^C)) \phi_0(o_t, I_t) \beta \widetilde{\textit{Emax}}_t \end{split}$$

```
where V^{L_w}(\Omega^P_{t+1}): value of parental leave V^U(\Omega_{t+1}): value of unemployment (w. prob \delta) \phi_0(o_t, l_t): job-offer prob.  E_{max_t} = \mathbb{E}_t [\max\{V^W_{t+1} + \eta^W_{t+1}, V^U_{t+1} + \eta^U_{t+1}, V^O_{t+1} + \eta^O_{t+1}\}]  E_{max_t} = \dots Also choose between leaving and staying (see p. 331).
```

### Recursive Formulation, working + not conceiving

• Value of **W**orking  $(I_t \in \{PT, FT\})$  and **N**ot **C**onceiving  $(b_t = 0)$ 

$$\begin{split} V^{W,NC}(\Omega_t) &= \max_{c_t} u(c_t, \bullet) \\ &+ \delta \beta \mathbb{E}_t [V^U(\Omega_{t+1})] \\ &+ (1 - \delta)(1 - \phi_0(o_t, I_t))\beta \textit{Emax}_t \\ &+ (1 - \delta)\phi_0(o_t, I_t)\beta \widetilde{\textit{Emax}}_t \end{split}$$

• **Fertility choice** is then (conditional on k/working)

$$b_t^{\star}(k) = \arg\max\{V^{k,C}(\Omega_t), V^{k,NC}(\Omega_t)\}$$

### Estimation Results

- Simulated method of moments (SMM)
  - Weighting matrix: diagonal, inverse of variance of empirical moments. 763 moments (Table 2)
- Estimate 88 parameters (allowing for unobserved types)

$$\begin{split} u_{ii} &= \frac{(c_{ii}/\bar{c})^{(1-\gamma c)} - 1}{1 - \gamma c} \exp \left[ \gamma_{PI}^{1} I_{l_{e}=PI} + \left( \gamma_{U}^{1} + f_{i}^{L} \right) I_{l_{e}=U} \right. \\ &+ \left( \gamma_{\text{OLF}}^{1} + f_{i}^{L} \right) I_{l_{e}=\text{OLF}} \right] \exp \left( \gamma_{NC} I_{n_{e}>0} \right) \\ &+ \left[ \gamma_{N}^{1} (f_{i}^{C}) I_{n_{e}=1} + \gamma_{N}^{2} (f_{i}^{C}) I_{n_{e}=2} \right] \cdot \exp \left( \gamma_{NH} I_{n_{e}>0k;h_{e-1}} \right) \\ &\cdot \exp (\gamma_{V)}^{l_{i_{e}=c}} \exp \left( \gamma_{\text{OLF}} + \gamma_{A,\text{OLF}}^{1} I_{\text{age}_{e}^{c}=[0,5]} \right. \\ &+ \gamma_{A,\text{OLF}}^{2} I_{\text{age}_{e}^{c}=[4,6]} + \gamma_{A,\text{OLF}}^{3} I_{\text{age}_{e}^{c}=[7,9]} \right)^{l_{i_{e}=AS}} \\ &\cdot \exp \left( \sum_{i=1}^{3} \gamma_{i_{e},PI} I_{a_{e}=i_{e}} + \gamma_{A,PI}^{1} I_{\text{age}_{e}^{c}=[7,9]} \right)^{l_{i_{e}=FI}} \\ &\cdot \exp \left( \sum_{i=1}^{3} \gamma_{i_{e},W} I_{a_{e}=i_{e}} \right)^{l_{i_{e}=FI}} + \eta_{ii}^{C} b_{ii} + \eta_{ii}^{NC} (1-b_{ii}). \end{split}$$

# **Estimation Results**

TABLE 3

Parameter	Routine	Abstract	Manual
	A. Atrophy Rates Parameters (Annual Depreciation Rates)		
At 3 years of uninterrupted work experience	06% (1e-5%)	11% (2e-5%)	03% (2e-5%)
At 6 years of uninterrupted work experience	50% (.11%)	-6.90% (.17%)	-3.45% (.24%)
At 10 years of uninterrupted work experience	61% (14.2%)	-2.65% (.01%)	-3.08% $(.18%)$
	B. Wage Equation Parameters		
Log wage constant	3.39 (.0038)	3.6 (.0054)	3.32 (.0059)
Years of uninterrupted work experience	.1 (3.3e-05)	.09	.123
Years of uninterrupted work experience, squared	00382 (3e-06)	0021 (4.1e-06)	00463 (6.4e-06)
	C. Amenity Value of Occupations		
Utility of work if children	0	056 (.001)	014 (.0005)
Utility of part-time work if children	0	(.001) 42 (.003)	08 (.007)

### Outline

Model and Mechanisms

Simulation Results

3 Simple Model

- Career costs of children
   NPV difference in income of women
   between baseline and alternative model
- Alternative model: no children (known by all)

#### Simulation Results: Career Costs of Children

- Career costs of children
   NPV difference in income of women
   between baseline and alternative model
- Alternative model: no children (known by all)

TABLE 6

CAREER COST OF CHILDREN: PERCENTAGE LOSS IN NET PRESENT VALUE
OF INCOME AT AGE 15. WITH AND WITHOUT FERTILITY

	Percentage Loss Compared to Baseline
Total cost	-35.3%
	A. Oaxaca Decomposition of Total Cost
Labor supply contribution Wage contribution	-27% -8.5%
	B. Oaxaca Decomposition of Wage Contributions
Contribution of atrophy Contribution of other factors	-1.8% -6.7%
Contribution of occupation Contribution of other factors	$^{-1.6\%}_{-7\%}$

Nore.—The career costs are evaluated using simulations and comparing the estimated model with a scenario in which the woman knows ex ante that she cannot have children. The costs are computed as the net present value of female incomes, including all wages, unemployment benefits, and maternity benefits in the calculations. The discount factor is set to 0.95 annually. Initial occupation is the one in the no-fertility scenario.

#### Simulation Results: Career Costs of Children

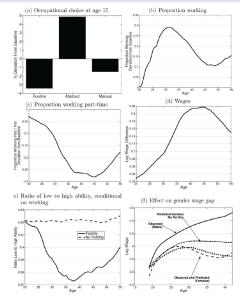


Fig. 3.-Effect of no fertility. The different panels display the difference in outcomes between a baseline scenario and one in which a woman knows that she is infertile.

### Counterfactual reform: Pro-fertility

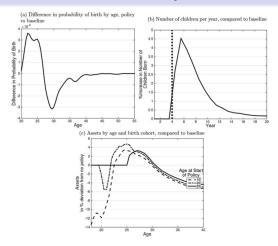


Fig. 4.—Effect of child premium. Panel a shows the effect of the policy (cash transfer of 6,000 at birth) byage on the probability of giving birth, comparing the policy to the baseline. In the policy scenario, women learn at age 15 about the policy. Panel b depicts the aggregate effect of the policy, byear, in an overlapping generation economy. The graph aggregates each year the behavior of women aged 15-60. Each year an new cohort of 15-year-olds enters the economy and the cohort who is 60 exits. The policy starts in year 4. Panel c displays the perentage change in assets as a function of age, compared to a baseline without transfer. The birth cohort who is 15 at the start of the policy can adjust right away their behavior. The cohorts who are 20 or 25 when the policy starts do not anticipate the policy.

### Outline

Model and Mechanisms

Simulation Results

3 Simple Model

# Extending our simple model

• We can extend the simple dynamic model of Keane (2016) Random arrival of a child,  $n_t \in \{0,1\}$  Dis-utility from work depend on children

- We can extend the simple dynamic model of Keane (2016) Random arrival of a child,  $n_t \in \{0, 1\}$  Dis-utility from work depend on children
- Bellman equation

$$V_{t}(n_{t}, a_{t}, k_{t}) = \max_{c_{t}, h_{t}} \frac{c_{t}^{1+\eta}}{1+\eta} - \beta(n_{t}) \frac{h_{t}^{1+\gamma}}{1+\gamma} + \rho \mathbb{E}_{t}[V_{t+1}(n_{t+1}, a_{t+1}, k_{t+1})]$$
s.t.
$$n_{t+1} = \begin{cases} n_{t} + 1 & \text{with prob. } p(n_{t}) \\ n_{t} & \text{with prob. } 1 - p(n_{t}) \end{cases}$$

$$a_{t+1} = (1+r)(a_{t} + (1-\tau_{t})w_{t}h_{t} - c_{t})$$

$$k_{t+1} = k_{t} + h_{t}$$

### Extending our simple model

• Endogenous wages (as before)

$$w_t = w\left(1 + \alpha k_t\right)$$

Simple Model 0000

Endogenous wages (as before)

$$w_t = w \left( 1 + \alpha k_t \right)$$

Probability of a child arriving

$$p(n_t) = \begin{cases} p_n & \text{if } n_t = 0 \\ 0 & \text{if } n_t = 1 \end{cases}$$

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**Dis-utility** from working

$$\beta(n_t) = \beta_0 + \beta_1 n_t$$

such that *original model is nested* if  $\beta_1 = 0$ .

Simple Model 0000

### Extending our simple model

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such that original model is nested if  $\beta_1 = 0$ .

Expected value is

$$\mathbb{E}_{t}[V_{t+1}(n_{t+1}, a_{t+1}, k_{t+1})] = p(n_{t})V_{t+1}(n_{t}+1, a_{t+1}, k_{t+1}) + (1 - p(n_{t}))V_{t+1}(n_{t}, a_{t+1}, k_{t+1})$$

See notebook...

Simple Model 0000

• **Next time:** Labor supply of couples.

Remember: Assignment!

#### Literature:

Borella, De Nardi and Yang (forthcoming): "Are Marriage-Related Taxes and Social Security Benefits Holding Back Female Labor Supply?"

Read before lecture.

Focus on "working-stage of couples" and removal of joint taxation

#### Reading guide:

Section 1: Introduction, Read

Section 2+3: Taxation of Couples in the US (short). *Motivation, key*.

Section 4: Model. Key, but complex. Get the idea. Focus on "working-stage of couples". Think about how children enter.

Section 5: Estimation, Skim.

Section 6: "Validation", short. Labor supply elasticities, read.

Section 7: Counterfactual simulations. Key - Read with focus on 7.1.

Section 8: Sensitivity/robustness. Can drop.

### References I

- ADDA, J., C. DUSTMANN AND K. STEVENS (2017): "The Career Costs of Children," Journal of Political Economy, 125(2), 293–337.
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