Acoustic far-field prediction using robotized measurements and the boundary elements method

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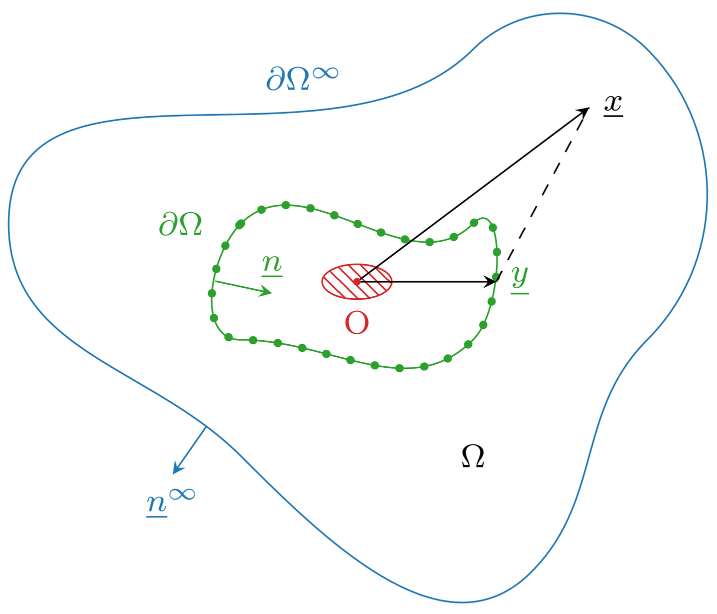
Introduction

Near-field acoustic holography (NAH) has proven to be an useful tool for the identification of sound fields generated by unknown vibro-acoustic sources. The underlying idea behind this method is to extrapolate the seeked acoustic quantities based on a set of localised near-field measurements and an assumed wave behaviour model, using an appropriate reconstruction algorithm [1]. We propose an original acoustic far-field prediction procedure, based on autonomous 3D pressure measurements, carried out with a robotic arm, and on a numerical solution derived from the boundary elements method (BEM).

The boundary elements method (BEM)

Problem modeling, integral operator and varitational formulation



Let $\partial\Omega$ define a closed surface confining an acoustic source O, such that the sationnary *Helmholtz wave equation* with the *Sommerfeld radiation condition* stand for the seeked acoustic pressure field p :

$$\begin{cases} \Delta p(\underline{x}) + k^2 p(\underline{x}) = 0 & \text{in } \Omega \\ p(\underline{x}) = p_0(\underline{x}) & \text{on } \partial\Omega \\ \lim\limits_{\partial\Omega^\infty \to \infty} \left(\frac{\partial}{\partial|x|} - ik \right) p(\underline{x}) = 0 & \end{cases} \quad (1)$$

Where $k \in \mathbb{R}$ is the acoustic wavenumber.

Introducing Helmholtz equation *free-field Green function*, $G(\underline{x}, \underline{y})$, [2] shows that the boundary trace of p can be written using the following integral operator :

$$\exists\, u : \partial\Omega \to \mathbb{C},\ \forall\, x \in \partial\Omega,\ p(\underline{x}) = \frac{1}{2}u(\underline{x}) + \int_{\partial\Omega} \left(\frac{\partial G(\underline{x}, \underline{y})}{\partial \underline{n}(\underline{y})} - ikG(\underline{x}, \underline{y}) \right) u(\underline{y}) d\sigma(\underline{y}) \quad (2)$$

Hence, the strong formulation (1) is equivalent to find p satisfying the weak formulation :

$$\forall\, v : \partial\Omega \to \mathbb{C},\ \int_{\partial\Omega} \left(p_0(\underline{x}) - \frac{1}{2}u(\underline{x}) \right) v(\underline{y}') d\sigma(\underline{y}') = \iint_{\partial\Omega\times\partial\Omega} \left(\frac{\partial G(\underline{x}, \underline{y})}{\partial \underline{n}(\underline{y})} - ikG(\underline{x}, \underline{y}) \right) u(\underline{y}) v(\underline{y}') d\sigma(\underline{y}\underline{y}') \quad (3)$$

Numerical resolution and convergence propeties

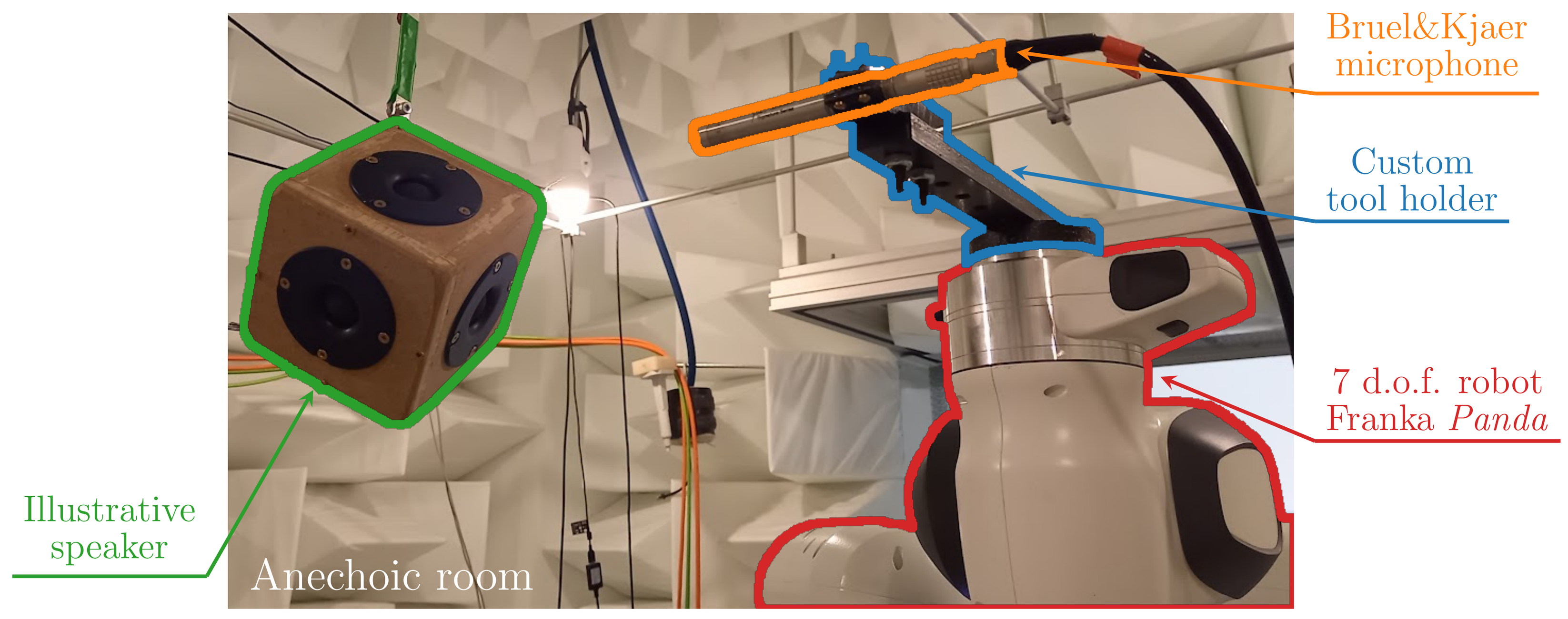
Given a triangular and regular mesh of $\partial\Omega$, and using P_0 Lagrange surfacic elements, (3) can be stated and solved as matrix equations, provided that actual information (e.g. measurements) about p is given at each triangle centroid.

Equation (2) can then easily be used to reconstruct and predict the studied sound field p at any point of Ω, with an l_2 error decreasing as fast as the squared mesh resolution h [3] i.e. as fast as the number of eventual number of measurements.

Both resolution and prediction steps were implemented using FreeFEM *BEM library [5]*

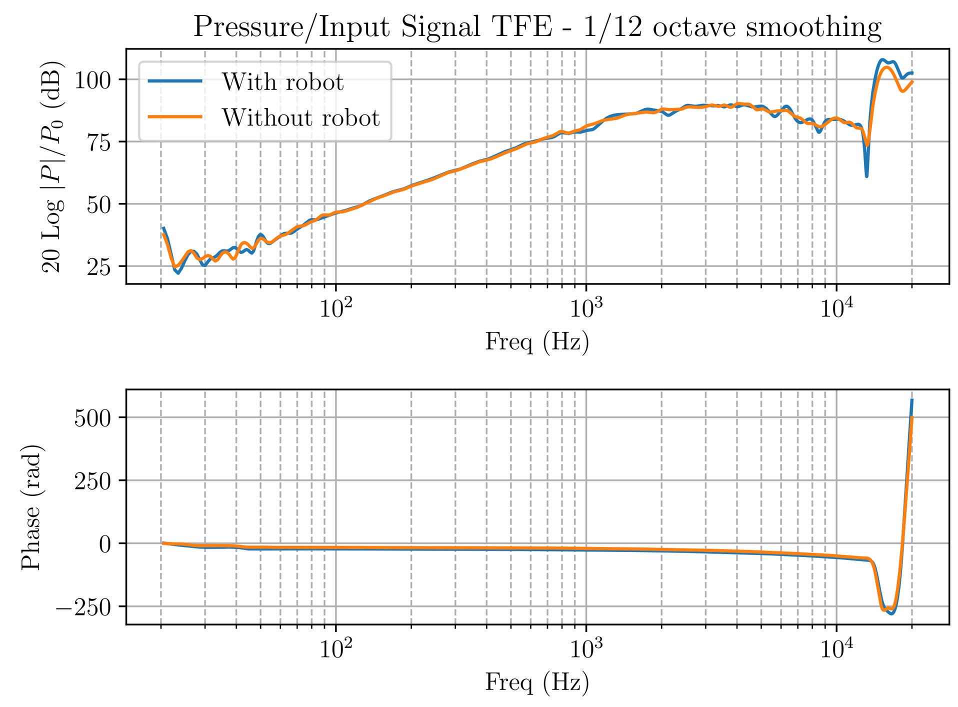
Robotized measurements

Experimental setup



Validity limits of the measurements

→ Reflections and scattering caused by the robot



The actual impact of the robot was assessed in 6 control configurations, in which measurements *with* and *without* the robot were performed.

The results obtained showed that measurements between 50 Hz and 1000 Hz remain below a 1 dB difference compared to the robotless reference.

All results were obtained using a 10 s logsweep signal sampled at 96 kHz, Welch's method and a 12^{th} octave smoothing

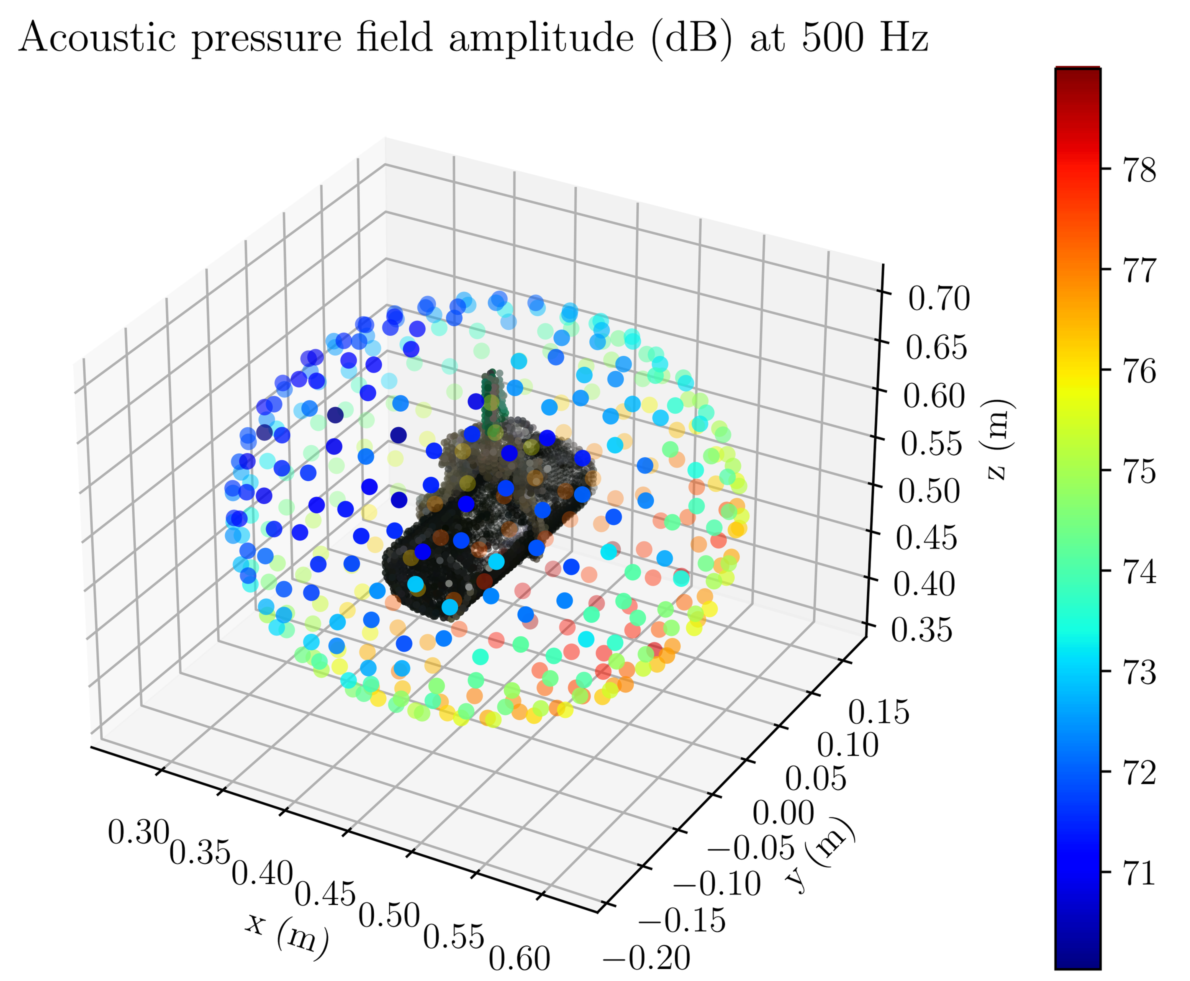
→ Flawed positioning accuracy of the robot

Using the complete calibration procedure presented in [4], the positioning accuracy of the robotic arm was increased to $\pm 2\ mm$, hence allowing measurements to be performed each centimeter with no risk of overlapping.

→ Sound source repetability and stationarity

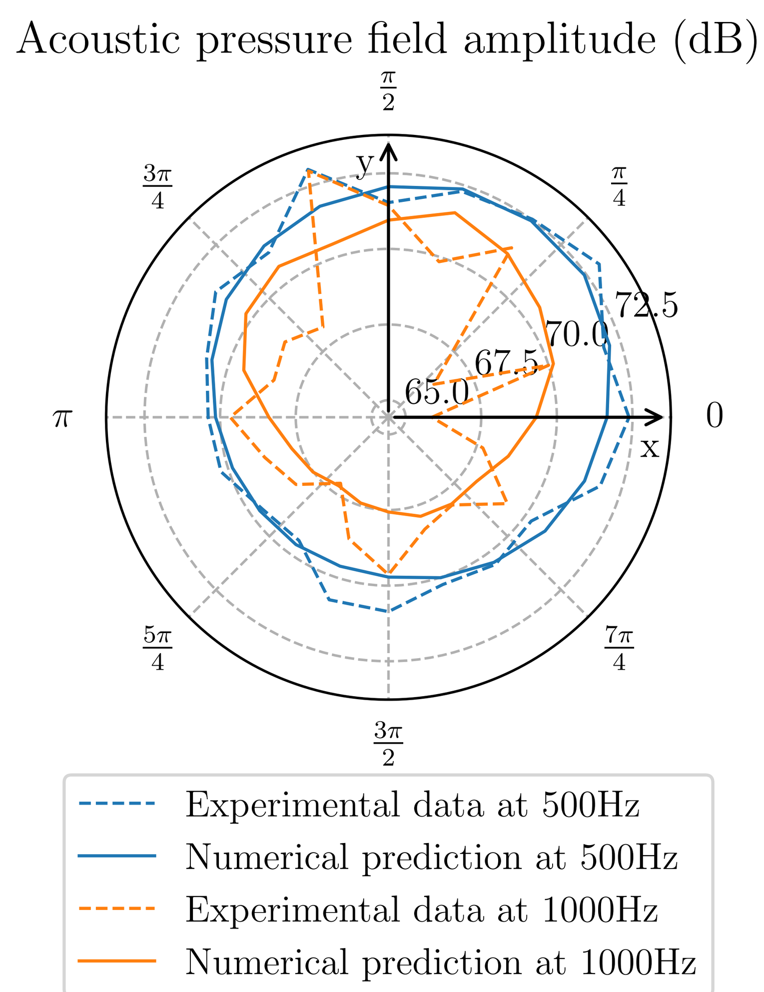
Experiments and results

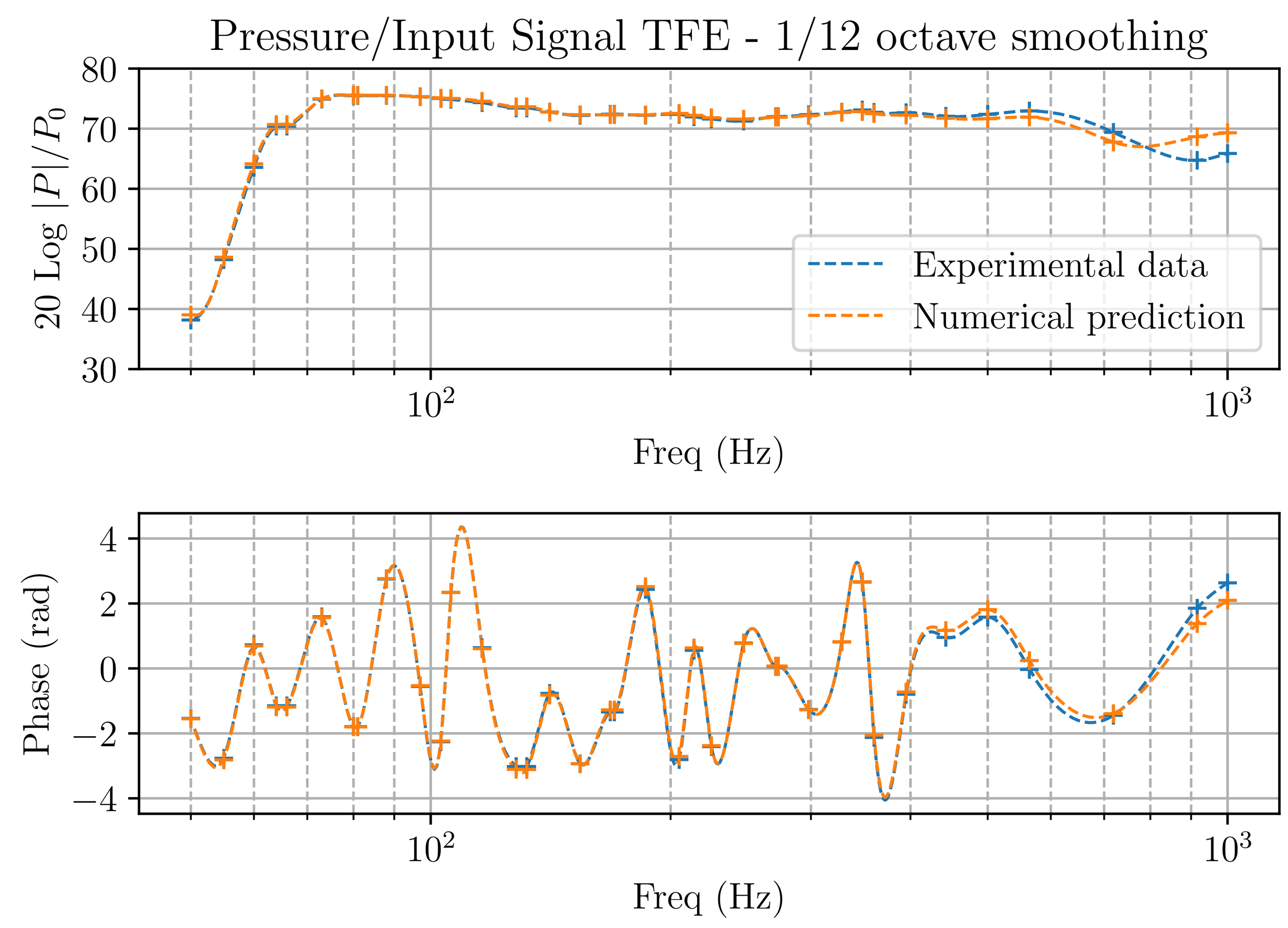
Acoustic pressure measurements



372 measurements carried out with a JBL flip 2 on a spherical mesh of diameter 35 cm and resolution 0.5 cm (total duration : ±2 h)

Prediction results





Sampled prediction results obtained on 20 measurements located on a circular mesh of radius 25 cm at $z = 0$ (left) and detailed data for $\phi = 0$ (right)

References

[1] T. Shi, J. S. Bolton, et W. Thor, "Acoustic far-field prediction based on near-field measurements by using several different holography algorithms", *The Journal of the Acoustical Society of America*, vol. 151, n. 3, p. 2171-2180, 2022.

[2] S. A. Sauter et C. Schwab, "Boundary Element Methods", vol. 39. in Springer Series in Computational Mathematics, Springer, 2011.

[3] S. N. Chandler-Wilde et al., "Numerical-asymptotic boundary integral methods in high-frequency acoustic scattering", *Acta Numerica*, vol. 21, p. 89-305, 2012.

[4] C. Pascal et al., "A ROS-based kinematic calibration tool for serial robots: the unburdeing of a crucial task", *IROS 2023*, 2023 (Submitted for publication).

[5] F. Hecht, "New development in FreeFem++", *Journal of numerical mathematics*, vol. 20, n. 3-4, p. 251-266, 2012.

Perspectives

- → **Investigate high frequency prediction errors**;
- → **Implement and evaluate new reconstruction algorithms**, such as spacial Fourier transform based methods, equivalent elementary sources decomposition methods, or BEM with P_1 elements;
- → **Improve and further automate the measurement process** : investigate robot induced noise canceling solutions, increase robustness and versatility.