

# Computational Modeling - Week 4 - Assignment 2 - Part 1

## Binomial Distributions – Teacher's Rate of Cognitive Science Knowledge

Caroline Casey

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Link to Github: <https://github.com/Carolinecasey17/ComputationalModels.Portfolio2.Part1.git>

### Data:

Riccardo: 3 correct answers out of 6 questions

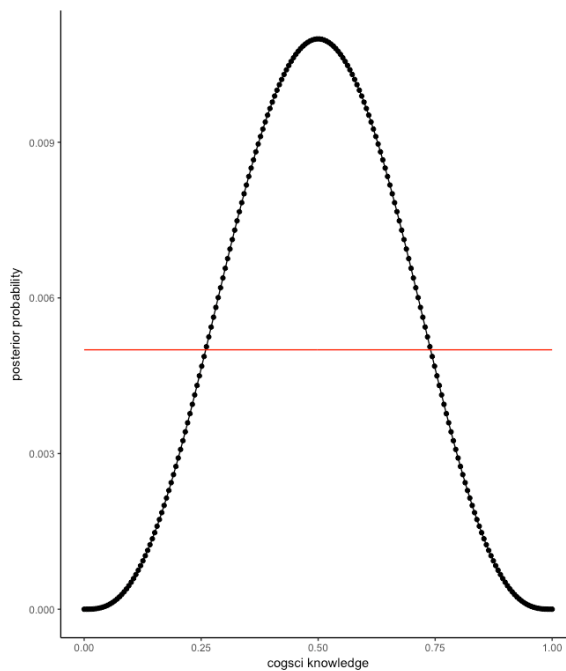
Kristian: 2 correct answers out of 2 questions (then he gets bored)

Josh: 160 correct answers out of 198 questions (Josh never gets bored)

Mikkel: 66 correct answers out of 132 questions

### 1) What's Riccardo's estimated knowledge of Cognitive Science? What is the probability he knows more than chance (0.5)?

When implementing a grid approximation with a uniform prior, and calculating the posterior we see that there is 50 % probability of Riccardo knowing more than chance (0.5), which is very unsure. This is illustrated in *Figure 1*.



*Figure 1*

2) Estimate all the teachers' knowledge of CogSci. Who's best? Use grid approximation. Comment on the posteriors of Riccardo and Mikkel.

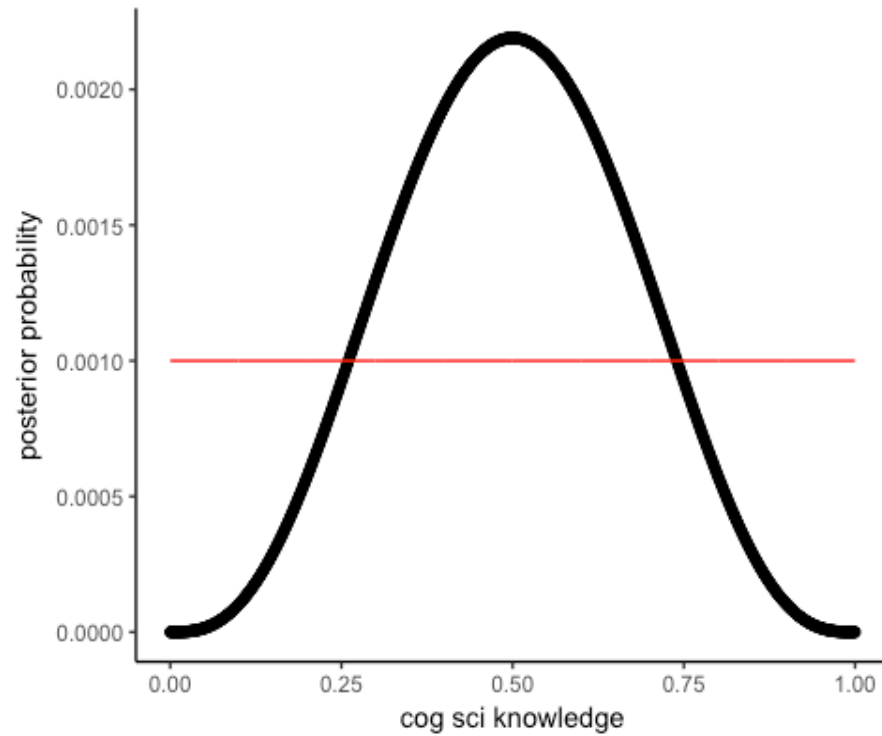
2a. Produce plots of the prior, and posterior for each teacher.

## **RICCARDO:**

Posterior density interval:

|0.5      0.5|  
0.3683684   0.6116116

The highest posterior density interval for Riccardo is from 0.37 to 0.61 Cognitive Science knowledge (50 % interval).



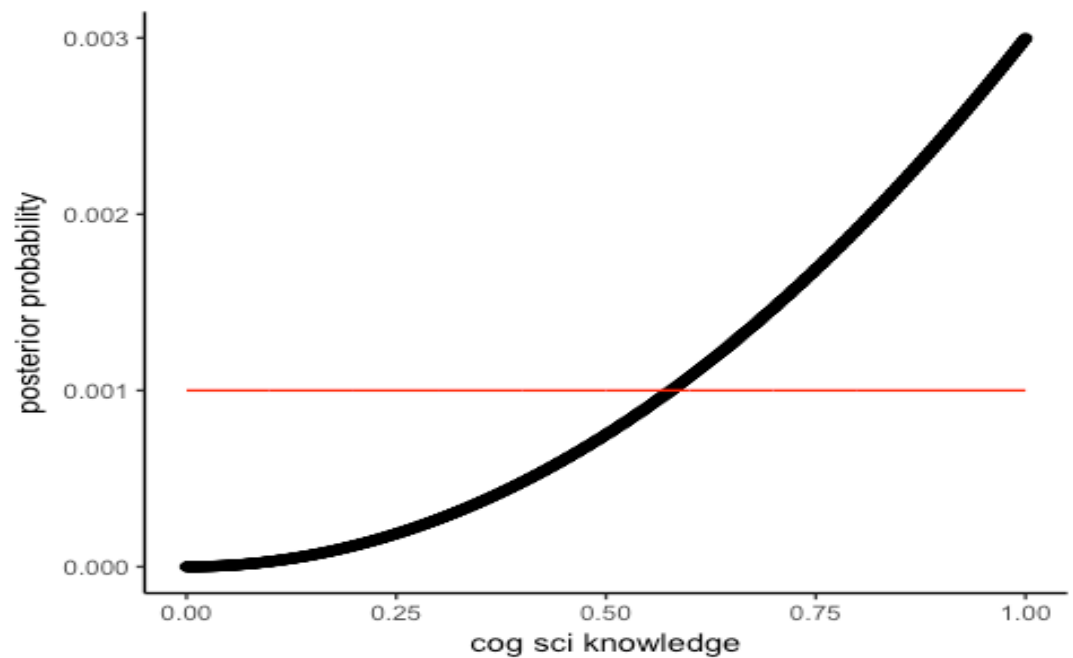
**KRISTIAN:**

Posterior density interval:

|0.5 0.5|

0.7937938 1.0000000

The highest posterior density interval for Kristian is from 0.79 to 1 Cognitive Science knowledge (50 % interval).



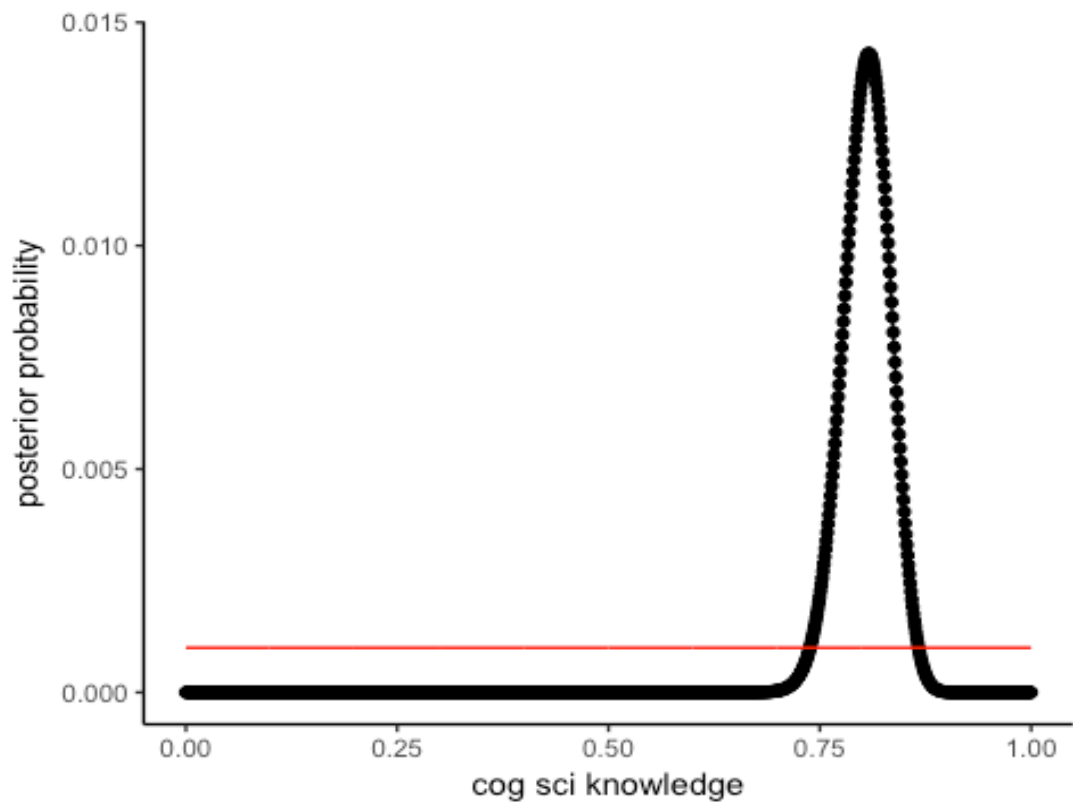
**JOSH:**

Posterior density interval:

|0.5 0.5|

0.7867868 0.8238238

The highest posterior density interval for Josh is from 0.78 to 0.82 Cognitive Science knowledge (50 % interval).



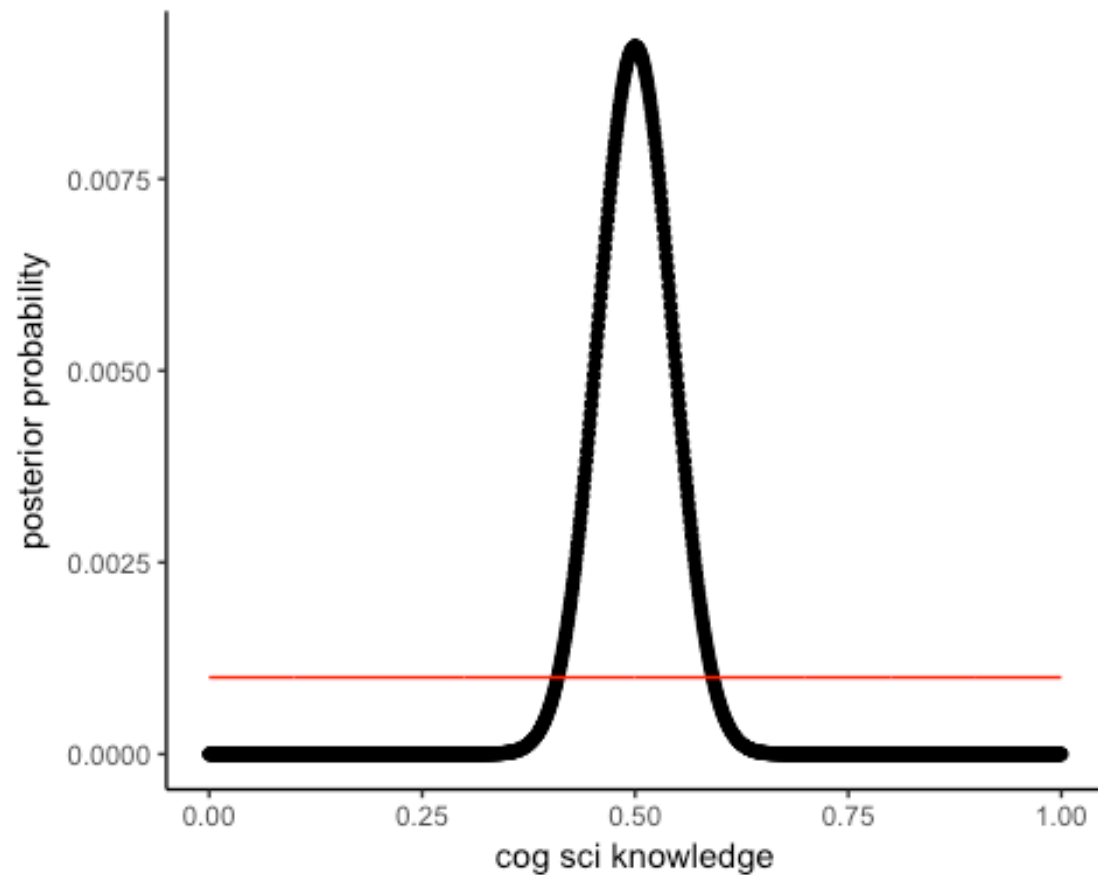
**MIKKEL:**

Posterior density interval:

|0.5    0.5|

0.4694695   0.5265265

The highest posterior density interval for Mikkel is from 0.47 to 0.53 Cognitive Science knowledge (50 % interval).



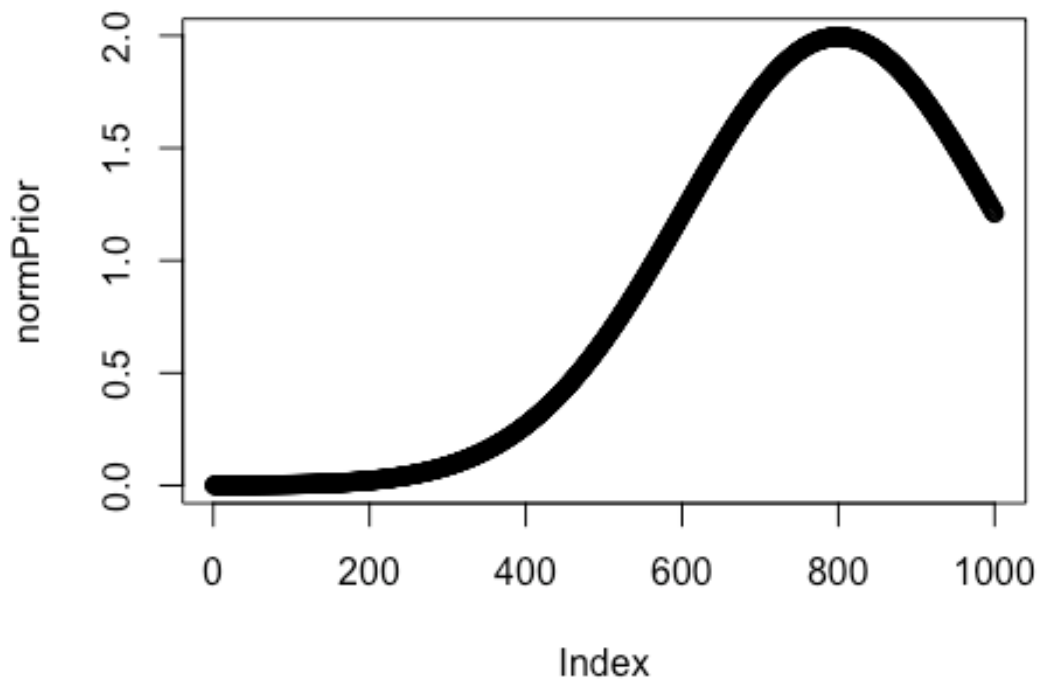
Riccardo and Mikkel both have maximum a posteriori (MAP) values of 50%, however there is much more uncertainty regarding Riccardo's estimate. This is due to Riccardo answering less questions than Mikkel, increasing the uncertainty of the distribution. The 50% most likely values for Riccardo's cog sci knowledge (HPDI) is between 0.37 and 0.61. For Mikkel, these values lie between 0.47 and 0.53.

The 'best' teacher is Kristian. His MAP value is 1. However, this is incredibly hard to determine as he only answered two teachers, so there is very little data to work off. So, there cannot be determined much from his scores.

3) Change the prior. Given your teachers have all CogSci jobs, you should start with a higher appreciation of their knowledge: the prior is a normal distribution with a mean of 0.8 and a standard deviation of 0.2. Do the results change (and if so how)?

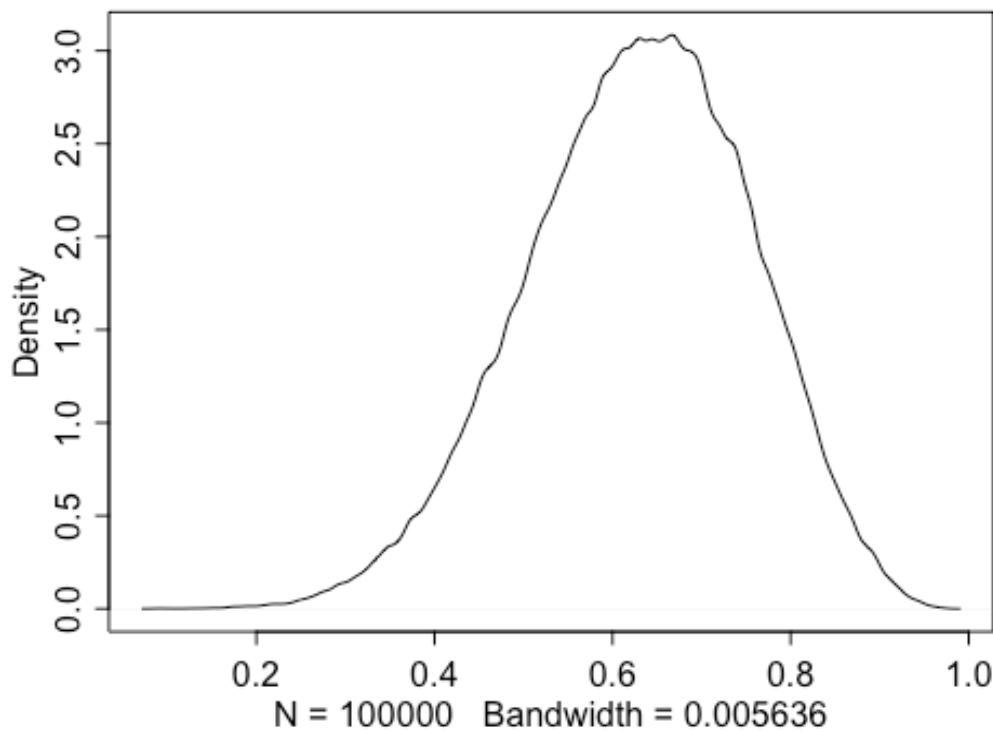
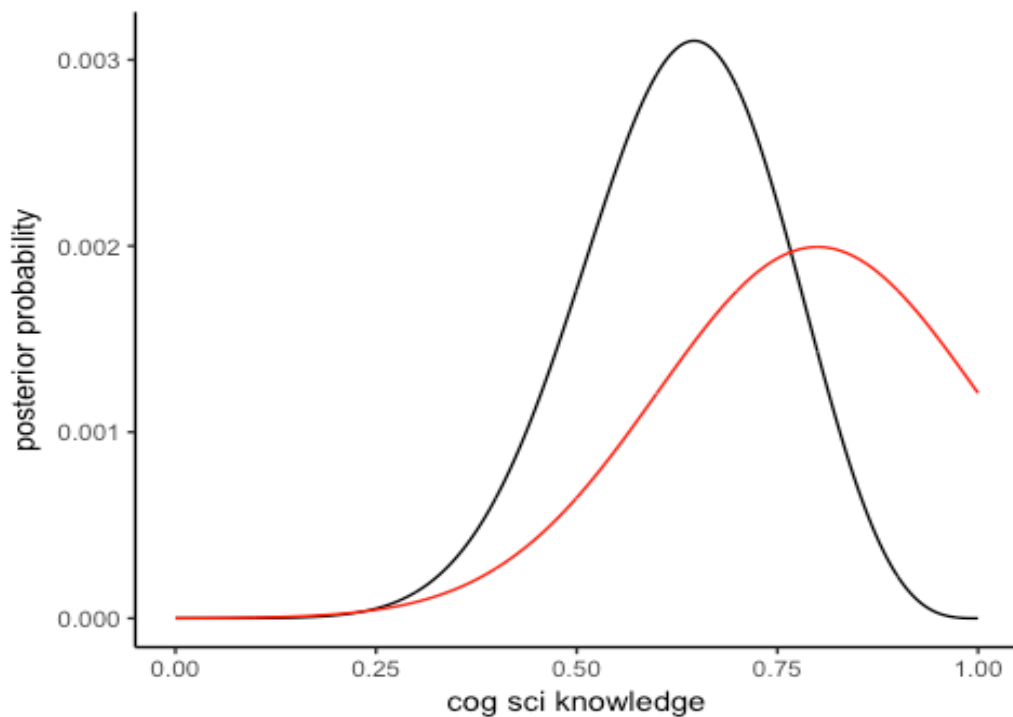
3a. Produce plots of the prior and posterior for each teacher.

Firstly, we make a normally distributed prior with mean 0.8 and sd 0.2.



Secondly, we produce plots of the prior and posterior for each teacher:

## RICCARDO:



### Sampling and HPDI after changed prior:

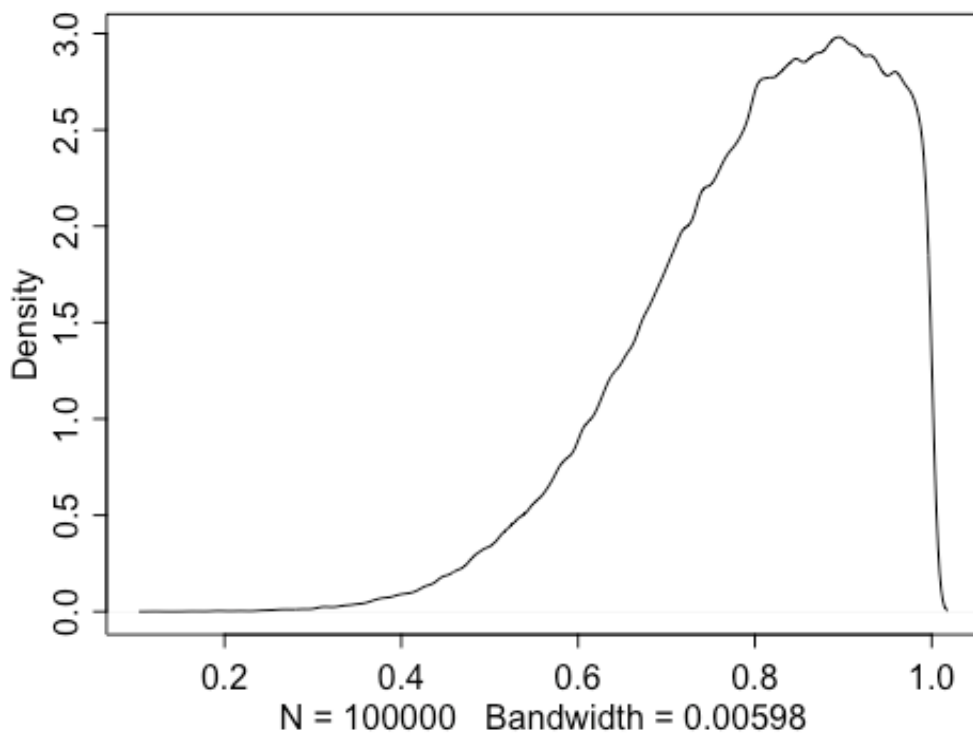
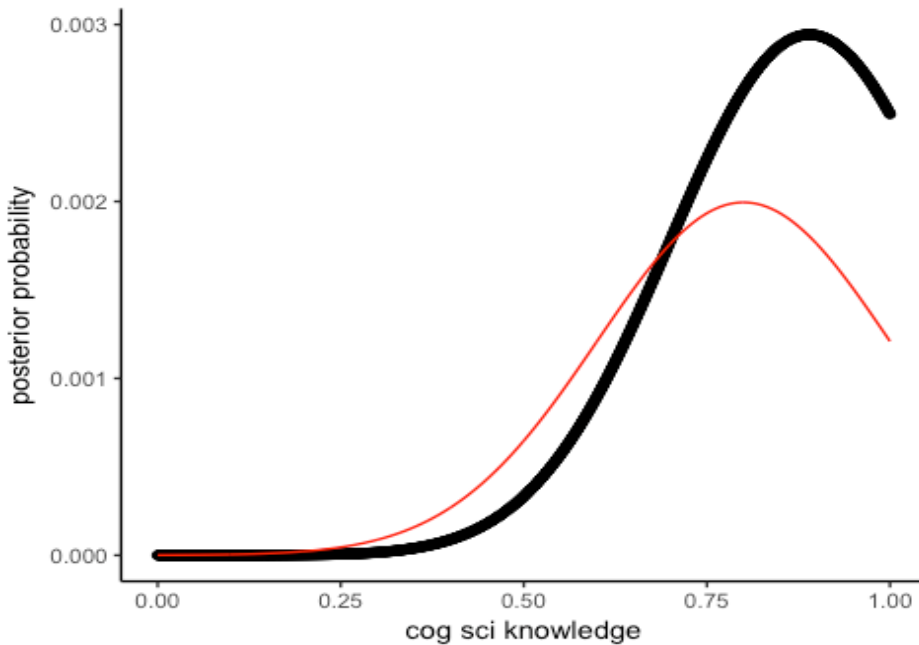
Posterior density interval:

|0.5      0.5|

0.5505506    0.7217217

The highest posterior density interval for Riccardo is from 0.55 to 0.72 Cognitive Science knowledge (50 % interval).

**KRISTIAN:**



**Sampling and HPDI after  
changed prior:**

Posterior density interval:

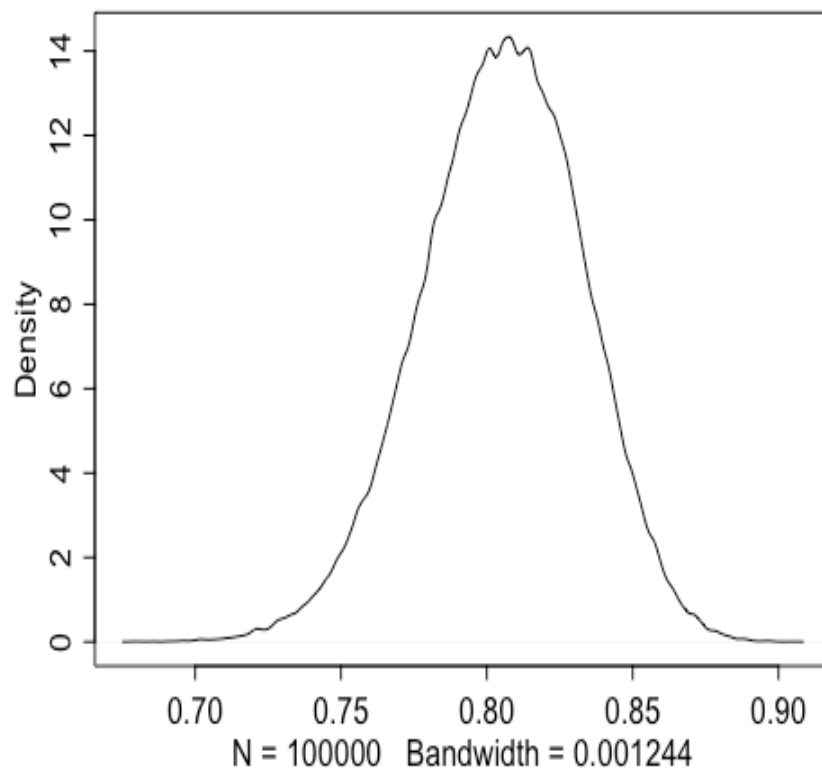
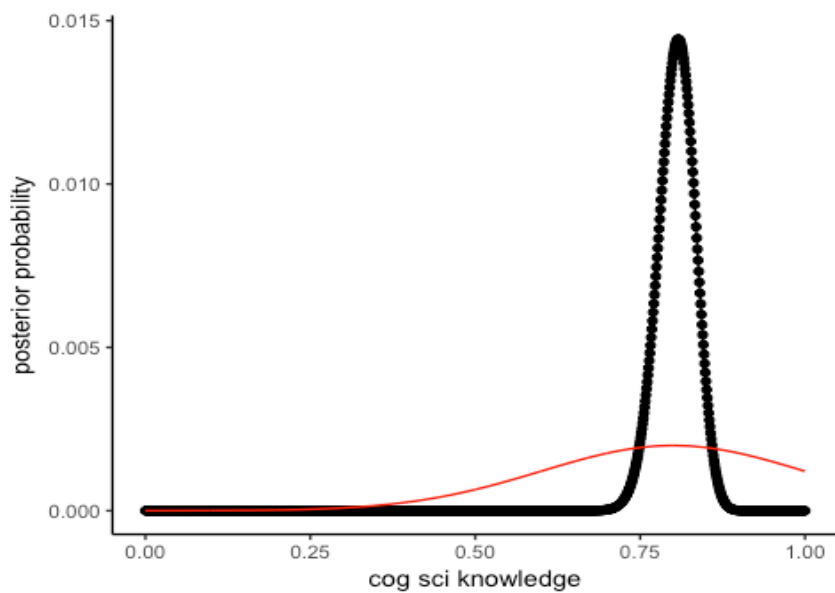
|0.5      0.5|

0.7867868    0.9629630

The highest posterior density  
interval for Kristian is from  
0.79 to 0.96 Cognitive Science  
knowledge (50 % interval).



**JOSH:**

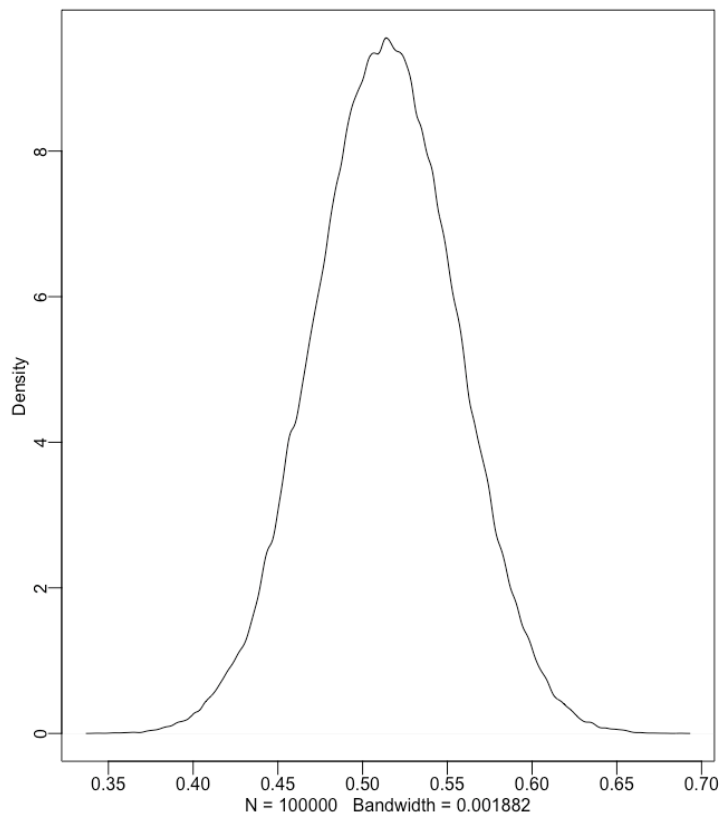
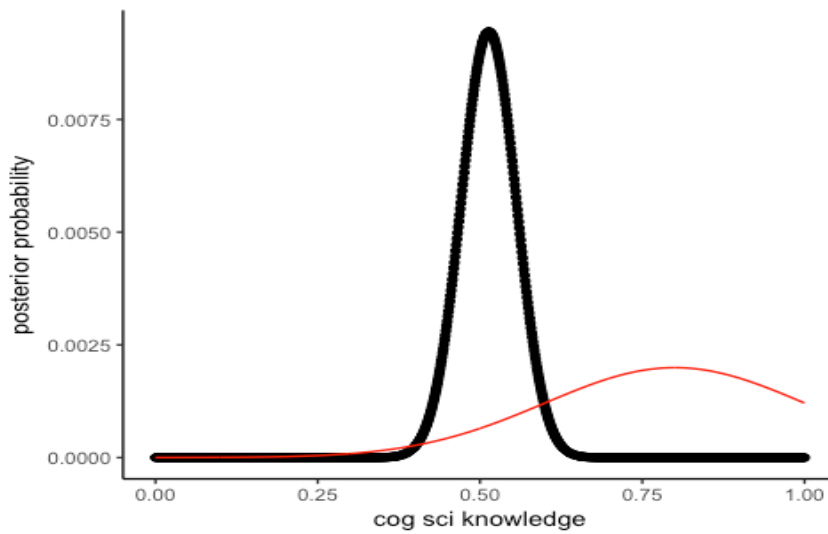


**Sampling and HPDI after  
changed prior:**

|0.5      0.5|  
0.7867868   0.8238238

The highest posterior density  
interval for Josh is from 0.78 to  
0.82 cog sci knowledge (50 %  
interval) (unchanged).

**MIKKEL:**



**Sampling and HPDI after  
changed prior:**

|0.5 0.5|

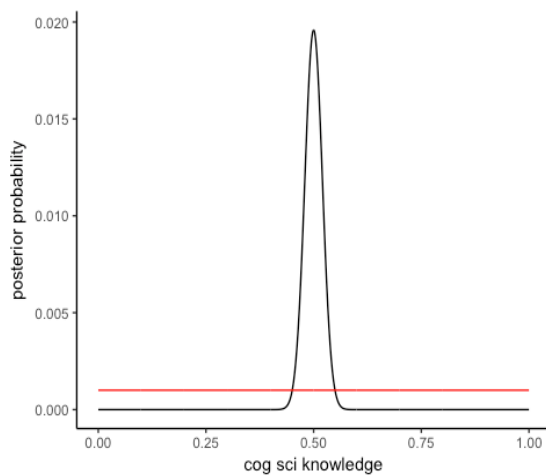
0.4824825 0.5385385

The highest posterior density  
interval for Mikkel is from 0.48 to  
0.54 Cognitive Science knowledge  
(50 % interval) (largely  
unchanged).

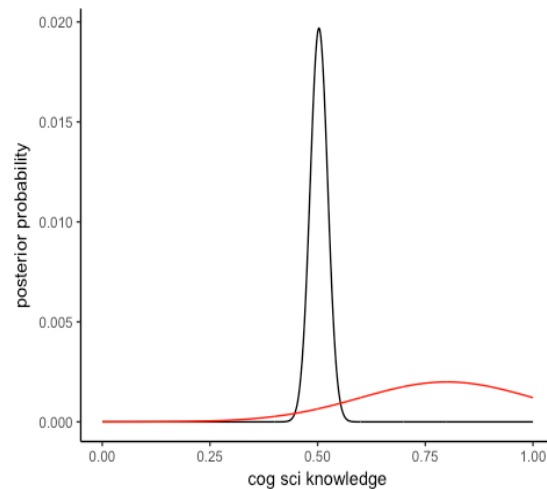
Overall, in cases where there is a lot of uncertainty (especially for Riccardo and Kristian due to low sample size), the posterior is pulled towards the prior quite a lot. In cases where there is lower uncertainty (due to having more data), there is not much of a change.

- 4) You go back to your teachers and collect more data (multiply the previous numbers by 100). Calculate their knowledge with both a uniform prior and a normal prior with a mean of 0.8 and a standard deviation of 0.2. Do you still see a difference between the results? Why?

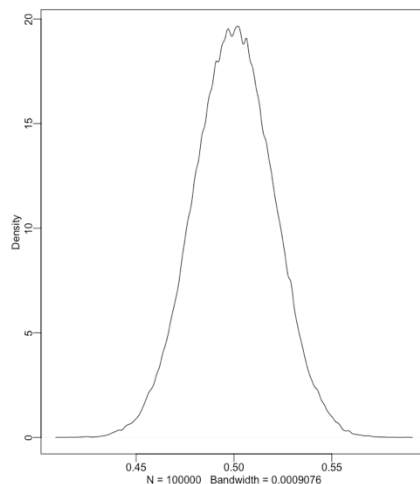
#### Riccardo with flat prior and Riccardo with normal distributed prior



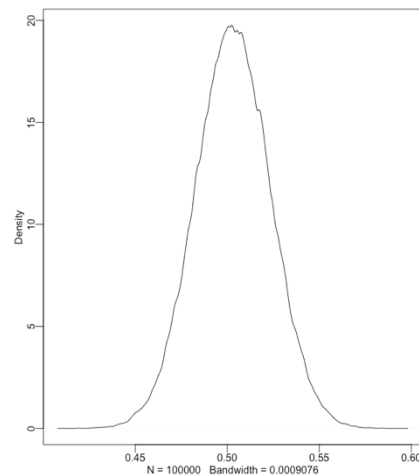
*flat prior*



*normal distributed prior*



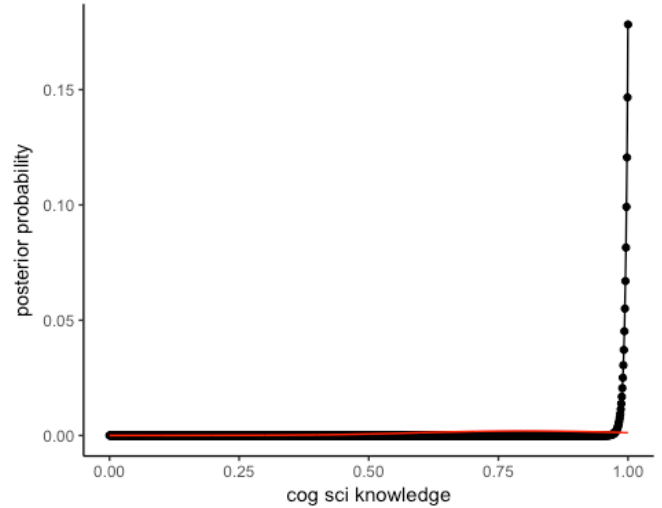
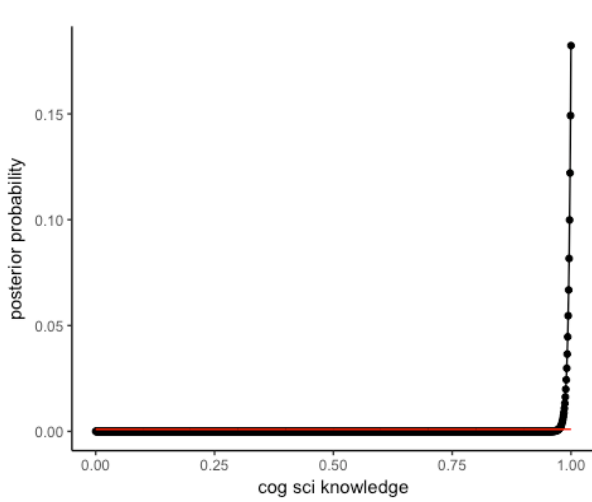
*Density plot flat prior*



*Density plot normal prior*

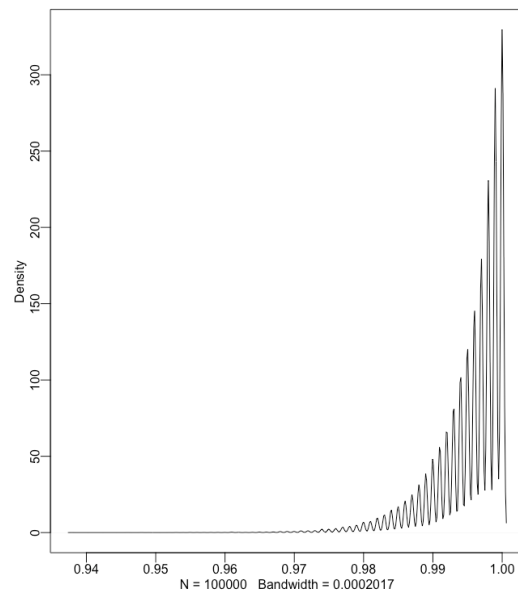
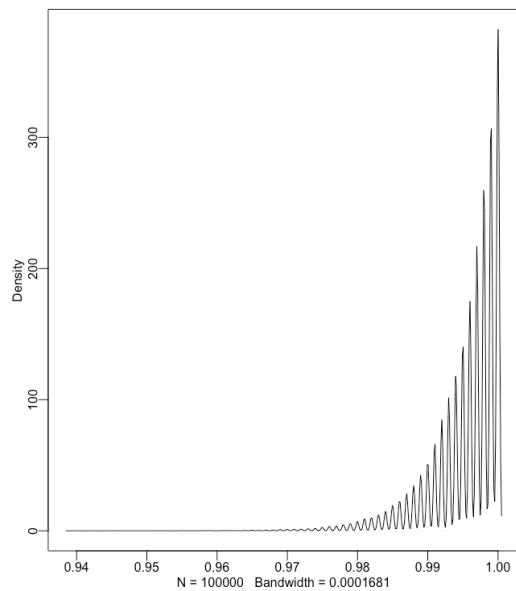
The highest posterior density interval (50 % interval) for Riccardo with flat prior is from 0.48 to 0.51 Cognitive Science knowledge, whereas with the normal distributed prior it is from 0.49 to 0.51, nearly the same.

### Kristian with flat prior and Kristian with normal distributed prior



*flat prior*

*normal distributed prior*

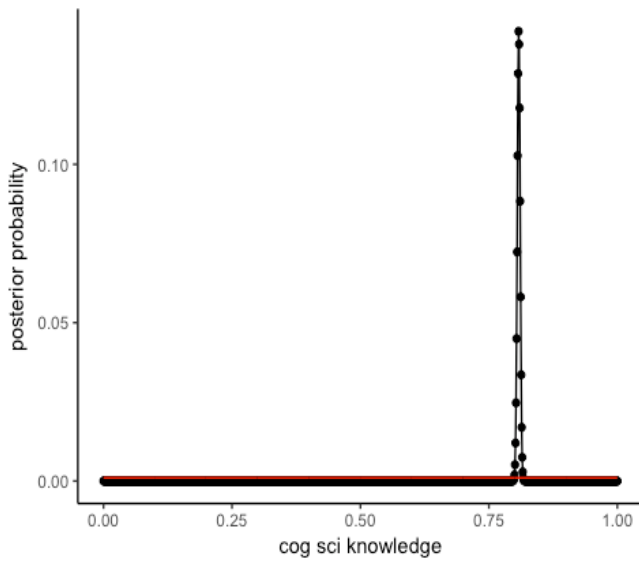


*Density plot with flat prior*

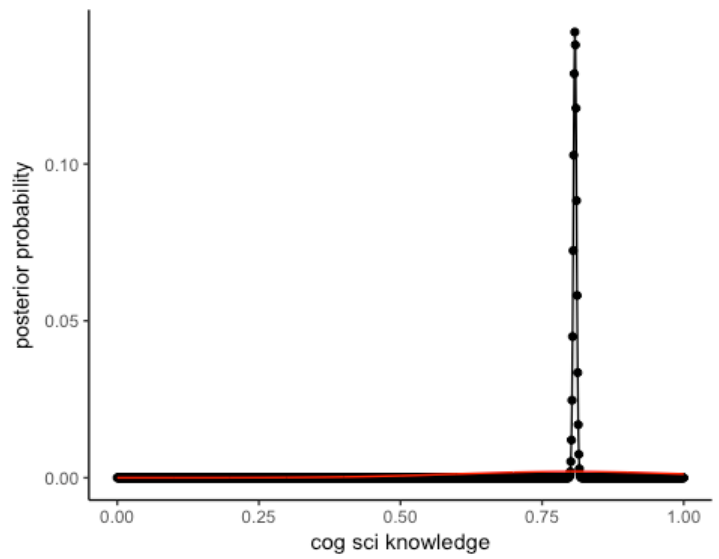
*Density plot with norm. prior*

The highest posterior density interval for Kristian is from 1 to 1 cogsci knowledge in a 50 % interval with both priors.

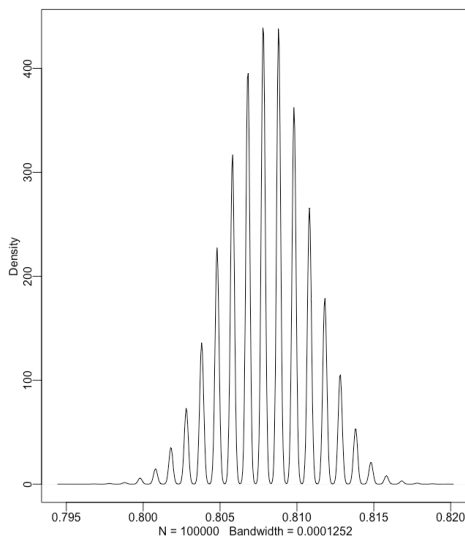
## Josh with flat prior and Josh with normal distributed prior



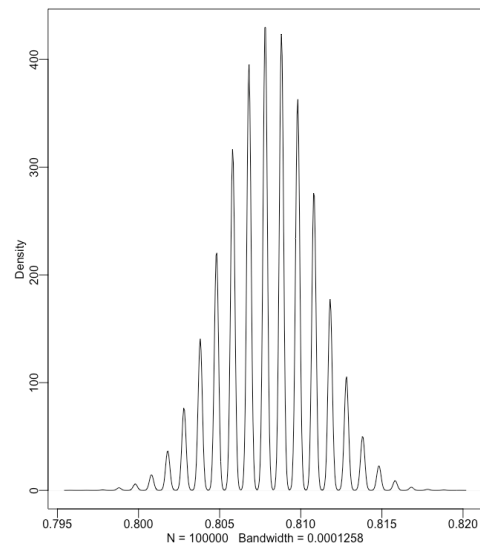
*flat prior*



*normal distributed prior*



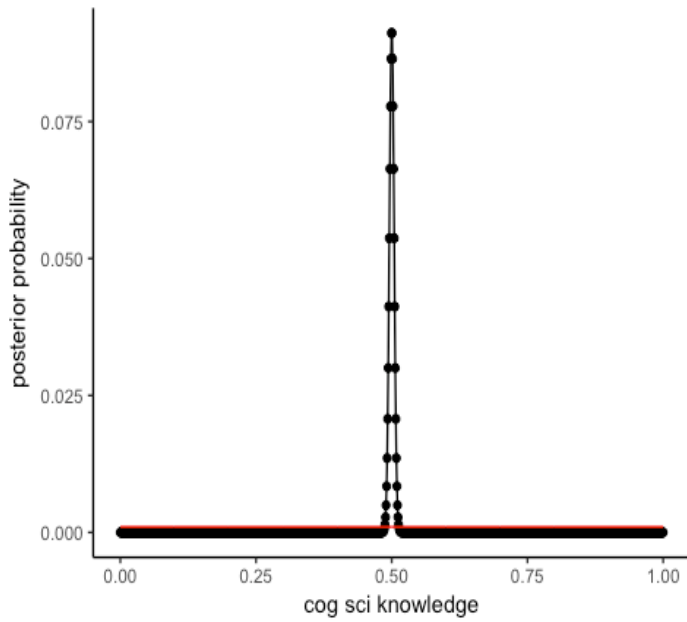
*Density plot with flat prior*



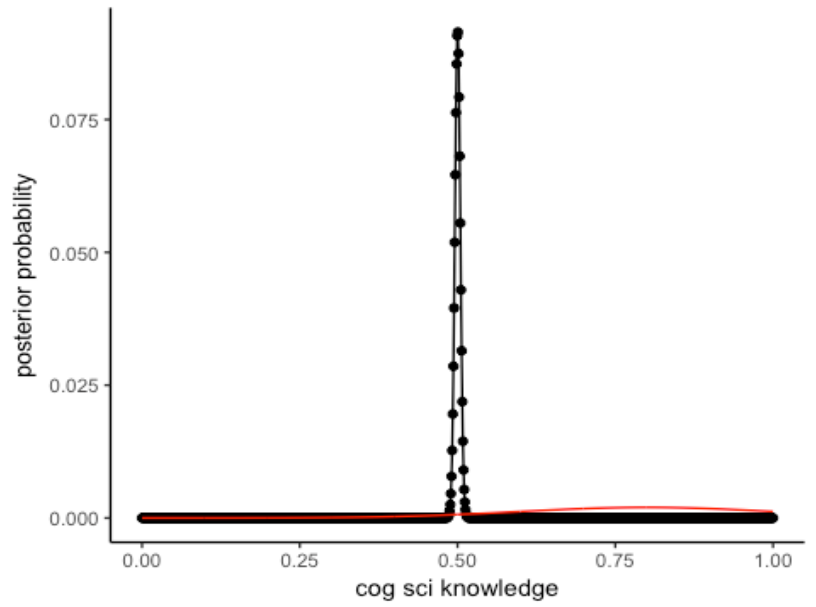
*Density plot with norm. prior*

The highest posterior density interval for Josh is from 0.81 to 0.81 cogsci knowledge for both priors.

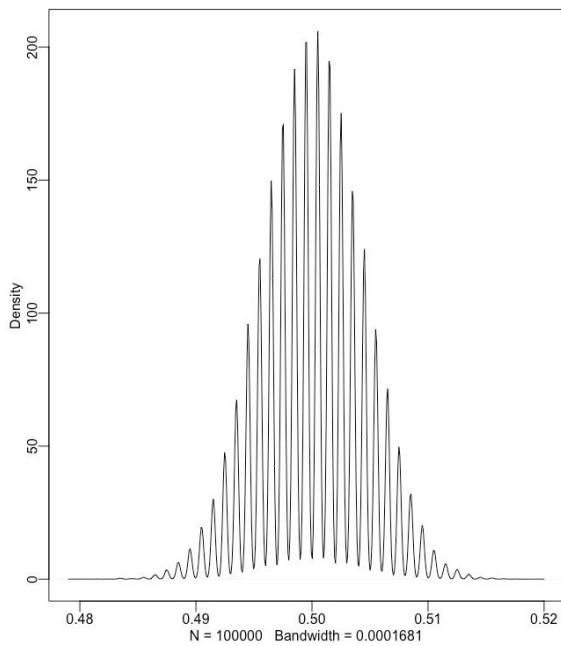
## Mikkel with flat prior and Mikkel with normal distributed prior



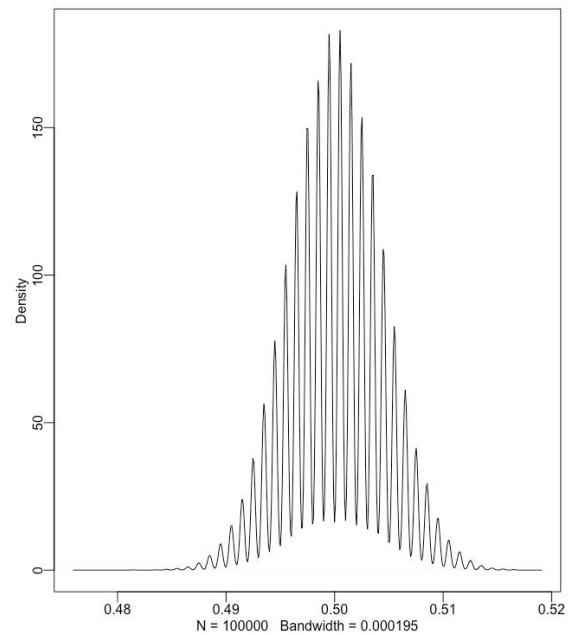
*flat prior*



*normal distributed prior*



*Density plot with flat prior*



*Density plot with norm. prior*

The highest posterior density interval for Mikkel is from 0.50 to 0.50 cogsci knowledge (50 % interval) with both priors.

Conclusively, the larger the sample size, the less of an effect the prior has.

5) Imagine you're a skeptic and think your teachers do not know anything about CogSci, given the content of their classes. How would you operationalize that belief?

To take a more skeptical approach, I would create a prior with a chance mean (e.g. 0.5) and very low SD, maybe (0.1 or 0.2).