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Dynamic asset allocation for a bank under CRRA and HARA framework

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Abstract

This paper analyzes an optimal investment and management strategy for a bank under constant relative risk aversion (CRRA) and hyperbolic absolute risk aversion (HARA) utility functions. We assume that the bank can invest in treasuries, stock index fund and loans, in an environment subject to *stochastic interest rate* and *inflation uncertainty*. The interest rate and the expected rate of inflation follow a correlated Ornstein–Uhlenbeck processes and the risk premia are constants. Then we consider the portfolio choice under a power utility that the bank's shareholders can maximize expected utility of wealth at a given investment horizon. Closed form solutions are obtained in a dynamic portfolio optimization model. The results indicate that the optimal proportion invested in treasuries increases while the optimal proportion invested in the loans progressively decreases with respect to time.

Keywords: Bank asset allocation; Basel II CAR; CRRA utility; HARA utility; stochastic interest rate.

JEL Classifications: C02; C61; GE50; G23

1. Introduction

Optimal portfolio allocation for banking institutions remains a problem when managing assets and liabilities under risk. Merton (1969, 1971) constructed an explicit solution for optimal investment and consumption under the assumption

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that the stock price follows a geometric Brownian motion (GBM) and individual preferences are of specific types. Since then, there has been continued interest to develop optimal asset allocation methods under stochastic interest rates and to incorporate the specific features of a bank. This will in turn motivate banks to invest in assets with an acceptable level of risk to generate high returns.

A dynamic portfolio position is important for a bank's risk management strategy. Most banks select an initial loan portfolio at the beginning of a loan period and often do not actively manage this portfolio until the event of possible default. Another motivation for analyzing a bank's optimal asset allocation is to capture the failures that spark due to risk management strategies and regulatory prescripts implemented to mitigate risk. The current prescription for banks are to carry out Basel Accord on capital adequacy requirements, which mandates that all major international banks hold capital in proportion to their perceived risks. However, an internal model may be used by banks to make an assessment of their portfolio risk when determining the capital requirement.

An important problem in asset allocation is to characterize the optimal rebalancing pattern of assets over a time period. Mulaudzi *et al.* (2008) analyzes an optimal allocation problem for bank funds with treasuries and loans under a risk and regret is considered in a theoretical framework. Fouche *et al.* (2005) formulated an optimal bank valuation problem via optimal choices of loan rate and demand which leads to maximal deposits, provisions for deposit withdrawals as well as bank profitability subject to cash flow, loan demand, financing and balance sheet constraints. In recent years a number of studies focused on an optimal investment strategy when some model coefficients are stochastic. Stochastic interest rates have been applied by Witbooi *et al.* (2011), using Cox–Ingersoll–Ross (CIR) model together with Cox and Huang (1989) methodology to obtain an optimal banking portfolio consisting of three assets such as, treasuries, securities and loans. They obtained an explicit solution for the optimal equity allocation strategy that optimizes the terminal utility of the bank's shareholders under a power utility function. Bajeux-Besnainou *et al.* (1998, 2003) applied Vasicek (1977) model to capture stochastic features of the model parameters and Deelstra *et al.* (2000) with CIR model in order to obtain an explicit optimal investment strategy of an investor under power utility function. Contrarily to Vasicek model, the CIR model is unable to capture negative interest rate and it is less manageable when the inflation uncertainty is not log-normally distributed. However, Bajeux-Besnainou *et al.* (1998) study does not consider inflation uncertainty.

From a shareholder's point of view, utilizing more capital will increase the value of the bank and indeed will earn higher returns on equity. On the other hand, from the regulator's perspective, banks should increase their buffer capital to ensure the safety and unassailability in the case where earnings may end up lower than the expected level. The Basel Committee on Banking Supervision (2004) regulates and supervises the international banking industry, by imposing minimal capital requirements and other measures. In recent years a number of authors have addressed the role of Basel I and Basel II, investment strategies, higher capital requirements, lower loan requirements, and balance sheets in the banking industry. A partial listing includes Ferguson (2003), Jacques (2008), Jones (2000), Berger and Udell (1994), Haubrich and Wachtel (1993), Jacques and Nigro (1997), Keeton (1994), Borio *et al.* (2001), Lowe (2002) and Basel (2004). However, maintaining a minimum capital adequacy ratio will guarantee that banks are prepared to absorb a reasonable level of loss before becoming insolvent. Hence, minimum CAR will help to promote the stability and effectiveness of the banking industry. In this study, the CAR in question is the risk based Basel II CAR given by $X = \frac{C}{b_{rw}}$, where C represents the *total capital* and b_{rw} the total *risk-weighted assets* (TRWAs) of the bank, respectively.

In particular, in this paper we focus on the optimal asset allocation strategy for a bank under constant relative risk aversion (CRRA) and hyperbolic absolute risk aversion (HARA) utility functions. The bank can continuously invest in treasuries, stock index fund and loans in an environment of *stochastic interest rate* and *inflation uncertainty*. We consider the portfolio choice of a power utility when the bank's shareholders can maximize expected utility of wealth at a given investment horizon. Closed form solutions are obtained in a dynamic portfolio optimization model. We then address the problem of compliance to minimum capital adequacy ratio (CAR) and under assumptions about retained earnings, loan-loss reserves, and market and shareholder-bank ownerships. We then construct a continuous-time model of the Basel II CAR via the total risk-weighted assets (TRWAs) and bank capital in a stochastic interest rate and inflation uncertainty setting. The stochastic features of the total bank capital and TRWAs are considered under the Basel II paradigm.

The remainder of this paper is organized as follows: Secs. 2 and 3 contain the description of the investment opportunities, market setting and model setup. In Sec. 4, we introduce the optimization problem and examines the optimal portfolio allocation of the bank subject to CRRA and HARA. The TRWAs and consequently the Basel II CAR are derived toward the end of Sec. 4. Section 5 concludes the paper.

2. Formulation of the Banking Model

We consider a complete and frictionless financial market in a continuous time framework over a fixed investment time interval $[0, T]$. We work within a filtered probability space $(\Omega, \mathcal{F}, P)$, where P is the reference probability measure and Ω denotes the information structure and satisfies the usual conditions. The mathematical model for a continuous-time market allows at least two types of financial assets (treasuries and stock index fund) to be bought and sold without incurring any transaction costs or restriction on short sales. Issuing of a loan is considered to be a third investment opportunity for the bank.

To capture the operation and management strategies of banks, we need to consider the balance sheet, which records the bank's assets (use of funds) and bank's liabilities (source of funds). As in Witbooi *et al.* (2011) and Mukuddem-Petersen and Petersen (2006), we define the balance sheet of a commercial bank at time t as

$$M(t) + S(t) + L(t) = D(t) + B(t) + C(t), \tag{1}$$

where, M, S, L, D, B and C are the treasuries, securities, loans, deposits, borrowing and bank capital, respectively. Each of these variables will be regarded as a function of $\Omega \times \mathbb{R}_+ \to \mathbb{R}_+$.

2.1. Treasuries, securities and loans

The amount of money that is set aside is known as the bank *reserves*. These funds are not used to lend to customers or to meet day-to-day currency withdrawals. *Treasury securities* or *treasuries* and bonds are issued by the national treasury as a means of borrowing money to meet government expenditures that has not been covered by tax revenues. *Marketable securities* are stocks and bonds that can be swiftly converted into cash, hence are highly liquid assets. We suppose that a commercial bank raises funds to invest in a risky asset, in this case a *loan*. The interest rate on the loan is denoted by $r(t)$.

2.2. Total bank capital

Banks can raise their capital by selling new equity, retaining earnings, issuing debt or building up loan reserves. By nature the dynamics of bank capital is stochastic due to uncertainty related to debt and shareholder contributions. However, in theory the bank can decide on the rate at which debt and equity is raised.

According to Mukuddem-Petersen and Petersen (2006), the bank capital can be portioned into so-called *Tier* 1, *Tier* 2 and *Tier* 3 capital, i.e.,

$$C(t) = C_{T1}(t) + C_{T2}(t) + C_{T3}(t).$$

Tier 1 capital is the book value of its stocks, $E(t)$ plus retained earnings, $E_r(t)$. Tier 2 and Tier 3 capital (collectively known as *supplementary capital*) is the sum of subordinate debt, $S_D(t)$ and loan-loss reserves $R_L(t)$. As a result, we have

$$C_{T1}(t) = E(t) + E_r(t), \tag{2}$$

and

$$C_{T2}(t) + C_{T3}(t) = S_D(t) + R_L(t). \tag{3}$$

Therefore, the total bank capital can be expressed as

$$C(t) = E(t) + E_r(t) + S_D(t) + R_L(t). \tag{4}$$

The market value of subordinate debt at time t may be given by

$$S_D(t) = S_D(0) \exp\left(\int_0^t r(u)du \right).$$

2.3. Dynamics of total capital

We assume that the bank holds capital in $n + 1$ categories, n of which are referred to as bank equity. Then the return on the ith bank equity is defined as

$$de_i(t) = e_i(t)\left[\left(r(t) + \sum_{j=1}^{n} \sigma_{ij}\vartheta_j \right) dt + \sum_{j=1}^{n} \sigma_{ij} d\hat{W}_j(t) \right], \quad i = 1, 2, \ldots, n. \tag{5}$$

The co-variance matrix and the market price of risk are given by $\Psi = (\sigma_{ij})_{i,j=1}^{n}$ and $\vartheta = (\vartheta_1, \ldots, \vartheta_n)^T$, respectively and are assumed to be constants. T is the transpose of a vector or matrix. At time t we assume that the bank capital is being converted into loan and marketable securities at the rate of $\rho_x(t) = \rho X(t)dt$ for a constant ρ. Here, $X(t)$ represents the total asset portfolio of the bank.

Greenbaum and Thakor (2010) argues that excessive high capital requirements may result in banks taking on more risk and may lead to a bank acquiring higher levels of equity on order to become compliant. The upshot of such practices includes reduced liquidity and erosion of discipline in the bank's operation while defeating the purpose of the regulatory requirements. Therefore, capital requirement should be pitched at an appropriate level and banks should operate as near as possible to the minimum required level of capital. Due to the nondynamic character of retained earnings and loan-loss reserves, these aspects are not considered to be active constituents of bank capital. This implies $dE_r(t) = dR_L(t) = 0$, $\forall\, t$.

Hence, the C-dynamics may be expressed as:

$$\begin{aligned} dC(t) &= C(t) \sum_{i=1}^{n} w_i(t) \frac{de_i(t)}{e_i(t)} + \left(1 - \sum_{i=1}^{n} w_i(t) \right) C(t) \frac{dS_D(t)}{S_D(t)} - \rho X(t)dt \\ &= C(t)[(r(t) + w^T(t)\Psi\vartheta) + w^T(t)\Psi d\hat{W}(t)]dt - \rho X(t)dt, \end{aligned} \tag{6}$$

where $w^T(t)$ are the proportions invested in securities. The diffusion term $w^T(t)$ $\Psi d\hat{W}(t)$ in (6) establishes a correlation between bank capital and total risk-weighted assets.

3. Financial Market Setting

3.1. Treasuries

As the first of three assets, we consider the treasury $M(t)$ which evolves according to

$$dM(t) = M(t)r(t)dt, \quad M(0) = 1, \tag{7}$$

where $r(t)$ is the instantaneous nominal interest rate at time t and evolves as an Ornstein–Uhlenbeck process

$$dr(t) = a_r(\bar{r} - r(t))dt + \sigma_r dW_r(t), \tag{8}$$

where $W_r(t)$ is a standard Brownian motion, $a_r, \bar{r}$ and σ_r are strictly positive constants and correspond to the degree of mean-reversion, the long-run mean and the volatility of the interest rates.

3.2. Stock index fund

The evolution of the price process of the stock index fund $S(t)$ is governed by the following stochastic differential equation (SDE):

$$dS(t) = S(t)[(r(t) + \xi_S)dt + \sigma_1 dW(t) + \sigma_2 dW_r(t)], \tag{9}$$

where ξ_S is the risk premium of the stock fund, σ_1 and σ_2 are positive volatility parameters, $W(t)$ and $W_r(t)$ are two orthogonal Brownian motions.

3.3. Loans

Any loan is essentially an interest rate contingent claim and by Itô's lemma the dynamics of the loan price $L(t)$ are assumed to follow according to the SDE

$$dL(t) = L(t)[(r(t) + \xi_L)dt + \sigma_L dW_r(t)]. \tag{10}$$

We assume that the bank grants loans at the interest rate on loans or loan rate as a sum of instantaneous interest rates, the market price of risk and the default risk premium. Here, $\xi_L = \xi_r \sigma_L + \delta$. As in Merton (1974), the default risk premium, δ, is the credit spread charged by the bank and it is the function of the probability of default, *PD*, and the loss given default of the loan, *LGD* (Spread = PD $*$ LGD). We assume that the loans available for the customer has a constant duration on the contingent claim D. This implies $\sigma_L := \sigma_r D$. According to Vasicek (1977) analysis $\sigma_L = \frac{\sigma_r(1-e^{-a_r,\tau})}{a_r}$, where $\frac{1-e^{-a_r,\tau}}{a_r}$ can be defined as the elasticity of the loan with respect to the short interest rate and referred to as the duration of the interest contingent claim D.

In order to capture the inflation factor into our model, we consider the influence of the instantaneous expected interest of inflation π.

The evolution of the price level of commodity $P(t)$ is governed by the following SDE:

$$dP(t) = P(t)[(\pi(t)dt + \sigma_\pi dW_\pi(t))], \tag{11}$$

The instantaneous expected interest rate of inflation π also follows an Ornstein-Uhlembeck process

$$d\pi(t) = b_\pi(\bar{\pi} - \pi(t))dt + \sigma_\pi dW_\pi(t), \tag{12}$$

where the parameters $b_\pi, \bar{\pi}$ and σ_π are constant parameters and correspond to the degree of mean-reversion, the long-run mean and the volatility of the instantaneous volatility of the inflation.

Then the risk premium of the loan fund condition to inflation is defined as

$$\xi_L = \theta_r \sigma_L \pi. \tag{13}$$

Subsequently, the associated risk premium of the stock-index fund under inflation will be

$$\xi_S = (\theta_S \sigma_1 + \theta_r \sigma_2)\pi, \tag{14}$$

where θ_S is the market price of the stock market risk.

3.4. Growth optimal portfolio

It is well documented in finance and mathematics literature that the growth optimal portfolio (or the logarithmic portfolio) will give a maximal expected growth rate over any time horizon. In order to investigate the growth optimal portfolio under a self-financing assumption, we define the volatility matrix $\sigma = \begin{pmatrix} \sigma_1 & \sigma_2 \\ 0 & \sigma_L \end{pmatrix}$ and the variance–covariance matrix as $\Sigma = \sigma\sigma^T = \begin{pmatrix} \sigma_1^2 + \sigma_2^2 & \sigma_2\sigma_L \\ \sigma_2\sigma_L & \sigma_L^2 \end{pmatrix}$. Then the inverse of the variance–covariance matrix is $\Sigma^{-1} = (\sigma_1^2\sigma_L^2)^{-1}\begin{pmatrix} \sigma_L^2 & -\sigma_2\sigma_L \\ -\sigma_2\sigma_L & \sigma_1^2 + \sigma_2^2 \end{pmatrix}$. Let $H(t)$ be the price of the growth optimal portfolio.

According to Merton (1992), $H(t)$ can be defined as

$$H(t) = \exp\left\{\int_0^t r(s)ds + \int_0^t \underline{\theta}^T(s, r(s))dW(s) + \frac{1}{2}\int_0^t \|\underline{\theta}(s, r(s))\|^2 ds\right\} \quad \text{or}$$

$$H(T) = H(t)\exp\left\{\int_t^T r(s)ds + \int_t^T \underline{\theta}^T(s, r(s))dW(s) + \frac{1}{2}\int_t^T \|\underline{\theta}(s, r(s))\|^2 ds\right\}, \tag{15}$$

where $\underline{\theta}^T$ denotes the transposition of the vector, and $\|\cdot\|$ denotes the Euclidean norm in $\mathbb{R}^2$.

The weight of the logarithmic portfolio can be defined as

$$\underline{w} = \begin{pmatrix} w_S \\ w_L \end{pmatrix} = \Sigma^{-1}\underline{\xi} = \Sigma^{-1}\begin{pmatrix} \xi_S \\ \xi_L \end{pmatrix}$$

$$= \frac{1}{\sigma_1^2\sigma_L^2}\begin{pmatrix} \sigma_L^2 & -\sigma_2\sigma_L \\ -\sigma_2\sigma_L & \sigma_1^2 + \sigma_2^2 \end{pmatrix}\begin{pmatrix} \theta_S\sigma_1\pi + \theta_r\sigma_2\pi \\ \theta_r\sigma_L\pi \end{pmatrix}, \tag{16}$$

where w_S, w_L are the weight of the stock-index fund and loans in the growth-optimal portfolio (logarithmic portfolio). Using (13) and (14), Eq. (16) can be summarized as

$$\begin{pmatrix} w_S \\ w_L \end{pmatrix} = \begin{pmatrix} \dfrac{\theta_S\pi}{\sigma_1} \\ \dfrac{\theta_r\sigma_1\pi - \theta_S\sigma_2\pi}{\sigma_1\sigma_L} \end{pmatrix}. \tag{17}$$

From, the above discussion, we know that

$$\underline{w} = \Sigma^{-1}\underline{\xi} = (\sigma\sigma^T)^{-1}\underline{\xi} = (\sigma^T)^{-1}\underline{\theta} = (\sigma^{-1})^T\underline{\theta}. \tag{18}$$

Hence, $\underline{w}^T\underline{\xi} = \underline{\theta}^T\sigma^{-1}\underline{\xi} = \underline{\theta}^T\underline{\theta} = \|\underline{\theta}\|^2$ and $\underline{w}^T\sigma = \underline{\theta}^T\sigma^{-1}\sigma = \underline{\theta}^T$.

Therefore, (15) can be formulated as

$$H(T) = H(t)\exp\left\{\int_t^T r(u)du + \underline{\theta}^T[\underline{W}(T) - \underline{W}(t)] + \frac{1}{2}\|\underline{\theta}\|^2(T-t)\right\}. \tag{19}$$

4. Optimal Strategies for the Bank

In accordance with the asset allocation theory, the shareholders of a bank expect higher returns on their capital investments while minimizing their risk. As a result, bank management need to allocate the shareholders equity in order to maximize the expected utility with continuous revisions of portfolio in the investment horizon.

4.1. Optimal strategies of CRRA shareholders

We describe the utility function $U : (x_0, +\infty) \to \mathbb{R}$, is increasing, strictly concave, continuously differentiable and satisfy the Inada condition $\lim_{x\to+\infty} U'(x) = 0,\ \lim_{x\to x_0} U'(x) = +\infty$, where $x_0 \in \mathbb{R} \cup \{-\infty\}$. When these conditions are satisfied, we define the constant relative risk aversion (CRRA) utility as

$$U[X(t)] = \begin{cases} \dfrac{X(t)^{1-\gamma}}{1-\gamma}, & \gamma < 1 \\ \ln X(t), & \gamma = 1 \end{cases} \tag{20}$$

where γ denotes the relative risk aversion that is a constant and $\frac{-U''[X(t)]}{U''[X(t)]} = \gamma$.

In order to obtain a closed form solution of the optimal portfolio, we transform the dynamic problem into a static problem. As a result we obtain the optimal weights via the equation of the optimal portfolio value. Therefore,

$$\frac{X(0)}{H(0)} = E\left[\frac{X(T)}{H(T)}\right], \tag{21}$$

where E represents the conditional expectation. Setting $H(0)$ as the initial value of the logarithmic portfolio, we assume $H(0) = 1$. Hence, $E[\frac{X(T)}{H(T)}] = X(0)$.

Therefore, the optimal terminal wealth of the CRRA shareholder can be expressed as

$$\max\left[\frac{X(T)^{1-\gamma}}{1-\gamma}\right] \text{subject to } E\left[\frac{X(T)}{H(T)}\right] = X(0). \tag{22}$$

Then the static problem can be solved via the Lagrangian theory. Denote $\bar{x} = \inf\{x \in \mathbb{R}_+ : U(x) > -\infty\}$ which follows the inverse of the right derivation of the power utility function, namely $I(y) = (U'_+)^{-1}(y)$ maps $(0, U'(\bar{x}^+))$ on to $(x, +\infty)$. According to Cox and Huang (1989) the terminal wealth $X^*(T)$ can be defied as $X^* = I[\frac{\kappa}{H(t)}]$, where κ is the Lagrangian multiplier of the budget constraint. From (20) we obtain $I(y) = (U'_+)^{-1}(y) = y^{\frac{1}{\gamma}}$. Hence, $X^*(T)$ can be written as $X^*(T) = [\frac{H(T)}{\kappa}]^{\frac{1}{\gamma}}$ where $\kappa = \frac{X(0)}{E[H(T)^{\frac{1}{\gamma-1}}]}$.

Through the Ornstein–Uhlenbeck process given by (8), we obtain

$$d[e^{a_r,s}(r(s) - \bar{r})] = e^{a_r,s}[dr(s) + a_r(r(s) - \bar{r})ds] = \sigma_r e^{a_r,s} dW_r$$

$e^{a_r,s}(r(s) - \bar{r}) - (r(0) - \bar{r}) = \int_0^s \sigma_r e^{a_r,u} dW_r(u)$, and the solution of $r(s)$ is obtained as

$$r(s) = (1 - e^{-a_r,s})\bar{r} + e^{-a_r,s} r(0) + e^{-a_r,s} \int_0^s \sigma_r e^{a_r,u} dW_r(u). \tag{23}$$

It is known that $r(s)$ is stationary and mean-reverted and $r(s) \sim N(\bar{r}, \frac{\sigma_r^2}{2a_r})$. Hence, $\int_t^T r(s)ds$ is normally distributed. According to (23)

$$\begin{aligned}\int_t^T r(s)ds &= \int_t^T (1 - e^{-a_r, s})\bar{r}ds + r(0)\int_t^T e^{-a_r, s}ds + \int_t^T e^{-a_r, s}\sigma_r e^{a_r, u}dW_r(u)ds \\ &= \bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r, (T-t)} + \sigma_r \int_t^T \frac{(1-e^{-a_r, s})}{a_r}dW_r(u). \quad (24)\end{aligned}$$

Hence, the third term of (24) is a martingale and the expectation of (24) becomes

$$E\left[\int_t^T r(s)ds\right] = \bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r, (T-t)}. \quad (25)$$

Proposition 1. *The variation of* (24) *can be derived as*

$$\mathrm{Var}\left[\int_t^T r(s)ds\right] = \frac{\sigma_r}{a_r}\left[(T-t) + \frac{4e^{-a_r(T-t)} - e^{-2a_r(T-t)}}{2\alpha_r}\right].$$

Proof.

$$\begin{aligned}\mathrm{Var}\left[\int_t^T r(s)ds\right] &= E\left[\int_t^T r(s)ds\right]^2 - \left[E\int_t^T r(s)ds\right]^2 \\ &= E\Bigg[\bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r(T-t)} \\ &\quad + \sigma_r\int_t^T \frac{(1-e^{-a_r, s})}{a_r}dW_r(u)\Bigg]^2 - \left[E\int_t^T r(s)ds\right]^2 \\ &= E\left[\bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r(T-t)}\right]^2 \\ &\quad + E\left[\sigma_r\int_t^T \frac{(1-e^{-a_r, s})}{a_r}dW_r(u)\right]^2 - \left[E\int_t^T r(s)ds\right]^2 \\ &= \frac{\sigma_r}{a_r}E\left[\sigma_r\int_t^T \frac{(1-e^{-a_r, s})}{a_r}dW_r(u)\right]^2.\end{aligned}$$

From Itô isometry theorem we have

$$\begin{aligned}E\left[\sigma_r\int_t^T \frac{(1-e^{-a_r, s})}{a_r}dW_r(u)\right]^2 &= E\int_t^T (1-e^{-a_r, s})^2 ds \\ &= (T-t) + \frac{4e^{-a_r(T-t)} - e^{-2a_r(T-t)}}{2\alpha_r},\end{aligned}$$

where $\alpha = \bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r(T-t)}$.

Hence,

$$\mathrm{Var}\left[\int_t^T r(s)ds\right] = \frac{\sigma_r}{a_r}\left[(T-t) + \frac{4e^{-a_r(T-t)} - e^{-2a_r(T-t)}}{2\alpha_r}\right].$$

Therefore, □

$$\int_t^T r(s)ds \sim N\Bigg(\bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r(T-t)},$$
$$\frac{\sigma_r}{a_r}\left[(T-t) + \frac{4e^{-a_r(T-t)} - e^{-2a_r(T-t)}}{2\alpha_r}\right]\Bigg). \tag{26}$$

Allying with Vasicek (1977) analysis and replicating τ by $T-t$, we rewrite (26) as

$$\int_t^T r(s)ds \sim N\left(\bar{r}\tau - \frac{(r(0)-\bar{r})(1-Da_r)}{a_r}, (\tau - D)\frac{\sigma_r}{a_r} - \frac{\sigma_r D^2}{2} + \frac{3\sigma_r}{2a_r^2}\right). \tag{27}$$

Let $(\underline{y}, Y)$ denote as the portfolio strategy and $Y(T) \equiv H(T)^n$ *a.s.*, and the bank's portfolio weights are given by x_S^*, x_L^* *and* x_T^*. Since $\frac{Y(t)}{H(t)}$ is a martingale we define

$$\frac{Y(t)}{H(t)} = E\left[\frac{Y(T)}{H(T)}\right] = E[H(T)^{n-1}]. \tag{28}$$

Substituting, (20) into (28) yields

$$\begin{aligned} Y(t) &= H(t)E\Bigg[H(t)^{n-1} \\ &\quad \times \exp\left\{\int_t^T r(u)du + \underline{\theta}^T[\underline{W}(T) - \underline{W}(t)] + \frac{1}{2}\|\underline{\theta}\|^2(T-t)\right\}^{n-1}\Bigg] \\ &= H(t)^n E\Bigg[(n-1) \\ &\quad \times \exp\left\{\int_t^T r(u)du + \underline{\theta}^T[\underline{W}(T) - W(t)] + \frac{1}{2}\|\underline{\theta}\|^2(T-t)\right\}^{n-1}\Bigg]. \end{aligned} \tag{29}$$

Furthermore, explicitly, we can express (29) as

$$Y(t) = H(t)^n E(e^{G_n}). \tag{30}$$

Since, the terms of G_n are lognormal, we express (30) as

$$Y(t) = H(t)^n \exp\left[E(G_n) + \frac{1}{2}\mathrm{Var}(G_n)\right], \tag{31}$$

where

$$\begin{aligned} E(G_n) &= (n-1)\left\{E\left[\int_t^T r(u)du\right] + \frac{1}{2}\|\underline{\theta}\|^2(T-t)\right\} \\ &= (n-1)\left[\bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r(T-t)} + \frac{1}{2}\|\underline{\theta}\|^2(T-t)\right]. \end{aligned} \tag{32}$$

$$\begin{aligned}
\mathrm{Var}(G_n) &= (n-1)^2\mathrm{Var}\left\{\int_t^T r(u)du + \underline{\theta}^T[\underline{W}(T) - \underline{W}(t)]\right\}\\
&= (n-1)^2\left\{\mathrm{Var}\left[\int_t^T r(u)du\right] + \|\underline{\theta}\|^2(T-t)\right.\\
&\quad \left. + 2\mathrm{Cov}\left[\int_t^T r(u)du, \underline{\theta}^T[\underline{W}(T) - \underline{W}(t)]\right]\right\}\\
&= (n-1)^2\left\{\mathrm{Var}\left[\int_t^T r(u)du\right] + \|\underline{\theta}\|^2(T-t)\right.\\
&\quad + 2E\left[\int_t^T r(u)du, \underline{\theta}^T[\underline{W}(T) - \underline{W}(t)]\right]\\
&\quad \left. - 2E\left[\int_t^T r(u)du\right]E[\underline{\theta}^T[\underline{W}(T) - \underline{W}(t)]]\right\}. \qquad (33)
\end{aligned}$$

Then

$$\begin{aligned}
&E\left(\int_t^T r(u)du \cdot \underline{\theta}^T[\underline{W}(T) - \underline{W}(t)]\right)\\
&\quad = \underline{\theta}^T E\left\{\left[\bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r(T-t)} + \sigma_r\int_t^T \frac{(1-e^{-a_r,s})}{a_r}dW_r(u)\right]\right.\\
&\qquad \left. \times \left[\int_t^T dW_r(u)\right]\right\}\\
&\quad = \underline{\theta}^T\sigma_r E\int_t^T \frac{(1-e^{-a_r,s})}{a_r}dW_r(u)\int_t^T dW_r(u) = \frac{\underline{\theta}^T\sigma_r}{a_r}\left[\int_t^T 1 - e^{-a_r,u}du\right]\\
&\quad = \frac{\underline{\theta}^T\sigma_r}{a_r}\left[\left((T-t) + \frac{e^{-a_r,(T-t)}}{a_r}\right)\right]. \qquad (34)
\end{aligned}$$

Since

$$E(\underline{\theta}^T[\underline{W}(T) - \underline{W}(t)]) = 0. \qquad (35)$$

Substituting (35) and (34) into (33) yields

$$\begin{aligned}
\mathrm{Var}(G_n) &= (n-1)^2\left\{\|\underline{\theta}\|^2(T-t) + \frac{\sigma_r(T-t)(1+2\underline{\theta}^T)}{\alpha_r}\right.\\
&\quad \left. + \frac{\sigma_r[4e^{-a_r(T-t)} - e^{-2a_r(T-t)} + 4\underline{\theta}^T e^{-a_r(T-t)}]}{2\alpha_r}\right\}. \qquad (36)
\end{aligned}$$

Using (32) and (36), we rewrite (31) as

$$Y(t) = H(t)^n \exp\left\{(n-1)[\bar{r}(T-t)] - \bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r(T-t)} + \frac{1}{2}\|\underline{\theta}\|^2(T-t)\right\} \exp\left\{\frac{(n-1)^2}{2}\left\{\|\underline{\theta}\|^2(T-t) + \frac{\sigma_r(T-t)(1+2\underline{\theta}^T)}{\alpha_r} + \frac{\sigma_r[4e^{-a_r(T-t)} - e^{-2a_r(T-t)} + 4\underline{\theta}^T e^{-a_r(T-t)}]}{2\alpha_r}\right\}\right\}. \tag{37}$$

On the other hand (37) can be expressed as the function of $H(t), J(n-1), K(\frac{1}{a_r} - \frac{\sigma_L}{a_a}), L(r)$ and a time function $\Theta(t)$.

$$Y(t) = H(t)^n \Theta(t) \exp\left[J(n-1)K\left(\frac{1}{a_r} - \frac{\sigma_L}{a_r}\right)L(r)\right]. \tag{38}$$

Applying Itô formula to $Y(t)$ and denoting $[\cdot]dt$ as deterministic factors, we have

$$\frac{dY(t)}{Y(t)} = n\frac{dH(t)}{H(t)} + J(n-1)K\left(\frac{1}{a_r} - \frac{\sigma_L}{a_r}\right)dr + [\cdot]dt. \tag{39}$$

Substituting (8) and (10) into (39), we obtain

$$\frac{dY(t)}{Y(t)} = n\frac{dH(t)}{H(t)} + J(n-1)K\left(\frac{1}{a_r} - \frac{\sigma_L}{a_r}\right)\frac{\sigma_r}{\sigma_L}\frac{dL(t)}{L(t)} + [\cdot]dt. \tag{40}$$

Here $J(n-1)$ is equal to $(n-1)$, $K(\frac{1}{a_r} - \frac{\sigma_L}{\sigma_r})$ is equal to $(\frac{1}{a_r} - \frac{\sigma_L}{\sigma_r})$, and let $n = \frac{1}{\gamma} a.s.$

Proposition 2. *Bank shareholders optimal investment strategy under CRRA is deterministic and given by*

$$\begin{cases} x_S^* = \dfrac{1}{\gamma}w_S \\ x_L^* = \left(1 - \dfrac{1}{\gamma}\right)\left(\dfrac{\sigma_r}{\sigma_r\sigma_L} - 1\right) + \dfrac{1}{\gamma}w_L \\ x_M^* = 1 - \dfrac{1}{\gamma}(w_S + w_L) - \left(1 - \dfrac{1}{\gamma}\right)\left(\dfrac{\sigma_r}{\sigma_r\sigma_L} - 1\right) \end{cases}.$$

The logarithmic portfolio under Proposition 2, has a constant weight strategy on two assets similar to Merton (1971) results. The optimal investment strategy contains the optimal level of stock, interest rate risk and inflation uncertainty. The desired level of stock market risk subject to inflation uncertainty is obtained by holding the stock index. The desired amount of interest rate risk under inflation uncertainty is achieved by an appropriate position in the loans. With stochastic interest rates the bank shareholders hold a constant weight strategy in stock index while gradually over time re-allocating the remaining wealth from the loan portfolio into treasuries portfolios. Furthermore, the optimal asset strategy is quite sensitive to the risk aversion parameter γ.

4.2. Optimal strategies of HARA shareholders

We define the family of hyperbolic absolute risk aversion (HARA) utility functions as

$$U_{\mathrm{HARA}}(Y) = \frac{\gamma}{\beta(1-\gamma)}\left[\frac{\beta(Y-Y^*)}{\gamma+\eta}\right]^{1-\gamma}. \tag{41}$$

Generally, HARA family has the following three types:

(i) If $\eta = 0$, we will obtain the power utility function.
(ii) If $\eta = 1$ and when $\gamma \to +\infty$, we obtain the exponential utility function $U_{\exp}(Y) = -\frac{1}{\beta}\exp[-\beta(Y-Y^*)], \beta > 0$.
(iii) If $\gamma \to 1$, we obtain a modified version of a HARA function $U_m(Y) = \frac{\gamma}{\beta(1-\gamma)}\{[\frac{\beta(Y-Y^*)}{\gamma+\eta}]^{1-\gamma} - 1\}$ which optimization problem admits the same solution as (35) and one could obtain the extended logarithmic utility function $U_{\log}(Y) = \frac{1}{\beta}\log[\beta(Y-Y^*)+\eta], \beta > 0, \eta > 0$.

In order to carry out our analysis, we set $\beta = 1$, $\eta = 0$ and obtain the power utility function

$$U_{\mathrm{HARA}}(Y) = \frac{\gamma}{1-\gamma}\left(\frac{Y-Y^*}{\gamma}\right)^{1-\gamma}, \tag{42}$$

where γ is the sensitivity of risk aversion and Y^* is a constant.

Then for HARA utility functions with increasing risk aversion $\gamma > 0$ will result in Y^* to be a minimum level of wealth, whereas with decreasing risk aversion $\gamma < 0$, Y^* will be a maximum level of wealth. Note that when $\gamma = 0$, Y^* will be unbounded. The optimal terminal wealth of a HARA bank shareholder can be expressed as

$$\max E\left[\frac{\gamma}{1-\gamma}\left(\frac{Y-Y^*}{\gamma}\right)^{1-\gamma}\right] \quad \text{subject to } E\left[\frac{Y(T)}{H(T)}\right] = Y_0. \tag{43}$$

Then the optimal terminal wealth $Y^*(T)$ under HARA utility function can be obtained

$$Y^*(T) = \gamma\left[\frac{H(T)}{\kappa'}\right]^{\frac{1}{\gamma}} + Y^*, \tag{44}$$

where κ' is the Lagrangian multiplier and $\kappa' = E\left[H(T)^{1-\gamma}\left(\frac{\gamma}{Y_0}\right)^{\gamma}\right]$.

Let $(\underline{y}', Y')$ denote the portfolio strategy of HARA utility functions

$$Y'(T) = \left[\frac{Y^* H(T)}{Y^*}\right]^n, \quad a.s., \tag{45}$$

and portfolio weights are given by $x_S'^{**}, x_L'^{**}$ and $x_T'^{**}$. Let $\varepsilon = \frac{Y^*}{Y^{**}}$, then ε is a function of $H(T)$, since Y^* is a constant. Observing (43) and using the definition of ε, we define $\varepsilon = H(T)^\gamma$. Hence, $Y'(t)$ can be rewritten as $Y'(T) = [\varepsilon H(T)]^n = [H(T)]^{n(\gamma+1)}$.

Hence

$$\frac{Y'(t)}{H(t)} = E\left[\frac{Y'(T)}{H(T)}\right] = E\left[\frac{(H(T))^{n(\gamma+1)}}{H(T)}\right], \tag{46}$$

and $Y'(t)$ can be written as

$$Y'(t) = H(t)^{n(\gamma+1)}[n(\gamma+1)-1]E(e^{G_n}), \tag{47}$$

where

$$\begin{aligned} E(e^{G_n}) = \exp&\left\{(n-1)[\bar{r}(T-t)] - \bar{r}(T-t) - \frac{r(0)-\bar{r}}{a_r}e^{-a_r(T-t)} + \frac{1}{2}\|\underline{\theta}\|^2(T-t)\right\} \\ &\times \exp\left\{\frac{(n-1)^2}{2}\left\{\|\underline{\theta}\|^2(T-t) + \frac{\sigma_r(T-t)(1+2\underline{\theta}^T)}{\alpha_r}\right.\right. \\ &\left.\left. + \frac{\sigma_r[4e^{-a_r(T-t)} - e^{-2a_r(T-t)} + 4\underline{\theta}^T e^{-a_r(T-t)}]}{2\alpha_r}\right\}\right\}. \end{aligned} \tag{48}$$

Then (47) can be written as

$$Y'(t) = H(t)^{n(\gamma+1)}\Theta(t)\exp\left[J[n(\gamma+1)-1]K\left(\frac{1}{a_r} - \frac{\sigma_L}{a_r}\right)L(r)\right]. \tag{49}$$

Applying Itô formula to $Y'(t)$ and denoting $[\cdot]dt$ as deterministic factors, we have

$$\frac{dY'(t)}{Y'(t)} = n(\gamma+1)\frac{dH(t)}{H(t)} + J[n(\gamma+1)-1]K\left(\frac{1}{a_r} - \frac{\sigma_L}{a_r}\right)dr + [\cdot]dt. \tag{50}$$

Substituting (8) and (10) into (50), we obtain

$$\frac{dY'(t)}{Y'(t)} = n(\gamma+1)\frac{dH(t)}{H(t)} + J[n(\gamma+1)-1]K\left(\frac{1}{a_r} - \frac{\sigma_L}{a_r}\right)\frac{\sigma_r}{\sigma_L}\frac{dL(t)}{L(t)} + [\cdot]dt. \tag{51}$$

Here $J[n(\gamma+1)-1]$ is equal to $n(\gamma+1)-1$, $K(\frac{1}{a_r} - \frac{\sigma_L}{\sigma_r})$ is equal to $(\frac{1}{a_r} - \frac{\sigma_L}{\sigma_r})$, and let

$$n = \frac{1}{\gamma} a.s.$$

Hence, (51) can be expressed as

$$\frac{dY'(t)}{Y'(t)} = \frac{\gamma+1}{\gamma}\frac{dH(t)}{H(t)} + \frac{1}{\gamma}\left(\frac{\sigma_r}{a_r\sigma_L} - 1\right)\frac{dL(t)}{L(t)} + [\cdot]dt. \tag{52}$$

Proposition 3. *Bank shareholders optimal investment strategy under HARA is given by*

$$\begin{cases} x_S^* = \dfrac{\gamma+1}{\gamma}w_S \\ x_M^* = \dfrac{\gamma+1}{\gamma}w_L - \dfrac{1}{\gamma^2}\left(\dfrac{\sigma_r}{\sigma_r\sigma_L} - 1\right) \\ x_T^* = 1 - \dfrac{\gamma+1}{\gamma}(w_S + w_L) + \dfrac{1}{\gamma^2}\left(\dfrac{\sigma_r}{\sigma_r\sigma_L} - 1\right) \end{cases}.$$

The logarithmic portfolio under Proposition 3 has constant weight that is similar to Merton (1971) results. With stochastic interest rates the bank shareholders will hold a constant weight strategy in stock index, while gradually (over time) re-allocating the remaining wealth from the loan portfolio to a treasuries portfolio. Furthermore, the optimal asset strategy is quite sensitive to the risk aversion.

Remark 1. The TRWAs at time t, $b_{rw}(t)$ can be described by the following SDE:

$$\frac{db_{rw}(t)}{b_{rw}(t)} = 0 \times (1 - w_S(t) - w_L(t))\frac{dB(t)}{B(t)} + 0.2 \times w_S(t)\frac{dS(t)}{S(t)} + 0.5 \times w_L(t)\frac{dL(t)}{L(t)},$$

which can be simplified as

$$\begin{aligned}\frac{db_{rw}(t)}{b_{rw}(t)} &= [0.2w_S(t)(r(t)+\xi_S) + 0.5w_L(t)(r(t)+\xi_L)]dt \\ &\quad + [0.2w_S(t)\sigma_1]dW(t) + [0.2w_S(t)\sigma_2 + 0.5w_L(t)\sigma_L]dW_r(t).\end{aligned} \tag{53}$$

Proposition 4 (Explicit SDE for the Basel II CAR). *Suppose that the dynamics of total bank capital $C(t)$ and total risk-weighted assets $b_{rw}(t)$ are given by* (6) *and* (53), *respectively. Then the dynamics of the Basel II capital adequacy ratio $X(t)$ of a bank is governed by the following SDE*:

$$dX(t) = X(t)(\alpha_1 - \alpha_2 - \alpha_3)dt + X(t)(\beta_1 d\hat{W}(t) - \beta_2 dW(t) - \beta_3 dW_r(t)), \tag{54}$$

where

$$\alpha_1 = r(t) + w^T(t)\Psi\vartheta, \alpha_2 = \rho X(t) + w^T(t)\Psi(0.2w_S\sigma_1),$$

$$\begin{aligned}\alpha_3 &= 0.2w_S(t)(r(t)+\xi_S)+(0.2w_S(t)\sigma_1)^2+(0.2w_S(t)\sigma_2)^2\\&\quad+(0.5w_L(t)(r(t)+\xi_L))+(0.5w_L(t)\sigma_L)^2,\\\beta_1 &= w^T(t)\Psi,\quad \beta_2=0.2w_S\sigma_1,\quad \beta_3=0.2w_S\sigma_2+0.5w_L\sigma_L.\end{aligned}$$

Proof. In this proof, we derive (54) by mainly using the general Itô formula.

Let $f(b_{rw}(t))=\frac{1}{b_{rw}(t)}$. Then

$$\begin{aligned}df(b_{rw}(t)) &= f(t)dt+f'(b_{rw}(t))dY_1(t)+\frac{1}{2}f''(b_{rw}(t))[db_{rw}(t)]^2\\&= 0dt-\frac{db_{rw}(t)}{b_{rw}^2(t)}+\frac{[db_{rw}(t)]^2}{b_{rw}^3(t)}\\&= -\frac{1}{b_{rw}(t)}\{[0.2w_S(t)(r(t)+\xi_S)+0.5w_L(t)(r(t)+\xi_L)]dt\\&\quad+[0.2w_S\sigma_1]dW(t)+[0.2w_S\sigma_2+0.5w_L\sigma_L]dW_r(t)\}\\&\quad+\frac{1}{b_{rw}(t)}\{[0.2w_S(t)\sigma_1]^2dt+[0.2w_S(t)\sigma_2+0.5w_L(t)\sigma_L]^2dt\}.\end{aligned}$$

Through algebraic manipulation and re-arranging the drift and diffusion part of $df(b_{rw}(t))$, we obtain

$$\begin{aligned}df(b_{rw}(t)) &= -\frac{1}{b_{rw}(t)}\{[0.2w_S(t)(r(t)+\xi_S)+(0.2w_S(t)\sigma_1)^2+(0.2w_S(t)\sigma_2)^2\\&\quad+(0.5w_L(t)(r(t)+\xi_L))+(0.5w_L(t)\sigma_L)^2]dt\\&\quad+[0.2w_S\sigma_1]dW(t)+[0.2w_S\sigma_2+0.5w_L\sigma_L]dW_r(t)\}.\end{aligned}$$

The CAR is expressed as: $X(t)=\frac{C(t)}{b_{rw}(t)}=C(t)f(b_{rw}(t))$.

Now we apply general Itô's product rule to $X(t)$, in order to find an expression for

$$\begin{aligned}dX(t) &= d(f(b_{rw}(t)C(t))=f(b_{rw}(t))dC(t)+C(t)df(b_{rw}(t))+dC(t)df(b_{rw}(t))\\&= \frac{C(t)}{b_{rw}(t)}\{[(r(t)+w^T(t)\Psi\vartheta)+w^T(t)\Psi d\hat{W}(t)]-\rho X(t)dt\}\\&\quad-\frac{C(t)}{b_{rw}(t)}\{[0.2w_S(t)(r(t)+\xi_S)+(0.2w_S(t)\sigma_1)^2+(0.2w_S(t)\sigma_2)^2\\&\quad+(0.5w_L(t)(r(t)+\xi_L))+(0.5w_L(t)\sigma_L)^2]dt\\&\quad+[0.2w_S\sigma_1]dW(t)+[0.2w_S\sigma_2+0.5w_L\sigma_L]dW_r(t)\}.\\&\quad-\frac{C(t)}{b_{rw}}w^T(t)\Psi(0.2w_S\sigma_1)dt.\\dX(t) &= X(t)(\alpha_1-\alpha_2-\alpha_3)dt+X(t)(\beta_1 d\hat{W}(t)-\beta_2 dW(t)-\beta_3 dW_r(t)),\end{aligned}$$

where we have defined $\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2$ and β_3 in this proposition. □

4.3. Comparative statistics

In Sec. 4.1, we provided a solution to the optimal strategy and the Basel II CAR. The optimal equity investment strategy shows that over a chosen time horizon T, the optimal proportion invested in treasuries increases and the optimal proportion invested in the loan decreases while the proportion invested in securities remain constant. This in turn demonstrates that the evolution of an optimal strategy is actually affected by realisation of the stochastic variables that characterise the economy.

5. Conclusion

This paper analyzes an optimal investment and management strategy of a bank under CRRA and HARA utility functions when investing in treasuries, stock index fund and loans in an environment of stochastic interest rate and inflation uncertainty. We consider the portfolio choice under a power utility that the bank's shareholders can maximize expected utility of wealth at a given investment horizon. We derive an explicit SDE for the dynamics of the Basel II CAR as a quotient of the total bank capital and TRWAs under *stochastic interest rate* and *inflation uncertainty*. The results show that under CRRA and HARA utility functions the weight of stock is constant. However, as time goes by, the weight of wealth shifts away from loan portfolio toward treasuries.

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