Excess Volatility of Corporate Bonds

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October 4, 2010

Abstract

This paper examines the connection between the return volatilities of corporate bonds, equities, and Treasuries under the Merton model with stochastic interest rates. Constructing empirical volatilities using bond returns over daily, weekly, and monthly horizons, we find that empirical bond volatilities are too high to be explained by equity and Treasury volatilities. Furthermore, the results are robust to using credit default swaps rather than corporate bonds to measure volatility in the credit market. At the daily return horizon, the excess volatility of corporate bonds is related to known liquidity proxies. However, this relation disappears at the monthly horizon even though corporate bonds continue to be excessively volatile. Thus, there appears to be a disconnect between corporate bonds and equities that goes beyond the illiquidity of corporate bonds.

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1 Introduction

We examine the relation between corporate bond and equity volatility of the same firm through a Merton (1974) model with stochastic interest rates, finding that empirically observed corporate bond volatilities are higher than those implied by equity volatilities. This excess volatility exists whether we use daily, weekly, or monthly returns. At a daily return horizon, the mean difference in annualized return volatilities is 16.31% while at a monthly return horizon, the difference in annualized volatilities is smaller, but still statistically and economically significant at 3.40%.

Our motivation mainly draws from two sources. First, we aim to better understand how securities in different markets are related, focusing on empirical evidence from return volatilities. Consider, for example, a firm that has a very low probability of default. For bonds of a typical maturity on such a firm, we would expect to see a low volatility, comparable to the volatility on a government bond of similar maturity. A much riskier firm would have a component of volatility common to equity. However, precisely pinning down how these volatilities should be related requires a model. To connect the diverging information from the three markets – corporate bond, equity and Treasury bond – into one unified framework, we adopt the Merton model with stochastic interest rates. In the model, corporate bond volatility has two contributions: random fluctuations in firm value and random fluctuations in the risk-free interest rate. While Treasury bond volatility can be directly estimated using Treasury bond returns, we can only infer, via the Merton model, the firm's asset volatility from estimates of the equity volatility of the firm. Consequently, the two important inputs of the model come from the volatility estimates of equity and of Treasury bond returns. In addition, we also rely on firm-level balance sheet information to estimate the other firm-level characteristics that are important to the model. By implementing a structural model of default with empirical equity and Treasury volatilities as inputs, we are able to compare bond volatilities to a benchmark, making quantitative rather than purely qualitative assessments. Overall, our calibrations indicate that relative to equity volatility and the Merton model, corporate bond volatility is too high. This result holds whether we use daily, weekly, or monthly returns to estimate volatility and also across different years, bond ratings, and bond durations.

Our second main motivation is to put a horizon dimension in examining the disconnect between bonds and stocks. We find a dramatic difference between corporate bond volatility measured at daily and monthly return horizons, reflecting the lack of liquidity in the corporate bond market. In fact, Huang and Huang (2003) suggest that a significant part of empirical yield spreads may be attributed to a premium for illiquidity. Chen, Lesmond, and Wei (2007) and Bao, Pan, and Wang (2010) have also provided evidence that yield spreads are related to illiquidity. By using daily returns, we are able to examine how much of a disconnect there is when a strong illiquidity effect is included. We then contrast this with monthly returns, which have less of an illiquidity effect and may reflect a fundamental disconnect. At this return horizon, we continue to find a significant excess volatility in the corporate bond market, with a mean excess volatility of 3.40%, which is approximately 40% of the empirically observed bond volatility.

To better understand the source of the discrepancy between empirical and model corporate bond volatilities, we examine both the cross-sectional and time-series relations between empirical and model volatilities. The horizon result for corporate bonds, the finding that excess volatility is particularly high when daily returns are used, suggests that liquidity is an important component in understanding excess volatility. As the Merton model is not designed to capture liquidity, any volatility that stems from the illiquidity of the corporate bond market will be reflected in the excess volatility. Indeed, we find direct evidence of a liquidity effect as proxies for liquidity such as amount outstanding, age, and quoted bid-ask spread help to explain excess volatility in the cross-section when daily returns are used. Thus, much of the large excess volatility for daily bond returns comes from the illiquidity of the corporate bond market.

Compared to the excess volatility when daily bond returns are used, the excess volatility when monthly bond returns are used is much smaller. Though smaller, this excess volatility is still statistically and economically significant. When monthly returns are used, the proxies for liquidity are statistically insignificant in explaining excess volatility. This suggests that the disconnect between empirical and model corporate bond volatilities goes beyond the illiquidity of corporate bonds. Furthermore, we also consider credit default swaps, which are considered to be much more liquid than corporate bonds, and continue to find excess volatility in the credit market relative to the equity market. Consistent with our findings in the corporate bond market, this suggests that fundamentals play an important role in the excess volatility of the credit market.

Though the Merton model cannot explain the level of volatilities in credit markets, it may play a role in helping us to characterize the time-series variation of corporate bond volatilities as there is significant co-movement between empirical and model volatilities. We first focus on the aggregate, constructing the mean empirical and model volatilities from daily CDS returns each month. These mean volatilities co-move quite closely with a correlation of 0.9478. At the individual bond and CDS-level, we run panel regressions of empirical bond volatilities on model bond volatilities with bond fixed-effects, finding a positive relation between empirical and model volatilities and within-group R^2 values that range from 11.71% to 38.06%. Thus, despite the Merton model's inability to capture the correct level of bond volatilities, it is useful in helping to capture the time-series variation of empirical corporate bond volatilities.

In the structural model dimension, a number of papers have examined the ability of structural models to explain the prices of corporate bonds, with most studies concluding that structural models overvalue corporate debt (predicting yield spreads that are too low), though there are some exceptions. In addition to the Huang and Huang (2003) paper, empirical studies include papers by Jones, Mason, and Rosenfeld (1984) and Eom, Helwege, and Huang (2004).[1] The focus of these papers is on the levels of prices rather than returns or volatilities. Focusing on the Merton model and a sample of 27 firms from 1975 to 1981, Jones, Mason, and Rosenfeld (1984) are perhaps the first to implement a structural model of default empirically and find that model prices are on average higher than empirical prices. Huang and Huang (2003) calibrate a number of structural models at the ratings level to match historical default probabilities, finding that model yield spreads are too low compared to empirical yield spreads. In contrast, Eom, Helwege, and Huang (2004) find that in their sample of 182 data points, the Merton model underpredicts empirical corporate bond yield spreads, but other structural models actually overpredict yield spreads. Thus, the general consensus is that structural models of default do poorly in explaining the level of bond prices.

Though structural models of default have typically been used to price corporate bonds, they also have implications for the relation between equities and corporate bonds. For example, Schaefer and Strebulaev (2008) find that Merton model hedge ratios perform well in relating bond returns to stock returns. This insightful result is a stark contrast to previous results on the failures of structural models of default and reflects the fact that both equity and bond returns are too high to be explained by fundamentals, but they are too high in the correct proportion. A natural conclusion from this result is that it may be fruitful to examine whether there is a common source of mispricing in the two markets.[2] Unlike Schaefer and Strebulaev (2008), we do find a disconnect between the bond and equity markets in our examination of volatilities. In addition, Vassalou and Xing (2004) and Campbell, Hilscher, and Szilagyi (2007) also relate bond and equity markets, though more indirectly. Both papers examine the relation between default probabilities and equity returns. Using a structural model approach, Vassalou and Xing (2004) find some evidence of a positive default premium while Campbell, Hilscher, and Szilagyi (2007) find lower returns for high default firms using a reduced-form model.

[1] A number of other papers have related evaluations of the Merton model including Crosbie and Bohn (2003), Leland (2004), and Bharath and Shumway (2008) which use structural models to forecast default probabilities.

[2] In fact, Bhamra, Kuehn, and Strebulaev (2009) examine the credit spread and equity premium puzzles in a unified framework.

The rest of the paper is organized as follows. Section 2 outlines the empirical specification. Section 3 summarizes the data and the empirical volatility estimates. Section 4 details the model implied volatility. Section 5 summarizes the main empirical results of our paper. Section 6 reports further examinations of excess volatility. Section 7 concludes.

2 Empirical Specification

2.1 The Merton Model

We use the Merton (1974) model to connect the equity and corporate bonds of the same firm.

Let V be the total firm value, whose risk-neutral dynamics are assumed to be

$$\frac{dV_t}{V_t} = (r_t - \delta)\, dt + \sigma_v\, dW_t^Q\,, \tag{1}$$

where W is a standard Brownian motion, and where the payout rate δ and the asset volatility σ_v are assumed to be constant.

We adopt a simple extension of the Merton model to allow for a stochastic interest rate.[3] This is important for our purposes because a large component of the corporate bond volatility comes from the Treasury market. Specifically, we model the risk-free rate using the Vasicek (1977) model:

$$dr_t = \kappa\,(\theta - r_t)\ dt + \sigma_r\, dZ_t^Q\,, \tag{2}$$

where Z is a standard Brownian motion independent of W, and where the mean-reversion rate κ, long-run mean θ and the diffusion coefficient σ_r are assumed to be constant.

Following Merton (1974), let us assume for the moment that the firm has, in addition to its equity, a single homogeneous class of debt, and promises to pay a total of K dollars to the bondholders on the pre-specified date T. Equity then becomes a call option on V:

$$E_t = V_t\, e^{-\delta\tau}\, N(d_1) - K\, e^{a(\tau)+b(\tau)\, r_t}\, N(d_2), \tag{3}$$

where $\tau = T - t$, $N(\cdot)$ is the cumulative distribution function for a standard normal, $d_1 = d_2 + \sqrt{\Sigma}$,

$$d_2 = \frac{\ln(V/K) - a(\tau) - b(\tau) r_t - \delta\tau - \frac{1}{2}\Sigma}{\sqrt{\Sigma}}, \tag{4}$$

[3]See Shimko, Tejima, and van Deventer (1993).

$$\Sigma = \tau(\sigma_v^2 + \frac{\sigma_r^2}{\kappa^2}) + \frac{2\sigma_r^2}{\kappa^3}(e^{-\kappa\tau} - 1) - \frac{\sigma_r^2}{2\kappa^3}(e^{-2\kappa\tau} - 1), \tag{5}$$

and where $a(\tau)$ and $b(\tau)$ are the exponents of the discount function of the Vasicek model:

$$b(\tau) = \frac{e^{-\kappa\tau} - 1}{\kappa}; \quad a(\tau) = \theta\left(\frac{1 - e^{-\kappa\tau}}{\kappa} - \tau\right) + \frac{\sigma^2}{2\kappa^2}\left(\frac{1 - e^{-2\kappa\tau}}{2\kappa} - 2\frac{1 - e^{-\kappa\tau}}{\kappa} + \tau\right). \tag{6}$$

Note that a Merton model extended to have Vasicek interest rates simply has $e^{-r\tau}$ replaced by $e^{a(\tau)+b(\tau)r_t}$ and $\sigma_v^2\tau$ replaced by Σ.

2.2 From Equity Volatility to Asset Volatility

We first use the Merton model to link the firm's asset volatility to its equity volatility. Let σ_E be the volatility of instantaneous equity returns. In the model, the equity volatility is affected by two sources of random fluctuations:

$$\sigma_E^2 = \left(\frac{\partial \ln E_t}{\partial \ln V_t}\right)^2 \sigma_v^2 + \left(\frac{\partial \ln E_t}{\partial r_t}\right)^2 \sigma_r^2 . \tag{7}$$

Using equation (3), we can calculate the sensitivities of equity returns to the random shocks in asset returns and risk-free rates:

$$\frac{\partial \ln E_t}{\partial \ln V_t} = \frac{1}{1 - \mathcal{L}} \quad \text{and} \quad \frac{\partial \ln E_t}{\partial r_t} = \frac{-b(\tau)\,\mathcal{L}}{1 - \mathcal{L}},$$

where

$$\mathcal{L} = \frac{K}{V}\,\frac{N(d_2)}{N(d_1)}\,\exp\left(\delta\,\tau + a(\tau) + b(\tau)\,r_t\right) . \tag{8}$$

Combining the above equations, we have

$$\sigma_E^2 = \left(\frac{1}{1 - \mathcal{L}}\right)^2 \sigma_v^2 + \left(\frac{\mathcal{L}}{1 - \mathcal{L}}\right)^2 b(\tau)^2\,\sigma_r^2 . \tag{9}$$

As expected, the firm's equity volatility σ_E is closely related to its asset volatility σ_v. In addition, it is also affected by the Treasury volatility σ_r through the firm's borrowing activity in the bond market. This is reflected in the second term of equation (9), with $-b(T)\,\sigma_r$ being the volatility of instantaneous returns on a zero-coupon risk-free bond of the same maturity T. The actual impact of these two random shocks is further amplified through $\mathcal{L}$, which, for lack of a better expression, we refer to as the "modified leverage." Specifically, for a firm with a higher $\mathcal{L}$, a one unit shock to its asset return is translated to a larger shock to its equity return – this is the standard leverage effect. Moreover, as shown in the second term of equation (9), for such a highly "levered" firm, its equity return also bears more interest rate risk. Conversely, for an all-equity firm, $\mathcal{L} = 0$, and the interest-rate component diminishes to zero.

As is true in many empirical studies before us, a structural model such as the Merton model plays a crucial role in connecting the asset value of a firm to its equity value. Ours is not the first empirical exercise to back out asset volatility using observations from the equity market.[4] In the existing literature, there are at least two alternative ways to approximate K/V. In the approach pioneered and popularized by Moody's KMV, the Merton model is used to calculate $\partial E/\partial V$ as well as to infer the firm value V through equation (3). By contrast, we use the Merton model to derive the entire piece of the sensitivity or elasticity function $\partial \ln E/\partial \ln V$, as opposed to using only $\partial E/\partial V$ from the model and then plugging in the market observed equity value E for the scaling component. At a conceptual level, we believe that taking the entire piece of the sensitivity function from the Merton model is a more consistent approach. At a practical level, while the Merton model might have its limitations in the exact valuation of bonds and equities, it is still valuable in providing insights on how a percentage change in asset value propagates to percentage changes in equity value for a levered firm.[5]

In this respect, our reliance on the Merton model centers on the sensitivity measure. To the extent the Merton model is important in our empirical implementation, it is in deriving the analytical expressions that enter equation (9). In particular, we rely on the Merton model to tell us how the sensitivities or elasticities vary as functions of the key parameters of the model including leverage K/V, asset volatility σ_v, payout rate δ, and debt maturity T. When it comes to the actual calculations of these key parameters, we deviate from the Merton model as follows.

The key parameter that enters equation (9) is the ratio K/V, where K is the book value of debt and V is the market value of the firm. We calculate the book debt K using Compustat data, and approximate the firm value V by its definition $V = S + D$, where S is the market value of equity and D is the market value of debt. To estimate the market value of debt D, we start with the book value of debt K. To further improve on this approximation, we collect, for each firm, all of its bonds in TRACE, calculate an issuance weighted market-to-book

[4]See, for example, Crosbie and Bohn (2003), Eom, Helwege, and Huang (2004), Bharath and Shumway (2008), and Vassalou and Xing (2004).

[5]Particularly in light of the results by Schaefer and Strebulaev (2008) that a Merton model does well in relating corporate bond and equity returns and the Huang and Huang (2003) results that the levels of corporate bond yield spreads are too low, we feel that using the Merton model to provide model elasticities alone rather than model prices is the best use of the model.

ratio, and multiply K by this ratio.

Implicit in our estimation of the firm value V is the acknowledgment that firms do not issue discount bonds as prescribed by the Merton model. In particular, we deviate from the zero-coupon structure of the Merton model in order to take into account the fact that firms typically issue bonds at par. By adopting this empirical implementation, however, we do have to live with one internal inconsistency with respect to the relation between K and D, and central to this inconsistency is the problem of applying a model designed for zero-coupon bonds to coupon bonds.

The main implication of our choice of V is on the ratio of K/V, which in turn, affects the firm's actual leverage. We can therefore gauge the impact of our implementation strategy by comparing the market leverage implied by the Merton model with the empirically estimated market leverage. Our results show that with our choice of K/V, the two market leverage numbers, model implied vs. empirically estimated, are actually very close for the sample of firms considered in this paper. Closely related to this comparison is the alternative estimation strategy that infers K/V by matching the two market leverage ratios: model-implied and empirically estimated.[6] From our analysis, we expect this approach to yield K/V ratios that are close to ours.

Finally, two other parameters that enter equation (9) are the firm-level debt maturity T and the firm's payout ratio δ. Taking into account the actual maturity structure of the firm, we collect, for each firm, all of its bonds in FISD and calculate the respective durations. We let the firm-level T be the issuance-weighted duration of all the bonds in our sample. Effectively, we acknowledge the fact that firm's maturity structure is more complex than the zero-coupon structure assumed in the Merton model, and our issuance-weighted duration is an attempt to map the collection of coupon bonds to the maturity of a zero-coupon bond. In calculating the payout ratio δ, we aggregate the firm's equity dividends and debt coupon payments and scale the total payout by firm value V, with the details of calculating V summarized above.

2.3 Model-Implied Bond Volatility

The second step of our empirical implementation is to calculate, bond-by-bond, the volatility of its instantaneous returns, taking the inferred asset volatility σ_v from the first step as a key input. Again, we have to make a simplification to the Merton model to accommodate the bonds of varying maturities issued by the same firm. Specifically, we rely on the Merton model to tell us, for any given time τ, the value of payments at τ contingent on $V_\tau > K$.

[6] We thank Hayne Leland for pointing this out and for extensive discussions on this issue.

Compared to taking the Merton model literally, which would imply no default between time 0 and the maturity date T, we find this to be a more realistic adoption of the model.[7]

Equipped with the term structure of default probabilities implied by the Merton model, we can now price defaultable bonds issued by each firm. Consider a τ-year bond paying semi-annual coupons with an annual rate of c. Assuming a face value of \$1, the time-t price of the bond is

$$B_t = \sum_{i=1}^{2\tau} \frac{c}{2} E_t^Q \left[\exp\left(-\int_t^{t+i/2} r_s ds \right) \mathbf{1}_{\{V_{t+i/2}>K\}} \right] + E_t^Q \left[\exp\left(-\int_t^{T} r_s ds \right) \mathbf{1}_{\{V_T>K\}} \right] \quad (10)$$
$$+ \sum_{i=1}^{2\tau} \mathcal{R} \left\{ E_t^Q \left[\exp\left(-\int_t^{t+i/2} r_s ds \right) \mathbf{1}_{\{V_{t+(i-1)/2}>K\}} \right] - E_t^Q \left[\exp\left(-\int_t^{t+i/2} r_s ds \right) \mathbf{1}_{\{V_{t+i/2}>K\}} \right] \right\}$$

where $\mathcal{R}$ is the risk-neutral expected recovery rate of the bond upon default. The first two terms in equation (10) collect the coupon and the principal payments taking into account the probabilities of survival up to each payment. The third term collects the recovery of the bond taking into account the probability of default happening exactly within each six-month period. The solutions to these expectations along with the full bond pricing formula are given in Appendix A.

Let σ_D^{Merton} be the volatility of the instantaneous returns of the defaultable bond. The model-implied bond volatility can be calculated as

$$\left(\sigma_D^{\text{Merton}}\right)^2 = \left(\frac{\partial \ln B_t}{\partial \ln V_t} \right)^2 \sigma_v^2 + \left(\frac{\partial \ln B_t}{\partial r_t} \right)^2 \sigma_r^2 \, . \quad (11)$$

The asset-sensitivity, $\partial \ln B_t / \partial \ln V_t$, arises from the sequence of (present value adjusted) risk-neutral default probabilities while the Treasury-sensitivity, $\partial \ln B_t / \partial r_t$ arises both explicitly from the sequence of Vasicek discount functions and implicitly from the sequence of risk-neutral default probabilities.

It might be instructive to consider a τ-year zero-coupon bond, since its calculation can be further simplified to

$$\frac{\partial \ln B_t}{\partial \ln V_t} = \frac{n(d_2)\,(1-\mathcal{R})}{N(d_2) + (1 - N(d_2))\,\mathcal{R}} \frac{1}{\sqrt{\Sigma}} \quad \text{and} \quad \frac{\partial \ln B_t}{\partial r_t} = b(\tau) \left(1 - \frac{\partial \ln B_t}{\partial \ln V_t} \right) ,$$

where $n(\cdot)$ is the probability distribution function of a standard normal. As expected, with full recovery upon default, $\mathcal{R} = 1$, the bond is equivalent to a treasury bond and its asset-sensitivity is zero and its Treasury-sensitivity becomes $b(\tau)$. The asset-sensitivity becomes

[7] A more self-consistent approach is to use the Black and Cox (1976) model, which generates a term structure of default probability that is the complementary first passage time distribution.

more important with increasing loss given default, $1-\mathcal{R}$, as well as with increasing firm leverage K/V. From this example, we can also see the importance of allowing for a stochastic risk-free rate, as the Treasury volatility is an important component in the defaultable bond volatility.

In calculating the model-implied bond volatility, we take advantage of the model-implied term structure of survival probabilities but avoid treating the defaultable bond as one large piece of zero-coupon bond with face value of K and maturity of T. This calculation is similar to the reduced-form approach of Duffie and Singleton (1999), except for the fact that our term structure of survival probabilities come from a structural model while theirs derives from a stochastic default intensity.

3 Data and Construction of Volatility Estimates

3.1 The TRACE Data Set

The bond pricing data for this paper are obtained from FINRA's TRACE (Transaction Reporting and Compliance Engine). This data set is a result of recent regulatory initiatives to increase the price transparency in the secondary corporate bond markets. FINRA, formerly NASD,[8] is responsible for operating the reporting and dissemination facility for over-the-counter corporate trades. Trade reports are time-stamped and include information on the clean price and par value traded, although the par value traded is truncated at \$1 million for speculative grade bonds and at \$5 million for investment grade bonds.

The cross-sections of bonds in our sample vary with the expansion of coverage by TRACE. On July 1, 2002, the NASD began Phase I of bond transaction reporting, requiring that transaction information be disseminated for investment grade securities with an initial issue of \$1 billion or greater. At the end of 2002, the NASD was disseminating information on approximately 520 bonds. Phase II, implemented on April 14, 2003, expanded reporting requirements, bringing the number of bonds to approximately 4,650. Phase III, implemented on February 7, 2005, required reporting on approximately 99% of all public transactions.

3.2 The Bond Sample

We use the transaction-level data from TRACE to construct bond return volatility for non-financial firms. First, we construct daily bond returns as follows. For any day t, we keep the

[8] In July 2007, the NASD merged with the regulation, enforcement, and arbitration branches of the New York Stock Exchange to form the Financial Industry Regulatory Authority (FINRA).

Table 1: **Bond Sample Summary Statistics**

Our Sample

	2002		2003		2004		2005		2006		2007		2008	
	mean	med	mean	med	mean	med	mean	med	mean	med	mean	med	mean	med
Num Bonds	144		266		342		445		477		504		572	
Maturity	8.62	7.32	8.17	6.31	7.58	5.83	7.23	5.51	7.13	5.31	8.04	5.37	8.66	5.26
Amt	1,513	1,325	1,085	1,054	978	821	868	694	892	738	953	754	1,039	826
Rating	6.58	7.00	6.07	6.17	6.26	6.25	7.51	7.00	7.78	7.00	7.98	7.25	7.93	7.00
Age	1.82	1.38	2.68	1.87	3.21	2.52	3.75	3.20	3.88	3.56	4.04	3.71	4.08	3.29
Trades	365	210	215	154	172	126	156	113	145	107	135	99	212	126
Volume	226	164	139	88	92	53	68	40	61	37	62	37	65	39
Turnover	13.89	11.88	10.95	8.54	8.38	6.33	7.40	5.36	6.76	4.81	6.31	4.38	6.00	4.41
Per Days Traded	95.64	97.65	94.88	98.82	93.93	98.63	93.15	96.86	92.72	96.13	91.65	94.52	92.14	94.88
Avg Trd Size	876	720	651	514	542	399	463	328	462	318	500	344	399	284
Firms	50		71		94		143		158		166		173	
Mktcap	39.71	21.26	43.53	22.97	43.30	25.92	34.20	14.83	34.37	14.40	38.47	16.70	33.06	14.65

US Corporates in FISD

	2002		2003		2004		2005		2006		2007		2008	
	mean	med	mean	med	mean	med	mean	med	mean	med	mean	med	mean	med
Num Bonds	20,956		21,775		23,606		25,871		27,983		31,118		30,517	
Maturity	7.30	4.51	7.39	4.57	7.56	4.70	7.58	4.81	7.33	4.53	7.02	4.09	7.15	4.13
Amt	172	64	173	54	169	46	162	30	160	24	160	16	172	12
Rating	7.60	7.00	7.52	6.92	7.12	6.50	6.89	6.00	6.61	5.75	6.15	5.00	6.60	5.50
Age	4.59	3.99	4.26	3.27	3.87	2.26	3.67	2.10	3.65	2.34	3.53	2.47	3.75	2.82

#Bonds and *#Firms* are the average numbers of bonds and firms per month. *Maturity* is the bond's time to maturity in years. *Amt* is the bond's amount outstanding in millions of dollars. *Rating* is a numerical translation of Moody's rating: 1=Aaa and 21=C. *Age* is the time since issuance in years. *#Trades* is the bond's total number of trades in a month. *Volume* is the bond's total trading volume in a month in millions of dollars of face value. *Turnover* is the bond's monthly trading volume as a percentage of the amount outstanding. *%Traded* is the percentage of business days in a month when the bond is traded. *Trd Size* is the average trade size of the bond in thousands of dollars of face value. *Mkt Cap* is the equity market capitalization in billions of dollars. The reported std and median are the time-series averages of cross-sectional values. Firms for which we cannot later calculate a model-implied σ_v and their corresponding bonds are omitted from the summary statistics under "Our Sample".

last observation of the day for the bond and calculate the log return on day t as:

$$R_t = \ln\left(\frac{P_t + AI_t + C_t}{P_{t-1} + AI_{t-1}}\right),$$

where P_t is the clean price as reported in TRACE, AI_t is the accrued interest, and C_t is the coupon paid at t if day t is a scheduled coupon payment day. We use FISD to get bond-level information on coupon rates and maturities. Accrued interest is calculated using the standard 30/360 convention. Returns are only calculated for day t if there is a price available for both t and $t-1$. Given the daily return data, we next construct time-series of monthly bond volatilities by taking the standard deviation of the daily bond returns (if there are at least 10 bond returns in a month) and annualizing. The sample of bonds that survive this calculation form the basis of our bond sample. In addition, to excluding very infrequently traded bonds, we include bonds for which we can construct monthly volatilities for at least 75% of its presence in TRACE.

Table 1 summarizes our bond sample. The number of bonds increases throughout our sample period largely due to the coverage expansion of TRACE. Compared with the universe of U.S. corporate bonds documented in FISD, our sample contains only a small number of bonds. In terms of size, however, these bonds are orders of magnitude larger than the median size bond in FISD. For example, at the beginning of our sample in 2002, the median bond size is $1,325 million in our sample, compared with $64 million in FISD. In the early sample, this is largely due to the limited coverage of TRACE, but overall, our sample construction biases toward picking more frequently traded bonds, which are typically larger.

The average maturity of the bonds in our sample is about 7.8 years, similar in magnitude but slightly higher than the average maturity of 7.3 years for the bond universe in FISD. While the cross-sectional median maturity is close to 5.7 years in our sample, it is only around 4.5 years in the FISD sample. These observations are consistent with a relatively higher degree of cross-sectional dispersion of bond maturity in FISD. In the early sample period, the bonds in our sample are noticeably younger than those in the FISD sample, although this difference disappears toward the later sample period. The representative bonds in our sample are investment grade, with a median rating of roughly 7 (Moody's A3).

Given that TRACE is transaction-level data, we can further collect trading information for the bonds in our sample. For example, in 2002, an average bond is traded on average 365 times a month with $226 million of average trading volume and 13.89% turnover. Over time, this set of numbers decrease quite significantly, reflecting the coverage expansion of TRACE to include smaller and less frequently traded bonds. By 2008, an average bond was traded on average 212 times a month with $65 million of average trading volume and 6%

turnover. Also, the average trade size is \$876 thousand in 2002 and \$399 thousand in 2008, reflecting the inclusion of smaller trades. Overall, compared with the entire TRACE sample, our sample is biased toward bonds that are more frequently traded. For example, on over 95% of the business days, a median bond in our sample is traded at least once on that day.

Merging our bond sample with CRSP and Compustat by bond issuer, the firm-level summary statistics are reported in Table 1. The average number of firms in our sample is 50 in 2002 and grows to 173 in 2008. By equity market capitalization, the firms whose bonds are in our sample are typically large, with an average market capitalization of \$39.71 billion in 2002 and \$33.06 billion in 2008.

3.3 Bond Return Volatility $\hat{\sigma}_D$

The direct outcome of our sample construction is a monthly time-series of bond return volatility, $\hat{\sigma}_D$, for cross-sections of bonds. Building on the same bond sample, we also use weekly bond returns to construct a quarterly time-series of bond volatility, and monthly bond returns to construct a yearly time-series.[9]

The first panel of Table 2 summarizes the empirically estimated bond volatility, $\hat{\sigma}_D$, using daily, weekly, and monthly returns. Moving across the three return horizons, the magnitude of $\hat{\sigma}_D$'s, all annualized, decreases markedly. Specifically, the sample mean of $\hat{\sigma}_D$ is 21.77% when estimated using daily returns, contrasted with 12.59% using weekly returns, and 8.10% using monthly returns. The time-series averages of the cross-sectional median of $\hat{\sigma}_D$ exhibit a similar pattern: 17.41% at daily, 9.78% at weekly, and 7.05% at monthly frequency.

Implicit in this pattern are strong negative auto-covariances of daily and weekly bond returns. Given that we are using transaction prices to construct bond returns, bid-ask bounce could be a natural candidate for such negative autocorrelations.[10] This provides initial evidence for liquidity issues in the corporate bond market driving volatility when short horizon returns are used. We examine this in more detail below.

To emphasize the cross-sectional variation of the empirical bond volatility, we sort our sample on a set of bond- and firm-level characteristics into quartiles each year and report the means of contemporaneous $\hat{\sigma}_D$ for each quartile. Specifically, while $\hat{\sigma}_D$ is calculated monthly for daily bond returns, we sort annually using averaged monthly data. As reported in Table 3, bonds with smaller amount outstanding, longer maturity, and poorer rating are more volatile. Bonds issued by firms with higher equity volatility, higher leverage, and higher payout ratios are also more volatile. The relation of $\hat{\sigma}_D$ to bond trading variables such as

[9]Like the case for daily returns, we require at least 10 weekly bond returns in a quarter to form a quarterly estimate of bond volatility, and at least 10 monthly bond returns in a year to form a yearly estimate.

[10]See, for example, Niederhoffer and Osborne (1966) and Roll (1984).

Table 2: **Volatility Estimates**

	Daily Returns			Weekly Returns			Monthly Returns		
	mean	med	sd	mean	med	sd	mean	med	sd
Empirical Bond Volatility $\hat{\sigma}_D$									
2003	20.70	17.87	13.00	11.66	10.41	6.96	9.06	8.80	5.00
2004	16.93	13.88	11.86	9.13	7.58	6.54	5.94	5.80	2.94
2005	16.64	13.38	12.77	9.05	7.16	7.23	5.93	4.97	4.28
2006	15.69	12.95	10.91	8.08	6.73	5.87	5.72	4.64	4.47
2007	18.16	15.56	12.45	9.87	8.24	8.10	5.79	4.92	3.27
2008	36.24	27.84	27.39	23.14	17.22	18.62	15.81	13.19	11.12
Full	21.77	17.41	15.59	12.59	9.78	9.83	8.10	7.05	5.18
Empirical Equity Volatility $\hat{\sigma}_E$									
2003	27.63	25.73	12.40	26.03	24.85	11.98	26.76	26.71	9.08
2004	22.05	18.84	12.34	22.17	18.63	12.13	18.69	16.55	8.09
2005	25.87	20.67	21.12	26.37	21.33	21.17	25.27	21.89	19.69
2006	25.15	20.35	18.35	25.92	21.14	18.41	24.62	19.07	19.27
2007	27.86	23.90	17.77	27.77	23.64	16.70	23.15	20.23	11.31
2008	59.84	50.05	34.57	59.39	49.05	35.78	47.67	37.17	30.56
Full	33.83	28.25	20.34	33.72	27.51	20.85	28.37	23.60	16.33
Empirical 7-Year Treasury Bond Volatility									
2003	7.06	7.32	1.39	7.81	7.65	0.84	8.78	N/A	N/A
2004	5.74	5.61	1.12	5.17	5.37	1.21	6.39	N/A	N/A
2005	4.38	4.45	0.61	4.29	4.28	0.18	4.83	N/A	N/A
2006	3.74	3.82	0.63	3.69	3.59	0.35	3.32	N/A	N/A
2007	5.32	5.33	2.01	4.78	4.74	1.32	5.36	N/A	N/A
2008	9.59	9.47	2.42	7.17	7.48	2.30	8.00	N/A	N/A
Full	6.10	5.50	2.42	5.52	4.84	1.84	6.11	5.88	2.04
Model-Implied Asset Volatility σ_v^{Merton}									
2003	18.39	17.43	10.65	17.52	16.89	10.59	17.30	16.39	7.83
2004	15.16	13.45	10.76	15.31	13.43	10.03	13.18	12.41	6.24
2005	17.41	13.81	18.29	17.63	14.17	17.19	17.07	13.20	17.15
2006	17.18	13.78	15.60	17.72	14.60	14.53	16.86	13.68	13.46
2007	19.45	16.99	15.18	19.10	16.85	13.23	15.66	14.08	7.29
2008	46.77	37.46	33.89	45.87	35.34	34.17	31.08	23.93	25.77
Full	24.45	19.88	18.50	24.20	19.28	18.38	19.00	15.61	12.96
Model-Implied Bond Volatility σ_D^{Merton}									
2003	5.80	6.10	2.56	6.09	6.60	2.48	6.27	7.33	2.56
2004	4.42	4.76	2.01	3.90	4.23	1.97	4.56	4.91	2.11
2005	3.66	3.68	2.63	3.63	3.64	2.84	3.77	3.95	2.34
2006	3.18	3.16	2.16	3.18	3.12	2.38	3.12	2.77	2.98
2007	4.48	4.53	2.34	4.26	4.12	2.57	4.29	4.37	2.12
2008	9.57	9.19	4.49	7.97	7.31	4.56	6.98	6.37	4.33
Full	5.45	5.50	2.87	4.95	4.93	3.01	4.70	4.95	2.74

All volatility estimates are annualized and expressed in percentages. The empirical bond and equity return volatilities are constructed using daily, weekly, and monthly bond and equity returns, respectively. The model-implied asset and bond return volatilities are backed out from equations (9) and (11), respectively, using the equity return volatility $\hat{\sigma}_E$ as inputs. The reported med and std are the time-series averages of cross-sectional medians and standard deviations. For empirical Treasury bond volatility, the reported numbers are time-series medians and standard deviations. The full sample includes data from July 2002 through 2008.

Table 3: **Volatility Estimates by Firm or Bond Characteristics**

	Daily Returns				Weekly Returns				Monthly Returns			
	low	Q2	Q3	high	low	Q2	Q3	high	low	Q2	Q3	high
Empirical Bond Volatility $\hat{\sigma}_D$												
Bond Amt	25.99	20.58	19.56	19.82	14.40	11.62	11.51	12.18	7.98	8.61	7.66	8.33
Bond Maturity	13.76	18.58	22.79	32.88	7.52	10.63	13.37	19.28	3.77	6.61	9.31	12.79
Rating	17.26	18.42	23.18	32.29	8.84	9.97	12.94	21.33	6.08	7.10	8.82	11.84
Equity Vol	18.94	19.85	20.94	29.18	9.98	10.89	11.27	19.29	7.00	7.74	7.34	10.35
Firm Leverage	16.92	21.88	22.01	28.27	8.83	12.08	12.30	18.68	6.24	7.80	9.11	9.44
Firm Payout	17.58	18.48	23.31	29.72	9.63	10.24	12.46	19.50	7.12	7.11	8.37	10.00
Bond Turnover	25.65	18.72	19.43	24.21	14.34	10.14	11.20	15.18	8.45	6.75	7.19	10.00
%Days Traded	22.35	23.38	21.56	19.93	13.58	13.37	12.34	11.13	9.26	8.51	7.16	7.42
Empirical Equity Volatility $\hat{\sigma}_E$												
Equity Mkt Cap	53.31	33.49	28.57	26.12	54.27	33.52	27.20	24.91	44.18	26.02	23.09	19.82
Firm Leverage	27.48	30.37	31.34	51.23	26.29	29.79	30.73	52.15	21.47	25.07	25.68	41.57
Model-Implied Asset Volatility σ_v^{Merton}												
Equity Mkt Cap	34.15	22.74	21.74	21.95	34.77	23.24	20.43	20.82	25.82	17.00	17.00	16.03
Firm Leverage	24.77	24.11	20.32	30.06	23.63	23.57	20.13	30.84	19.20	19.60	16.30	21.00
Equity Vol	14.40	20.27	25.67	42.74	13.37	18.72	25.19	43.53	10.09	14.65	18.70	32.89
Rating	16.98	20.14	23.05	32.38	16.10	19.10	22.35	33.13	13.51	15.39	17.43	26.47
Model-Implied Bond Volatility σ_D^{Merton}												
Bond Amt	5.28	5.45	5.49	5.63	4.80	4.96	5.01	5.10	4.68	4.81	4.69	4.65
Firm Leverage	4.61	5.31	5.71	6.53	4.02	4.68	5.24	6.21	4.10	4.38	5.17	5.22
Bond Maturity	3.25	5.17	6.47	6.97	2.87	4.62	5.99	6.36	2.15	4.40	5.96	6.34
Equity Vol	4.57	5.25	5.44	6.90	3.97	4.55	4.92	6.66	4.21	4.12	4.25	6.26
Firm Payout	4.90	5.37	5.58	6.13	4.36	4.73	4.93	6.02	4.43	4.17	4.84	5.44
Bond Vol $\hat{\sigma}_D$	3.52	5.24	6.17	7.00	2.91	4.64	5.60	6.86	2.32	4.23	5.52	6.75
Rating	4.68	4.95	5.79	7.03	4.04	4.37	5.12	6.91	3.73	4.29	5.04	6.36

All volatility estimates are annualized and expressed in percentages. The empirical bond and equity return volatilities are constructed using daily, weekly, and monthly bond and equity returns, respectively. The model-implied asset and bond return volatilities are backed out from equations (9) and (11), respectively, using equity return volatility $\hat{\sigma}_E$ as inputs. The sample is sorted by the respective variables from low to high into quartiles: Q1 (low), Q2, Q3, and Q4 (high). For bond ratings: Q1=Aaa&Aa, Q2=A, Q3=Baa and Q4=Junk.

turnover and the frequency of its trading are not as clear. Finally, it is interesting to notice that moving across measurement horizons from daily to monthly returns, the cross-sectional patterns hold quite well for most characteristics, while the overall magnitude decreases in a dramatic fashion. The one important exception is that there is no clear pattern in $\hat{\sigma}_D$ using monthly returns when we sort by amount outstanding. As amount outstanding is a proxy for liquidity, this is consistent with liquidity being an important concern for shorter return horizons, but much less of a concern at longer horizons.

3.4 Equity Return Volatility $\hat{\sigma}_E$

The equity return volatility, from which the asset volatility for the firm can be backed out, is one key input to our structural model. The equity sample used to construct the equity volatility mirrors the bond sample summarized in Table 1. For each firm whose bonds enter our bond sample, we use CRSP daily, weekly, and monthly returns to form monthly, quarterly, and yearly estimates of equity volatility.

The second panel of Table 2 summarizes the empirical equity return volatility $\hat{\sigma}_E$. When we average across firms and time, the annualized equity volatility of our sample is 33.83% when estimated using daily equity returns, 33.72% using weekly returns, and 28.37% using monthly returns. Compared with the dramatic pattern of decreasing empirical bond volatility with increasing measurement horizon, this is a strong indication of the relative importance of liquidity in the bond and equity markets. The cross-sectional variation of the empirical equity volatility is reported in the second panel of Table 3. As expected, smaller firms are more volatile, and so are more leveraged firms.

Overall, our volatility measures are lower than those reported for U.S. equities, in part due to the fact that firms in our sample are typically larger firms. Another important driver is that the early part of our sample period is a relatively low volatility period, though the end of our sample is a high volatility period. We maintain a contemporaneous sample of empirical bond and equity volatilities so as to capture the time-variation in asset volatility and its impact on both the bond and equity volatilities. Nevertheless, the model is set in a constant volatility setting. So a lower than average equity volatility would have a more permanent impact on our estimate of the firm asset volatility than it otherwise would in a stochastic volatility setting. We will consider the robustness of our results with respect to this limitation of the model.

4 Model-Implied Volatility

4.1 Parameter Calibration of the Merton Model

The parameters that govern the dynamics of the risk-free rate are calibrated as follows. First, we use the daily time-series of three-month T-bill rates from 1982 through 2008 to calibrate the long-run mean parameter θ and the rate of mean reversion parameter κ. Specifically, we set $\theta = 5.19\%$, so that the long-run mean equals its time-series average; $\kappa = 0.1704$, so that the daily autocorrelation of the model matches the sample autocorrelation.[11] Second, we set the volatility parameter σ_r so that, for a 7-year Treasury coupon bond,[12] the model-implied volatility of its instantaneous returns matches the sample volatility. More specifically, to parallel our treatment of the empirical bond and equity volatilities, we use daily 7-year Treasury coupon-bond returns to form monthly estimates of Treasury bond volatilities, and weekly returns to form quarterly estimates and monthly returns to form yearly estimates. The Treasury volatility estimates are reported in the third panel of Table 2. When averaged across time, the annualized volatility estimates remain fairly stable regardless of the measurement horizons, again, contrasting with the pattern in corporate bonds.

The random fluctuations of the Treasury rate are an important component in corporate bonds. Our choice of the risk-free parameters, particularly σ_r, have a big impact on the model-implied bond volatility. The median maturity of the bonds in our sample is close to 6 years while our mean is close to 8 years. We choose σ_r to match the volatility of a 7-year Treasury coupon bond so that on average, the Treasury component of the bond volatility is matched to its sample counterpart. While σ_r varies over time in our calibrations, its time-series average is around 1.6%. In contrast, the volatility coefficient σ_r estimated from the time-series of three-month T-bill rates is only at 1.3%, which would under-estimate the Treasury component of corporate bond volatility. Implicit in this difference in σ_r is the fact that the simple one-factor model of Vasicek cannot match the term structure of Treasury bond volatility well. Our approach is to force the model to match well near the 7-year maturity, a medium-term bond which has a maturity between the median and mean maturities of the corporate bonds in our sample.

Apart from the asset volatility σ_v, which is to be inferred from the model, the firm-level parameters to be calibrated are the payout ratio δ, leverage K/V, and maturity T. For each firm, its leverage K/V is calculated as follows. First, we calculate the book value of the firm's debt, K, to be the sum of long-term debt and debt in current liabilities using Compustat

[11] To be more precise, we should match the risk-neutral value of κ since we are using the model for pricing purposes.

[12] We use the Federal Reserve's 7-year TCM yield to construct returns on a par Treasury coupon bond.

data. Second, we set $V = S + D$ to be the firm value with S equaling the market value of the equity and D equaling the market value of the debt. Given that firms typically issue bonds at par, the market value of the debt should be close to the book value. To improve on this approximation, however, we collect, for each firm, all of its bonds covered by TRACE and calculate an issuance weighted market-to-book ratio. We then approximate the market value of the debt by multiplying K by the market-to-book ratio.

In calibrating the firm-level debt maturity T, we take into account the actual maturity structure of the firm and collect all of the firm's bonds in FISD and calculate the respective durations. We let the firm-level T be the issuance-weighted duration of all the bonds in our sample. Effectively, we acknowledge the fact that firm's maturity structure is more complex than the zero-coupon structure assumed in the Merton model, and our issuance-weighted duration is an attempt to map the collection of coupon bonds to the maturity of a zero-coupon bond. Finally, we calculate the firm's payout ratio δ by adding its annual dividends and annual coupon payments, all scaled by the total dollar payout by the firm value V.

Table 4: **Firm-Level Parameters of the Merton Model**

	Firm T			Leverage K/V			Modified Leverage			Payout Ratio δ		
	mean	med	std	mean	med	std	mean	med	std	mean	med	std
2003	6.26	6.32	2.43	30.49	26.14	17.09	32.15	27.11	19.09	3.36	3.25	1.50
2004	6.07	5.81	2.21	25.72	21.24	17.06	26.08	20.51	18.48	3.15	2.82	1.63
2005	5.73	5.74	2.04	32.57	29.39	19.74	30.32	26.25	19.90	3.48	3.22	1.74
2006	5.77	5.74	1.96	32.76	29.07	21.11	28.50	25.55	18.02	3.47	3.26	1.80
2007	6.01	5.80	1.96	30.10	28.20	17.67	28.72	26.15	17.65	3.17	3.05	1.40
2008	5.61	5.42	1.89	47.24	37.93	61.71	34.36	30.52	18.62	4.54	4.15	3.05
Full	5.85	5.74	2.04	33.98	28.97	32.65	30.04	26.68	18.69	3.57	3.30	2.04

Firm-level parameters from calibrations using monthly returns. Firm T is the amount outstanding-weighted duration of a firm's bonds. Leverage K/V is the sum of long-term debt and debt in current liabilities from Compustat divided by the value of the firm. Modified leverage is as defined in equation (8). The payout ratio is the sum of a firm's coupon and dividend payments scaled by firm value. Reported full sample medians and standard deviations are for the panel.

With the exception of leverage K/V, the other firm-level parameters are constant in the model. We do, however, take its time-variation into account and update the firm level parameters at the appropriate frequencies. Table 4 summarizes the firm-level parameters. The average leverage K/V of the firms in our sample is about 33.98%, the average firm T is 5.85 years, and the average average payout ratio 3.57%. Overall, these firm-level variables are stable over time, and are only mildly skewed.

4.2 Model-Implied Asset Return Volatility $\sigma_v^{\textbf{Merton}}$

For each firm in our sample, we back out its asset volatility σ_v^{Merton} using the Merton model via equation (9). The risk-free parameters as well as the firm-level model parameters including leverage K/V, payout ratio δ, and firm T are calibrated as described in Section 4.1. The fourth panel of Table 2 summarizes the model-implied asset volatility using the firm-level parameters, the estimated equity volatility $\hat{\sigma}_E$, and the bond volatility σ_r as inputs. Among the inputs, however, the key variable is equity volatility, which remains quite stable across measurement horizons. The model-implied asset volatility inherits this pattern. In addition, it also inherits a relatively low asset volatility from the relatively low equity volatility.

The cross-sectional variation of the model-implied asset volatility is reported in the third panel of Table 3. It shows that smaller firms have higher asset volatilities. On the other hand, firms whose bonds are speculative grade have markedly higher volatilities than that of the investment grades. Within investment grade, there is some evidence of increasing asset volatility with decreasing credit rating. Finally, the relation of asset volatility to equity volatility is monotonically increasing, indicating that the cross-sectional variation in leverage, or more precisely the modified leverage $\mathcal{L}$, does not break the link between the two.

Finally, we should mention one bias in our sample regarding the calculation of model-implied asset volatility. Although it is clear from equation (9) that, for each firm with a fixed set of parameters, the equity volatility will be the deciding factor in backing out the asset volatility, the interest rate volatility component does plays a role. For firms with high leverage, its equity volatility should have a component tied to the volatility of the risk-free rate. But for some highly levered firms in our sample, their empirical equity volatility is too low to account for the risk-free interest-rate volatility component. In such cases, an asset volatility cannot be backed out from equation (9), and we exclude the firm and their bonds from our sample. The frequency of such incidents is not rare, and happens for about 15% of the firms and 16% of the bond-years in our original sample. We discuss such cases in more detail in Section 5.2.3. It should be noted that such cases are already evidence of volatility being de-linked in the equity, bond, and Treasury markets.

4.3 Model-Implied Bond Return Volatility $\sigma_D^{\textbf{Merton}}$

For each bond in our sample, we calculate its model-implied volatility σ_D^{Merton} using the Merton model via equation (11). The risk-free parameters as well as the firm-level model parameters are the same as before, except that we take the model-implied asset volatility σ_v^{Merton} as a key input. Moreover, we no longer need the firm maturity T. Instead, the respective bond maturity is used in calculating the return volatility for coupon bonds and the loss given default is set at 50%.[13]

The last panel of Table 2 summarizes the model-implied bond volatility, σ_D^{Merton}. It is interesting to note that while the sample mean of $\hat{\sigma}_D$ is 21.77%, 12.59%, and 8.10% when estimated using daily, weekly, and month bond returns, the sample mean of σ_D^{Merton} is 5.45%, 4.95%, and 4.70%, respectively. Of course, this lack of variation across horizons is not surprising given that a key input in estimating σ_D^{Merton} is the equity return volatility $\hat{\sigma}_E$, which is relatively stable when various horizon returns are used. It does, however, reflect an interesting disconnect between the bond and equity market. We will compare $\hat{\sigma}_D$ and σ_D^{Merton} more closely in the next section.

The cross-sectional variation of σ_D^{Merton} is summarized in the last panel of Table 3. Quite intuitively, bonds with higher firm leverage, longer maturity, and poorer rating are more volatile. In fact, among all the variables used in the sorting procedure, bond maturity is among the most effective variables in generating a spread in σ_D^{Merton}. In other words, the duration risk is an important component in the cross-sectional determinants of σ_D^{Merton}. Another very effective variable is the bond volatility $\hat{\sigma}_D$ estimated directly from the data. The fact that $\hat{\sigma}_D$ and σ_D^{Merton} line up in the expected direction is encouraging for the model. In addition, the fact that the spread is even wider when sorted by the empirical bond volatility σ_D^{Merton} estimated using monthly bond returns is even more telling. It indicates that the model-implied bond volatility, which includes no information about the potential liquidity problems in corporate bonds, lines up better cross-sectionally with the empirical bond volatilities that are estimated using longer horizon returns and are less subject to liquidity contamination. Sorting by equity volatility $\hat{\sigma}_E$ generates cross-sectional variation in σ_D^{Merton}, although the effect is somewhat muted.[14] Finally, the relation of σ_D^{Merton} to the size of the bond, however, is not as clear as it is the case for $\hat{\sigma}_D$.

5 Bond Volatility: Empirical vs. Model

5.1 Level of Return Volatility

Table 5 summarizes the main result of our paper. Specifically, there is a strong discrepancy between the bond volatility $\hat{\sigma}_D$ estimated directly using bond return data and the bond volatility σ_D^{Merton} implied by the Merton model. Using daily returns to construct the volatility

[13]Huang and Huang (2003) use a recovery rate of 51.31%.

[14]It should be noted that σ_D^{Merton} is not necessarily monotonic in σ_E. For example, while the first term equation (11), which measures the defaultable bond's exposure to firm risk, is clearly increasing in σ_v, the second term, however, is decreasing in σ_v. For long duration bonds, the second term, which captures the risk-free interest rate exposure, could outweigh the first.

estimates, the sample means of $\hat{\sigma}_D$ and σ_D^{Merton} are 21.77% and 5.45%, respectively. The full sample mean of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ is 16.31% with a robust t-stat of 10.62 (clustered by month and firm). The economic magnitude of such a discrepancy is quite large, and it indicates a volatility component in corporate bonds that is disconnected from the equity volatility of the same issuer or the interest rate volatility in Treasury bonds. While the magnitude of this discrepancy is larger for 2008 (26.67%) than for the rest of the sample (13.51%), the discrepancy remains economically significant for the pre-2008 period and is actually more statistically significant.

To better understand this large excess volatility component, we examine $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ across various measurement horizons.[15] When the volatility estimates are constructed using weekly returns, the sample mean of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ shrinks to 7.64% with a robust t-stat of 4.37. Moving to monthly horizon, the difference is further reduced to 3.40% with a robust t-stat of 2.60. Putting aside the fact that even at the monthly level the discrepancy is still significant statistically and large economically, the dramatic reduction in $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ across measurement horizons indicates that the disconnect is most severe at shorter horizons. Indeed, this horizon result is driven mostly by the short-term behavior of corporate bond returns. Specifically, the sample means of the empirical bond volatility $\hat{\sigma}_D$ are 21.77%, 12.59%, 8.10%, respectively, when measured using daily, weekly, and monthly bond returns. By contrast, the empirical equity volatility $\hat{\sigma}_E$ remains relatively stable across the different measurement horizons as does the model-implied bond volatility. Implicit in this horizon result is a high degree of negative auto-covariances in short-horizon bond returns, accentuating a severe liquidity component in the corporate bond market.

To exclude the possibility that our results are driven by a few bonds with extreme values of $\hat{\sigma}_D$, we examine the time-series average of the cross-sectional medians of the volatility estimates. Specifically, the medians of $\hat{\sigma}_D$ are 17.41%, 9.78%, and 7.05% for daily, weekly, and monthly measurement horizons respectively; the medians of σ_D^{Merton} are 5.50%, 4.93%, and 4.95% respectively; and the medians of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ are 11.99%, 4.90%, and 2.01%, respectively. The patterns are similar to those reported for the sample mean results. In other words, the excess volatility in corporate bonds is most severe when measured with short-horizon returns, and then dramatically decreases near monthly returns.

As shown in Table 5, the pattern of excess volatility is quite robust, whether the sample is split by bond type, by year, by credit rating, or by bond duration. While our full sample includes only straight bonds and callable bonds, we also report the results for convertible

[15] It should be mentioned that the model-implied volatility σ_D^{Merton} is derived for instantaneous returns. As such, when we move on to calculate volatility of bond returns over monthly horizons, the approximation error would increase.

Table 5: **Data Estimated vs. Model Implied Bond Volatility**

	$\hat{\sigma}_D - \sigma_D^{Merton}$								
	Daily Returns			Weekly Returns			Monthly Returns		
	#obs	mean	t-stat	#obs	mean	t-stat	#obs	mean	t-stat
Full Sample*	32,132	16.31	10.62	10,953	7.64	4.37	2,332	3.40	2.60
Straight	10,394	15.04	6.02	3,501	6.81	3.44	759	2.74	3.41
Callable Only	21,738	16.92	12.56	7,452	8.03	4.71	1,573	3.71	2.45
Convertible	2,400	30.21	15.61	817	17.09	7.35	170	12.19	3.93
Putable Not Conv	297	29.71	7.16	100	14.92	3.71	26	6.51	3.34
By Year									
2003	3,196	14.90	17.64	1,024	5.57	11.86	199	2.80	9.08
2004	4,104	12.51	18.07	1,443	5.22	11.05	320	1.39	11.76
2005	5,336	12.99	18.41	1,817	5.42	12.10	438	2.17	15.13
2006	5,722	12.51	19.83	1,948	4.90	14.92	465	2.60	16.71
2007	6,048	13.68	18.51	2,099	5.60	12.38	446	1.49	13.28
2008	6,859	26.67	4.92	2,344	15.17	2.08	464	8.82	18.16
By Rating									
Aaa	3,309	12.94	5.36	1,083	5.20	4.97	234	3.02	3.85
Aa	3,207	11.79	10.23	1,066	4.42	7.88	204	1.58	2.95
A	11,785	13.39	11.59	3,956	5.49	6.58	901	2.81	2.27
Baa	7,799	17.60	11.71	2,704	8.04	6.14	586	3.78	2.72
Ba	2,463	18.04	13.45	881	9.20	4.37	175	4.82	3.41
B	2,115	21.42	6.37	722	9.32	4.49	131	4.60	5.98
By Duration									
≤ 2	5,428	9.57	10.13	1,901	4.26	6.08	463	1.61	4.12
2 - 4	7,657	13.47	8.52	2,680	6.11	3.41	595	2.15	2.50
4 - 6	7,803	16.69	7.50	2,641	8.36	2.87	573	3.17	2.52
6 - 8	5,920	17.00	13.03	1,903	7.58	5.42	309	3.64	2.38
> 8	4,723	26.98	15.30	1,627	12.96	7.83	351	8.01	2.64
Using Excess (Bond - Treasury) Returns									
	29,653	20.15	13.18	10,103	10.77	6.36	2,140	5.99	4.78
Factoring in Bid/Ask Spreads									
	31,055	15.04	10.03	10,583	7.18	4.13	2,250	3.24	2.55

The full sample includes straight and callable bonds, excluding convertibles and putables. Convertibles and putables are excluded from all tests except those reported under Convertible, and Putable Not Convertible. $\hat{\sigma}_D$ is estimated using daily, weekly, and monthly bond returns, respectively. σ_D^{Merton} is the model-implied bond volatility. #obs is bond month for daily, bond quarter for weekly, and bond year for monthly. The t-stat's are calculated using robust standard errors, clustered by time and by firm, with the by-year results at the monthly horizon being the only exception.

and putable bonds separately. It is expected that, except for straight bonds, the model-implied bond volatility will be off in capturing the real bond volatility. In particular, the model would under-estimate the volatility for a convertible bond. Indeed, we find a much higher level of excess volatility for convertible bonds. The fact that the putable bonds also have higher excess volatility, however, is puzzling, although the sample is quite small. Given that the callability feature is mostly tied to the random fluctuation of interest rates and effectively shortens the duration of a callable bond, one would expect the model to over-estimate the volatility in a callable bond, and therefore generate a lower degree of excess volatility. Table 5, however, shows that the excess volatility is slightly higher for callable bonds than for straight bonds. This comparison, however, fails to factor in bond-level characteristics such as maturity and rating, which could be important in driving the excess volatility results. Indeed, the average maturity of the callable bonds in our sample is 8.94 years compared with 5.56 years for the straight bonds. Their credit ratings are also on average one or two notches below the straight bonds. We will revisit this issue in Section 6, when we examine the difference of these two samples after controlling for other bond level characteristics including maturity and rating.

The improved performance of the model when monthly returns are used to calculate volatility is perhaps not surprising in light of the illiquidity in the corporate bond market. At a daily return horizon, the illiquidity of corporate bonds plays an important role in volatility. At a monthly horizon, underlying fundamentals should play a much more important role and this is reflected in Table 3 where sorting on amount outstanding, a proxy for bond liquidity, generates a spread in bond volatilities estimated from daily returns, but not in bond volatilities estimated from monthly returns. Though the model performs much better using monthly horizon returns, an excess volatility of 3.40% when the average empirically estimated volatility is 8.10% is still quite economically significant, suggesting that the disconnect between equity and corporate bond return volatilities goes above and beyond a simple difference in liquidity.

Overall, our results indicate that while at the daily and weekly measurement horizons, the Merton model cannot even begin to generate the kind of bond volatility observed in the data due to the liquidity problems in corporate bonds, the model does improve at the monthly horizon. We explore the connection between excess volatility and short-horizon returns and also the CDS market (which was more liquid at least in the early part of our sample) in the next few sections in an attempt to better understand the sources of the disconnect between volatilities.

5.2 Further Considerations

5.2.1 Stochastic Interest Rates

The simple term-structure model employed in this paper is an issue of concern. A proper account of the risk-free volatility is important because small fluctuations in the risk-free interest rate will be magnified by the duration of the bond to a sizeable volatility. Because of this, we calibrate the volatility coefficient σ_r in the risk-free rate model so that the model generates the empirically observed level of volatility for a 7-year Treasury coupon bond. Effectively, we force the model to match well near the 7-year maturity, as the median maturity in our bond sample is close to 6 years and the mean is close to 8 years. This, however, still does not fully capture the entire term-structure of interest rate volatility.[16]

To account for this, we work with excess bond returns to avoid relying on a term structure model. We calculate excess bond returns by subtracting contemporaneous Treasury bond returns of a similar maturity from the corporate bond returns.[17] We then subtract off the model-implied default component of bond volatility. Comparing the volatility measured by bond excess returns to the model-implied excess bond volatility, we find that the results are similar to our main result. For the daily, weekly, and monthly measurement horizons, the sample means of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ are respectively 20.15%, 10.77%, and 5.99% with robust t-stats of 13.18, 6.36, and 4.78; the time-series averages of the cross-sectional medians are 15.82%, 8.01%, and 4.77%.

Overall, the excess bond volatility puzzle is somewhat deepened here with the adoption of the model-free approach. The main reason is that in working with the Merton model with stochastic interest rates, we inherit the model-implied correlation between the corporate bond and Treasury bond. In the model-free approach, the empirical correlation is used. As it is implicit in the current result, the empirical link is weaker than that prescribed by the model,[18] consequently making the excess volatility puzzle even larger in magnitude. Thus, the bond volatility inferred in equation (10) would be even lower if empirical sensitivities were substituted for model-implied sensitivities.

[16]We are particularly concerned with underestimating the sensitivity of corporate bonds to Treasury bonds and thus, underestimating the component of corporate bond volatility due to Treasury bond volatility.

[17]The return horizons are matched at daily, weekly, and monthly, respectively. We use 1-, 2-, 5-, 7-, 10-, 20-, and 30-year Treasury returns as the basis of our extrapolation to get the target maturity.

[18]This is consistent with Schaefer and Strebulaev (2008) who find that the empirical sensitivity of corporate bonds to Treasuries is significantly lower than suggested by the Merton model with stochastic interest rates.

5.2.2 Bid-Ask Bounce

Given the importance of liquidity as a potential explanation of our results, we further try to correct the bond volatility measure by factoring in the effect of bid-ask bounce. Following Roll (1984) and assuming that transactions at bid and ask are equally likely, we can map an observed bid-ask spread to its impact on return volatility. For all of the bonds in our sample, we collect monthly bid-ask spreads from Bloomberg Terminals, and calculate the associated "bid-ask bounce" contribution to the bond return volatility.[19]

The last panel in Table 5 reports $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$, where $\hat{\sigma}_D$ is the square-root of the difference between the empirical bond variance minus the variance generated by the bid-ask bounce. While decreasing somewhat from the previous results, the magnitude of the discrepancy remains similar to our main result. In other words, the excess bond volatility documented here cannot be explained by quoted bid-ask bounce alone. We further address this issue by considering the mid price of credit default swaps in section 5.3.

5.2.3 Firms with Missing Asset Volatility

We use the Merton model to back out asset volatility from equity and Treasury volatilities via equation (9). For most firms, the key input of this calculation is equity volatility, with interest rate volatility relegated to playing only a minor role. It is for this reason, many of the existing studies do not include Treasury volatility in the calculation. For firms with higher than usual leverages, however, this Treasury component becomes too large to be ignored. If the high leverage is further coupled with a higher than usual payout ratio δ, it would result in a high level of modified leverage $\mathcal{L}$. And from equation (9), we see that for such a firm, its equity volatility would collect a component amplified by $\mathcal{L}/(1-\mathcal{L})$ from the Treasury volatility. If in practice, the equity volatility for such a firm is not large enough to account for this component alone, then we run into the problem of not being able to back out asset volatility from equation (9) as there is no asset volatility that can generate a model equity volatility as low as the empirical equity volatility.

Indeed, for 15% of the firms-years originally in our sample based on monthly returns, we run into this problem of missing asset volatility.[20] This pool of firms has an average leverage

[19]Bloomberg typically provides bid-ask quotes from various dealers and we use the Bloomberg Generic (BGN) Quote, which reflects consensus market quotes. BGN quotes are available for a larger number of bonds in our subsample and typically have a longer time series than quotes by other dealers. These are the same types of quotes collected by Chen, Lesmond, and Wei (2007). It should also be noted that our adjustment is one-sided, since the bid-ask spread in the equity market might also have an impact as it finds its way to σ_D^{Merton}.

[20]Though these firm-years and their corresponding bond-years satisfy our data-filtering criteria, they are not included in our summary statistics tables.

(K/V) of 127%, far larger than the sample average of 34% for firms with non-missing asset volatility. They have an average payout ratio of 10.82%, more than three standard deviations from the sample average of 3.57%. On the other hand, their firm-T is on average 4.9 years, not that different from and slightly lower than the sample average. Their equity volatility is on average 48.32%, which is indeed higher than the sample average of 28.37% for firms for which an asset volatility can be backed out, but not high enough to account for their interest rate exposure.

This problem of missing asset volatility affects 16% of the bond-years in our original sample. Using monthly bond returns, we construct empirical bond volatility for this sample of bonds and find an average bond volatility of 20.11% and a median volatility of 13.26%, which are markedly higher than the average of 8.10% and median of 7.05% for the sample of bonds without the missing asset volatility problem. To get a sense of the possible differences in model and empirical bond volatilities with these bonds included, for firms with missing σ_v^{Merton} we calculate the σ_v^{Merton} that gives the lowest possible model equity volatility.[21] This typically generates a much higher (model-implied) equity volatility than empirically observed due to the high leverage of such firms. Specifically, the mean difference between the lowest possible model equity volatility and the empirically observed equity volatility across bond-years in our sample is 37.65%, indicating a disconnect between firm-level parameters and volatility in the equity market for such firms. We then use these σ_v estimates to estimate model-implied bond volatilities. Including such cases, the mean $\hat{\sigma}_D - \sigma_D^{Merton}$ is now 3.39%, though it is no longer statistically significant. It is important, however, to note that this difference is based on a model equity volatility that is much higher than empirically observed equity volatility and this is again consistent with a disconnect in volatility. In particular, the cases in which we cannot back out σ_v^{Merton} are evidence of equity volatility being too low.

5.2.4 Conditional vs. Unconditional Asset Volatility

One limitation of our modeling approach is the tension between conditional versus unconditional volatility. Using conditional volatility sharpens the contemporaneous connection among the equity, Treasury, and corporate bond volatilities, capturing the link both across firms and across time. Indeed, our approach leans heavily toward this conditional approach by constructing monthly estimates of volatility using daily returns, and yearly estimates using monthly returns. The limitation, however, is that when the Merton model is used to calculate the sensitivity coefficients in equations (7) and (11), the conditional rather than

[21]Setting σ_v to 0 may actually generate a larger model-implied equity volatility than a non-zero σ_v. From equation (9), holding $\mathcal{L}$ fixed, a higher σ_v generates a higher σ_E. However, $\mathcal{L}$ is a function of σ_v such that a higher σ_v could generate a lower $\mathcal{L}$ that results in a lower model-implied σ_E.

unconditional volatility is plugged into the model. This has the implicit implication that if the current volatility is low, it will stay low for the entire life of the firm or the entire maturity of the bond. The early part of our sample is a low equity volatility period while the later part is high equity volatility so equity hedge ratios may be underestimated in the early part of the sample and overestimated in the later part. The best resolution of this tension is to have a stochastic volatility model. Given that it will dramatically increase the complexity of the problem without the benefit of additional insights, we examine the robustness of our main result by using the following "hybrid" approach.

We first obtain unconditional estimates of equity and Treasury bond volatilities using monthly equity and Treasury bond returns going as far back into history as possible. We then plug in the unconditional volatilities into equation (7) to obtain an unconditional version of the asset volatility. Averaged across all firms, this unconditional asset volatility is 24.52% with a cross-sectional median of 22.16%, which are higher than the the model-implied asset volatilities in the early part of our sample period, but lower than in 2008. Armed with the unconditional asset volatility, we can now fix the problem with respect to the model-implied sensitivity coefficients in equations (7) and (11). Given that the horizon of the firm is typically long (firm-T is on average 6 years), and the rate of mean reversion of equity volatility is relatively fast, calculating the sensitivity coefficients using an unconditional approach seems reasonable. Applying this hybrid approach, we find only minor effects on our main results. For the monthly measurement horizon, the excess volatility measure $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ is on average 3.13% with a t-stat of 2.66. In other words, excess volatility remains significant both statistically and economically even at the monthly horizon.

5.3 Credit Default Swaps

While our previous analysis suggests that the disconnect in volatilities between equities and corporate bonds of the same firm is driven by both differences in liquidity in the two markets (in particular, the illiquidity of the corporate bond market) and also a disconnect in fundamentals (as highlighted by the excess volatility of corporate bonds even when monthly returns are used), we further study the disconnect due to fundamentals by making use of credit default swaps. Following Longstaff, Pan, Pedersen, and Singleton (2010), we construct a synthetic floating rate corporate bond as a risk-free floating rate bond plus writing a CDS contract. Details of this construction are in Appendix B.

Using credit default swaps in our analysis provides three important advantages. First, the CDS market and particularly the 5-year CDS market was historically considered to be quite liquid and to accurately measure the credit risk of underlying firms.[22] For example, Longstaff, Mithal, and Neis (2005) use credit default swaps as their measure of credit risk when examining liquidity in the corporate bond market. Second, we use quoted spreads from Datastream, which in turn uses Credit Market Analysis (CMA) as its data source. This allows us to use mid prices and avoid having volatility driven by bid-ask bounce as with transaction-level data. Finally, a synthetic floating rate bond is much less sensitive to the interest rate model used in pricing than a comparable fixed coupon bond, due to its short duration.

Table 6: **Volatility of Synthetic Floating Rate Bonds using CDS**

Panel A: Daily Returns										
Period	#obs	Empirical Volatility			Model Volatility			Difference		
		mean	med	sd	mean	med	sd	mean	t-stat	med
2004	1,077	2.10	0.92	3.83	0.90	0.41	1.86	1.20	4.41	0.44
2005	1,572	3.04	1.22	7.14	1.11	0.33	2.92	1.93	5.33	0.76
2006	1,730	2.26	0.96	4.25	0.98	0.28	2.26	1.27	6.27	0.55
2007	1,865	3.16	1.20	5.39	1.35	0.44	2.68	1.81	6.24	0.64
2008	1,838	8.76	3.50	18.77	6.28	4.48	5.91	2.48	2.57	-0.31
Full	8,082	4.08	1.56	7.88	2.29	1.19	3.13	1.79	6.25	0.42
Panel B: Monthly Returns										
Period	Obs	Empirical Volatility			Model Volatility			Difference		
		mean	med	sd	mean	med	sd	mean	t-stat	med
2004	80	1.53	0.66	3.51	0.72	0.45	1.12	0.82	2.75	0.17
2005	130	2.91	1.03	4.33	1.22	0.36	3.11	1.69	8.26	0.62
2006	132	2.10	0.88	2.89	0.91	0.25	2.17	1.18	6.18	0.50
2007	142	3.46	1.40	3.80	1.04	0.42	2.15	2.42	8.99	0.94
2008	131	8.40	4.57	13.82	5.88	2.41	6.94	2.52	2.56	1.66
Full	615	3.85	1.71	5.67	2.04	0.78	3.10	1.81	5.40	0.78

Empirical, model-implied, and difference in volatilities for synthetic five-year floating bonds constructed using credit default swaps. In Panel A, empirical volatilities are estimated each month using daily returns. Panel B uses monthly returns. #obs is the number of firm-months or firm-years. Medians and standard deviations are the time-series averages of the cross-sectional statistics. The t-stat's are calculated using robust standard errors, clustered by time and by firm, with the exception of year-by-year estimates in Panel B which are robust to heteroskedasticity.

We use the same firms for which we have estimates of corporate bond volatility, finding that the synthetic floating rate bonds exhibit a mean excess volatility of 1.79% when daily returns are used and 1.81% when monthly returns are used. Both excess volatilities are

[22] The effect of counterparty risk in CDS spreads has received much press since the start of the financial crisis, but Arora, Gandhi, and Longstaff (2007) find that the economic impact of counterparty risk is small as a 400 basis point increase in a dealer's credit spread translates into only a one basis point decline in the spread that a dealer charges for selling credit protection. Arora, Gandhi, and Longstaff (2007) attribute this tiny effect to the standard practice of collateralizing swap liabilities.

strongly statistically significant and their economic significance can be highlighted by the fact that the mean empirical volatilities for daily and monthly returns are 4.06% and 3.85%, respectively. A feature of our results on CDS is that, unlike corporate bonds, the excess volatility is stable whether we use daily or monthly returns. This is due to our use of mid prices rather than transaction prices and the fact that the CDS market is is much more liquid than the corporate bond market. Futhermore, the excess volatility using synthetic floating rate bonds is much more similar to the monthly return results for corporate bonds. Remarkably, the excess volatility is about 45% of empirical volatility, similar to the excess volatility of monthly bond returns. The similarity in excess volatility between CDS-based synthetic bond returns and monthly corporate bond returns is consistent with much of the excess volatility in daily corporate bond returns being related to liquidity with the excess volatility in monthly corporate bond returns largely being driven by fundamentals.

Given the turmoil in the financial sector in 2008, one might argue that credit default swaps poorly reflected fundamentals during this period as financial firms are often the counterparties in CDS contracts. We find that with 2008 omitted, the excess volatility of the synthetic floating rate bonds is 1.59% (t-stat = 9.02) for daily returns and 1.62% (t-stat = 4.56) for monthly returns. In addition, we report year-by-year excess volatilities in Table 6, finding that the average excess volatility is significant for each year.

Thus, our evidence is consistent with volatilities in the credit market being higher than expected when compared to equity and Treasury markets when a Merton model is used. As the credit default swap market is not subject to many of the frictions in the corporate bond market (particularly when high frequency corporate bond prices are used), this excess volatility is consistent with a fundamental part that cannot simply be explained by the illiquidity of the corporate bond market. Instead, these results provide a contrast to the Schaefer and Strebulaev (2008) findings that while the Merton model does poorly with the levels of prices, it does reasonably well in relating the dynamics of prices in the equity and corporate bond markets.

6 Further Examination of Excess Volatility

6.1 Volatility in the Time-Series

To shed further light on the discrepancy between empirical and model bond volatilities, we first examine how well the model-implied bond volatility co-moves with empirically observed bond volatilities. In particular, we are concerned with whether Merton model-implied bond volatilities can help to characterize the time-series variation of empirical bond volatilities despite being unable to match the level of empirical volatilities. To examine this time-series dimension,[23] we start by examining monthly estimates of volatilities using daily returns from the synthetic floating rate bond constructed from CDS.[24] In Figure 1, we plot the time-series of the cross-sectional means and medians of $\hat{\sigma}_{CDS} - \sigma_{CDS}^{Merton}$, $\hat{\sigma}_{CDS}$, and σ_{CDS}^{Merton}. The cross-sectional means of $\hat{\sigma}_{CDS}$ and σ_{CDS}^{Merton} have a time-series correlation 0.9478. This correlation drops to 0.6529 if 2008 is excluded. It seems that model volatility moved quite well with empirical volatility in the CDS market even towards the end of 2008. This suggests that in the aggregate, fundamental volatilities in the credit market did not move in a particularly extreme way relative to equities late in 2008, at least in our sample of non-financial firms. Consistent with year-by-year excess volatility results for corporate bonds and credit default swaps presented earlier, this suggests that the excess volatility is not driven solely by the financial crisis of 2008.

In Table 7, we examine this time-series relation at the bond level by running panel regressions of empirical bond volatilities on model volatilities with bond fixed-effects. Regardless of whether we use volatilities of daily or monthly returns, we find that empirical bond volatilities are strongly statistically and economically related to model bond volatilities. Since the included bond fixed-effects take out the average level of bond volatilities for each bond, this reflects the model volatility being able to explain fluctuations in empirical bond volatilities over time. Furthermore, we find that the within-group R^2 is higher when volatilities are calculated from monthly returns, regardless of whether corporate bonds or CDS are used. For corporate bonds, a natural explanation is that volatility at a daily level includes a large liquidity component. For credit default swaps, the explanation for a lower R^2 when daily returns are used is not as clear, though it is possible that this reflects differences in how information is impounded in equity and credit markets.[25] Overall, our results suggest that the volatility of corporate bonds co-move with the volatility of equities and Treasuries in a sensible way and that the Merton model can capture this co-movement qualitatively even though it cannot quantitatively explain the typical level of corporate bond volatilities.

[23] As shown in Table 3, $\hat{\sigma}_D$ and σ_D^{Merton} do line-up cross-sectionally.

[24] We use volatilities based on daily returns as this provides a monthly time series of volatilities. Further, we use the synthetic floating rate bonds rather than corporate bonds as the corporate bond volatilities from daily returns are likely related to illiquidity.

[25] In particular, if stocks lead bonds in incorporating firm-specific information, this could affect volatility estimates at shorter horizons. The existing empirical evidence is mixed as Kwan (1996) finds that stocks lead bonds in reflecting firm-specific information while Hotchkiss and Ronen (2002) find that bond and stock markets are similarly efficient.

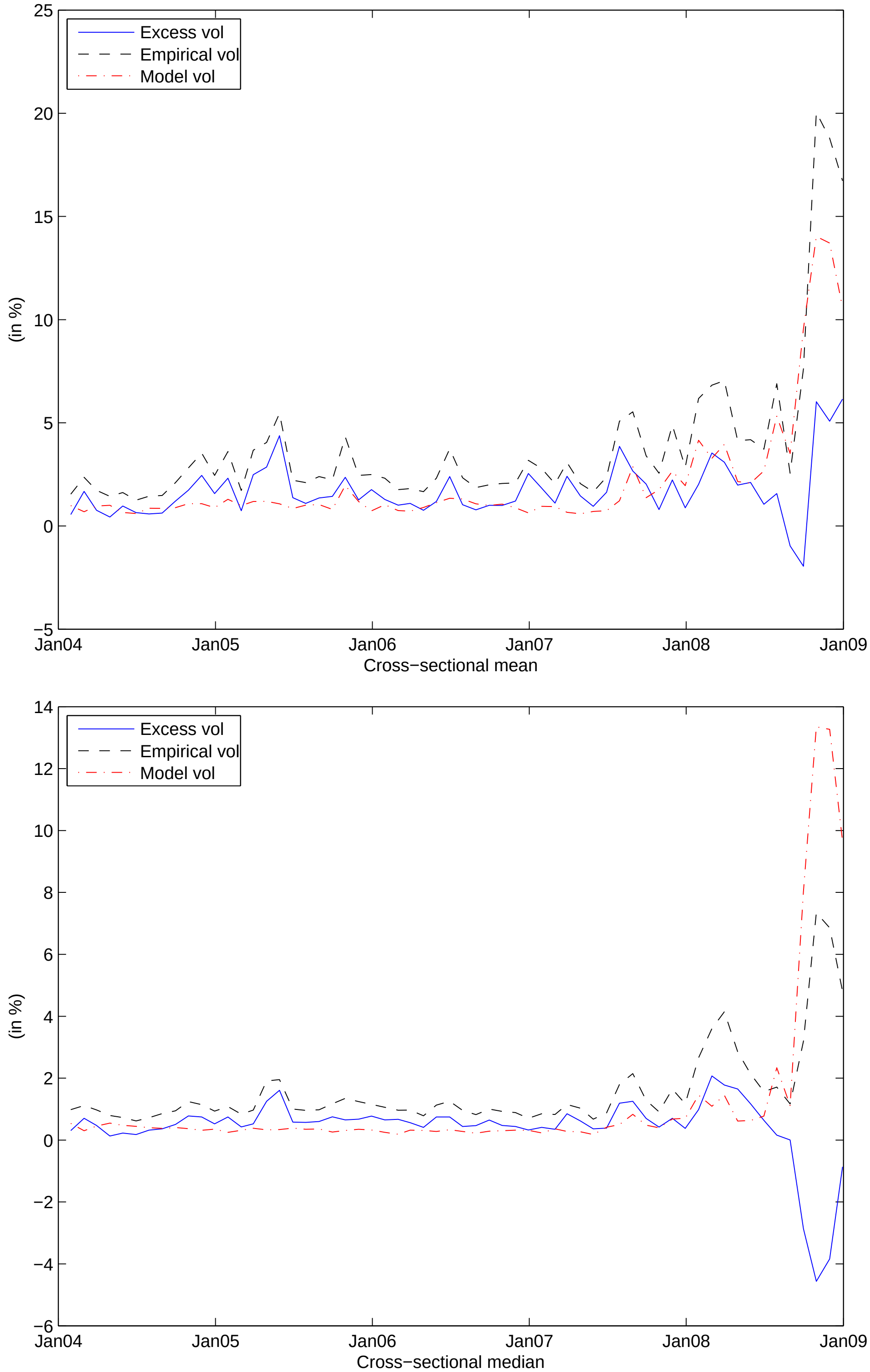

Figure 1: Time-series of empirical and model volatilities for synthetic floating rate bonds. The top panel is the cross-sectional mean for each period and the bottom panel is the cross-sectional median.

Table 7: **Time-Series Relation between Empirical and Model Volatilities**

	$\hat{\sigma}_D$	$\hat{\sigma}_D$	$\hat{\sigma}_{CDS}$	$\hat{\sigma}_{CDS}$
σ_D^{Merton}	1.73	0.96		
	[7.94]	[8.60]		
σ_{CDS}^{Merton}			1.11	1.03
			[11.68]	[16.39]
Return Horizon	D	M	D	M
Obs	32,132	2,332	8,082	615
R-sqd	51.92	72.02	47.45	73.00
Within-group R-sqd	11.71	20.77	16.90	38.06

Reported are panel regressions with bond fixed-effects. The dependent variable is the empirical bond volatility of either corporate bonds ($\hat{\sigma}_D$) or synthetic floating rate bonds ($\hat{\sigma}_{CDS}$). The dependent variables are model volatilities. T-stats with standard errors clustered by time are reported.

6.2 Volatility in the Cross-Section

A natural explanation for the fact that annualized excess bond volatilities drop dramatically when using monthly returns rather than daily returns is the fact that the corporate bond market is much less liquid than both the equity and Treasury markets. Our sample, which includes mostly large firms, is likely to have particularly liquid equity. As equity and Treasury volatility are important inputs into the calculation of model bond volatilities, the model bond volatility will have very little influence from liquidity while the empirical volatility (when using short-horizon returns) is likely to have a large influence. A cross-sectional implication is that bonds with a higher excess volatility may be less liquid bonds. Thus, we examine the cross-sectional determinants of $\hat{\sigma}_D - \sigma_D^{Merton}$, using proxies for liquidity as explanatory variables. We use three proxies for liquidity from prior literature:[26] (1) the amount outstanding of a bond, (2) the age of a bond, and (3) the volatility that can be generated by a bond's quoted bid-ask spread.[27]

We find that excess volatility is related to our three liquidity proxies when daily bond returns are used, but not when monthly returns are used. When daily returns are used, excess volatility is more severe in bonds with a smaller amount outstanding (at a 10% significance level). Given that the amount outstanding proxies for the amount of a bond that is available for trading,[28] bonds with a smaller amount outstanding are thought to be less liquid. In

[26]See Houweling, Mentink, and Vorst (2005) for a survey on the proxies used to measure bond liquidity.

[27]In particular, this is $\sqrt{252}\sqrt{\frac{1}{2}(\ln a)^2 + \frac{1}{2}(\ln b)^2 - (\ln a)(\ln b)}$, with $\sqrt{252}$ replaced by $\sqrt{12}$ when monthly returns are used.

[28]Nashikkar, Mahanti, Subrahmanyam, Chacko, and Mallik (2008) develop a latent liquidity measure based on the idea that bonds with a smaller amount available for trading are less liquid.

Table 8: **Cross-sectional Determinants of Excess Volatility**

	Bond	Bond	Bond	Bond	CDS	CDS
ln(Amt)	-1.82	-1.80	0.32	0.45		
	[-1.92]	[-1.90]	[1.28]	[1.63]		
Maturity	0.67	0.66	0.27	0.27		
	[8.72]	[10.24]	[11.82]	[12.10]		
Age	1.12	0.96	0.08	0.08		
	[3.68]	[4.74]	[1.59]	[1.57]		
Rating	0.32	0.22	0.08	0.05		
	[2.22]	[1.78]	[1.17]	[0.81]		
Leverage	0.041	0.028	0.025	-0.002	0.067	0.052
	[1.33]	[1.01]	[2.37]	[-0.17]	[4.28]	[4.55]
Equity Vol	0.26	0.10	0.05	0.04	0.19	0.11
	[4.65]	[3.73]	[0.94]	[1.34]	[5.12]	[1.55]
B/A Vol	0.67	0.67	0.81	0.44		
	[5.40]	[4.37]	[1.40]	[0.59]		
Callable	-2.09	-1.77	-0.63	-0.75		
	[-1.62]	[-1.93]	[-2.05]	[-2.73]		
$\hat{\sigma}_{CDS} - \sigma_{CDS}^{Merton}$		0.68		0.65		
		[7.27]		[9.60]		
Return Horizon	D	D	M	M	D	M
Obs	30,500	23,276	2,212	1,735	8,082	615
R-sqd	38.45	55.01	40.48	60.82	16.76	18.03

Reported are panel regressions with time fixed-effects. The dependent variable is the difference between empirical bond volatilities, $\hat{\sigma}_D$, and model bond volatilities, σ_D^{Merton}, (both in %) in the first four columns. In the last two columns, the dependent variable is the difference between empirical and model volatilities for synthetic floating rate bonds constructed from credit default swaps. The return horizon is denoted in the table and all volatilities are annualized. Standard errors are clustered by firm. Convertible and putable bonds are excluded from the regression, and Callable is one for callable bonds and zero otherwise. Maturity is the number of years to maturity, Age is number of years since issuance, Amt is the amount outstanding in \$m, Ratings are coded as 1 for Aaa and 21 for C, Leverage is the market leverage in decimals, and equity vol is in %. B/A Vol is the volatility that can be generated by quoted bid-ask spreads and is in %.

addition, a bond that is a year older has an excess volatility that is on average 112 basis points higher. This is consistent with a liquidity explanation as newly issued bonds are thought to be more liquid, while older bonds have a larger fraction held by buy-and-hold investors. Finally, an increase in the bid-ask spread implied volatility of 1 percentage point corresponds to a 67 basis point increase in excess volatility.

In contrast to the results based on daily bond returns, the excess volatility when monthly returns are used are not significantly related to the liquidity proxies examined. In particular, the sign on the amount outstanding flips while both age and bid-ask spread implied volatility are statistically insignificant. One variable that is particularly stable in both return horizons is the excess synthetic floater volatilities constructed from CDS. Taking CDS as measuring fundamental credit risk, this suggests that the excess bond volatility has a fundamental component that is stable across return horizons and an additional liquidity component when higher frequency returns are used. As shown in columns 5 and 6 of Table 8, the fundamental component is related to firm-level characteristics, such as a firm's market leverage.

Overall, it appears that liquidity is an important contributor to the cross-sectional variation in the disconnect between corporate bonds and stocks at short horizons, but is not an important contributor at longer horizons. This is consistent with underlying firm-level fundamentals contributing to the excess volatility in the credit market, relative to the equity and Treasury markets.

7 Conclusion

In recent years, there has been increasing research activity on the empirical performance of structural models of default, focusing largely on the ability (or more often inability) of such models to match credit spreads. Moreover, much of this line of research, while intriguing and informative, has used calibrations at the level of credit ratings or applications of models to a small number of observations. With the availability of high frequency data from TRACE, we are able to examine structural models more closely.

Our paper's contribution is to examine the (dis)connect in return volatilities between equities and bonds through a Merton model with stochastic interest rates. In particular, we find that empirical bond volatilities are too high to be explained by the Merton model with empirical equity and Treasury volatilities as inputs. By considering different measurement horizons (which are differentially affected by liquidity) and augmenting our evidence with CDS data and additional examinations of this excess volatility, we are able to assess whether the disconnect between equities and corporate bonds can be attributed solely to liquidity or whether there is a disconnect in fundamentals. While liquidity is an important driver in the disconnect when short-horizon bond returns are used, our results suggest that there is still a portion of the disconnect due to an inability to adequately link the fundamentals of equities and corporate bonds through the Merton model. This adds further to the debate on the ability of structural models to adequately characterize the relation between assets in different markets, a debate that has been advanced by Schaefer and Strebulaev (2008) who find that Merton model hedge ratios do well in relating equity and corporate bond returns.

Specifically, we use corporate bond returns from July 2002 to December 2008 and find that there is an overwhelming amount of excess volatility in corporate bonds when daily returns are used. This excess volatility is related to variables known to be related to bond liquidity. While this excess volatility decreases as we use weekly and monthly return horizons, it remains both economically and statistically significant, suggesting that liquidity alone cannot explain this disconnect. Excess volatility when monthly returns are used is not significantly related to liquidity proxies, further suggesting that the disconnect goes beyond illiquidity in the corporate bond market. Furthermore, there also appears to be excess volatility when CDS spreads are used, even in periods when the CDS market was liquid and counterparty risk was not considered to be important. By implementing a structural model of default, we are able to make these quantitative conclusions about the level of volatilities, though we see some evidence of empirical and model volatilities lining-up. Empirical corporate bond volatilities co-move in the time-series with a model bond volatility for which equity and Treasury volatilities are important inputs. In cross-sectional sorts, we find that bonds with higher empirical corporate bond volatilities also tend to have higher model bond volatilities.

Overall, our main conclusion is that there is a pattern of excess volatility in the credit markets that goes beyond the illiquidity of corporate bonds and remains a puzzle. As the excess volatility puzzle is robust across different return horizons, for different bond ratings and durations, across different years in our sample, and in the credit default swap market, this raises a number of questions for further research. Since liquidity alone cannot explain the disconnect, what fundamental risks can be incorporated that would explain the high empirical corporate bond volatility? Such a task is difficult in that it would require a risk that does not simultaneously increase equity volatility. Could the disconnect reflect something else such as market segmentation which cannot be captured by the standard structural models of default? We leave these questions to future research.

Appendix

A Model Bond Prices

To calculate corporate bond prices in our setting, it is important to calculate:

$$E^Q\left[\exp\left(-\int_0^{T_2} r_s ds\right)\mathbf{1}_{\{V_{T_1}>K\}}\right] \tag{12}$$

where $T_2 \geq T_1$

$$E^Q\left[\exp\left(-\int_0^{T_2} r_s ds\right)1_{\{V_{T_1}>K\}}\right] = \exp\left(a(T_2)+b(T_2)r_0\right)N\left(d_3\right) \tag{13}$$

where

$$d_3 = \frac{\ln\left(\frac{V}{K}\right) - a(T_1) - b(T_1)r_0 - \delta T_1 - \frac{1}{2}\Sigma + \sigma_r^2\frac{b(T_2-T_1)}{\kappa}\left(-b(T_1)+\frac{\exp(-2\kappa T_1)-1}{2\kappa}\right)}{\sqrt{\Sigma}}$$

$$\Sigma = T_1\left(\sigma_v^2+\frac{\sigma_r^2}{\kappa^2}\right)+\frac{2\sigma_r^2}{\kappa^3}\left(e^{-\kappa T_1}-1\right)-\frac{\sigma_r^2}{2\kappa^3}\left(e^{-2\kappa T_1}-1\right)$$

Proof. It can be shown that the above equation satisfies the PDE for arbitrage-free prices:

$$g\cdot r = g_t + g_v(r-\delta)V + \frac{1}{2}g_{vv}V^2\sigma_v^2 + g_r\kappa(\theta-r)+\frac{1}{2}g_{rr}\sigma_r^2 \tag{14}$$

$$g = \exp(a(T_2) + b(T_2)r_0)N(d_3)$$
$$g_v = \frac{Dn(d_3)}{V\sqrt{\Sigma}}, \text{where } D = \exp\left(a(T_2) + b(T_2)r_0\right)$$
$$g_{vv} = -\frac{Dd_3n(d_3)}{V^2\Sigma} - \frac{Dn(d_3)}{V^2\sqrt{\Sigma}}$$
$$g_r = Db(T_2)N(d_3) - \frac{Dn(d_3)b(T_1)}{\sqrt{\Sigma}}$$
$$g_{rr} = D(b(T_2))^2N(d_3) - 2Db(T_2)n(d_3)\frac{b(T_1)}{\sqrt{\Sigma}} - Dn(d_3)d_3\frac{(b(T_1))^2}{\Sigma}$$
$$g_t = -DN(d_3)(\theta\kappa b(T_2) + \frac{\sigma_r^2}{2}(b(T_2))^2 - e^{-\kappa T_2}r)$$
$$- \frac{Dn(d_3)}{\sqrt{\Sigma}}(-\delta - \theta\kappa b(T_1) - \sigma_r^2(b(T_1))^2 + e^{-\kappa T_1}r - \frac{1}{2}\sigma_v^2$$
$$+ \sigma_r^2\frac{b(T_2 - T_1)}{\kappa}(e^{-\kappa T_1} - e^{-2\kappa T_1})) + Dn(d_3)\frac{d_3}{2\Sigma}(\sigma_v^2 + \sigma_r^2(b(T_1))^2)$$

After some algebra, we can verify that g satisfies the PDE.

Boundary condition:

$T_1 \to 0$

$$E^Q\left[\exp\left(-\int_0^{T_2} r_s ds\right)\mathbf{1}_{\{V_{T_1}>K\}}\right] \to \begin{cases} 0 & \text{if V} < \text{K} \\ \exp\left(a(T_2 - T_1) + b(T_2 - T_1)r_{T_1}\right) & \text{if V} > \text{K} \end{cases} \qquad \blacksquare$$

Special cases include:

1. $T_1 = T_2$

   $$E^Q\left[\exp\left(-\int_0^{T} r_s ds\right)\mathbf{1}_{\{V_T>K\}}\right] = \exp\left(a(T) + b(T)r_0\right)N(d_2)$$

   where d_2 is as defined in the text of the paper

2. $K = 0$ (no default)

   $$E^Q\left[\exp\left(-\int_0^{T} r_s ds\right)\mathbf{1}_{\{V_T>K\}}\right] = \exp\left(a(T) + b(T)r_0\right)$$

Finally, our bond pricing formula (at t = 0) is:

$$B = \sum_{i=1}^{2T} \frac{c}{2} \exp\left(a\left(\frac{i}{2}\right) + b\left(\frac{i}{2}\right) r\right) N\left(d_2\left(\frac{i}{2}\right)\right) + \exp\left(a(T) + b(T)r\right) N\left(d_2(T)\right) \quad (15)$$
$$+ \sum_{i=1}^{2T} \exp\left(a\left(\frac{i}{2}\right) + b\left(\frac{i}{2}\right) r\right) \left[N\left(d_3\left(\frac{i-1}{2}\right)\right) - N\left(d_2\left(\frac{i}{2}\right)\right)\right] \mathcal{R}$$

For a zero-coupon bond where the payment contingent on default is paid at maturity, the bond price at t = 0 is:

$$B = \exp\left(a(T) + b(T)r\right) N\left(d_2(T)\right) + exp\left(a(T) + b(T)r\right)\left(1 - N\left(d_2(T)\right)\right) \mathcal{R}$$

B Synthetic Floating Rate Bond

B.1 Empirical Volatility

We follow Longstaff, Pan, Pedersen, and Singleton (2010) in constructing a synthetic floating rate corporate bond as a risk-free floating rate bond plus writing a CDS contract.[29] This bond pays quarterly coupon payments equal to the prevailing 3-month interest rate at the previous coupon date (divided by 4) plus $\frac{s}{4}$, where s is the annual CDS premium. Specifically, the synthetic floating rate bond consists of three positions:

1. Risk-free floating rate bond paying quarterly
2. Inflow of $\frac{s}{4}$ each quarter if the underlying remains solvent
3. Outflow of (1 - $\mathcal{R}$) if the underlying defaults

The initial price of this synthetic bond is its face value as a risk-free floater is worth its face value at all ex-coupon dates by arbitrage arguments and the initial CDS spread is set so that the values of (2) and (3) cancel.

To calculate returns, allow one day to elapse. Suppose now that the prevailing CDS spread is $\hat{s}$ and that the prevailing CDS spread for a (5 year - 1 day) CDS that has payments aligned with the above 5 year CDS is the same as the prevailing 5 year CDS spread. We are left to determine the changes in the value of our positions:

[29]See also Duffie and Singleton (2003).

(1) To calculate the value of the risk-free floater, note that at the next coupon date, a coupon of $\frac{r_0}{4}$ will be paid and the ex-coupon price of the floater will be its face value. Thus, discount $1 + \frac{r_0}{4}$ at the prevailing interest rate.

(2) & (3) The value of the $\frac{s}{4}$ inflow versus the (1 - $\mathcal{R}$) outflow is the value of a stream of $\frac{s-\hat{s}}{4} = -\frac{\Delta s}{4}$. This is equal to $-\frac{\Delta s}{4}$ times the value of a risky (5 year - 1 day) annuity paying quarterly. As in Longstaff, Pan, Pedersen, and Singleton (2010), the discount rate is the Treasury rate plus the prevailing CDS spread. Finally, add in the accrued CDS premium, properly discounted.

B.2 Model Volatility

The model bond volatility is calculated by noting the three positions that comprise the synthetic floater and applying the formulas derived in Appendix A. In particular, the value of the stream of CDS premia is:

$$\sum_{i=1}^{4T} exp\left(a\left(\frac{i}{4}\right) + b\left(\frac{i}{4}\right) r\right) N\left(d_2\left(\frac{i}{4}\right)\right)\frac{s}{4} \tag{16}$$

The value of the outflow contingent on default is:

$$\sum_{i=1}^{4T} exp\left(a\left(\frac{i}{4}\right) + b\left(\frac{i}{4}\right) r\right)\left[N\left(d_3\left(\frac{i-1}{4}\right)\right) - N\left(d_2\left(\frac{i}{4}\right)\right)\right](1-\mathcal{R}) \tag{17}$$

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