Generalized Disappointment Aversion, Long-Run Volatility Risk and Asset Prices *

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Abstract

We propose an asset pricing model where preferences display generalized disappointment aversion (Routledge and Zin, 2009) and the endowment process involves long-run volatility risk. These preferences, which are embedded in the Epstein and Zin (1989) recursive utility framework, overweight disappointing results as compared to expected utility, and display relatively larger risk aversion for small gambles. Our endowment process has only one of the two sources of long-run risks proposed by Bansal and Yaron (2004) (BY): the volatility risk. Through a matching procedure of the endowment process with a Markov switching model, we derive closed-form solutions for all returns moments and predictability regressions. The model produces asset returns moments and predictability patterns in line with the data. Compared to BY we generate: i) more predictability of excess returns by price-dividend ratios; ii) less predictability of consumption growth rates by price-dividend ratios. Differently from the BY model, our results do not depend on a value of the elasticity of intertemporal substitution greater than one: similar results may be obtained with values lower than one. Our results are not due to an overparametrization of preferences either: a simple model where risk aversion comes only from pure disappointment aversion generates similar implications.

Keywords: Equilibrium Asset Pricing, Disappointment Aversion, Long-run Risks, Equity Premium, Risk-free Rate Puzzle, Predictability of returns

JEL Classification: G1, G12, G11, C1, C5

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1 Introduction

The Consumption-based Capital Asset Pricing Model (CCAPM) has recently been revived by models of long-run risks (LRR)[1]. Bansal and Yaron (2004) (hereafter BY) explain several asset market stylized facts by a model with a small long-run predictable component driving consumption and dividend growth and fluctuating economic uncertainty measured by consumption volatility, together with Epstein and Zin's (1989) Kreps-Porteus (hereafter KP) preferences, a recursive utility certainty equivalent that separates risk aversion from intertemporal substitution. These preferences play a crucial role in the long-run risks model. In a canonical expected utility risk, only short-run risks are compensated, while long-run risks do not carry separate risk premia. With KP preferences, long-run risks earn a positive risk premium as long as investors show preference for early resolution of uncertainty. Routledge and Zin (2009) extended the KP certainty equivalent to represent generalized disappointment aversion preferences (GDA). GDA preferences distort the probability weights of expected utility by overweighting outcomes below a threshold determined as a fraction of the certainty equivalent. They show that that recursive utility with GDA risk preferences generates effective risk aversion that is countercyclical and therefore produce a large equity premium even with a simple autoregressive model consumption-growth process[2].

In fact, the existence of a long-run risk component in expected consumption growth is a source of debate. If a very persistent predictable component exists in consumption growth, as proposed by BY, it is certainly hard to detect it as consumption appears very much as a random walk in the data[3]. In Campbell and Cochrane (1999), consumption growth is modeled as an i.i.d. process. There is also some discussion about the persistence of consumption growth volatility. Calvet and Fisher (2007) find empirical evidence of much longer duration volatility shocks than in BY, creating the potential of a more important contribution of volatility risk to explaining asset pricing stylized facts.

In this paper, we revisit the LRR model with GDA preferences. In the LRR model, two key mechanisms are at play. The first one is related to the presence of a slow long-run

[1]The extensive literature about the equity premium puzzle and other puzzling features of asset markets are reviewed in a collection of essays in Mehra (2008). See also Campbell (2000, 2003), Cochrane and Hansen (1992), Kocherlakota (1996), and Mehra and Prescott (2003).

[2]Effective risk aversion refers to the risk aversion of an expected utility agent that will price risk in the same way as a disappointment-averse agent.

[3]Bansal (2007) cites several studies that provide empirical support for the existence of a long-run component in consumption. Bansal, Gallant and Tauchen (2007) and Bansal, Kiku and Yaron (2007) test the LRR model using the efficient and generalized method of moments, respectively. Hansen, Heaton and Li (2008) and Bansal, Kiku and Yaron (2009) present evidence for a long-run component in consumption growth using multivariate analysis.

component in the mean of consumption and dividend growth rates. The second channel channel reflects time-varying economic uncertainty, and is captured by fluctuating conditional volatility of consumption. As BY show clearly in their paper, the first channel is essential with KP preferences to achieve an equity premium commensurate with historical data. Given the debate about the nature of the consumption process, we start by restricting the LRR model to a random walk model with heteroscedastic innovations to investigate whether fluctuations in economic uncertainty are sufficient with GDA preferences to reproduce observed asset pricing stylized facts.

This benchmark model reproduces asset pricing stylized facts and predictability patterns put forward in the previous literature, both in population and in finite sample. The equity premium and the risk-free rate are very closely matched, as well as the volatility of the price-dividend ratio and of returns. The price-dividend ratio predicts excess returns at various horizons, while by the very nature of the assumed process consumption and dividend growth rates are unpredictable, as observed in the data. The intuition is straightforward.

In the simplest representation of GDA preferences, where the only source of risk aversion is disappointment aversion (the utility function is otherwise linear with a zero curvature parameter and an infinite elasticity of intertemporal substitution), the stochastic discount factor has only two values in each state of the economy at time t. The SDF for disappointing outcomes is α^{-1} times the SDF for non-disappointing outcomes, where α measures the degree of disappointment aversion. For say a value of α equal to 0.33, the value of the SDF will be three times higher in the disappointing states than in the non-disappointing ones. This will create a sizable negative covariance between the pricing kernel and the return on a risky asset, making the risk premium sizable. If states are persistent, as it is the case in the LRR model, then the stochastic discount factor distribution will change gradually, implying persistent and predictable conditional expected returns. As argued by Fama and French (1988), this type of process for expected returns generate mean reversion in asset prices. Therefore, the price-dividend ratio today should be a good predictor of returns over several future periods.

An important question is to establish whether the results obtained with a random walk consumption and GDA preferences are maintained when we introduce a long-run risk in expected consumption growth. We verify that all the statistics reproduced for the GDA preferences are very close to what we obtained with the random walk model. This confirms that volatility risk is the main economic mechanism behind the asset pricing results. Adding a risk in the expected consumption growth does not affect much the GDA investor. In contrast, the results are changing for KP in several dimensions with respect to the simple random walk, reaffirming the essential role played by the small long-run predictable component in expected consumption growth in the BY model.

BY as most recent models rely on parameter calibration for consumption and dividend processes, as well as preferences, to derive asset pricing implications from the model. The solution technique to solve for asset valuation ratios is based on loglinear approximations. Since the GDA utility is non-differentiable at the kink where disappointment sets in, one cannot rely on the same approximation techniques to solve the model. In this paper, we propose a methodology that provides an analytical solution to the LRR model with GDA preferences and a fortiori with KP preferences, yielding formulas for the asset valuation ratios in equilibrium. The key to this analytical solution is to use a Markov Switching process for consumption and dividends that matches the LRR specification. In addition, we report analytical formulas for the population moments of equity premia as well as for the coefficients and R^2 of predictability regressions that have been used to assess the ability of asset pricing models to reproduce stylized facts.

Analytical formulas allow us to avoid using simulation techniques to reproduce stylized facts as it is usually done in the literature. A good recent example is a recent paper by Beeler and Campbell (2009). They assess the long-run risks model in various dimensions using simulations to reproduce both population and finite-sample moments of the distribution of returns. For population statistics, they use a single simulation run of 1.2 million months. This means that evidence in support of the models are almost invariably based on a given set of parameters that reproduce the stylized facts. The cost of producing results limits the potential for sensitivity analysis and model assessment is based on the plausibility of the chosen parameters.

Thanks to our analytical formulas, we are able to conduct a thorough comparative analysis between models by varying the preference and endowment parameters. We will produce graphs that exhibit the sensitivity of asset pricing statistics or predictability regressions R^2 to a large set of values for key parameters such as the persistence in volatility or expected consumption growth. This provides an excellent tool to measure the robustness of model implications.

This is of particular importance since the choice of parameters is a source of lively debate. Take the value of the elasticity of intertemporal substitution. Bansal and Yaron (2004) report empirical evidence in favor of a value greater than 1[4] but mention that Hall (1988) and Campbell (1999) estimate an IES below 1. They also argue that in the presence of time-varying volatility, there is a severe downward bias in the point estimates of the IES. While the argument is correct in principle, Beeler and Campbell (2009) simulate the

[4]They cite Hansen and Singleton (1982) and Attanasio and Weber (1989), among others.

BY model and report no bias if the riskless interest rate is used as an instrument[5]. One important result with GDA preferences is that the IES value of 1 is not pivotal for the asset pricing implications, for the random walk as well as the LRR model. Both moment fitting and predictability results remain intact with values of the IES below 1. The main effect of setting the IES below 1 is of course to increase the level and the volatility of the risk-free rate but these moments remain in line with the data.

We also conduct a sensitivity analysis with respect to the risk parameters. We investigate the simplest specification among disappointment averse preferences. We set the threshold at the certainty equivalent, as in the original disappointment aversion model of Gul (1991), and we do not allow for any curvature in the stochastic discount factor, except for the disappointment kink. In other words, if disappointment aversion were not present, the stochastic discount factor would be equal to the constant time discount parameter. This pure disappointment aversion model reproduces rather well the stylized facts in population, especially predictability of returns. Routledge and Zin (2009) stress the importance of GDS for obtaining a counter-cyclical price of risk in their Mehra-Prescott economy. Since we have a richer endowment process, there is not such a stark contrast between DA and GDA preferences in our model.

There is also an active debate about the predictability of returns by the dividend yield. Econometric and economic arguments fuel the controversy about the empirical estimates of R^2 in predictive regressions of returns or excess returns over several horizons on the current dividend yield. Some claim that the apparent predictability is a feature of biases inherent to such regressions with persistent regressors, others that it is not spurious since if returns were not predictable, dividend growth should, by accounting necessity, be predictable, which is not the case in the data[6]. Therefore, evidence that a consumption-based asset pricing model is able to reproduce these predictability patterns based on data would certainly clarify the debate.

Our comparison of the BY model with KP preferences to our GDA model puts forward interesting results in terms of predictability. For excess return predictability we arrive at a surprising result. While the random walk model generated some predictability in population, the full LRR model does not produce any predictability at all in population. For consumption growth, the LRR model with KP preferences overpredicts strongly in population, with R^2 in the order of 20%.

To summarize, while persistence of expected consumption growth appears fundamental

[5] They confirm the presence of a bias (negative estimate of the IES) when the equity return is used and attribute it to a weak instrument problem.

[6] See in particular Valkanov (2003), Stambaugh (1999), Cochrane (2008) and the special issue of the Review of Financial Studies about the topic of predictability of returns.

for the moment matching ability of the LRR model with KP preferences, disappointment aversion relies mostly on the persistence of consumption volatility. When preferences are disappointment averse, the main driver of the asset pricing matching ability of the model is the persistent consumption growth volatility. The GDA model performs well even when coupled with a random walk model for consumption and dividend growth, provided that its volatility is persistent. However, the sensitivity of results to the volatility persistence parameter with GDA preferences is not as drastic as with the persistence of expected consumption growth with KP preferences in the BY model.

Disappointment aversion preferences were introduced by Gul (1991) to be consistent with the Allais Paradox. They differ from expected utility by introducing an additional weight to outcomes that are below the certainty equivalent. Routledge and Zin (2004) (RZ) generalized these preferences by allowing the disappointment threshold to be placed at an arbitrary proportion of the certainty equivalent. Disappointment averse preferences are endogenously state-dependent through the certainty equivalent threshold and, therefore, are apt to produce counter-cyclical risk aversion. Investors may become more averse in recessions if the probability of disappointing outcomes is higher than in booms.

Models with exogenous reference levels, such as Campbell and Cochrane (1999) and Barberis, Huang ans Santos (2001), generate counter-cyclical risk aversion and link it to return predictability. Investors will be willing to pay a lower price in bad states of the world, implying higher future returns. In Lettau and Van Nieuwerburgh (2008), predictability empirical patterns can be explained by changes in the steady-state mean of the financial ratios. These changes can be rationalized by a LRR model with GDA preferences.

Bernartzi and Thaler (1995) are also using asymmetric preferences over good and bad outcomes to match the equity premium, but instead of using an intertemporal asset pricing framework with preferences defined over consumption streams, they start from preferences defined over one-period returns based on Kahneman and Tversky (1979)'s prospect theory of choice. By defining preferences in this way directly over returns, they avoid the challenge of reconciling the behavior of asset returns with aggregate consumption.

Recently, Ju and Miao (2009) have embedded a model of smooth ambiguity aversion in a recursive utility framework. While ambiguity aversion implies attaching more weight to bad states, as in disappointment aversion, the mechanism is very different. An ambiguity averse decision maker will prefer consumption that is more robust to possible variations in probabilities. They fear stocks because they build pessimistic views about consumption growth realizations.[7]

[7]Ambiguity aversion increases the conditional equity premium when there is uncertainty about the current state of the economy (and its future prospects). However, various versions of the ambiguity model

Following the seminal paper by Hamilton (1989), Markov switching models have been used in the consumption-based asset pricing literature to capture the dynamics of the endowment process. While Cechetti, Lam and Mark (1990) and Bonomo and Garcia (1994) estimate univariate models for either consumption or dividend growth, Cechetti, Lam and Mark (1993) estimate a homoscedastic bivariate process for consumption and dividend growth rates, and Bonomo and Garcia (1993, 1994) a heteroscedastic one. Recently, Lettau, Ludvigson and Wachter (2008), Bhamra, Kuehn, and Strebulaev (2009), and Ju and Maio (2009) have also estimated such processes. Calvet and Fisher (2007) estimate multifractal processes with Markov switching in a large number of states setting in a consumption-based asset pricing model. Apart from capturing changes in regimes, another distinct advantage of Markov switching models is to provide a flexible statistical tool to match other stochastic processes such as autoregressive processes as in Tauchen (1986). In this paper we match the heteroscedastic autoregressive models for consumption and dividend growth rates in Bansal and Yaron (2004), based on the parameter configuration in Bansal, Kiku and Yaron (2007), with a four-state Markov switching model. Recently, Chen (2008) has approximated the dynamics of consumption growth process of the BY LRR model using a discrete-time Markov and the quadrature method of Tauchen and Hussey (1991) in a model to explain credit spreads.

This paper extends considerably the closed-form pricing formulas provided in Bonomo and Garcia (1994) and Cecchetti, Lam and Mark (1990) for the Lucas (1978) and Breeden (1979) CCAPM model. Bonomo and Garcia (1993) have studied disappointment aversion in a bivariate Markov switching model for consumption and dividend growth rates and solved numerically the Euler equations for the asset valuation ratios. For recursive preferences, solutions to the Euler equations have been mostly found either numerically or after a log linear approximate transformation. However, Chen (2008) and Bhamra, Kuehn, and Strebulaev (2009) use a Markov chain structure for consumption growth to solve analytically for equity and corporate debt prices in an equilibrium setting with Kreps-Porteus preferences, while Calvet and Fisher (2007) focused on the equity premium[8].

Recently, some papers have also proposed to develop analytical formulas for asset pricing models. Abel (1992, 2008) calculate exact expressions for risk premia, term premia, and the premium on levered equity in a framework that includes habit formation and consumption externalities (keeping up or catching up with the Joneses). The formulas are derived

have difficulty reproducing predictability patterns and magnitudes.

[8]These papers have been developed contemporaneously and independently from the first version of the current paper titled "An Analytical Framework for Assessing Asset pricing Models and Predictability", presented in May 2006 at the CIREQ and CIRANO Conference in Financial Econometrics in Montreal and discussed by Motohiro Yogo.

under lognormality and an i.i.d. assumption for the growth rates of consumption and dividends. We also assume log-normality but after conditioning on a number of states and our state variable captures the dynamics of the growth rates. Eraker (2008) produces analytic pricing formulas for stocks and bonds in an equilibrium consumption CAPM with Epstein-Zin preferences, under the assumption that consumption and dividend growth rates follow affine processes. However, he uses the Campbell and Shiller (1988) approximation to maintain a tractable analytical form of the pricing kernel. Quite recently, Gabaix (2008) proposed a class of linearity-generating processes that ensures closed-form solutions for the prices of stocks and bonds. This solution strategy is based on reverse-engineering of the processes for the stochastic discount factors and the asset payoffs.

The rest of the paper is organized as follows. Section 2 sets up the preferences and endowment processes. Generalized disappointment averse preferences and the BY long-run risks model for consumption and dividend growth are presented. In section 3, we describe a matching-moment procedure for the LRR model based on a Markov Switching process, solve for asset prices and derive formulas for predictive regressions. Section 4 explains how endowment and preference parameters are chosen for the benchmark random-walk model of consumption and dividends. We also explore the asset pricing and predictability implications of the model. A thorough sensitivity analysis is conducted in Section 5 for preference parameters and persistence in consumption volatility. Section 6 provides a comparison with the LRR model of Bansal and Yaron (2004). Section 7 concludes.

2 An Asset Pricing Model with GDA Preferences and LRR Fundamentals

Our primary goal in this section is to formulate a model that includes both a long-run risk specification for consumption and dividends, and recursive preferences. In BY, where a long-run risk asset pricing model is developed, the Kreps-Porteus (KP) recursive preferences have an expected utility certainty equivalent that disentangles risk aversion from intertemporal substitution. In this paper the certainty equivalent is extended to represent generalized disappointment aversion preferences (GDA)recently introduced by Routledge and Zin (2009). These preferences generalize the former disappointment aversion specification of the recursive utility family introduced by Epstein and Zin (1989) and studied empirically by Bekaert, Hodrick and Marshall (1997) and Bonomo and Garcia (1993) in the context of asset pricing, and Ang, Bekaert and Liu (2005) for portfolio allocation.

GDA preferences distort the probability weights of expected utility by overweighting outcomes below a threshold determined as a fraction of the certainty equivalent. Two parameters are added with respect to the KP specification, one that determines the threshold at which the investor gets disappointed as a percentage of the certainty equivalent, and another one that sets the magnitude of disappointment incurred by the investor below this threshold. GDA admits both KP and simple disappointment aversion as particular cases. In the latter case, the threshold is set right at the certainty equivalent. In an economy with a simple autoregressive endowment-growth process, Routledge and Zin (2009) show that recursive utility with GDA risk preferences generates effective risk aversion that is countercyclical[9]. The economic mechanism at play is an endogenous variation in the probability of disappointment in the representative investors intertemporal consumption-saving problem that underlies the asset-pricing model. We extend their investigation to a more complex long-run risk model for consumption and dividends combined with GDA preferences.

2.1 Generalized Disappointment Aversion

RZ generalized Gul's (1991) disappointment aversion preferences and embedded them in the recursive utility framework of Epstein and Zin (1989). Formally, let V_t be the recursive intertemporal utility functional:

$$\begin{aligned} V_t &= \left\{(1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta\left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1 & (2.1) \\ &= C_t^{1-\delta}\left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{\delta} \quad \text{if } \psi = 1, & (2.2) \end{aligned}$$

where C_t is the current consumption, δ (between 0 and 1) is the time preference discount factor, ψ (greater than 0) is the elasticity of intertemporal substitution and $\mathcal{R}_t\left(V_{t+1}\right)$ is the certainty equivalent of the random future utility conditional on time t information.

With GDA preferences the certainty equivalent function $\mathcal{R}(.)$ is implicity defined by:

$$\frac{\mathcal{R}^{1-\gamma}}{1-\gamma} = \int_{(-\infty,\infty)} \frac{V^{1-\gamma}}{1-\gamma} dF(V) - \left(\alpha^{-1}-1\right)\int_{(-\infty,\kappa\mathcal{R})}\left(\frac{(\kappa\mathcal{R})^{1-\gamma}}{1-\gamma} - \frac{V^{1-\gamma}}{1-\gamma}\right) dF(V) \quad \kappa \leq 1. \tag{2.3}$$

When α is equal to one, $\mathcal{R}$ becomes the certainty equivalent corresponding to expected utility while V_t represents the Kreps-Porteus preferences. When $\alpha < 1$, outcomes lower than $\kappa\mathcal{R}$ receive an extra weight $(\alpha^{-1}-1)$, decreasing the certainty equivalent. Thus, α is interpreted as a measure of disappointment aversion, while the parameter κ is the percentage of the certainty equivalent $\mathcal{R}$ such that outcomes below it are considered disappointing[10]. Formula 2.3 makes clear that the probabilities to compute the certainty equivalent are redis-

[9]Effective risk aversion refers to the risk aversion of an expected utility agent that will price risk in the same way as a disappointment-averse agent.

[10]Notice that the certainty equivalent, besides being decreasing in γ, is also increasing in α and decreasing in κ (for $\kappa \leq 1$). Thus α and κ are also measures of risk aversion, but of a different type than γ.

tributed when disappointment sets in, and that the threshold determining disappointment is changing over time.

With Kreps-Porteus preferences, Hansen, Heaton and Li (2008) derive the stochastic discount factor in terms of the continuation value of utility of consumption, as follows:

$$M_{t,t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{\frac{1}{\psi}-\gamma}. \tag{2.4}$$

If $\gamma = 1/\psi$, equation (2.4) corresponds to the stochastic discount factor of an investor with time-separable utility with constant relative risk aversion, where the powered consumption growth values short-run risk as usually understood. The ratio of future utility V_{t+1} to the certainty equivalent of this future utility $\mathcal{R}_t\left(V_{t+1}\right)$ will add a premium for long-run risk.

For GDA preferences, long-run risk enters in an additional term capturing disappointment aversion[11], as follows:

$$\begin{aligned} M_{t,t+1} &= \delta^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{\frac{1}{\psi}-\gamma} \\ &\times \frac{\left[1+\left(\alpha^{-1}-1\right) I\left(\left(\frac{V_{t+1}}{\mathcal{R}_t V_{t+1}}\right)^{1-\gamma} < \kappa^{1-\gamma}\right)\right]}{\left[1+\kappa^{1-\gamma}\left(\alpha^{-1}-1\right) E_t I\left(\left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{1-\gamma} < \kappa^{1-\gamma}\right)\right]} \end{aligned}$$

Generalized disappointment aversion kicks in whenever the ratio of future utility to its certainty equivalent is less than the threshold κ. For disappointment aversion, this threshold is one. This representation makes clear that a small but persistent shock to the mean of consumption growth may have a very small impact through the single immediate consumption growth, but a very large one through the lifetime utility ratio.

However, the most common representation of recursive preferences with the Kreps-Porteus certainty equivalent characterizes the stochastic discount factor as a function of the return on an asset that yields aggregate consumption as payoff, which we call the wealth or market portfolio and denote R^m_{t+1}[12]. Under this representation, GDA preferences imply a stochastic discount factor given by:

$$M_{t,t+1} = z_{t+1}^{1-\gamma}\left(R^m_{t+1}\right)^{-1} \frac{\left[1+\left(\alpha^{-1}-1\right) I\left(z_{t+1}<\kappa\right)\right]}{\left[1+\kappa^{1-\gamma}\left(\alpha^{-1}-1\right) E_t I\left(z_{t+1}<\kappa\right)\right]}, \tag{2.5}$$

[11]Although Routledge and Zin (2009) do not model long-run risk, they discuss how its presence could interact with GDA preferences in determining the marginal rate of substitution.

[12]When a closed-form expression for the value function is available as in Hansen, Heaton, and Li (2009), this representation has the advantage of not requiring a direct measurement of the market portfolio return since it is fundamentally unobservable and has been proxied by a stock market index.

where:

$$z_{t+1} = \delta^{\frac{1}{1-\frac{1}{\psi}}} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1}{1-\psi}} \left(R^m_{t+1}\right)^{\frac{1}{1-\frac{1}{\psi}}}.$$

It is clear that when there is no disappointment aversion ($\alpha = 1$), the expression above reduces to the familiar Kreps-Porteus pricing kernel, which was used in BY:

$$M_{t,t+1} = \delta^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1-\gamma}{1-\psi}} \left(R^m_{t+1}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}. \tag{2.6}$$

This representation will be useful to clarify the role of disappointment aversion preferences with an otherwise linear utility function, that is $\gamma = 0$ and $\psi = \infty$. In this case, the stochastic discount factor becomes:

$$M_{t,t+1} = \delta \frac{\left[1 + \left(\alpha^{-1} - 1\right) I \left(R^m_{t+1} < \frac{\kappa}{\delta}\right)\right]}{\left[1 + \kappa \left(\alpha^{-1} - 1\right) E_t I \left(R^m_{t+1} < \frac{\kappa}{\delta}\right)\right]}.$$

Notice that in this case, the only source of risk aversion is disappointment aversion. For each state of the economy in t, the stochastic discount factor has only two values. The SDF for disappointing outcomes is α^{-1} times the SDF for non-disappointing outcomes. The probability of disappointment occurring is given by the likelihood that the return on the market portfolio is less than the ratio between κ and the time discount factor δ. Suppose for simplicity that κ is equal to δ. Then, disappointment occurs when the gross return is less than one, which means when a negative net return occurs. In that case, the variability of the SDF will depend on the distance between the two SDF values, determined by α, as on the respective likelihoods of positive and negative returns[13]. These likelihoods are conditional on the state at time t and therefore produce state-dependent risk aversion.

Suppose further for illustration purposes that these likelihoods are equal. Then, for say a value of α equal to 0.33, the value of the SDF will be three times higher in the negative-return states than in the nonnegative-return ones. This will create a sizable negative covariance between the pricing kernel and the return on a risky asset, making the risk premium sizable. In contrast, if the likelihood of disappointing outcomes is negligible, the covariance will be very small and the risk premia close to zero.

In the next sections we will provide evidence that this GDA pricing kernel has also the potential to generate return predictability by the dividend-price ratio. If states are persistent, as it is the case in the LRR case, then the stochastic discount factor distribution will change gradually, implying persistent and predictable conditional expected returns. As argued by Fama and French (1988), this type of process for expected returns generate

[13]Note that the return on the market portfolio also depends on the pricing kernel. Thus, the probability of a disappointing return on it depends also on α.

mean reversion in asset prices. Therefore, the price-dividend ratio today should be a good predictor of returns over several future periods.

2.2 A Long-Run Volatility Risk Benchmark Model for Consumption and Dividends

In the long-run risk model of BY, the consumption and dividend growth processes are evolving dynamically as follows:

$$
\begin{aligned}
\Delta c_{t+1} &= x_t + \sigma_t \epsilon_{c,t+1} \\
\Delta d_{t+1} &= (1-\phi_d)\,\mu_x + \phi_d x_t + \nu_d \sigma_t \epsilon_{d,t+1} \\
x_{t+1} &= (1-\phi_x)\,\mu_x + \phi_x x_t + \nu_x \sigma_t \epsilon_{x,t+1} \\
\sigma^2_{t+1} &= (1-\phi_\sigma)\,\mu_\sigma + \phi_\sigma \sigma^2_t + \nu_\sigma \epsilon_{\sigma,t+1},
\end{aligned} \tag{2.7}
$$

where c_t is the logarithm of real consumption and d_t is the logarithm of real dividends. In this characterization, x_t, the conditional expectation of the consumption growth, is modeled as a slowly reverting AR(1) process (ϕ_x smaller but close to one). Notice that $\phi_d x_t$ also governs the conditional expectation of the dividend growth, and ϕ_d is assumed to be greater than one - the leverage ratio on consumption growth. The volatility of consumption growth σ_t is also assumed to be a very persistent process (ϕ_σ smaller but close to one) with unconditional mean μ_σ. The innovations in the expected growth processes and in the volatility process are assumed to be independent.

In this LRR model, two key mechanisms are at play to determine asset prices. The first one relates to expected growth: both consumption and dividend growth rates contain a small long-run component in the mean. Shocks today have a very persistent effect on expected consumption growth far in the future. The second channel reflects time-varying economic uncertainty, and is captured by fluctuating conditional volatility of consumption. As BY show clearly in their paper, the first channel is essential with KP preferences to achieve an equity premium commensurate with historical data. By choosing a random walk benchmark model, we want to show that fluctuations in economic uncertainty are sufficient with generalized disappointment averse investors to generate a similar equity premium as well as most stylized facts looked at in the literature.

If a very persistent predictable component exists in consumption growth, as proposed by BY, it is certainly hard to detect it as consumption appears very much as a random walk in the data[14]. Campbell and Cochrane (1999) use a random walk model for consumption and

[14]See in particular Campbell (2003). Bansal and Yaron (2004) stress this difficulty of distinguishing

a heteroscedastic slowly mean-reverting surplus that is dynamically driven by consumption growth innovations that feeds into habit persistent preferences. More recently, Calvet and Fisher (2007) have proposed a model where consumption growth is i.i.d. and where the log dividend follows a random walk with state-dependent drift and volatility. They also extend the model to allow consumption growth to exhibit regime shifts in drift and volatility.

The model that we propose differs from these previous specifications in the sense that the drifts of consumption growth and dividend growth are constant while volatilities are time-varying[15]:

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_x + \sigma_t \varepsilon_{c,t+1} \qquad (2.8)\\
\Delta d_{t+1} &= \mu_x + \nu_d \sigma_t \varepsilon_{d,t+1}\\
\sigma^2_{t+1} &= (1-\phi_\sigma)\mu_\sigma + \phi_\sigma \sigma^2_t + \nu_\sigma \varepsilon_{\sigma,t+1}.
\end{aligned}
$$

As we will see in the next section combining GDA preferences with the random walk 2.8 or the LRR 2.7 models for fundamentals necessitates a solution technique that departs from the usual approximate solutions based on log linearization.

3 Solving a Long-Run Risks Model with GDA Preferences

The LRR model with KP preferences cannot be solved analytically. BY use Campbell and Shiller (1988) approximations to obtain analytical expressions that are useful for understanding the main mechanisms at work, but when it comes to generate numerical results they appeal to numerical simulations of the original model. A second type of approximation, proposed by Hansen, Heaton and Li (2008), is done around a unitary value for the elasticity of intertemporal substitution ψ[16]. However, since the GDA utility is non-differentiable at the kink where disappointment sets in, one cannot rely on the same approximation techniques to obtain analytical solutions of the model. In this paper we propose a methodology that provides an analytical solution to the LRR model with GDA preferences and a fortiori with KP preferences, yielding formulas for the asset valuation ratios in equilibrium. The key to this analytical solution is to use a Markov Switching process for consumption and

empirically a very persistent process from a random walk, even though the two models have drastically different asset pricing implications. Using multivariate analysis, Hansen, Heaton and Li (2008) and Bansal, Kiku and Yaron (2007) present some evidence for long-run components in consumption growth.

[15]We also depart from the LRR model of BY by allowing a correlation ρ between innovations in consumption growth and in dividend growth, as in Bansal, Kiku and Yaron (2007).

[16]A previous version of this paper (SSRN working paper 1109080) derived the approximate solutions for the recursive utility model using the Campbell-Shiller and the Hansen-Heaton-Li approximations and analyzed their respective accuracy for various sets of parameter values.

dividends that matches the LRR specification. In addition, we report analytical formulas for the population moments of equity premia as well as for the coefficients and R^2 of predictability regressions that have been used to assess the ability of asset pricing models to reproduce stylized facts.

3.1 A Matching-Moment Procedure for the Long-Run Risk Model

We will describe the matching procedure for the general LRR model in (2.7) since it will apply equally to the restricted version (2.8) that we set as our benchmark model. Let s_t be a Markov state process at time t. We postulate that the consumption and dividend growth processes evolve dynamically as a function of s_t as follows:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c(s_t) \;\; + \;\; (\omega_c(s_t))^{1/2}\,\varepsilon_{c,t+1} \\ \Delta d_{t+1} &= \mu_d(s_t) \;\; + \;\; (\omega_d(s_t))^{1/2}\varepsilon_{d,t+1}, \end{aligned} \tag{3.9}$$

where $\varepsilon_{c,t+1}$ and $\varepsilon_{d,t+1}$ follow a bivariate normal process with mean zero and correlation ρ.

Bonomo and Garcia (1996) proposed and estimated the specification (3.9, **??**) for the joint consumption-dividends process with a three-state Markov switching process to investigate if an equilibrium asset pricing model with different types of preferences can reproduce various features of the real and excess return series.[17]

We now explain how we match the multivariate process (2.7) with the MS process (3.9). The main features of the (2.7) process to be matched are:

1. The expected means of the consumption and dividend growth rates are a linear function of the same autoregressive process of order one denoted x_t;

2. The conditional variances of the consumption and dividend growth rates are a linear function of the same autoregressive process of order one denoted h_t;

3. The variables x_{t+1} and h_{t+1} are independent conditionally to their past;

4. The innovations of the consumption and dividend growth rates are correlated given the state variables.

[17] Cecchetti, Lam, and Mark (1990) use a two-state homoscedastic specification for a univariate process of the endowment process, and Bonomo and Garcia (1994) a heteroscedastic specification in order to investigate if an equilibrium model could reproduce the mean reversion in asset prices. Cecchetti, Lam, and Mark (1993) use a homoscedastic consumption-dividend process in order to try to match the first and second moments of asset returns. The authors use two models, one with a leverage economy, another with a pure exchange economy without bonds. In both instances, they are unable to replicate the first and second moments taken together. All the articles above use expected utility function.

In the MS case, the first characteristic of the LRR model implies that one has to assume that the expected means of the consumption and dividend growth rates are a linear function of the same Markov chain with two states given that a two-state Markov chain is an AR(1) process. Likewise, the second feature implies that the conditional variances of the consumption and dividend growth rates are a linear function of the same two-state Markov chain. According to the third feature, the two Markov chains should be independent. Consequently, we shall assume that the Markov chain has 4 states, two states for the conditional mean and two states for the conditional variance and that the transition matrix P is restricted such as the conditional means and variances are independent. Finally, the last feature is captured by the correlation parameter ρ. By combining the two states - high and low - in mean and in volatility we obtain four states, $s_t \in \{\mu_L\sigma_L, \mu_L\sigma_H, \mu_H\sigma_L, \mu_H\sigma_H\}$. The states evolve according to a 4 by 4 transition probability matrix P.

The details of the matching procedure are given in a technical appendix to this paper available upon request from the authors. We apply this matching procedure first to the restricted random walk version of the general LRR model defined in (2.8). We also apply it to the general LRR model in (2.7) in section 6.

While the matching procedure concerns unconditional moments of the consumption and dividend processes, we verify that the fit of the Markov-switching model is also adequate in finite samples. To assess the fit, we simulate 10,000 samples of the size of the original data for both the original consumption and dividend processes and the matching MS process, and compute empirical quantiles of several moments of the consumption and dividend processes[18]. The percentile values are very close between the two processes except for the volatilities[19].

3.2 Solving for Asset-Valuation Ratios

Solving the model means finding explicit expressions for the price-consumption ratio $P_{c,t}/C_t$ (where $P_{c,t}$ is the price of the unobservable portfolio that pays off consumption), the price-dividend ratio $P_{d,t}/D_t$ (where $P_{d,t}$ is the price of an asset that pays off the aggregate dividend), and finally the price $P_{f,t}/1$ of a single-period risk-free bond that pays for sure one unit of consumption. To obtain these three valuation ratios, we need expressions for $\mathcal{R}_t\left(V_{t+1}\right)/C_t$, the ratio of the certainty equivalent of future lifetime utility to current consumption, and for V_t/C_t, the ratio of lifetime utility to current consumption.

The Markov property of the model is crucial for deriving analytical formulas for these

[18]For space consideration, the results are reported in a technical appendix.

[19]The mean and median volatilities for consumption and dividend growth produced by the MS model are closer to the mean and median volatility values computed with the data than the original autoregressive processes of consumption and dividend growth.

expressions. In general, the Markov state s_t in (3.9) and (**??**) will arbitrarily have N possible values, say $s_t \in \{1, 2, .., N\}$, although 4 values as described in the previous section are sufficient to provide a good approximation of the BY long run risk model. Let $\zeta_t \in \mathbb{R}^N$ be the vector Markov chain equivalent to s_t and such that:

$$\zeta_t = \begin{cases} e_1 = (1, 0, 0, .., 0)^\top & \text{if } s_t = 1 \\ e_2 = (0, 1, 0, .., 0)^\top & \text{if } s_t = 2 \\ \dots & \\ e_N = (0, 0, .., 0, 1)^\top & \text{if } s_t = N, \end{cases}$$

where e_i is the $N \times 1$ column vector with zeroes everywhere except in the i^{th} position which has the value one, and $\top$ denotes the transpose operator for vectors and matrices.

We show in the appendix that the variables $\mathcal{R}_t\left(V_{t+1}\right)/C_t$, V_t/C_t, $P_{d,t}/D_t$, $P_{c,t}/C_t$ and $P_{f,t}/1$ are (non-linear) functions of the state variable s_t. However, since the state variable s_t takes a finite number of values, any real non-linear function $g(\cdot)$ of s_t is a linear function of ζ_t, that is a vector in $\mathbb{R}^N$. This property will allow us to characterize analytically the price-payoff ratios while other data generating processes need either linear approximations or numerical methods to solve the model. The structure of the endowment process implies that there will be one such payoff-price ratio per regime and this will help in computing closed-form analytical formulas. For these valuation ratios, we adopt the following notations:

$$\frac{\mathcal{R}_t\left(V_{t+1}\right)}{C_t} = \lambda_{1z}^\top \zeta_t, \quad \frac{V_t}{C_t} = \lambda_{1v}^\top \zeta_t, \quad \frac{P_{d,t}}{D_t} = \lambda_{1d}^\top \zeta_t, \quad \frac{P_{c,t}}{C_t} = \lambda_{1c}^\top \zeta_t, \text{ and } \frac{P_{f,t}}{1} = \lambda_{1f}^\top \zeta_t. \qquad (3.10)$$

Solving the GDA model amounts to characterize the vectors λ_{1z}, λ_{1v}, λ_{1d}, λ_{1c} and λ_{1f} as functions of the parameters of the consumption and dividend growth dynamics and of the recursive utility function defined above. In the appendix, we provide expressions for these ratios.

3.3 Analytical Formulas for Expected Returns, Variance of Returns and Predictability Regressions

Since the seminal paper of Mehra and Prescott (1985), reproducing the equity premium and the risk-free rate has become an acid test for all consumption-based asset pricing models. Follow-up papers added the volatilities of both excess returns and the risk-free rate, as well as predictability regressions where the predictor is most often the price-dividend ratio and the predicted variables are equity returns or excess returns or consumption and dividend growth rates.

Bansal and Yaron (2004) use a number of these stylized facts to assess the adequacy of their LRR model and Beeler and Campbell (2009) provide a thorough critical analysis of the BY LRR model for a comprehensive set of stylized facts. The methodology used in Beeler and Campbell (2009) to produce population moments from the model rests on solving a loglinear approximate solution to the model and on a single simulation run over 1.2 million months (100,000 years). This simulation has to be run for each configuration of preference parameters considered. Typically, as in most empirical assessments of consumption-based asset pricing models, they consider a limited set of values for preference parameters and fix the parameters of the LRR model at the values chosen by Bansal and Yaron (2004) or Bansal, Kiku and Yaron (2007). Therefore, it appears very useful to provide analytical formulas for statistics used to characterize stylized facts in the literature.

Given expressions for the asset valuation ratios, it is easy to develop formulas for expected (excess) returns and unconditional moments of (excess) returns, formulas for predictability of (excess) returns and consumption and dividend growth rates by the dividend-price ratio, and formulas for variance ratios of (excess) returns. These analytical formulas, given in the appendix, will allow us to assess the sensitivity of the results to wide ranges of the parameters of the LRR model and to several sets of preference parameter values.

Therefore, we provide in the appendix the formulas for population coefficients of determination in regressions of returns aggregated over a number of periods on the current dividend-price ratio, as it is common in the asset pricing literature to run such predictive regressions. Similar regressions can be run with cumulative excess returns, consumption growth or dividend growth as the dependent variable[20].

We will compare these model-produced statistics to the corresponding empirical quantities computed with a data set of quarterly consumption, dividends and returns data for the US economy (1930:1 to 2007:4). The empirical first and second moments of asset prices and the empirical predictability results are reported first in the second column of Table 2 and then repeated for convenience of comparison in all relevant tables[21].

4 The Benchmark Model of Random Walk Consumption and Dividends and GDA Preferences: Calibration and Asset Pricing Implications

In this section we explain in detail how we chose the parameters for the fundamentals and for the preferences. Then, based on these calibrated values, we look at the asset pricing

[20] Papers in the literature on consumption-based asset pricing usually compute by simulation small sample and large sample statistics (see for example Cecchetti, Lam and Mark (1990) and Bonomo and Garcia (1994) in the older literature, Beeler and Campbell (2009) in the most recent one. The large sample statistics aim at approximating the population statistics, while the small sample statistics aim at reproducing stylized facts obtained with a finite sample of data.

[21] Stylized facts show a strong predictability of (excess) returns by the dividend-price ratio, which increases with the horizon. Although a vast literature discusses whether this predictability is actually present or not because of several statistical issues, we will sidestep the various corrections suggested since we are looking for a model that rationalizes the observed stylized facts.

implications in terms of matching moments and predictability. We conclude the section by interpreting the results through an SDF analysis.

4.1 Choosing Parameters for Consumption and Dividends Risks

To calibrate this process, we start with the parameters of the long-run risk model (2.7)) chosen by Bansal, Kiku and Yaron (2007), that is $\mu_x = 0.0015$, $\phi_d = 2.5$, $\nu_d = 6.5$, $\phi_x = 0.975$, $\nu_x = 0.038$, $\sqrt{\mu_\sigma} = 0.0072$, $\nu_\sigma = 0.28 \times 10^{-5}$ and $\rho = 0.39985$, except that we set phi_σ at a less persistent value of 0.995 instead of 0.999[22]. The later value implies a half-life of close to 58 years. The value 0.995 corresponds to the value estimated by Lettau, Ludvigson and Wachter (2008)[23]. It implies a half-life of 11.5 years which is still long but more reasonable than 58 years.

From this long-run risk model, we set $\phi_x = 0$ and $\nu_x = 0$ to obtain the random walk model and we adjust the other parameters when necessary such that consumption and dividend growth means, variances and covariance remain unchanged from the original model. The random walk model is then calibrated with $\mu_x = 0.0015$, $\nu_d = 6.42322$, $\sqrt{\mu_\sigma} = 0.0073$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.28 \times 10^{-5}$ and $\rho = 0.40434$.

We then apply the matching procedure described in section (2.2) to recover the parameters of the corresponding Markov switching process with two states in volatility. The calibrated MS random walk parameters are reported in Panel A of Table 1. The unconditional probability of being in the low volatility state is close to 80%. The volatilities of consumption and dividend are roughly multiplied by three in the high volatility state compared with the low volatility state.

For comparison purposes, we also matched the full LRR model calibrated in BKY (2007), except for the persistence of volatility. The calibrated MS LRR parameters are reported in Panel B of Table 1. We have now four states, two in the means and two in the volatilities, as explained in section 2.2. We verify that the volatilities and the corresponding probabilities are relatively robust with respect to the random walk model. Moreover, we see that introducing two mean states does not alter much the values of parameters associated with the volatility states. This LRR extended set of MS parameters will be used in Section **??** to compare the model of Bansal and Yaron (2004) with KP preferences to our model with GDA preferences.

[22]The calibration with $phi_\sigma = 0.999$ is currently the reference model in long-run risk; two recent papers by Beeler and Campbell (2009) and Bansal, Kiku and Yaron (2009) use it. We will look at its implications with GDA preferences in the robustness section.

[23]They estimate a two-state Markov switching process for quarterly consumption growth and found transition probabilities of 0.991 and 0.994 for the high and low states respectively. The equivalent persistence parameter is 0.991+0.994-1=0.9850 for quarterly frequency, or 0.995 for monthly frequency.

4.2 Choosing parameters for GDA Preferences

We need to choose values for the five preference parameters δ, ψ, γ, α and κ. For the time preference parameter δ we follow Bansal, Kiku and Yaron (2007) and use 0.9989 for a monthly frequency, which corresponds to 0.9869 at an annual frequency or a marginal rate of time preference of 1.32%[24].

The value of the elasticity of intertemporal substitution is a source of debate. In the literature on long-run risk, Bansal and Yaron (2004) and Lettau, Ludvigson and Wachter (2008) adopt a value of 1.5. In their models, ψ must be greater than 1 for a decline in volatility to raise asset prices. Empirically, some researchers have found that the IES is relatively small and often statistically not different from zero[25]. Others have found higher values of ψ using cohort or household level data[26] Bansal and Yaron (2004) also argue that in the presence of time-varying volatility, there is a severe downward bias in the point estimates of the IES. Beeler and Campbell (2009) simulate the BY model and report no bias if the riskfree interest rate is used as an instrument. In this benchmark model we follow the literature and keep a value of 1.5 for ψ. However, we will look at the sensitivity of results to values of ψ lower than one in section 5.

The remaining parameters all act on effective risk aversion. The parameter γ representing risk aversion in Kreps-Porteus EZ utility is set at 10 in Bansal and Yaron (2004) and at a very high value of 30 in Lettau, Ludvigson and Wachter (2008). Since the disappointment aversion parameters α and *kappa* interact with γ to determine the level of effective risk aversion of investors, we certainly need to lower γ. To guide our choice for γ and α together we rely on Epstein and Zin (1991). In this paper, they estimate a disappointment aversion model (κ=1) by GMM with two measures of consumption. The values estimated for γ and α are 1.98 and 0.38 for nondurables consumption, and 7.47 and 0.29 for nondurables and services. With these estimated parameters they cannot reject the disappointment aversion model according to the Hansen J-statistic of over-identifying restrictions at conventional levels of confidence. We choose an intermediate set of parameters, that is $\gamma = 2.5$ and $\alpha = 0.3$. Finally, we have to choose the parameter κ that sets the disappointment cut-off. Routledge and Zin (2009) discuss the value of this parameter in connection with the autocorrelation of consumption growth in a simple two-state Markov chain. In order to generate counter-cyclical risk aversion, they state that a value less than one for κ is needed when there is a negative autocorrelation of consumption growth and a value greater than

[24] Lettau, Ludvigson and Wachter (2008) and Routledge and Zin (2009) use a value of 0.970 or a marginal rate of 3%.

[25] See among others Campbell and Mankiw (1989) and Campbell (2003)

[26] See among others Attanasio and Weber (1993) and more recently Vissing-Jorgensen and Attanasio (2003).

one when the autocorrelation is positive[27]. The economic mechanism behind this this link is the substitution effect. In our random walk process, the autocorrelation of consumption growth is zero but we have a consumption volatility risk that triggers a precautionary savings motive. Moving κ below one makes investors less precautious, which drives the equilibrium interest rate upwards. We finally choose $\kappa = 0.989$ for matching our stylized facts.

Another way to compare the level of risk aversion is to draw indifference curves for the same gamble for an expected utility model and a disappointment aversion model. Figure 1 plots indifference curves for a hypothetical gamble with two outcomes with equal probability for GDA preferences calibrated as just described and for expected utility preferences with coefficient of relative risk aversion 5 and 10. While GDA preferences exhibit higher risk aversion than both expected utility preferences for small gambles, the same is not true for larger gambles. When the size of the gamble is about 20%, the GDA indifference curve crosses the expected utility indifference curve with risk aversion equal to 10, becoming less risk averse for larger gambles. For higher gamble sizes it approaches the expected utility with relative risk aversion equal to 5.

4.3 Asset Pricing Implications

In terms of asset pricing implications we will look at a set of moments for returns and price-dividend ratios, namely the expected value and the standard deviation of the equity premium, the risk-free rate and the price-dividend ratio. The moments are population moments and are computed with the analytical formulas discussed in section 3.3 and reported in the appendix. In the literature, these population moments are usually obtained by running a very long single simulation[28]. Having an analytical formula will make it a lot easier to study the sensitivity of results to changes in the parameter values.

We also report the median of the finite-sample distribution and the p-value of the statistic computed with the data with respect to the finite-sample distribution. To generate the distribution in finite sample, we choose a sample size of 938 months as in the data sample we used to estimate the stylized facts. We then simulate the random walk model 10,000 times and report the percentile of the cross-sectional distribution of the model's finite-sample statistics that corresponds to the value of this statistic in the data. This percentile can be interpreted as a p-value for a one-sided test of the model based on the data statistic.

[27] In their Table III, they set κ to 0.9692 for an autocorrelation of -0.14 and to 1.0431 for an autocorrelation of 0.20.

[28] For example in Beeler and Campbell (2009) population moments are calculated from a single simulation run of 1.2 million months.

We also consider several predictability regressions by the price-dividend ratio, for excess returns, consumption growth and dividend growth. Again the R^2 and the regression coefficients are computed analytically with the formulas reported in the previous section. There are, therefore, population statistics. These will also be compared with finite sample results with reported medians and p-values.

4.3.1 Matching the Moments

The asset pricing results for the benchmark RW process are reported in Panel A of Table 2. We consider a set of moments for returns and price-dividend ratios, namely the expected value and the standard deviation of the equity premium, the real risk-free rate and the price-dividend ratio. The first column corresponds to the statistics computed in the data. We use a sample starting in 1930 as in Bansal, Kiku and Yaron (2007) and Beeler and Campbell (2009), and extend it till 2007. The reported statistics are annualized moments based on quarterly data estimation. The computed values are close to the usual values found for these statistics with an equity premium mean close to 7% and a volatility of roughly 20%. The real interest rate is close to 1% and its volatility is in the order of 4%. Finally, the mean of the price-dividend ratio is close to 30 and the volatility of the dividend yield is about 1.5%.

The population values produced by the benchmark model with the random walk model described in section 4.1 and the preference parameters set in section 4.2 are reported in the second column of Table 2. Except for the volatility of the real interest rate, which is about half the value computed in the data, and the somewhat low level of the expected price-dividend ratio relative to the data[29], all other population moments are very much in line with the data statistics. Given the random walk process for consumption in the benchmark model, it means that for an investor with GDA preferences, it is the macroeconomic uncertainty that solely explains the high equity premium and a low risk-free rate. In the high volatility state, which happens about 20% of the time in the benchmark case, the required premium is much higher than in the low volatility state. It is also the variation of the price-dividend ratio over the two states of volatility that gives enough variability to the dividend yield to match what is observed in the data.

In finite samples, the model is rejected for the standard deviation of the risk-free rate. The model produces too low standard errors compared to the data. As we will see in the robustness section, it is due in part the higher-than-one value of the elasticity of intertemporal

[29]However, the level we obtain with our calibration is not out of line with values found in the sample until the year 2000 where it reached a peak of close to 90 and stayed relatively high afterwards. The median of 24.95 is closer to our population mean of 23.30.

substitution. A value greater than one implies that the investor perceives consumption at two different times as substitutes and does not want to borrow from the future to smooth out volatile consumption. This results in a low and less volatile riskless interest rate. The p-value for the expected price-dividend ratio confirms that the value of 30.57 is out of the realm of the finite-sample distribution produced by the model. The median of the finite-sample distribution is indeed close to the population moment and close to the median of the price-dividend ratio in the data. For the four other moments, the p-values indicate that the empirical values are quite close to the center of the finite-sample distribution.

4.3.2 Predictability

The predictability results for the benchmark RW process are reported in Panels B, C and D of Table 2. In panel B, we reproduce the predictability of excess returns. For both the R^2 statistics and the slopes of the regression of excess returns on the dividend-price ratio, we reproduce the increasing pattern over the horizons from 1 year to five years. In terms of magnitude we are a bit over the data values. However, our small-sample R^2 medians are very close to the data with p-values of about 50%.

We included the predictability of consumption growth and dividend growth even though in the random walk model there is no predictability in population. We wanted to illustrate that in finite samples we may reject wrongly the model. For the R^2 statistic of consumption growth and dividend growth predictability, the p-values are all below the 20% level. This shows the value of computing population values implied by a model.

4.4 Understanding the results through the SDF

To better understand why the generalized disappointment benchmark model explains well the stylized facts, we have a closer look at the underlying stochastic discount factor. As we showed before in the description of the endowment matching, the MS endowment process we are using has two states in volatility σ_L and σ_H. Table 1 reports the transition probability matrix between the states. Both variance states are very persistent, with the transitions from the high state to the low state occurring more frequently than the reverse.

Table 5 reports the moments of the SDF of the GDA benchmark specification in the two states. These are the mean and the variance of the state-conditional distributions of the SDF. The state with low variance has a higher probability mass associated with a non-disappointing outcome. Therefore, the mean SDF in this state is low (0.99820) and not very variable (0.11481), resulting in a low risk premium. The state with high variance is the one with a more variable SDF (0.66929), a higher SDF mean (1.00304) and a corresponding higher risk premium. The switching between the low and high variance states produce state-dependent risk aversion, which is essential for predictability.

Another way to understand our results is to see how they change when we vary either the preferences or the stochastic processes of the fundamentals. This is the object of the next section.

5 Sensitivity Analysis

We start by looking at a set of specific preferences in the family of disappointment aversion preferences and at the Kreps-Porteus preferences used in Bansal and Yaron (2004) and other ensuing papers. We then replace the random walk fundamentals by the long-run risks dynamics given by 2.7. In this sensitivity analysis, we report results obtained with specific values of the parameters but we also illustrate with graphs the sensitivity of results to variations in the parameters over large intervals.

5.1 Sensitivity to Changes in Preferences

In this section we want to show what will be the implications of different calibrations for the preference parameters. First, we reproduce tables similar to Table 2 for three specific configurations of interest, namely a similar GDA than the benchmark case but with an EIS lower than 1 (that we will call GDA1), another with $\kappa = 1$, a pure disappointment aversion model, with linear preferences ($\gamma = 1$) and infinite EIS ($\psi = \infty$) (called DA0), which will isolate the role of disappointment aversion alone, and finally the Kreps-Porteus preferences ($\alpha = 1$), which are until now the preferences that have been used exclusively in the long-run risk model, with the acronym KP. Second, we will produce graphs showing the sensitivity of asset pricing and predictability implications to intervals of values for preference parameters.

5.1.1 Specific Configurations of Preferences

Table 3 reports the population and finite sample p-values for moments and predictability associated with the three specifications GDA1, DA0, and KP.

(*a*) EIS lower than one - GDA1 The value of the elasticity of intertemporal substitution ψ is a matter of debate. Bansal and Yaron (2004) argue for a value larger than 1 for this parameter since it is critical for reproducing the asset pricing stylized facts. Their main argument is that the presence of fluctuating consumption volatility leads to a serious downward bias in the estimates of the IES using the instrumental variable (IV) regression approach pursued in Hall (1988). Beeler and Campbell (2009) simulate the long-run risks model to see whether the downward bias is important in IV estimates of ψ and conclude that there is no downward bias when the riskless interest rate is used as instrument, but that there is a poor finite-sample performance of IV regressions with stock returns as instrument, reflecting a weak instrument problem. They add that the high volatility of the real interest rate is hard to reconcile with an IES greater than 1. We have seen in Table 2 that it is indeed one dimension over which the GDA model was not performing well.

Given this debate over the value of ψ, we set the elasticity of substitution at ($\psi = 0.75$). We maintain for the other parameters the same values as in the benchmark model. It can be seen in the second column of Table 3 that the random walk model with this GDA1 configuration of preferences can reproduce almost as well the asset pricing stylized facts. Therefore, we can see that the EIS is not pivotal for the results. It does affect however the level and the volatility of the riskless interest rate. Since the investor perceives consumption at two different dates as complementary, he wants to borrow from the future to smooth out volatile consumption. This implies a higher (1.97% instead of 0.93% with GDA) and a more volatile (3.25% instead of 2.34%) interest rate. The higher interest-rate mean is reflected in Table 5 by the fact that the mean of the SDF spread in the most frequent low-volatility state is smaller for GDA1 than for GDA. A wider spread between the conditional means of the SDFs for GDA1 than for GDA explains the higher volatility of the interest rate.

One dimension over which GDA1 performs less well than GDA is the volatility of the dividend-price ratio, which falls to 1.04 instead of 1.38. This translates into higher coefficients in the return predictability regressions but the patterns and the finite sample values are very similar to the ones obtained with GDA. The finite sample results for consumption and dividend growth predictability are the same as with GDA.

Generalized disappointment aversion preferences shed a new light on the debate about the value of the intertemporal elasticity of substitution in long-run risks models. The need for an elasticity higher than one to match asset pricing moments is a feature of the Kreps-Porteus preferences that have been used until now.

***(b)* Pure Disappointment Aversion - DA0** The specification denoted DA0 is the simplest one among disappointment averse preferences. First, as $\kappa = 1$, the threshold is the certainty equivalent. Furthermore, other than the kink, the stochastic discount factor has no curvature, as $\gamma = 0$ and $\psi = \infty$. In other words, if disappointment aversion were not present ($\alpha = 1$) the stochastic discount factor would be equal to the constant time discount factor δ. This simplistic specification of the GDA preferences will allow us to gain intuition about the potential for such a pure disappointment aversion model, that does not use the curvature engendered by the other preference parameters, to replicate the asset pricing and predictability stylized facts we analyzed with GDA.

The results reported in Table3 show that DA0 does in fact reproduce rather well the stylized facts in population, especially predictability of returns. When we compare the moments obtained with DA0 to the ones generated by GDA1, two weak points emerge. The average price-dividend ratio is too low and the volatility of the dividend-price ratio is too high. The equity premium is also higher than in the data. These deteriorating statistics are brought about by an enlarged set of disappointing outcomes when κ is increased from 0.989 to one. In fact, we could bring some improvement by reducing the intensity of disappointment aversion α. This would lower the equity premium, increase the price-dividend ratio and decrease the volatility of the dividend-price ratio, bringing them more in line with the data. We chose to keep it at the same value of 0.5 for comparison purposes. The other drawback of such simplistic preferences is that the risk-free rate is constant. Indeed, with $\kappa = 1$ the conditional expectation of the SDF in **??** is equal to *delta*, the time-discount parameter.

The results for predictability of excess returns are even better than with GDA1. The patterns obtained for population statistics are maintained in finite sample and the p-values associated with the R^2 and the coefficients of the return predictability regressions are close to the median.

Routledge and Zin (2009) stress the importance of generalized disappointment aversion for obtaining counter-cyclical price of risk in their Mehra-Prescott economy. In their setting, disappointment aversion alone cannot generate enough variation in the distribution of the stochastic discount factor, leading to a similar conditional equity premium in both states. Since they have two possible outcomes, one is necessarily above the certainty equivalent and the other is below. Then, for each state there is always one disappointing outcome. With generalized disappointment aversion it is possible to place the kink at a certain fraction of the certainty equivalent such that for one of the states both results are non-disappointing. Then, there is disappointment only in the bad state, engendering a counter-cyclical equity premium.

Since we have a richer endowment process, with an infinite number of possible outcomes, there is not such a stark contrast between DA and GDA preferences in our model. For each state there will always be a very large number of disappointing outcomes for both types of preferences. The probability of disappointment may change with the state even with DA preferences, generating predictable time-variation in returns. When DA is combined with $\gamma = 0$ and $\psi = \infty$ the risk free rate becomes constant and equal to δ as mentioned above. This does not imply a constant risk premium, since the conditional covariance between the SDF and the equity return is state-dependent.

(*c*) Kreps-Porteus Preferences The Kreps-Porteus preferences are a key ingredient in the long-run risks model of Bansal and Yaron (2004). Recall that in the latter a small persistent component adds risk in expected consumption growth. Here we evaluate whether volatility risk alone is enough to replicate the stylized facts. We use the preference parameter values used in BY. It is clear that volatility risk alone is not sufficient to generate statistics in line with the data. The equity premium is very small, 1.42% compared to 7.25% in the data, the expected price-dividend ratio is much too high and the volatility of the dividend price ratio is practically zero. The last two facts translate into very high and negative slope coefficients and low R^2 in the predictability regressions of excess returns. Beeler and Campbell (2009) argue that high persistence in volatility is essential to reproduce the results. We see clearly here that KP preferences with a heteroskedastic random walk consumption is not enough to reproduce the moments and explain predictability.

5.1.2 Sensitivity to Preference Parameters

It is customary for studies on consumption-based asset pricing models to stop here since we have found configurations of parameters that reproduce the stylized facts we selected to evaluate the model. The main reason is that often the model is hard to solve and that very long simulations are necessary to compute population statistics. The recent study of Beeler and Campbell (2009), which looks comprehensively at the LRR model with KP preferences, uses simulations of length of 1.2 million months to compute population return moments and predictability statistics.

We gauge the sensitivity of the statistics to changes in preference parameters through graphs. We keep the value of the risk aversion parameter γ to 2.5 and vary the disappointment aversion parameter α, the elasticity of intertemporal substitution ψ and the kink parameter κ. In Figure 2, we study the implications of the changes in α in three horizontal panels for expected excess returns, the risk-free rate and the price-dividend ratio respectively, where κ varies between 0.980 and 0.990. We look at three values for ψ, 0.75, 1 and 1.5, which results in three sets of three panels.

The equity premium increases with κ and decreases with α. Increasing α makes the agent less averse to disappointment and therefore prices will be higher and risky returns lower. The parameter κ acts in the opposite direction. When it gets closer to 1, there are more outcomes that makes the investor disappointed. As the elasticity of intertemporal substitution increases, it produces a rather small increase in the level of the equity premium.

The risk-free rate goes down as aversion to disappointment and the set of disappointing outcomes increase, that is when α is decreasing and κ is increasing. The effect of $\kappa$is much more pronounced since the curves fan out as we lower κ, especially for $\psi = 1.5$. The effect of ψ on the risk-free rate is important since it affects directly intertemporal trade-offs in terms of consumption. Below the value of 1 the investor sees consumption at two different times as complementary and wants to borrow from the future, resulting in a higher level of the risk-free rate, while with a value above 1 consumption today and tomorrow are perceived as substitutes and the equilibrium risk-free rate is lower.

Finally, the expected price-dividend ratio decreases with disappointment aversion, with the main factor being κ, since the curves bunch up as κ gets closer to 1. Decreasing ψ lowers the level of the expected price-dividend ratio and makes it less sensitive to changes in α.

In Figure 3, we apply a similar sensitivity analysis, with identical changes in the parameters, to the predictability of excess returns at one, three and five-year horizons. The main conclusion is that predictability necessitates a large amount of disappointment aversion. It appears to be consistent with the data for lower values of α and higher values of κ. Changing ψ does not affect much predictability since both the levels and the slopes are identical across graphs. These features apply to all horizons, with the additional remark that the differences in effects are amplified as the horizon lengthens.

5.2 Sensitivity to Persistence in Consumption Volatility

A key parameter in our benchmark model is the persistence of consumption volatility. How sensitive are our results to this parameter? Bansal, Kiku and Yaron (2007) chose an extreme value of 0.999 while we reduced it to 0.995 based on a more reasonable value for the half-life of a shock to volatility. In Figures 4 and 5, we plot the sensitivity of the asset pricing statistics and predictability statistics, respectively, to variations in the persistence parameter of consumption volatility ϕ_σ. In Figure 4 we observe that all asset pricing statistics for KP preferences, while out of line with the data, remain roughly insensitive to variations of ϕ_σ from 0.9 to 1. For GDA, the patterns are similar across the three specifications. The biggest changes occur in the volatility of the dividend yield that goes towards zero as we approach 0.9. Otherwise, the other statistics remain pretty much the same as we vary ϕ_σ from 0.9 to 1. In Figure 5, the patterns in R^2 for all preference specifications are similar. Their values decrease steeply as ϕ_σ approaches 0.9. As we mentioned before, KP preferences show some predictability but the values of the slopes become unrealistic (they do not show in the graphs for 3 and 5 years). One can see that the magnitude of predictability for the GDA specifications depends very much on the value of ϕ_σ, but that some predictability remains for a sizable range of values. It should be stressed that the curves for GDA and GDA1 are very similar both in terms of asset pricing moments and predictability statistics, except for the volatility of the risk-free rate, which is higher for GDA1 as explained before. What the graph tells us in this case is that the difference remains uniform across the values of ϕ_σ between 0.9 and 1.

6 Comparison with the Long-Run Risks Model of Bansal-Yaron (2004): Risks in Both Expected Consumption Growth and Consumption Volatility

The long-run risks model introduced by Bansal and Yaron (2004) features two main sources of risk, a risk in expected consumption growth and a risk in volatility of consumption. We saw that our benchmark model, featuring only the second risk, could explain the stylized facts when combined with GDA preferences but not with the Kreps-Porteus preferences chosen by BY. An important question is to establish whether the results obtained with the random walk consumption and GDA preferences are robust to introducing a long-run risk in expected consumption growth. We will also derive the asset pricing implications in population of the BY model with KP preferences. In the LRR model, the persistence of expected consumption growth is the key parameter. Therefore, we will assess the sensitivity of results with respect to this parameter.

6.1 Asset Pricing Implications

In Table 4, we report results for the long-run risk model of Bansal and Yaron (2004) defined in (2.7), with the calibration chosen in Bansal, Kiku and Yaron (2007) and used also by Beeler and Campbell (2009). The calibrated values of the parameters are: $\mu_x = 0.0015$, $\phi_d = 2.5$, $\nu_d = 6.5$, $\phi_x = 0.975$, $\nu_x = 0.038$, $\phi_\sigma = 0.995$, $\sqrt{\mu_\sigma} = 0.0072$, $\nu_\sigma = 0.28 \times 10^{-5}$ and $\rho = 0.39985$. The main difference with our benchmark random walk process is the presence of a persistent component in the mean of the consumption and dividend growth processes. Note also that in this calibration the volatility persistence parameter is lower (0.995) than in the LRR calibration of Bansal, Kiku and Yaron (2007)(0.999). We apply to this calibrated set of parameters the matching procedure described in section 2.2 to obtain the equivalent set of parameters for the MS model in (3.9). The MS matching parameters are reported in Panel B of Table 1. We have two states for the means (mu_L and mu_H) and two states for the volatility (σ_L and σ_H), that combine into four states: $\{\mu_L\sigma_L, \mu_L\sigma_H, \mu_H\sigma_L, \mu_H\sigma_H\}$. In the low state both consumption and dividend growth means are negative, while they are positive and between 2.5 % and 3 % annually in the high state. The estimated volatilities are close to what we obtained in the random walk model. Overall, we are in the high mean-low variance 70% of the time and 18% of the time in the high mean-high variance state. The low mean state occurs about 10% of the time, mostly with the low volatility state.

We report moments and predictability statistics for the benchmark GDA model and the four specifications GDA, GDA1, DA0 and KP analyzed with the benchmark random walk model. Two main conclusions can be drawn. First, all the statistics reproduced for the GDA or DA preferences are very close to what we obtained with the random walk model. This confirms that volatility risk is the main economic mechanism behind the asset pricing results. Adding a risk in the expected consumption growth does not affect much the GDA investor. Second, the results are changing for KP in several dimensions. The moments are now closer to the data, except still for the volatility of the riskless interest rate and of the dividend-price ratio. This confirms the essential role played by the small long-run predictable component in expected consumption growth in the BY and BKY models. For excess return predictability we arrive at a surprising result. While the random walk model generated some predictability in population, the full LRR model does not produce any predictability at all in population. In finite sample, we can reject the model in this dimension at a 10% level of confidence.

For consumption growth, the LRR model with KP preferences overpredicts strongly in population, with R^2 in the order of 20%, but the distribution in finite sample is such that we cannot reject the model in this dimension at the 5% level. The p-values for the R^2 are 0.07, 0.08 and 0.14 respectively at the one, three and five-year horizon. It should be stressed that the GDA and DA models give statistics and p-values that do not differ too much from the KP model. It is therefore hard to differentiate between the models in finite sample. In population, the difference is clear and the KP model produces too much predictability in consumption growth.

For dividend growth, the LRR model with KP preferences overpredicts a bit compared to the three disappointment specifications but again it is hard to distinguish between the models based on finite-sample p-values.

6.2 Sensitivity to Persistence of Expected Consumption Growth

We illustrate through graphs the sensitivity of the asset pricing and predictability statistics to large variations in the persistence of expected consumption growth (ϕ_x) in Figures 6 and 7 respectively. We start with the robustness of asset pricing moments in Figure 6. We exhibit 6 graphs, one for each moment. All the curves associated with GDA are almost parallel straight lines to the horizontal axis showing that the computed moments are insensitive to the expected growth persistence parameter. For DA0, the patterns are a bit different for values of ϕ_x close to 1 but settle to straight lines as we reduce ϕ_x. For KP, as already mentioned, the parameter ϕ_x is key. All results obtain for values close to 1, emphasizing the essential role of a very persistent component in expected consumption growth. The pattern of the expected price dividend ratio for KP is particularly striking, increasing steeply from a low value of 20 for the benchmark BY value of 0.975 to values greater than 100 as we just move away from it.

In Figure 7 we explore the implications for predictability of variations in ϕ_x. We show two sets of six graphs, that is three horizons and two statistics (R^2 and slope) for the prediction of excess returns and consumption growth. In each graph, we plot the three specifications of disappointment averse preferences and KP. All three disappointment averse specifications exhibit predictability patterns of excess returns consistent with what is observed in the data, while it is not the case for KP. Predictability stays close to zero over the whole set of values of ϕ_x for KP, increasing a bit when the value of the persistence parameter decreases but we know that the moments are no longer matched for these values. For consumption and dividend growth, the benchmark ϕ_x produces too much predictability when it gets close to 1. Otherwise it is flat at zero. Here again, we cannot reproduce the low predictability of the consumption growth and the dividend growth and the moments at the same time.

We can conclude from this sensitivity analysis that the source of long-run risk, whether in the mean or the volatility of consumption growth, needs to be persistent for the agent's preferences to operate in a way consistent with the observed data. For KP preferences in the Bansal and Yaron (2004) model, we see a strong tension as ϕ_x, the persistence of expected consumption growth, moves away from 1. The ability to reproduce asset pricing moments deteriorates quickly while the predictability statistics improve. For the GDA preferences that we advocate in this paper, the persistence in the volatility of consumption growth ϕ_σ is key for reproducing the predictability stylized facts but results are not as sensitive to this persistence than they were with KP preferences for the persistence of expected consumption growth. The means of the equity and risk-free returns are pretty insensitive to ϕ_σ, while their volatilities decrease but not drastically as ϕ_σ moves away from one. It is really for the volatility of the dividend-price ratio that the persistence of volatility is very important, since it is decreasing fast as the value of ϕ_σ is approaching 0.9.

7 Volatility and Stock Prices

Volatility risk is the key source of risk in our long-run risk model with GDA preferences. Both Beeler and Campbell (2009) and Bansal, Kiku and Yaron (2009) report evidence on the relation between asset prices and volatility of returns, consumption and dividends in the data and in the LRR model with KP preferences. We produce similar regressions of these volatility measures on the dividend price ratio.

To assess the predictive power of equity return volatility by the dividend-price ratio, we first compute the AR(1) regression:

$$R_{t+1} = a_R + b_R R_t + U_{R,t+1} \tag{7.11}$$

where R_{t+1} denotes the gross return on equity. We measure volatility as a moving sum of the squared residuals and consider the predictive regression[30]:

$$\sum_{j=1}^{h} U_{R,t+j}^2 = a\left(h\right) + b\left(h\right)\frac{D_t}{P_t} + \eta_{t+h}\left(h\right). \tag{7.12}$$

We explain in Appendix D how we compute population coefficients and R^2 for the latter regression. We run similar regressions and derive equivalent formulas for the volatility of consumption growth and dividend growth.

Regression results are reported in Table 7. In the data, we observe high predictability of consumption volatility at horizons of 3 to 5 years, with R^2 close to 50%. The sign of the relation is positive, that is high dividend-price ratio predicts a high volatility of consumption. This predictability is also captured in finite samples by all models, although the sign of the relation is negative for KP. Looking at p-values, no model can be rejected at conventional levels for most horizons. This reiterates the difficulty to discriminate between models in finite samples. When we look at the population values of the R^2 we find of course that this predictability disappears as it should be given the assumed stochastic process for the volatility of consumption. The predictability in finite samples is due to the persistence of both variables.

Predictability patterns are similar for dividend growth volatility but in the data the level of the R^2 at an horizon of 3 to 5 years is about half the size (about 20%) of the R^2 for consumption growth volatility.

Finally, predictability of return volatility by the dividend-price ratio is much weaker in the data, with a maximum of 18% at a five-year horizon. Surprisingly, the sign of the relation comes out negative. In population the GDA models produce R^2 that match quite closely the data, but the relation is positive at it should be. In finite samples all models produce high R^2 compared to the data due to the high persistence of the source of risk. The KP model produces little predictability in population, but with the wrong sign and unreasonable magnitudes for the regression coefficients. This last fact is present in finite samples as well. Again, based on p-values, we cannot discriminate between models.

[30]We consider this measure instead of the absolute value chosen in Beeler and Campbell (2009) and Bansal, Kiku and Yaron (2009) since we can compute analytical formulas for the R^2 and the regression coefficients

8 Conclusion

We have examined an asset pricing model with long-run risk where preferences display generalized disappointment aversion (Routledge and Zin, 2009). Our benchmark endowment process had only one of the two sources of long-run risks proposed by Bansal and Yaron (2004) (BY): the volatility risk. The model produces asset returns moments and predictability patterns in line with the data. Differently from the BY model, our results do not depend on a value of the elasticity of intertemporal substitution greater than one: similar results may be obtained with values lower than one. Contrary to Routledge and Zin (2009), a a simple model where risk aversion comes only from pure disappointment aversion generates similar implications. Our results are also robust to a more elaborate endowment process where the expected consumption growth risk is also present. Notwithstanding the presence of this main source of risk in Bansal and Yaron's (2004), the persistent volatility of consumption growth strongly interacts with disappointment aversion to generate realistic moments and predictability patterns,

Disappointment aversion preferences introduce a kink in the utility function, raising a challenge to solve the asset pricing model. We propose a matching procedure that allows to solve analytically the model and to obtain closed-form formulas for asset-valuation ratios. We also compute population statistics for asset-return moments and predictability regression coefficients and R^2. These population values are usually produced with very long simulations, limiting the sensitivity analysis of the results to variations in the parameters of the model, both in terms of preferences and endowment.

While we have focused in this paper on the time-series implications of our generalized disappointment aversion model with long-run volatility risk, it will be fruitful to investigate whether this model can rationalize the evidence put forward by Tédongap (2008) about consumption volatility and the cross-section of stock returns. He shows that growth stocks have a lower volatility risk than value stocks and that, for most investment horizons, consumption volatility risk is more correlated with multiperiod returns on the Fama and French size and book-to-market sorted portfolios than consumption level risk.

Another natural extension will be to incorporate the recent LRR models with jumps in the expected growth and the volatility dynamics[31] since our framework is amenable to match processes with jumps. This extension will be particulary useful to explore several option pricing puzzles with closed-form formulas.[32]

[31] See in particular Drechsler and Yaron (2007) and Eraker and Shaliastovich (2008).

[32] Garcia, Luger and Renault (2003) and Chabi-Yo, Garcia and Renault (2008) have derived generalized Black-Scholes formulas in Kreps-Porteus recursive utility models with Markov-switching processes for consumption and dividend growth.

Appendix

Appendix A. In what follows, we will use the following notation. The transition probability matrix P of the Markov chain is given by

$$P^\top = [p_{ij}]_{1\leq i,j\leq N}, \quad p_{ij} = P(\zeta_{t+1} = e_j \mid \zeta_t = e_i). \tag{A.1}$$

We assume that the Markov chain is stationary with ergodic distribution and second moments given by:

$$E[\zeta_t] = \Pi \in \mathbb{R}_+^N, \quad E[\zeta_t \zeta_t^\top] = Diag(\Pi_1, .., \Pi_N) \text{ and } Var[\zeta_t] = Diag(\Pi_1, .., \Pi_N) - \Pi\Pi^\top,$$

where $Diag\,(u_1, .., u_N)$ is the $N \times N$ diagonal matrix whose diagonal elements are u_1,..,u_N. The time-varying variables $\mu_c(s_t)$, $\mu_d(s_t)$, $\omega_c(s_t)$, $\omega_d(s_t)$, and $\rho(s_t)$ defined in (3.9) are given by

$$\mu_c(s_t) = \mu_c^\top \zeta_t,\ \mu_d(s_t) = \mu_d^\top \zeta_t,\ \omega_c(s_t) = \omega_c^\top \zeta_t,\ \omega_d(s_t) = \omega_c^\top \zeta_t,\ \rho(s_t) = \rho^\top \zeta_t.$$

We define the vectors μ_{cd}, ω_{cd}, μ_{cc}, and ω_{cc} by

$$\mu_{cd} = -\gamma\mu_c + \mu_d,\ \omega_{cd} = \omega_c + \omega_d - 2\gamma\rho \odot \omega_c^{1/2} \odot \omega_d^{1/2},\ \mu_{cc} = (1-\gamma)\,\mu_c, \omega_{cc} = (1-\gamma)^2\,\omega_c, \tag{A.2}$$

where the vector operator $\odot$ denotes the element-by-element multiplication. The vector ι denotes the $N \times 1$ vector with all components equal to one. Likewise, Id is the $N \times N$ identity matrix.

Appendix B. This Appendix provides the formulas of the vectors λ that appear in (3.10). These vectors are computed in two steps. In the first one, we characterize the ratio of the certainty equivalent of future lifetime utility to current consumption and the ratio of lifetime utility to consumption. In the second step, we characterize the price-consumption ratio, the equity price-dividend ratio and the single-period risk-free rate. These characterizations are done by solving the Euler equation for different assets. One has

$$\frac{\mathcal{R}_t\,(V_{t+1})}{C_t} = \lambda_{1z}^\top \zeta_t \text{ and } \frac{V_t}{C_t} = \lambda_{1v}^\top \zeta_t,$$

where the components of the vectors λ_{1z} and λ_{1v} are given by:

$$\lambda_{1z,i} = \exp\left(\mu_{c,i} + \frac{1-\gamma}{2}\omega_{c,i}\right)\left(\sum_{j=1}^N p_{ij}^* \lambda_{1v,j}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{B.1}$$

$$\lambda_{1v,i} = \left\{(1-\delta) + \delta\lambda_{1z,i}^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \text{ if } \psi \neq 1 \text{ and } \lambda_{1v,i} = \lambda_{1z,i}^\delta \text{ if } \psi = 1, \tag{B.2}$$

while the matrix $P^{*\top} = \left[p^*_{ij}\right]_{1\le i,j\le N}$ is defined by

$$p^*_{ij} = p_{ij}\frac{1+\left(\alpha^{-1}-1\right)\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}-\left(1-\gamma\right)\omega_{c,i}^{1/2}\right)}{1+\left(\alpha^{-1}-1\right)\kappa^{1-\gamma}\sum_{j=1}^{N}p_{ij}\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}\right)}. \tag{B.3}$$

The second step leads to

$$\frac{P_{d,t}}{D_t} = \lambda_{1d}^\top\zeta_t, \quad \frac{P_{c,t}}{C_t} = \lambda_{1c}^\top\zeta_t \quad \text{and} \quad \frac{1}{R_{f,t+1}} = \lambda_{1f}^\top\zeta_t,$$

where the components of the vectors λ_{1d}, λ_{1c}, and λ_{1f} are given by

$$\lambda_{1d,i} = \delta\left(\frac{1}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}\exp\left(\mu_{cd,i}+\frac{\omega_{cd,i}}{2}\right)\left(\lambda_{1v}^{\frac{1}{\psi}-\gamma}\right)^\top P^{**}\left(Id-\delta A^{**}\left(\mu_{cd}+\frac{\omega_{cd}}{2}\right)\right)^{-1}e_i, \tag{B.4}$$

$$\lambda_{1c,i} = \delta\left(\frac{1}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}\exp\left(\mu_{cc,i}+\frac{\omega_{cc,i}}{2}\right)\left(\lambda_{1v}^{\frac{1}{\psi}-\gamma}\right)^\top P^{*}\left(Id-\delta A^{*}\left(\mu_{cc}+\frac{\omega_{cc}}{2}\right)\right)^{-1}e_i, \tag{B.5}$$

$$\lambda_{1f,i} = \delta\exp\left(-\gamma\mu_{c,i}+\frac{\gamma^2}{2}\omega_{c,i}\right)\sum_{j=1}^{N}\tilde{p}^*_{ij}\left(\frac{\lambda_{1v,j}}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}, \tag{B.6}$$

where the matrices $P^{**\top} = \left[p^{**}_{ij}\right]_{1\le i,j\le N}$, $\tilde{P}^{*\top} = \left[\tilde{p}^*_{ij}\right]_{1\le i,j\le N}$ are given by

$$p^{**}_{ij} = p_{ij}\frac{1+\left(\alpha^{-1}-1\right)\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}-\left(\rho_i\omega_{d,i}^{1/2}-\gamma\omega_{c,i}^{1/2}\right)\right)}{1+\left(\alpha^{-1}-1\right)\kappa^{1-\gamma}\sum_{j=1}^{N}p_{ij}\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}\right)}, \tag{B.7}$$

$$\tilde{p}^*_{ij} = p_{ij}\frac{1+\left(\alpha^{-1}-1\right)\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}+\gamma\omega_{c,i}^{1/2}\right)}{1+\left(\alpha^{-1}-1\right)\kappa^{1-\gamma}\sum_{j=1}^{N}p_{ij}\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right)-\mu_{c,i}}{\omega_{c,i}^{1/2}}\right)}, \tag{B.8}$$

while, for $u \in \mathbb{R}^N$, the matrix functions $A^*(u)$ and $A^{**}(u)$ are given by

$$A^*(u) = Diag\left(\left(\frac{\lambda_{1v,1}}{\lambda_{1z,1}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_1), ..., \left(\frac{\lambda_{1v,N}}{\lambda_{1z,N}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_N)\right) P^*, \tag{B.9}$$

$$\tag{B.10}$$

$$A^{**}(u) = Diag\left(\left(\frac{\lambda_{1v,1}}{\lambda_{1z,1}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_1), ..., \left(\frac{\lambda_{1v,N}}{\lambda_{1z,N}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_N)\right) P^{**}. \tag{B.11}$$

Appendix C. This Appendix provides the formulas of the expected returns and some of their properties. We define the return process, R_{t+1}, and aggregate returns over h periods, $R_{t+1:t+h}$, by

$$R_{t+1} = \frac{P_{d,t+1} + D_{t+1}}{P_{d,t}} = \left(\lambda_{2d}^\top \zeta_t\right)\left(\lambda_{3d}^\top \zeta_{t+1}\right)\exp(\Delta d_{t+1}) \quad \text{and} \quad R_{t+1:t+h} = \sum_{j=1}^{h} R_{t+j}, \tag{C.1}$$

with $\lambda_{2d} = 1/\lambda_{1d}$ and $\lambda_{3d} = \lambda_{1d} + \iota$. We also define the excess returns R^e_{t+1} and aggregate excess returns $R^e_{t+1:t+h}$, i.e., $R^e_{t+1} = R_{t+1} - R_{f,t+1}$ and $R^e_{t+1:t+h} = R_{t+1:t+h} - R_{f,t+1:t+h}$. One has

$$E[R_{t+j} \mid J_t] = \psi_d^\top P^{j-1}\zeta_t \text{ and } E\left[R^e_{t+j} \mid J_t\right] = (\psi_d - \lambda_{2f})^\top P^{j-1}\zeta_t, \quad \forall j \geq 2, \tag{C.2}$$

$$E[R_{t+1:t+h} \mid J_t] = \psi_{h,d}^\top \zeta_t \quad \text{and} \quad E\left[R^e_{t+1:t+h} \mid J_t\right] = (\psi_{h,d} - \lambda_{h,2f})^\top \zeta_t, \tag{C.3}$$

where $\lambda_{2f} = 1/\lambda_{1f}$ and

$$\psi_{d,i} = \lambda_{2d,i}\exp(\mu_{d,i} + \omega_{d,i}/2)\lambda_{3d}^\top P e_i, \quad i = 1, ..., N, \tag{C.4}$$

$$\psi_{h,d} = \left(\sum_{j=1}^{h} P^{j-1}\right)^\top \psi_d \quad \text{and} \quad \lambda_{h,2f} = \left(\sum_{j=1}^{h} P^{j-1}\right)^\top \lambda_{2f}. \tag{C.5}$$

The variance of returns over h periods is given by:

$$\begin{aligned} Var[R_{t+1:t+h}] = {} & h\theta_2^\top E\left[\zeta_t\zeta_t^\top\right] P^\top \theta_3. \\ & + h(\theta_1 \odot \theta_1)^\top E\left[\zeta_t\zeta_t^\top\right] P^\top(\lambda_{3d} \odot \lambda_{3d}) - h^2\left(\theta_1^\top E\left[\zeta_t\zeta_t^\top\right] P^\top \lambda_{3d}\right)^2 \\ & + 2\sum_{j=2}^{h}(h - j + 1)\theta_1^\top E\left[\zeta_t\zeta_t^\top\right] P^\top\left(\lambda_{3d} \odot \left(\left(P^{j-2}\right)^\top\left(\theta_1 \odot \left(P^\top \lambda_{3d}\right)\right)\right)\right), \end{aligned} \tag{C.6}$$

where

$$\theta_1 = \lambda_{2d} \odot (\exp(\mu_{d,1} + \omega_{d,1}/2), ..., \exp(\mu_{d,N} + \omega_{d,N}/2))^\top, \tag{C.7}$$

$$\theta_2 = (\theta_1 \odot \theta_1 \odot (\exp(\omega_{d,1}), ..., \exp(\omega_{d,N}))^\top) - (\theta_1 \odot \theta_1), \tag{C.8}$$

$$\theta_3 = \lambda_{3d} \odot \lambda_{3d}. \tag{C.9}$$

One can get similar formulas for excess returns.

Appendix D. This section deals with predictive regressions. When one runs a predictive regression, i.e., one regresses a variable $y_{t+1:t+h}$ onto a variable x_t and a constant, one gets

$$y_{t+1:t+h} = a\left(h\right) + b\left(h\right) x_t + \eta_{y,1,t+h}\left(h\right), \tag{D.1}$$

$$\text{with} \qquad b\left(h\right) = \frac{Cov\left(y_{t+1:t+h}, x_t\right)}{Var\left[x_t\right]} \text{ and } R^2\left(h\right) = \frac{\left(Cov\left(y_{t+1:t+h}, x_t\right)\right)^2}{Var\left[y_{t+1:t+h}\right] Var\left[x_t\right]}, \tag{D.2}$$

where $R^2\left(h\right)$ is the corresponding population coefficient of determination. Consequently, the characterization of the predictive ability of the dividend-price ratio for future expected returns requires the variance of payoff-price ratios, covariances of payoff-price ratios with aggregate returns and variance of aggregate returns. We show that

$$Var\left[\frac{D_t}{P_{d,t}}\right] = \lambda_{2d}^{\top} Var\left[\zeta_t\right] \lambda_{2d} \text{ and } Cov\left(R_{t+1:t+h}, \frac{D_t}{P_{d,t}}\right) = \psi_{h,d}^{\top} Var\left[\zeta_t\right] \lambda_{2d}, \tag{D.3}$$

and the variance of aggregate returns is given by (C.6). One gets similar formulas for excess returns, consumption and dividend growth processes.

We also considered in the paper the predictability of future realized variance of returns when one uses the dividend-price ratio as a predictor. For this purpose, we computed the realized variance as follows. We computed the population regression

$$R_{t+1} = a_R + b_R R_t + U_{R,t+1}, \tag{D.4}$$

where R_{t+1} denotes the gross return on equity:

$$R_{t+1} = \left(\lambda_{2d}^{\top}\zeta_t\right) \exp\left(\mu_d^{\top}\zeta_t + \left(\omega_d^{\top}\zeta_t\right)^{1/2} \varepsilon_{d,t+1}\right)\left(\lambda_{3d}^{\top}\zeta_{t+1}\right). \tag{D.5}$$

It follows that $U_{R,t+1} = R_{t+1} - b_R R_t - a_R$ where $b_R = Cov\left(R_{t+1}, R_t\right)/Var\left[R_t\right]$ and $a_R = E\left[R_{t+1}\right]\left(1 - b_R\right)$. We measured realized variance as a moving sum of these squared residuals and considered the predictive regression

$$\sum_{j=1}^{h} U_{R,t+j}^2 = a\left(h\right) + b\left(h\right)\frac{D_t}{P_t} + \eta_{t+h}\left(h\right). \tag{D.6}$$

Again, one has

$$b\left(h\right) = \frac{Cov\left(\sum_{j=1}^{h} U_{R,t+j}^2, D_t/P_t\right)}{Var\left[\sum_{j=1}^{h} U_{R,t+j}^2\right]} \text{ and } R^2\left(h\right) = \frac{\left(Cov\left(\sum_{j=1}^{h} U_{R,t+j}^2, D_t/P_t\right)\right)^2}{Var\left[\sum_{j=1}^{h} U_{R,t+j}^2\right] Var\left[D_t/P_t\right]}. \tag{D.7}$$

In the supplement appendix, all these formulas are given explicitly. Similar formulas for consumption growth and dividend growth volatilities are provided in the supplement appendix.

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Table 1: **Parameters of the Random Walk and the Long-Run Risks Markov-Switching Models.**
The long-run risks model defined in (**??**)-(**??**)is calibrated with $\mu_x = 0.0015$, $\phi_d = 2.5$, $\nu_d = 6.5$, $\phi_x = 0.975$, $\nu_x = 0.038$, $\sqrt{\mu_\sigma} = 0.0072$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.62547 \times 10^{-5}$ and $\rho_1 = 0.39985$. In Panel A, we report the parameters of the two-state monthly Markov-switching model of the form (3.9,**??**) such that $\mu_{c,1} = \mu_{c,2}$ and $\mu_{d,1} = \mu_{d,2}$. From the long run risk model, we set $\phi_x = 0$ and $\nu_x = 0$ to obtain a random walk model, and we adjust the other parameters when necessary such that consumption and dividend growth means, variances and covariance remain unchanged from the original model. The random walk model is then calibrated with $\mu_x = 0.0015$, $\nu_d = 6.42322$, $\sqrt{\mu_\sigma} = 0.0073$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.62547 \times 10^{-5}$ and $\rho_1 = 0.40434$. In Panel A, we report the parameters of the four-state monthly Markov-switching model of the form (3.9,**??**) that matches the full long-run risk model of Bansal, Kiku and Yaron (2007). In both panels, μ_c and μ_d are conditional means of consumption and dividend growths, ω_c and ω_d are conditional variances of consumption and dividend growths and ρ is the conditional correlation between consumption and dividend growths. $P^\top$ is the transition matrix across different regimes and Π is the vector of unconditional probabilities of regimes. Means and standard deviations are in percent.

Panel A	σ_L	σ_H
$\mu_c^\top$	0.15	0.15
$\mu_d^\top$	0.15	0.15
$\left(\omega_c^\top\right)^{1/2}$	0.46	1.32
$\left(\omega_d^\top\right)^{1/2}$	2.94	8.48
$\rho^\top$	0.40434	0.40434
	$P^\top$	
σ_L	0.99894	0.00106
σ_H	0.00394	0.99606
$\Pi^\top$	0.78868	0.21132

Panel B	$\mu_L\sigma_L$	$\mu_L\sigma_H$	$\mu_H\sigma_L$	$\mu_H\sigma_H$
$\mu_c^\top$	-0.19513	-0.19513	0.19393	0.19393
$\mu_d^\top$	-0.71283	-0.71283	0.25982	0.25982
$\left(\omega_c^\top\right)^{1/2}$	0.44071	1.31462	0.44071	1.31462
$\left(\omega_d^\top\right)^{1/2}$	2.86569	8.54824	2.86569	8.54824
$\rho^\top$	0.39985	0.39985	0.39985	0.39985
		$P^\top$		
$\mu_L\sigma_L$	0.97679	0.00103	0.02215	0.00002
$\mu_L\sigma_H$	0.00386	0.97397	0.00009	0.02209
$\mu_H\sigma_L$	0.00282	0.00000	0.99612	0.00105
$\mu_H\sigma_H$	0.00001	0.00281	0.00393	0.99325
$\Pi^\top$	0.08905	0.02386	0.69963	0.18746

Table 2: **(RW) Asset Prices and Predictability: Benckmark**
The entries of Panel A are model population values of asset prices. The expressions $E[R - R_f]$, $E[R_f] - 1$ and $E[P/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a(h) + b(h)\left(\frac{D}{P}\right)_{t-11:t} + \eta_{t+12h}(h)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	GDA	50%	PV
δ		0.9989		
γ		2.5		
ψ		1.5		
α		0.3		
κ		0.989		
Panel A. Asset Pricing Implications				
$E[R - R_f]$	7.25	7.21	6.14	0.61
$\sigma[R]$	19.52	19.33	16.90	0.45
$E[R_f] - 1$	1.21	0.93	1.39	0.62
$\sigma[R_f]$	4.10	2.34	1.84	1.00
$E[P/D]$	30.57	23.30	24.20	1.00
$\sigma[D/P]$	1.52	1.38	1.07	0.79
Panel B. Predictability of Excess Returns				
$R^2(1)$	7.00	12.04	7.44	0.48
$[b(1)]$	3.12	5.05	6.25	0.20
$R^2(3)$	14.67	28.35	17.27	0.46
$[b(3)]$	7.05	14.30	16.91	0.18
$R^2(5)$	27.26	38.00	22.47	0.56
$[b(5)]$	12.34	22.49	23.14	0.25
Panel C. Predictability of Consumption Growth				
$R^2(1)$	0.06	0	0.76	0.16
$[b(1)]$	-0.02	0	0.02	0.47
$R^2(3)$	0.09	0	1.67	0.13
$[b(3)]$	-0.05	0	0.07	0.46
$R^2(5)$	0.24	0	2.23	0.18
$[b(5)]$	-0.11	0	0.04	0.47
Panel D. Predictability of Dividend Growth				
$R^2(1)$	0.00	0	0.71	0.00
$[b(1)]$	0.04	0	0.11	0.49
$R^2(3)$	0.20	0	1.44	0.21
$[b(3)]$	-0.48	0	0.17	0.46
$R^2(5)$	0.08	0	1.75	0.14
$[b(5)]$	-0.37	0	-0.48	0.51

Table 3: **(RW) Asset Prices and Predictability: Robustness to Preference Parameters**

The entries of Panel A are model population values of asset prices. The expressions $E[R-R_f]$, $E[R_f]-1$ and $E[P/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a(h) + b(h)\left(\frac{D}{P}\right)_{t-11:t} + \eta_{t+12h}(h)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	GDA1	50%	PV	DA0	50%	PV	KP	50%	PV
δ		0.9989			0.9989			0.9989		
γ		2.5			0			10		
ψ		0.75			∞			1.5		
α		0.3			0.3			1		
κ		0.989			1			1		
Panel A. Asset Pricing Implications										
$E[R-R_f]$	7.25	6.12	5.00	0.69	10.32	9.56	0.12	1.42	1.16	0.98
$\sigma[R]$	19.52	18.04	15.75	0.27	19.14	16.94	0.00	16.38	13.96	0.05
$E[R_f]-1$	1.21	1.97	2.60	0.68	1.32	1.32	0.61	1.93	2.04	0.75
$\sigma[R_f]$	4.10	3.25	2.55	1.00		0.00	1.00	0.59	0.46	1.00
$E[P/D]$	30.57	22.05	22.74	1.00	13.10	13.59	1.00	470.66	467.82	0.00
$\sigma[D/P]$	1.52	1.04	0.81	1.00	2.32	1.80	0.44	0.01	0.00	1.00
Panel B. Predictability of Excess Returns										
$R^2(1)$	7.00	13.53	7.78	0.47	8.00	4.54	0.61	1.29	0.87	0.88
$[b(1)]$	3.12	6.70	7.98	0.18	2.38	3.08	0.51	-294.98	-253.63	0.65
$R^2(3)$	14.67	30.54	17.33	0.46	19.88	11.10	0.57	3.33	1.69	0.87
$[b(3)]$	7.05	18.94	20.94	0.18	6.73	8.43	0.42	-834.28	-712.09	0.65
$R^2(5)$	27.26	39.72	21.94	0.56	27.78	14.63	0.67	4.81	2.26	0.92
$[b(5)]$	12.34	29.79	28.89	0.24	10.58	11.35	0.54	-1312.45	-807.13	0.61
Panel C. Predictability of Consumption Growth										
$R^2(1)$	0.06	0	0.76	0.16	0	0.76	0.16	0	0.75	0.17
$[b(1)]$	-0.02	0	0.02	0.47	0	0.01	0.45	0	-3.39	0.52
$R^2(3)$	0.09	0	1.68	0.13	0	1.66	0.13	0	1.65	0.13
$[b(3)]$	-0.05	0	0.09	0.47	0	0.04	0.45	0	-14.53	0.52
$R^2(5)$	0.24	0	2.23	0.18	0	2.23	0.18	0	2.24	0.18
$[b(5)]$	-0.11	0	0.05	0.48	0	0.02	0.46	0	-9.98	0.51
Panel D. Predictability of Dividend Growth										
$R^2(1)$	0.00	0	0.72	0.00	0	0.71	0.00	0	0.70	0.00
$[b(1)]$	0.04	0	0.14	0.49	0	0.07	0.50	0	-24.52	0.52
$R^2(3)$	0.20	0	1.44	0.21	0	1.44	0.21	0	1.44	0.21
$[b(3)]$	-0.48	0	0.23	0.47	0	0.10	0.44	0	-47.28	0.51
$R^2(5)$	0.08	0	1.75	0.14	0	1.75	0.14	0	1.76	0.14
$[b(5)]$	-0.37	0	-0.63	0.51	0	-0.28	0.50	0	107.27	0.48

Table 4: **(LRR) Asset Prices and Predictability: Robustness to Endowment Dynamics**

The entries of Panel A are model population values of asset prices. The expressions $E\left[R-R_f\right]$, $E\left[R_f\right]-1$ and $E\left[P/D\right]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma\left[R\right]$, $\sigma\left[R_f\right]$ and $\sigma\left[D/P\right]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a\left(h\right) + b\left(h\right)\left(\frac{D}{P}\right)_{t-11:t} + \eta_{t+12h}\left(h\right)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	GDA	50%	PV	GDA1	50%	PV	DA0	50%	PV	KP	50%	PV
δ		0.9989			0.9989			0.9989			0.9989		
γ		2.5			2.5			0			10		
ψ		1.5			0.75			∞			1.5		
α		0.3			0.3			0.3			1		
κ		0.989			0.989			1			1		
Panel A. Asset Pricing Implications													
$E\left[R-R_f\right]$	7.25	8.60	7.54	0.46	6.92	5.83	0.62	11.47	10.68	0.06	6.69	6.33	0.65
$\sigma\left[R\right]$	19.52	19.35	17.91	0.56	18.04	16.84	0.62	20.78	19.01	0.52	18.11	16.22	0.65
$E\left[R_f\right]-1$	1.21	0.96	1.33	0.47	2.19	2.67	0.27	1.32	1.32	0.00	1.21	1.28	0.44
$\sigma\left[R_f\right]$	4.10	2.48	1.95	1.00	3.70	2.85	0.96	0.00	0.00	1.00	1.05	0.78	1.00
$E\left[P/D\right]$	30.57	17.70	18.23	1.00	18.06	18.48	1.00	11.93	12.28	1.00	22.50	22.56	1.00
$\sigma\left[D/P\right]$	1.52	1.56	1.18	0.68	1.11	0.86	1.00	2.59	1.95	0.42	0.48	0.29	1.00
Panel B. Predictability of Excess Returns													
$R^2\left(1\right)$	7.00	10.30	6.49	0.52	11.90	7.06	0.50	6.38	4.75	0.62	0.05	0.83	0.94
$\left[b\left(1\right)\right]$	3.12	4.13	4.72	0.30	5.86	6.37	0.23	2.05	2.48	0.61	0.86	2.45	0.53
$R^2\left(3\right)$	14.67	24.07	14.31	0.50	26.81	14.89	0.50	15.84	11.14	0.58	0.05	1.98	0.94
$\left[b\left(3\right)\right]$	7.05	11.62	12.55	0.25	16.52	17.15	0.21	5.71	7.00	0.50	1.54	6.05	0.52
$R^2\left(5\right)$	27.26	32.07	18.72	0.61	34.80	18.00	0.61	22.08	15.11	0.71	0.03	2.64	0.99
$\left[b\left(5\right)\right]$	12.34	18.21	18.75	0.29	25.92	25.01	0.24	8.88	10.43	0.63	1.41	8.26	0.56
Panel C. Predictability of Consumption Growth													
$R^2\left(1\right)$	0.06	1.68	2.70	0.10	1.30	2.68	0.10	2.96	2.87	0.09	16.39	5.49	0.07
$\left[b\left(1\right)\right]$	-0.02	-0.24	-0.28	0.68	-0.29	-0.37	0.68	-0.19	-0.19	0.70	-2.42	-2.00	0.81
$R^2\left(3\right)$	0.09	2.14	4.15	0.09	1.66	4.08	0.09	3.77	4.35	0.09	20.91	4.86	0.08
$\left[b\left(3\right)\right]$	-0.05	-0.54	-0.65	0.65	-0.67	-0.84	0.65	-0.43	-0.42	0.66	-5.52	-3.58	0.76
$R^2\left(5\right)$	0.24	1.85	4.87	0.14	1.44	4.90	0.14	3.27	4.95	0.14	18.09	4.31	0.14
$\left[b\left(5\right)\right]$	-0.11	-0.71	-0.77	0.60	-0.88	-1.09	0.61	-0.57	-0.49	0.61	-7.21	-3.65	0.71
Panel D. Predictability of Dividend Growth													
$R^2\left(1\right)$	0.00	0.31	1.66	0.00	0.24	1.64	0.00	0.55	1.63	0.00	3.05	1.52	0.00
$\left[b\left(1\right)\right]$	0.04	-0.60	-0.99	0.63	-0.73	-1.37	0.63	-0.48	-0.62	0.66	-6.05	-5.38	0.76
$R^2\left(3\right)$	0.20	0.51	3.31	0.14	0.40	3.22	0.14	0.90	3.31	0.14	4.96	2.63	0.17
$\left[b\left(3\right)\right]$	-0.48	-1.36	-2.15	0.59	-1.68	-2.92	0.59	-1.09	-1.41	0.58	-13.80	-9.90	0.71
$R^2\left(5\right)$	0.08	0.50	3.79	0.07	0.39	3.77	0.08	0.88	3.59	0.07	4.87	2.70	0.09
$\left[b\left(5\right)\right]$	-0.37	-1.78	-2.85	0.57	-2.19	-3.87	0.57	-1.42	-1.70	0.57	-18.03	-10.24	0.65

Table 5: **(RW) Conditional Moments of the Stochastic Discount Factor**
The entries of the table are the mean and the volatility of the stochastic discount factor in each state of the economy (i.e. low volatility and high volatility of aggregate consumption growth). The benchmark Random Walk dynamics is calibrated, with $\mu_x = 0.0015$, $\nu_d = 6.42322$, $\sqrt{\mu_\sigma} = 0.0073$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.62547 \times 10^{-5}$ and $\rho = 0.40434$.

	$\mu(M \mid \sigma_L)$	$\sigma(M \mid \sigma_L)$	$\mu(M \mid \sigma_H)$	$\sigma(M \mid \sigma_H)$
GDA	0.99820	0.11481	1.00304	0.66929
GDA1	0.99695	0.11394	1.00365	0.67033
DA0	0.99890	0.63412	0.99890	0.61151
KP	0.99814	0.21202	0.99935	0.13928

Table 6: **(RW) Conditional Probability Distribution Function of the Stochastic Discount Factor: DA0.**
The entries of the table are the probability density function of the stochastic discount factor conditional in each state of the economy, namely $g(M \mid J_t)$, when the representative investor has time separable preferences with $\gamma = 1/\psi = 0$. The distribution is concentrated on two points, $a_{i\cdot}$ and $a_{i\cdot}/\alpha$ where i is the state of the economy, and the table shows each point with the associated weight. The benchmark Random Walk model is calibrated, with $\mu_x = 0.0015$, $\nu_d = 6.42322$, $\sqrt{\mu_\sigma} = 0.0073$, $\phi_\sigma = 0.995$, $\nu_\sigma = 0.62547 \times 10^{-5}$ and $\rho_1 = 0.40434$.

i	$a_{i\cdot}$	$Prob(a_{i\cdot})$	$a_{i\cdot}/\alpha$	$Prob(a_{i\cdot}/\alpha)$
σ_L	0.625	0.744	2.085	0.256
σ_H	0.544	0.642	1.813	0.358

Table 7: **(RW) Predictability of Volatility**

The entries of the table are the R^2 and the slope of the regression

$$\frac{\sum_{j=1}^{12h} U^2_{Y,t+j}}{h} = a\left(h\right) + b\left(h\right)\left(12\frac{D_t}{P_t}\right) + \eta_{t+12h}\left(h\right),$$

where $U^2_{Y,t+1}$ is the square of the residuals of the regression

$$Y_{t+1} = a_Y + b_Y Y_t + U_{Y,t+1},$$

and where Y_{t+1} stands for returns R_{t+1}, consumption growth C_{t+1}/C_t and dividend growth D_{t+1}/D_t, in Panels A, B and C respectively.

	Data	GDA	50%	PV	GDA1	50%	PV	DA0	50%	PV	KP	50%	PV
δ		0.9989			0.9989			0.9989			0.9989		
γ		2.5			2.5			0			10		
ψ		1.5			0.75			∞			1.5		
α		0.3			0.3			0.3			1		
κ		0.989			0.989			1			1		
Panel A. Predictability of Returns Volatility													
$R^2\left(1\right)$	2.453	4.296	43.54	0.22	3.365	45.53	0.23	5.063	44.04	0.22	0.909	44.55	0.26
$\left[b\left(1\right)\right]$	-0.266	17.048	10.67	0.03	18.496	13.41	0.03	11.478	6.36	0.03	-1031.500	-1945.08	0.96
$R^2\left(3\right)$	11.843	10.789	57.09	0.32	8.454	57.61	0.33	12.490	57.32	0.32	2.315	53.60	0.35
$\left[b\left(3\right)\right]$	-0.395	16.071	9.44	0.06	17.435	11.71	0.06	10.819	5.60	0.06	-972.417	-1694.89	0.92
$R^2\left(5\right)$	17.922	15.492	51.93	0.36	12.154	51.44	0.37	17.700	51.81	0.36	3.360	45.96	0.39
$\left[b\left(5\right)\right]$	-0.383	15.169	8.32	0.08	16.457	10.39	0.08	10.212	4.95	0.08	-917.850	-1485.51	0.90
Panel B. Predictability of Consumption Growth Volatility													
$R^2\left(1\right)$	27.668	0.000	45.69	0.37	0.000	45.67	0.38	0.000	45.69	0.37	0.000	45.54	0.38
$\left[b\left(1\right)\right]$	0.004	0.014	1.42	0.04	0.018	1.86	0.04	0.008	0.84	0.04	-2.324	-301.00	0.96
$R^2\left(3\right)$	46.751	0.000	54.24	0.45	0.000	54.25	0.45	0.000	54.23	0.45	0.000	54.19	0.45
$\left[b\left(3\right)\right]$	0.004	0.005	1.24	0.09	0.006	1.62	0.09	0.003	0.73	0.09	-0.775	-260.19	0.92
$R^2\left(5\right)$	45.249	0.000	46.30	0.49	0.000	46.27	0.49	0.000	46.29	0.49	0.000	46.18	0.49
$\left[b\left(5\right)\right]$	0.003	0.003	1.09	0.11	0.004	1.43	0.11	0.002	0.65	0.11	-0.465	-227.51	0.89
Panel C. Predictability of Dividend Growth Volatility													
$R^2\left(1\right)$	20.556	0.008	45.29	0.35	0.008	45.26	0.35	0.008	45.29	0.35	0.008	45.12	0.35
$\left[b\left(1\right)\right]$	3.825	0.563	9.02	0.07	0.743	11.85	0.06	0.334	5.35	0.14	-96.375	-1911.83	0.96
$R^2\left(3\right)$	19.093	0.003	54.10	0.37	0.003	54.07	0.37	0.003	54.10	0.37	0.003	53.95	0.37
$\left[b\left(3\right)\right]$	3.013	0.188	7.91	0.14	0.248	10.38	0.13	0.111	4.69	0.20	-32.125	-1663.96	0.92
$R^2\left(5\right)$	20.147	0.002	46.14	0.40	0.002	46.11	0.40	0.002	46.14	0.40	0.002	45.99	0.40
$\left[b\left(5\right)\right]$	2.352	0.113	7.02	0.18	0.149	9.19	0.16	0.067	4.16	0.24	-19.275	-1458.28	0.90

Figure 1: **Indifference Curves for GDA Preferences**
Indifference curves over two outcomes x and y with the fixed probability $p = Prob(x) = 1/2$.

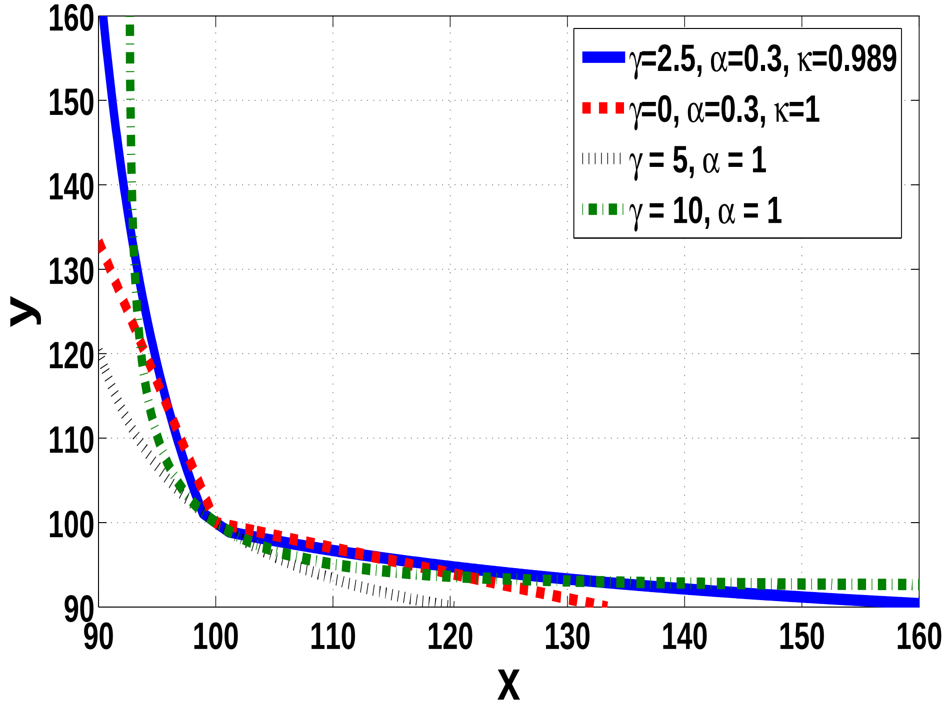

Figure 2: **(RW) Equity Premium, Risk-Free Rate and Valuation Ratio, GDA**
The figure displays population values of asset prices. The expressions $E[R - R_f]$, $E[R_f] - 1$ and $E[P/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The parameter of risk aversion is set to $\gamma = 2.5$.

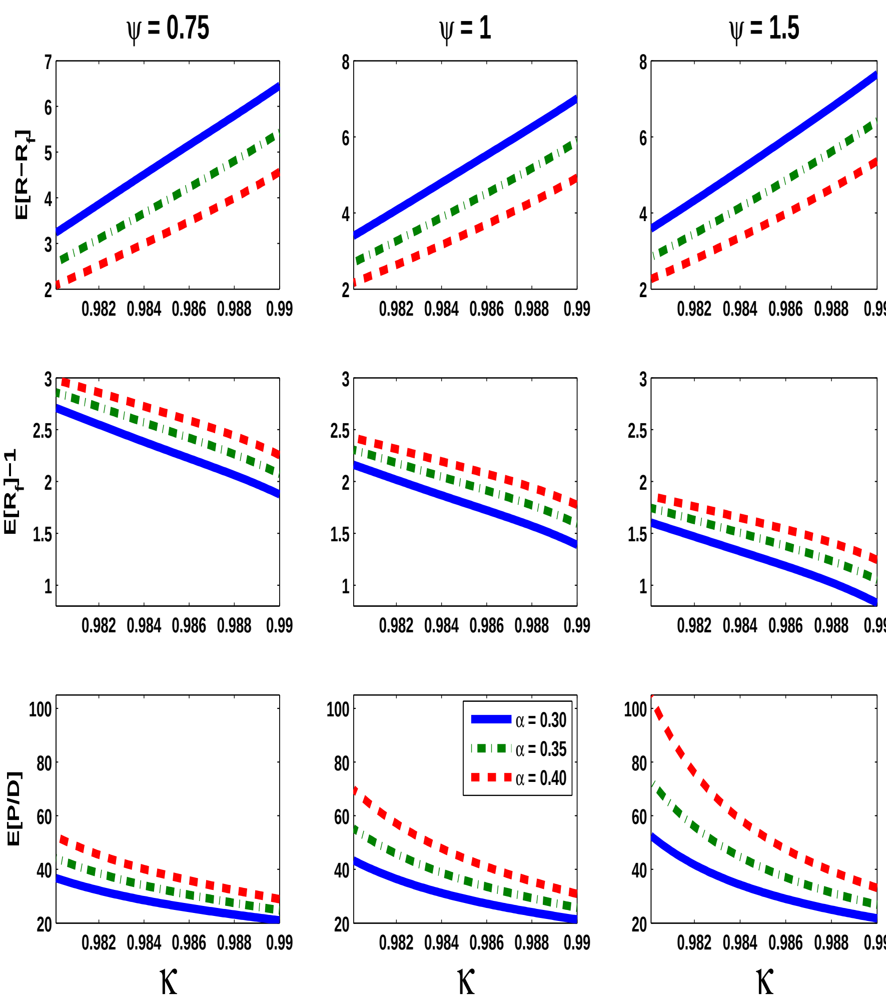

Figure 3: **(RW) Predictability of Excess Returns (R^2), GDA**
The figure shows the population R^2 of the monthly regression $R^e_{t+1:t+h} = a\,(h) + b\,(h)\,\frac{D_t}{P_{d,t}} + \eta_{t+h}\,(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The parameter of risk aversion is set to $\gamma = 2.5$.

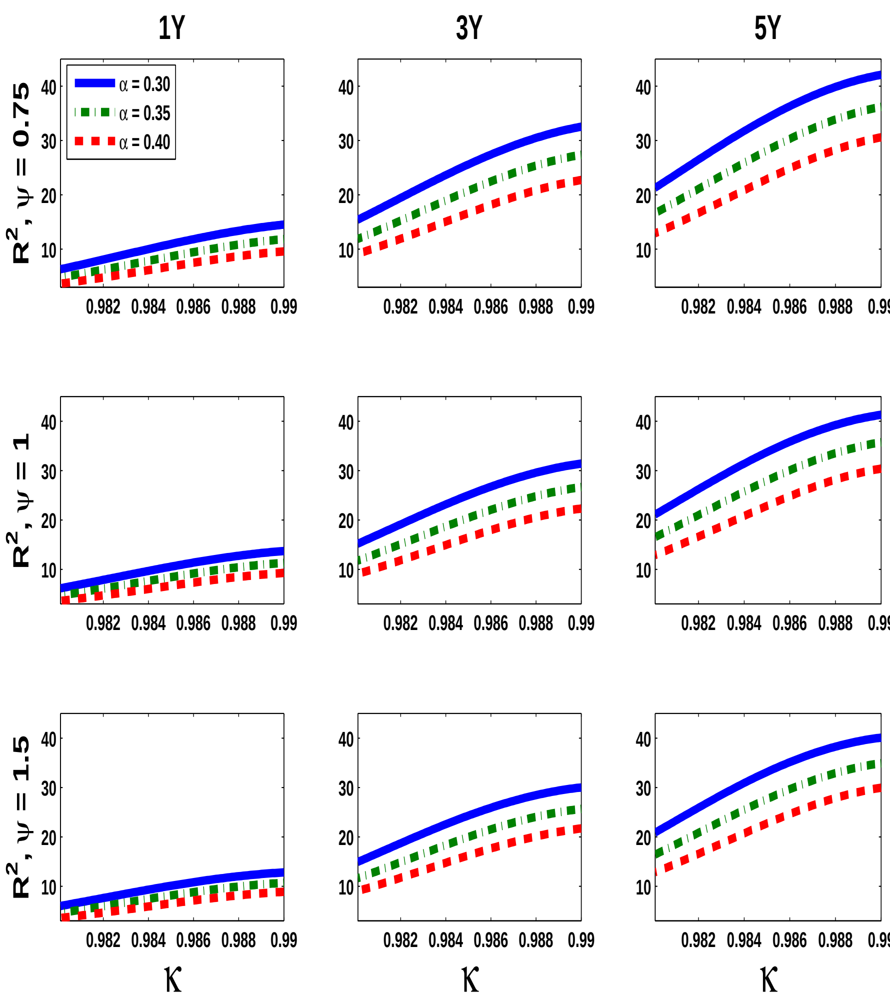

Figure 4: **(RW) Sensitivity of Asset Prices to the Persistence of Consumption Volatility: KP and GDA.**
The figure displays population values of asset prices as functions of the persistence of consumption volatility. The expressions $E\left[R-R_f\right]$ and $E\left[P_d/D\right]$ are respectively the annualized equity premium and mean price-dividend ratio. The expressions $\sigma\left[R-R_f\right]$ and $\sigma\left[D/P_d\right]$ are respectively the annualized standard deviations of the equity excess return and the equity dividend-price ratio.

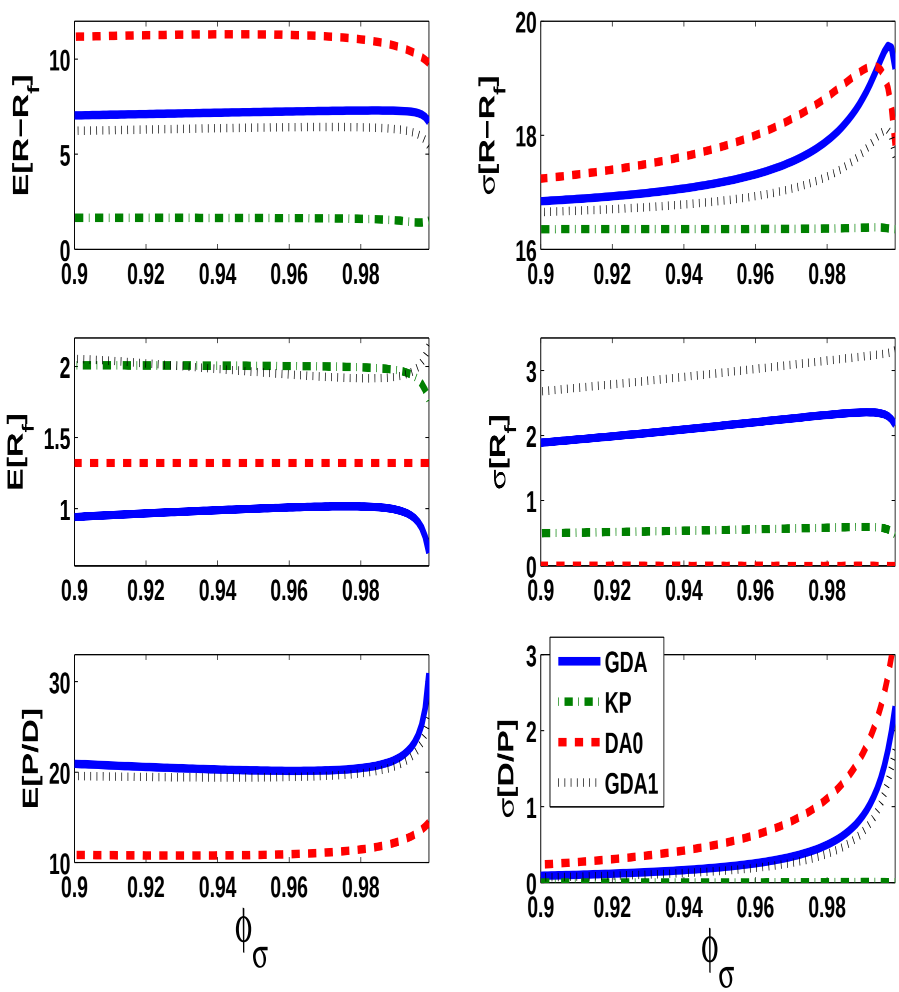

Figure 5: **(RW) Sensitivity of Excess Return Predictability to the Persistence of Consumption Volatility.**

The figure shows the population R^2 of the monthly regression $y_{t+1:t+h} = a(h) + b(h)\left(\frac{D}{P_d}\right)_{t-11:t} + \eta_{t+h}(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The variable y stands for excess returns $R - R_f$. The R^2 is plotted as a function of the persistence of consumption volatility.

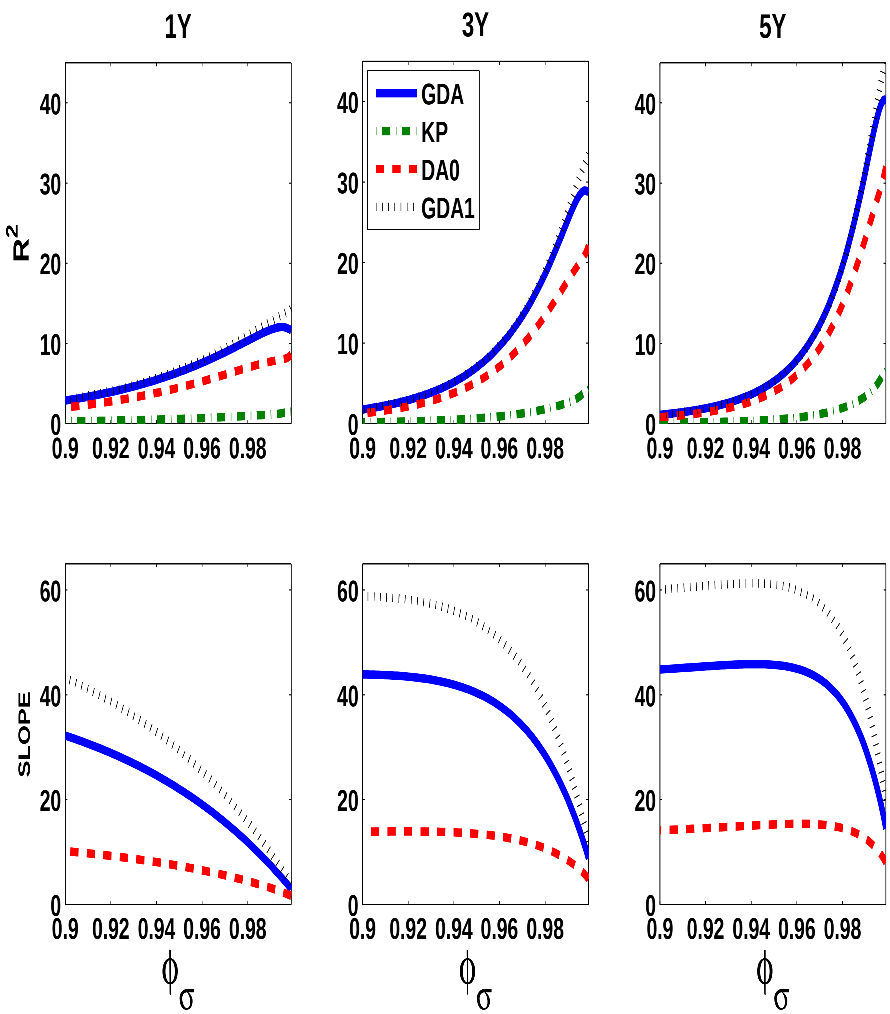

Figure 6: **(LRR) Sensitivity of Asset prices to the Persistence of Expected Consumption Growth: KP and GDA.**
The figure displays population values of asset prices as functions of the persistence of expected consumption growth. The expressions $E\left[R-R_f\right]$ and $E\left[P_d/D\right]$ are respectively the annualized equity premium and mean price-dividend ratio. The expressions $\sigma\left[R-R_f\right]$ and $\sigma\left[D/P_d\right]$ are respectively the annualized standard deviations of the equity excess return and the equity dividend-price ratio.

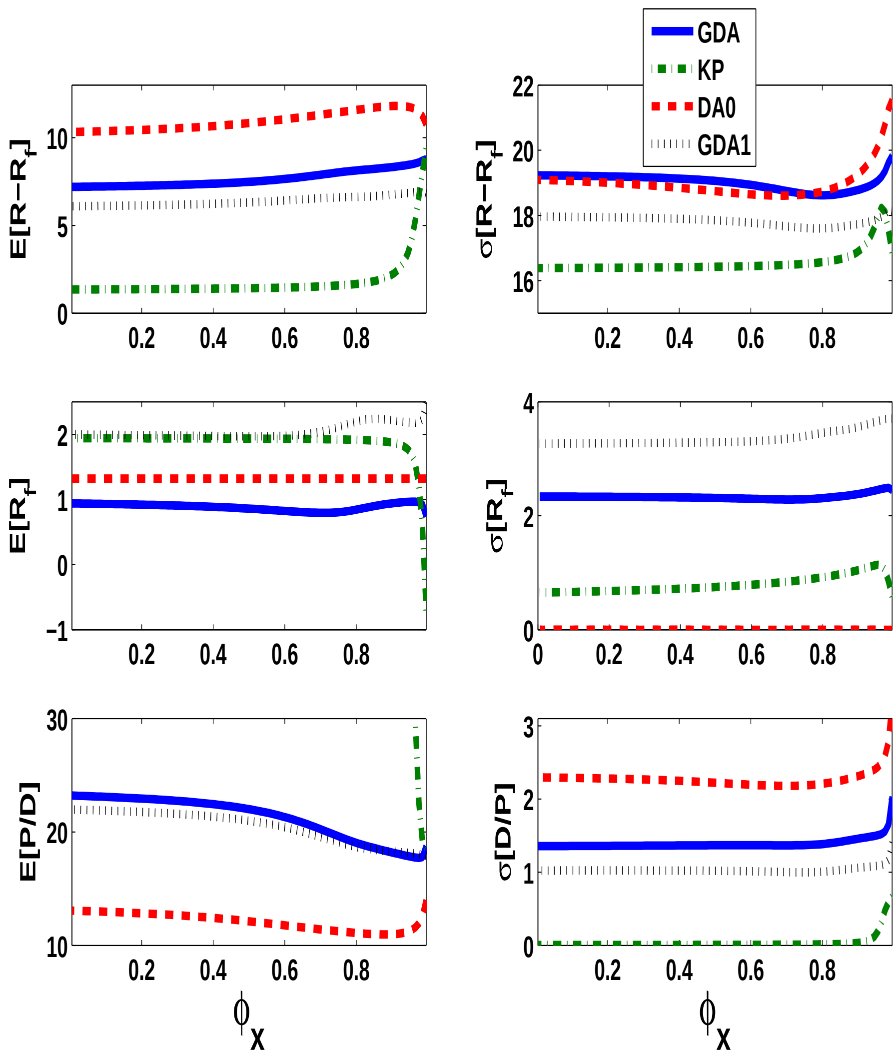

Figure 7: **(LRR) Sensitivity of Excess Return and Growth Rates Predictability to the Persistence of Expected Consumption Growth: KP and GDA.**
The figure shows the population R^2 of the monthly regression $y_{t+1:t+h} = a(h) + b(h)\left(\frac{D}{P_d}\right)_{t-11:t} + \eta_{t+h}(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The variable y stands for excess returns $R - R_f$ and consumption growth Δc. The R^2 is plotted as a function of the persistence of expected consumption growth.