The Journal of FINANCE
The Journal of THE AMERICAN FINANCE ASSOCIATION

THE JOURNAL OF FINANCE • VOL. LXVIII, NO. 1 • FEBRUARY 2013

Ex Ante Skewness and Expected Stock Returns

JENNIFER CONRAD, ROBERT F. DITTMAR, and ERIC GHYSELS*

ABSTRACT

We use option prices to estimate ex ante higher moments of the underlying individual securities' risk-neutral returns distribution. We find that individual securities' risk-neutral volatility, skewness, and kurtosis are strongly related to future returns. Specifically, we find a negative (positive) relation between ex ante volatility (kurtosis) and subsequent returns in the cross-section, and more ex ante negatively (positively) skewed returns yield subsequent higher (lower) returns. We analyze the extent to which these returns relations represent compensation for risk and find evidence that, even after controlling for differences in co-moments, individual securities' skewness matters.

MODELS IMPLYING THAT INVESTORS consider higher moments in returns have a long history in the literature. Researchers such as Rubinstein (1973) and Kraus and Litzenberger (1976, 1983) develop models of expected returns that incorporate skewness. In these models, the higher moments that are relevant for individual securities are co-moments with the aggregate market portfolio. More recent, empirical work provides evidence that higher moments of the return distribution are important in pricing securities. Consistent with the models' focus on co-moments, the tests in these papers ask whether a security's co-skewness or co-kurtosis with the market is priced; historical returns data are typically used to measure these co-moments. For example, Harvey and Siddique (2000) explore both skewness and co-skewness and test whether co-skewness is priced, and Dittmar (2002) tests whether a security's co-skewness and co-kurtosis with the market portfolio might influence investors' expected returns.

*Conrad is with the Department of Finance, Kenan-Flagler Business School, University of North Carolina at Chapel Hill; Dittmar is with Department of Finance, Stephen Ross School of Business, University of Michigan; Ghysels is with the Department of Finance, Kenan-Flagler Business School, and Department of Economics, University of North Carolina at Chapel Hill. An earlier version of the paper was circulated under the title "Skewness and the Bubble." All errors are the responsibility of the authors. We especially thank Cam Harvey for many insightful comments. We also thank Robert Battalio, Patrick Dennis, and Stewart Mayhew for providing data and computational code. We thank Rui Albuquerque, Andrew Ang, Leonce Bargeron, John Griffin, Paul Pfleiderer, Lukasz Pomorski, Feng Wu, and Chu Zhang for helpful comments and suggestions, as well as seminar participants at Babson College, Berkeley, Boston College, the 2009 China International Conference in Finance, Cornell University, Dartmouth College, Erasmus University, Financial Intermediation Research Society 2009 Conference, National University of Singapore, New York University Stern, the Ohio State University, Queen's University Belfast, Stanford University, the Universities of Amsterdam, Arizona, Michigan, Tilburg, Texas, and Virginia, and the 2010 European and Western Finance Association conferences. The paper was awarded the 2009 China International Conference in Finance Best Paper Award.

Other recent papers suggest that additional features of individual securities' payoff distribution may be relevant for understanding differences in assets' returns. For example, Ang et al. (2006, 2009) document that firms' idiosyncratic return volatility contains important information about future returns. The work of Barberis and Huang (2008) and Brunnermeier, Gollier, and Parker (2007), together with the empirical evidence presented in Mitton and Vorkink (2007) and Boyer, Mitton, and Vorkink (2010) imply that the skewness of individual securities may also influence investors' portfolio decisions. Xing, Zhang, and Zhao (2010) find that portfolios formed by sorting individual securities on a measure that is related to idiosyncratic skewness generate cross-sectional differences in returns. Green and Hwang (2009) use the approach of Zhang (2006) and find that IPOs with high expected skewness ("lottery" stocks) experience significantly greater first-day returns, followed by substantially greater negative abnormal returns in the subsequent 3 to 5 years.

We therefore have two strands in the existing literature: (1) models and empirical results that emphasize the importance of higher moments as they affect stochastic discount factors (SDFs), and (2) models and empirical evidence that focus on the higher moment characteristics of individual securities. In this second strand of the literature, researchers have proposed both behavioral and rational models. For example, Barberis and Huang (2008) argue that investors with cumulative prospect theory preferences demand securities with highly skewed payoffs, such as IPO stocks. Brunnermeier, Gollier, and Parker (2007) develop a model of optimal (as opposed to rational) beliefs that also predicts that investors will overinvest in the most highly (right-) skewed securities, with the consequence that those securities will have lower subsequent average returns. They also show that, while there is a rational expectations solution to their model, it represents a knife-edge case. Mitton and Vorkink (2007) introduce a rational model where investors have heterogeneous preferences for skewness and show that idiosyncratic skewness can impact prices.

In this paper, we examine the importance of higher moments using a new approach. We exploit the fact that, if option and stock prices reflect the same information, then it is possible to use options market data to extract estimates of the higher moments of the securities' (risk-neutral) probability density function. Our method has several advantages. First, option prices are a market-based estimate of investors' expectations. Many authors, including Bates (1991), Rubinstein (1994), and Jackwerth and Rubinstein (1996), argue that option market prices can capture the information of market participants. Second, the use of option prices eliminates the need for a long time series of returns to estimate the moments of the return distribution; this is especially helpful when trying to forecast the payoff distribution of relatively new firms or during periods where expectations, at least for some firms, may change relatively quickly. Third, options reflect a true ex ante measure of expectations; they do not give us, as Battalio and Schultz (2006) note, the "unfair advantage of hindsight." As Jackwerth and Rubinstein (1996, p. 1614) state, "not only can the nonparametric method reflect the possibly complex logic used by market participants to consider the significance of extreme events, but it also implicitly brings a much larger set of information . . . to bear on the formulation of probability distributions."

We begin with a sample of options on individual stocks, and test whether cross-sectional differences in estimates of the higher moments of an individual security's payoff extracted from options are related to subsequent returns. Consistent with the Ang et al. (2006, 2009) findings for physical measures of idiosyncratic volatility, we find a negative relation between risk-neutral volatility and subsequent returns. We also document a significant negative relation between firms' risk-neutral skewness and subsequent returns, that is, more negatively skewed securities have higher subsequent returns. In addition, we find a significant positive relation between firms' risk-neutral kurtosis and subsequent returns. These relations persist after controlling for firm characteristics, such as beta, size, and book-to-market ratios, and adjustment for the Fama and French (1993) risk factors.

We examine the extent to which these relations between risk-neutral higher moments and subsequent returns are determined by co-moments with the market portfolio. We measure co-moments using the approaches of Harvey and Siddique (2000) and Bakshi, Kapadia, and Madan (2003), and then decompose total moments into co-moments, such as co-skewness, and idiosyncratic moments. We find that the relation between idiosyncratic higher moments, particularly idiosyncratic skewness, and subsequent returns persists, even after controlling for differences in covariance, co-skewness, and co-kurtosis.[1]

Our results are consistent with models such as Brunnermeier, Gollier, and Parker (2007) and Barberis and Huang (2008), which predict that investors will trade off the benefits of diversification and skewness, holding more concentrated positions in skewed securities, and resulting in a negative relation between idiosyncratic skewness and expected returns. These results are also consistent with the empirical evidence in Mitton and Vorkink (2007), who examine the choices of investors in a sample of discount brokerage accounts and find that investors appear to hold relatively undiversified portfolios and accept lower Sharpe ratios for positively skewed portfolios and securities. These papers focus on physical moments of returns, in contrast to the risk-neutral moments that we examine. Consequently, we analyze the relation between our risk-neutral estimates of skewness and estimates formed from historical returns. We find a positive and statistically significant relation between these estimates. However, we find comparatively little evidence that the relation between risk-neutral moments and subsequent returns in our sample is driven by this relation, that is, after controlling for differences in physical moments, the predictive relation between risk-neutral moments and subsequent returns continues to hold. In contrast, after controlling for differences in risk-neutral moments, we find no clear pattern in returns for portfolios that differ in physical skew.

[1] In robustness checks, we also explore an SDF approach and consider several alternative specifications of the SDF, both parametric and nonparametric. We find results similar to those obtained from the decomposition of higher moments. These results are available in a companion document containing supplementary material: see the Internet Appendix, available online in the "Supplements and Datasets" section at http://www.afajof.org/supplements.asp.

The remainder of the paper is organized as follows. In Section I, we detail the method we employ for recovering measures of volatility, skewness, and kurtosis following Bakshi, Kapadia, and Madan (2003) and we discuss the data (and data filters) used in our analysis. In Section II, we focus on testing whether estimates of the ex ante higher moments of the payoff distribution obtained from options data are related to the subsequent returns of the underlying security. In Section III, we analyze the extent to which the relations between option-based ex ante higher moment sorts and subsequent returns are due to investors seeking compensation for higher co-moment risk, rather than idiosyncratic moments. In Section IV, we examine the relation between risk-neutral and physical distributions, and in particular the comparison of portfolio sorts based on skewness under both measures. We conclude in Section V.

I. Data and Computing Ex Ante Risk-Neutral Moments

We wish to examine the relation, if any, between features of the risk-neutral density function and the pricing of stocks. In this section we describe the data and the methods used to compute ex ante estimates of volatility, skewness, and kurtosis.

Our data on option prices are from Optionmetrics (provided through Wharton Research Data Services). We begin with daily option price data for all out-of-the-money calls and puts for all stocks from 1996 to 2005. Closing prices are constructed as midpoint averages of the closing bid and ask prices.

Data on stock returns are obtained from the Center for Research in Security Prices (again provided through Wharton Research Data Services). We employ daily and monthly returns from 1996 to 2005 for all individual securities covered by CRSP with common shares outstanding. Risk-free rates are the continuously compounded yield computed from the bank discount yields on secondary market 3-month Treasury bills taken from Federal Reserve Report H.15. Finally, we obtain balance sheet data for the computation of book-to-market ratios from Compustat and compute these ratios following the procedure in Davis, Fama, and French (2000).

We begin by calculating higher moments of firms' risk-neutral probability distributions. Intuitively, a risk-neutral probability distribution is computed so that today's fair (i.e., arbitrage-free) price of an asset is equal to the discounted expected value of the future payoffs of the asset, where the discount rate used is simply the risk-free rate. Thus, under the risk-neutral measure, all financial assets in the economy have the same expected rate of return, regardless of their risk. In contrast, if we use the actual (or physical) probability distribution of the asset's payoffs and assume that investors are risk-averse, assets that have more risk in their distribution of payoffs should have a greater expected rate of return (and so lower prices) than less risky assets. The relation between risk-neutral and physical probabilities therefore depends on the price of risk; risk-neutral probabilities subsume, or incorporate, the effects of risk, since the prices from which they are calculated embed investors' risk preferences.

Like the physical density, the risk-neutral density has first, second, third, and fourth moments, respectively, mean, variance, skewness, and kurtosis. All densities are extracted from options and are therefore conditional and for a given horizon. In a risk-neutral density, the mean should correspond to the risk-free rate at a given time with a particular maturity.

To estimate the higher moments of the (risk-neutral) density function of individual securities, we use the results in Bakshi and Madan (2000) and Bakshi, Kapadia, and Madan (2003). Bakshi and Madan (2000) show that any payoff to a security i can be constructed and priced using a set of option prices with different strike prices on that security. They define $V_{i,t}(\tau)$, $W_{i,t}(\tau)$, and $X_{i,t}(\tau)$ as the time t prices of τ-maturity quadratic, cubic, and quartic contracts, respectively. Bakshi, Kapadia, and Madan (2003) define these contracts as contingent claims with payoffs equal to future second, third, and fourth powers, respectively, of log price returns. The contracts are based on $C_{i,t}(\tau;K)$ and $P_{i,t}(\tau;K)$, which are the time t prices of European calls and puts written on the underlying stock with strike price K and expiration τ periods from time t. Expressions for $V_{i,t}(\tau)$, $W_{i,t}(\tau)$, and $X_{i,t}(\tau)$ appear in Appendix A as equations (A1), (A2), and (A3). Using the prices of these contracts, standard moment definitions imply that the risk-neutral moments can be calculated as

$$VAR^{Q}_{i,t}(\tau) = e^{r\tau}V_{i,t}(\tau) - \mu_{i,t}(\tau)^2 \tag{1}$$

$$SKEW^{Q}_{i,t}(\tau) = \frac{e^{r\tau}W_{i,t}(\tau) - 3\mu_{i,t}(\tau)e^{r\tau}V_{i,t}(\tau) + 2\mu_{i,t}(\tau)^3}{\left[e^{r\tau}V_{i,t}(\tau) - \mu_{i,t}(\tau)^2\right]^{3/2}} \tag{2}$$

$$KURT^{Q}_{i,t}(\tau) = \frac{e^{r\tau}X_{i,t}(\tau) - 4\mu_{i,t}(\tau)W_{i,t}(\tau) + 6e^{r\tau}\mu_{i,t}(\tau)^2V_{i,t}(\tau) - \mu_{i,t}(\tau)^4}{\left[e^{r\tau}V_{i,t}(\tau) - \mu_{i,t}(\tau)^2\right]^2}, \tag{3}$$

where

$$\mu_{i,t}(\tau) = e^{r\tau} - 1 - e^{r\tau}V_{i,t}(\tau)/2 - e^{r\tau}W_{i,t}(\tau)/6 - e^{r\tau}X_{i,t}(\tau)/24 \tag{4}$$

and r represents the risk-free rate. We follow Dennis and Mayhew (2000) and use a trapezoidal approximation to estimate the integrals in expressions (A1) to (A3) using discrete data.[2]

In Table I, we present descriptive statistics for the sample estimates of volatility, skewness, and kurtosis. We report medians as well as 5th and 95th percentiles over time and across securities for each year during the sample period. The results indicate that higher moments are important in describing the risk-neutral distribution. While we do not conduct formal tests for the statistical significance of departures from normality, it is clear that there are individual stocks that are strongly negatively skewed (with 5th percentiles of skewness always smaller than –3 and medians as small as –1.3) and fat-tailed

[2] We are grateful to Patrick Dennis for providing us with his code to perform the estimation.

Table I

Descriptive Statistics: Risk-Neutral Moments

A risk-neutral density is a probability in which today's fair (i.e., arbitrage-free) price of a derivative security is equal to the discounted expected value of the future payoff of the derivative, where the discount rate is the risk-free rate. All densities are extracted from options and are therefore conditional and for a given horizon. The mean should correspond to the risk free rate at a given time with a particular maturity. We calculate the risk-neutral moments following the procedure in Bakshi, Kapadia, and Madan (2003) using data on out-of-the-money (OTM) puts and calls. We require at least two OTM puts and two OTM calls to calculate the moments and restrict attention to options with prices in excess of $0.50 for which we have at least 10 quotes per month and are not expiring within 1 week. Finally, we eliminate any options that violate put-call parity restrictions and lie in the extreme 1% of the distribution of the risk-neutral moments. The sample consists of 3,722,700 option-day combinations over the time period January 1996 through December 2005. Entries to the table are the 5th percentile, median, and 95th percentiles of risk-neutral volatility, skewness, and kurtosis across securities by year.

	Volatility			Skewness			Kurtosis		
Year	P5	P50	P95	P5	P50	P95	P5	P50	P95
1996	11.09	24.20	43.91	−3.61	−0.30	0.92	1.23	3.75	18.59
1997	11.42	23.89	44.10	−4.04	−0.32	0.88	1.22	3.77	23.05
1998	12.27	24.76	48.01	−3.63	−0.30	1.06	1.24	3.96	21.85
1999	13.31	27.03	55.34	−3.88	−0.35	0.85	1.12	3.67	23.04
2000	15.49	30.55	61.91	−3.47	−0.42	0.86	1.09	3.68	20.55
2001	14.53	30.17	69.55	−3.28	−0.57	0.81	1.30	4.05	19.08
2002	13.81	27.54	69.26	−3.55	−0.63	0.89	1.41	4.50	22.11
2003	12.03	25.57	81.23	−4.46	−1.14	0.63	1.55	5.58	27.95
2004	10.52	23.81	74.79	−4.87	−1.20	0.77	1.71	6.78	33.38
2005	9.39	22.33	55.73	−5.48	−1.34	0.76	1.76	7.70	38.66

(with the 95th percentile of kurtosis in the cross-section above 18 and medians consistently above three). As Bakshi, Kapadia, and Madan (2003) point out, skewed risk-neutral distributions imply that the physical distribution is skewed, fat-tailed, or both. In either case, the underlying physical distribution is non-Gaussian.

There are clear patterns in the time series of these moments over the sample period, as well as evidence of interactions between them. Volatility peaks in 2000, during the height of the Internet bubble, then declines through 2005.[3] The median risk-neutral skewness is negative, indicating that the distribution is left-skewed; the median value stays relatively flat through 2000 after which it declines sharply, while the median kurtosis estimate increases during the same period, more than doubling from 2000 through 2005. In very broad terms, the results in Table I imply that the estimates of higher moments that we

[3] There is some disagreement over whether the 1998 to 2000 run-up in the prices of technology stocks represents a "bubble." In fact, part of the motivation for our analysis is to examine the possibility that the high valuations given to these securities are related to higher moments of their payoff distribution (see Section IV.B). Despite this disagreement, we will use the phrase "Internet bubble" throughout the paper as a convenient short-hand description of price behavior over this interval.

obtain using the Optionmetrics sample and the method of Bakshi, Kapadia, and Madan (2003) are related to price movements in the underlying market.

II. Ex Ante Higher Moments and the Cross-Section of Returns

Our focus in this section is on testing whether estimates of the ex ante higher moments of the payoff distribution obtained from options data are related to the subsequent returns of the underlying security.

A. Arbitrage Issues

Under the assumption that no-arbitrage rules hold between the options market and the underlying security prices, the information set contained in both cash and derivatives markets should be the same. Several authors have shown that information in option prices can provide valuable forecasts of features of the payoff distributions in the underlying market. For example, Bates (1991) examines option prices (on futures contracts) prior to the market crash of 1987 and concludes that the market anticipated a crash in the year, but not the 2 months, prior to the October market decline. He also shows that fears of a crash increased immediately after the crash itself.

Our sample period includes the Internet bubble. Some researchers have argued that option prices and equity prices diverged during this period. For example, Ofek and Richardson (2003) propose that the Internet bubble is related to the "limits to arbitrage" argument of Shleifer and Vishny (1997). This argument requires that investors could not, or did not, use the options market to profit from mispricing in the underlying market. Ofek and Richardson (2003) also provide empirical evidence that option prices diverged from the (presumably misvalued) prices of the underlying equity during this period. However, based on a different data set of option prices from that of Ofek and Richardson (2003), Battalio and Schultz (2006) conclude that shorting synthetically using the options market was relatively easy and cheap, and that short-sale restrictions are not the cause of persistently high Internet stock prices. A corollary to their results is that option prices and the prices of underlying stocks did not diverge during the Internet bubble and hence they argue that Ofek and Richardson's results may be a consequence of misleading or stale price quotes in their options data set. Note that, if option and equity prices do not contain similar information, then our tests should be biased against finding a systematic relation between estimates of higher moments obtained from option prices and subsequent returns in the underlying market.[4] However, motivated by the Battalio and Schultz results, we employ additional filters to try to ensure that our results are not driven by stale or misleading prices. In addition to

[4] Robert Battalio graciously provided us with the OPRA data used in their analysis; unfortunately, these data, provided by a single dealer, do not have a sufficient cross-section of data across calls and puts to allow us to estimate the moments of the risk-neutral density function in which we are interested.

eliminating option prices below 50 cents and performing robustness checks with additional constraints on option liquidity as mentioned above, we also remove options with less than 1 week to maturity and eliminate days in which closing quotes on put-call pairs violate no-arbitrage restrictions.

B. Portfolio Sorts

Each day, we sample the prices of out-of-the-money calls and puts on individual securities that have expiration dates that are closest to 0.083 years (1 month), 0.250 years (3 months), 0.500 years (6 months), and 1.000 years to maturity and midpoint bid and offer prices of $0.50 or greater. As documented in Dennis and Mayhew (2009), the procedure for calculating risk-neutral moments is most accurate when we have an equal number of puts and calls. If there are a greater number of puts than calls, we retain only the same number of puts as we have calls. The puts that we retain in this circumstance are those that are closest to, but out-of-the-money. We also require that there be trading volume in out-of-the-money options for that firm at the selected maturity on the trading day. We then use the Bakshi, Kapadia, and Madan procedure outlined in the previous section to estimate volatility, skewness, and kurtosis for horizons of 1, 3, 6, and 12 months. The resulting output of this procedure is a set of three risk-neutral moments (volatility, skewness, and kurtosis) for four horizons (1, 3, 6, and 12 months) for each firm on each day that data are available. At the daily frequency, a number of firms exhibit apparent outliers in measures of skewness and kurtosis, which appear to be a result of data errors. We remove observations in the top 1% and bottom 1% of the cross-sectional distribution of volatility, skewness, and kurtosis each day to mitigate the effect of these outliers. Finally, we delete observations on firms that have less than 10 trading days of observations in a given calendar month.

Following Bakshi, Kapadia, and Madan (2003), we average the daily estimates for each stock over time (in our case, the calendar quarter). Thus, each firm in our sample has a single observation for volatility, skewness, and kurtosis for each maturity (1, 3, 6, and 12 months) over the period starting in the first quarter of 1996 and ending in the fourth quarter of 2005. We rank each firm within the quarter on the basis of its maturity-dependent volatility, skewness, and kurtosis into terciles. The "extreme" terciles contain 30% of the sample, while the middle tercile contains 40% of the sample. This sorting procedure results in 12 rankings per firm-quarter, on the basis of three moments and four maturities. We then use the rank to form equally weighted portfolios over the subsequent calendar quarter, holding the moment tercile rank fixed. The result is a total of 36 portfolios ranked on horizon-dependent moments, with returns sampled at the monthly frequency over the period April 1996 through December 2005.

In Table II, we report results for portfolios sorted on the basis of estimated volatility, skewness, and kurtosis. We focus on two maturity bins, 3 (Panel A) and 12 (Panel B) months; all other maturities are discussed in the

Table II

Descriptive Statistics: Risk-Neutral Moment Portfolios

Panels A and B present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30th and 70th percentiles. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia, and Madan (2003); in Panel A we report results using options closest to 3 months to maturity, and in Panel B results with options closest to 12 months to maturity. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5×5 size and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average individual firm's risk-neutral volatility, skewness, and kurtosis of the stocks in the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. The final row of the table presents t statistics of the null hypothesis that the difference in the third and first terciles is zero. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
			Panel A: 3 Months to Maturity					
Volatility								
1	1.354	0.441	17.837	−1.284	10.764	0.916	15.739	0.349
2	0.987	0.088	28.513	−1.151	9.521	1.354	14.701	0.342
3	0.792	0.081	49.816	−1.273	6.453	1.873	14.114	0.370
3−1	−0.562	−0.360	31.979	0.011	−4.312	0.957	−1.625	0.020
t(3−1)	−0.713	−0.571	44.376	0.151	−5.220	27.246	−21.999	2.284
Skewness								
1	1.448	0.570	31.507	−2.814	15.437	1.246	15.636	0.331
2	1.042	0.205	32.261	−0.980	5.392	1.420	14.741	0.357
3	0.627	−0.216	31.140	0.026	7.327	1.408	14.170	0.370
3−1	−0.821	−0.786	−0.367	2.841	−8.110	0.162	−1.466	0.039
t(3−1)	−2.062	−2.079	−0.633	33.671	−6.456	5.627	−31.693	5.908
Kurtosis								
1	0.631	−0.247	37.250	−0.369	2.370	1.523	13.996	0.388
2	1.106	0.308	31.710	−0.955	5.449	1.386	14.770	0.351
3	1.355	0.458	26.134	−2.450	20.302	1.177	15.766	0.322
3−1	0.724	0.705	−11.116	−2.081	17.933	−0.346	1.770	−0.066
t(3−1)	2.011	2.119	−26.365	−24.244	14.378	−10.839	45.560	−8.732
			Panel B: 12 Months to Maturity					
Volatility								
1	1.338	0.441	17.488	−1.295	10.908	0.897	15.846	0.350
2	1.014	0.142	27.870	−1.115	9.316	1.355	14.686	0.340
3	0.772	0.004	47.956	−1.238	6.469	1.896	14.028	0.373
3−1	−0.566	−0.437	30.468	0.057	−4.439	0.998	−1.819	0.023
t(3−1)	−0.694	−0.679	50.135	0.789	−5.374	27.299	−26.073	2.375
Skewness								
1	1.446	0.548	30.156	−2.743	15.207	1.261	15.591	0.330
2	0.979	0.153	31.313	−0.974	5.480	1.414	14.754	0.355

(*Continued*)

Table II—*Continued*

Panel B: 12 Months to Maturity

Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
3	0.714	−0.123	30.689	0.019	7.323	1.400	14.197	0.373
3−1	−0.732	−0.671	0.533	2.761	−7.884	0.140	−1.394	0.043
t(3−1)	−1.864	−1.783	1.073	31.855	−6.286	5.085	−31.502	6.601
Kurtosis								
1	0.622	−0.223	36.600	−0.371	2.408	1.512	13.998	0.390
2	1.127	0.295	30.441	−0.951	5.533	1.387	14.782	0.352
3	1.337	0.459	25.416	−2.382	20.036	1.188	15.747	0.318
3−1	0.715	0.682	−11.184	−2.010	17.627	−0.324	1.749	−0.072
t(3−1)	2.055	2.120	−25.670	−23.057	14.085	−11.175	41.433	−9.975

Internet Appendix.[5] Specifically, we report the subsequent raw returns of the equally weighted moment-ranked portfolios over the next month in the column with the label "Mean." In the next column, we report the characteristic-adjusted return over the same month. To calculate the characteristic-adjusted return, we perform a calculation similar to that in Daniel et al. (1997). For each individual firm, we assess to which of the 25 Fama and French (1993) size- and book-to-market-ranked portfolios the security belongs. We subtract the return of that Fama and French (1993) portfolio from the individual security return and then average the resulting excess or characteristic-adjusted "abnormal" return across firms in the moment-ranked portfolio. In the next three columns of Table II, we report the average firm's risk-neutral volatility, skewness, and kurtosis estimates for each of the ranked portfolios. Finally, we report average betas, average (log) market values, and average book-to-market equity ratios of the securities in the portfolio.

Summary statistics in Panel A of Table II imply a strong negative relation between volatility and subsequent raw returns; for example, in the 3-month maturity options, the returns differential between high volatility (Portfolio 3) and low volatility (Portfolio 1) securities is –56 basis points per month; longer 12-month maturities in Panel B have differentials of –69 basis points per month. The magnitude of this difference is not statistically significant. We speculate that this result is more due to the relatively small sample size (117 monthly observations) than a meaningful difference in the return differential across maturities. Low (high) volatility portfolios tend to contain low (high) beta firms and larger (smaller) firms, while differences in book-to-market equity ratios across portfolios are relatively small and differ across maturity bins. We adjust for these differences in size and book-to-market equity ratio in the characteristic-adjusted return column. After adjusting for the differences in size and book-to-market equity observed across the volatility portfolios, the

[5] An Internet Appendix for this article is available online in the "Supplements and Datasets" section at http://www.afajof.org/supplements.asp.

return differentials are smaller. However, although the differential is reduced, it remains economically significant, with the lowest volatility portfolios earning –36 (–44) basis points, for 3- (12-)month maturity options, more than the highest volatility portfolios per month.

There is virtually no relation between volatility and skewness estimates in the sample. The relation between volatility and kurtosis is much stronger: as average volatility increases in the portfolio, kurtosis declines. Thus, the relation between volatility and returns may be confounded by the effect, if any, of other moments on returns; we examine this possibility later in Section II.C. Finally, the average number of securities in each portfolio indicates that the portfolios should be relatively well diversified. The top and bottom tercile portfolios average 92 firms, whereas the middle tercile portfolio averages 123 firms. Combined with the fact that we are sampling securities that have publicly traded options, this breadth should reduce the effect of outlier firms on our results.

Most interestingly, we see significant differences in Table II in returns across skewness-ranked portfolios. The raw returns differential is negative for 3- and 12-month maturities, at –82 and –73 basis points per month, respectively. That is, on average, in each maturity bin the securities with lower skewness earn higher returns in the next month, while securities with less negative or positive skewness earn lower returns. The differentials in raw returns are of the same order of magnitude and somewhat larger than those observed in the volatility-ranked portfolios, and the difference is statistically different from zero at the 10% level or better. Compared to the volatility-ranked portfolios, the skewness-ranked portfolios show relatively little difference in their betas, and comparable differences in their market value and book-to-market equity ratios. When we adjust for the size- and book-to-market characteristics of securities, the characteristic-adjusted returns hardly change, averaging –79 and –67 basis points per month, respectively, across the two maturity bins.[6]

In addition to the differences in returns, the table indicates that there is a negative relation between skewness and kurtosis. That is, kurtosis declines as we move across skewness-ranked portfolios. As in Panel A, interactions between other moments and returns could be masking or exacerbating the relation between skewness and returns. Consequently, in later tests we control for the relation of other higher moments to returns in estimating their effect.

[6] In a different application, Xing, Zhang, and Zhao (2010) find a positive relation between a skewness metric taken from option prices and the next month's returns. Their measure of skewness is the absolute value of the difference in implied volatilities of out-of-the-money call option contracts, where the strike price is constrained to be within the range of $0.8S$ to S, (where S is the current price of the underlying stock), and preferably in the range of $0.95S$ to S. Thus, their skewness measure is related to the slope of the volatility smile over a smaller range of strike prices. We conduct a Monte Carlo exercise using a Heston model with plausible parameter values, to compare the performance of our skewness metric to theirs in a setting where skewness is known. In this controlled environment we find that the slope estimate of skewness used by Xing, Zhang, and Zhao is relatively noisy (using a mean squared error metric) compared to the Bakshi, Kapadia, and Madan approach we use. The simulation details are reported in the Internet Appendix.

Both panels in Table II also report the results when securities are sorted on the basis of estimated kurtosis. Generally, we see a positive relation between kurtosis and subsequent raw returns; the return differential is economically significant, at approximately 72 basis points per month across each of the two maturities. As with the other moment-ranked portfolios, the effect is reduced after adjusting for book-to-market and market capitalization differences, but the differences are very slight and the effect remains highly economically significant across the two maturity bins and statistically significant at the 5% critical level. As in the other panels, we also observe patterns in the other estimated moments, with both volatility and skewness decreasing as kurtosis increases.[7]

C. Multivariate Sorts

We estimate the relation between higher moments and subsequent returns, while controlling for variation in other higher moments, using double and triple sorts.[8] In the double-sorting method, we sort firms into tercile portfolios based independently on volatility, skewness, and kurtosis. We then form portfolios based on the intersection of rankings of volatility and either skewness or kurtosis.[9] For each of the nine portfolios formed, we report subsequent returns. The results from sorting on volatility and skewness, for 3- and 12-month options, are reported in Panel A of Table III; the results from sorting on volatility and kurtosis are reported in Panel B. In each panel, the number of firms in each portfolio is reported in parentheses below the returns. Reading across the columns of Panel A in Table III, we observe that, holding volatility constant, skewness continues to be negatively related to subsequent returns for all levels of volatility and for both maturities. The effect is monotonic in four out of six cases and the return differential across the extreme skewness terciles varies from –19 to –100 basis points per month. This magnitude is roughly consistent with the magnitude of the return differential in univariate-sorted portfolios in Table II. The table also highlights the fact that the negative relation between risk-neutral volatility and subsequent returns is found primarily in the middle and high skewness firms.

[7] We also recalculated our results using value- rather than equal-weighting, and found similar differentials for skewness and kurtosis, although the sign of the volatility relation is reversed for options at some maturities. For example, in volatility-sorted portfolios, spreads for 1-, 3-, 6-, and 12-month options are 10, 39, –20, and 17 basis points per month, respectively. For skewness-sorted portfolios, spreads are consistently negative and of similar magnitude across maturities; for kurtosis-sorted portfolios, we continue to observe positive differentials, although they vary somewhat more in magnitude across option maturities.

[8] We explored the use of Fama and MacBeth (1973) regressions to estimate the effect of variation in individual higher moments, while holding other moments constant, at the individual firm level. Unfortunately, the number of firms combined with the noise in the moment estimates provided insufficient power to draw reliable inferences about the relations between moments and returns.

[9] Sorts on skewness and kurtosis resulted in portfolios that were too sparse to make inferences; empirically, it's rare to observe very skewed distributions (in either direction) that do not have high kurtosis.

Table III

Risk-Neutral Moment Double- and Triple-Sorted Portfolios

The table presents the results of multi-way sorts on risk-neutral moments. We independently sort firms into tercile portfolios based on volatility, skewness, and kurtosis, and then form portfolios on the intersection of volatility and either skewness or kurtosis. For each of the nine portfolios formed, we report the average of subsequent returns. The results from sorting on volatility and skewness, for 3- and 12-month options, are reported in Panel A, and the results from sorting on volatility and kurtosis are reported in Panel B. We present results from sorting on medians of volatility, skewness, and kurtosis independently in Panel C. In Panels A and B, the number of firms in each portfolio are reported in parentheses below the returns.

Panel A: Volatility-Skewness Sorts

	3 Months to Maturity				12 Months to Maturity		
	S1	S2	S3		S1	S2	S3
V1	1.409	1.420	1.048	V1	1.351	1.288	1.005
N	(52)	(39)	(32)	*N*	(53)	(39)	(31)
V2	1.044	0.902	0.765	V2	1.132	0.949	0.944
N	(33)	(25)	(34)	*N*	(34)	(25)	(33)
V3	1.374	0.650	0.369	V3	1.376	0.419	0.453
N	(37)	(28)	(27)	*N*	(36)	(28)	(28)

Panel B: Volatility-Kurtosis Sorts

	3 Months to Maturity				12 Months to Maturity		
	K1	K2	K3		K1	K2	K3
V1	1.144	1.346	1.438	V1	0.963	1.325	1.366
N	(34)	(37)	(52)	*N*	(33)	(38)	(52)
V2	1.190	0.513	0.946	V2	1.211	0.683	1.254
N	(37)	(29)	(26)	*N*	(37)	(29)	(26)
V3	0.349	1.409	1.213	V3	0.377	1.530	0.829
N	(51)	(27)	(14)	*N*	(52)	(26)	(14)

Panel C: Volatility-Skewness-Kurtosis Sorts

	V1S1K1	V1S1K2	V1S2K1	V1S2K2	V2S1K1	V2S1K2	V2S2K1	V2S2K2
Three Months to Maturity								
Mean	1.623	1.304	1.087	1.002	1.250	1.196	0.592	0.872
N	(8)	(71)	(52)	(22)	(25)	(49)	(68)	(11)
12 Months to Maturity								
Mean	1.424	1.209	1.110	1.069	1.301	1.268	0.621	0.626
N	(9)	(72)	(51)	(22)	(24)	(48)	(70)	(12)

In Panel B, where we sort on volatility and kurtosis, the results are also generally consistent with the univariate kurtosis sorts in Table II. In five out of six cases, holding volatility constant, the return differential in extreme kurtosis portfolios is positive, and varies between 4 and 86 basis points per month. In the sixth case (moderate volatility portfolios and 3-month options), the return differential is –24 basis points per month. The negative relation between risk-neutral volatility and subsequent returns occurs largely among firms with relatively low (tercile 1) or high (tercile 3) kurtosis.

We also perform a multivariate independent sort on all three higher moment estimates. Clearly, the number of firms in each of the portfolios would decline sharply if we form tercile portfolios for each moment, and many portfolios would be empty. As a consequence, for the triple sort, we sort into two portfolios only. These results are presented in Panel C of Table III. While the number of firms in some portfolios is relatively small, the results are fairly clear. If we hold skewness and kurtosis levels constant, we continue to see a negative relation between volatility and subsequent returns. In seven out of eight cases, the differential is negative, varying between –11 and –50 basis points per month. If we hold volatility and kurtosis constant, we continue to see a negative relation between skewness and subsequent returns, with the differential varying between –14 and –68 basis points per month. However, when we control for *both* volatility and skewness, the kurtosis effect does not appear to be stable. In six of the eight cases, increasing kurtosis is associated with a *decline* in returns, with the magnitude of the return differential varying from –32 to –3 basis points. In the remaining two cases, the effect of increasing kurtosis is positive (at 1 and 28 basis points per month, respectively).

Overall, the results in Tables II and III imply that, on average, higher moments in the distribution of securities' payoffs are related to subsequent returns. Consistent with the evidence in Ang et al. (2006), we see that securities with higher volatility have lower subsequent returns. We also find that securities with higher skewness have lower subsequent returns. Finally, while higher kurtosis is related to higher subsequent returns in individual sorts, this effect is not robust when controlling for variation in volatility and skewness.

The evidence that the relation between kurtosis and returns is relatively weak compared to skewness is consistent with the evidence in Chang, Christoffersen, and Jacobs (2009). In addition, the evidence that skewness in individual securities is negatively related to subsequent returns is consistent with the models of Barberis and Huang (2008) and Brunnermeier, Gollier, and Parker (2007). In their papers, they note that investors who prefer positively skewed distributions may hold concentrated positions in securities whose payoffs are more right-skewed—that is, investors may trade off skewness against diversification, since adding securities to a portfolio will increase diversification, but at the cost of reducing skewness. The preference for skewness will increase the demand for, and consequently the price of, securities with higher skewness and so reduce their expected returns. The magnitude of the differential for skewness that we find is consistent with the empirical results in Boyer, Mitton, and Vorkink (2010), who generate a cross-sectional model of expected skewness in physical distributions for individual securities and find that portfolios sorted on expected skew generate a return differential of approximately 67 basis points per month. Finally, our results in monthly returns are consistent with the longer-horizon results of Green and Hwang (2009), who find that while IPOs with high expected skewness have significantly greater first-day returns, they tend to earn significantly lower returns over the next 3 to 5 years.

Despite the fact that the negative relation between skewness and returns, for which we find evidence, is consistent with behavioral explanations as in Barberis and Huang (2008), it does not follow that we can rule out rational explanations. For example, in an analysis of investors who optimize over mean,variance, skewness, and kurtosis of returns, Chabi-Yo, Ghysels, and Renault (2010) show that allowing for heterogeneity in investors' preferences and beliefs can give rise to additional factors (beyond co-moments) in the pricing of nonlinear risks. Moreover, Mitton and Vorkink (2007) show that allowing for heterogeneity in investors' preferences for skewness can also lead to right-skewed securities having higher prices.

D. Factor-Adjusted Returns

In Table II, we adjust for the differences in characteristics across portfolios, following Daniel et al. (1997), by subtracting the return of the specific Fama and French (1993) portfolio to which an individual firm is assigned. However, Fama and French (1993) interpret the relation between characteristics and returns as evidence of risk factors. Consequently, we also adjust for differences in characteristics across our moment-sorted portfolios by estimating a time series regression of the High-Low portfolio returns for each moment on the three factors proposed in Fama and French (1993).

Results of this risk adjustment are reported in Table IV. The dependent variables in our regressions are the monthly returns from portfolios reformed each month (as in Table II), where the portfolios are either the tercile sorts or a long position in the portfolio of securities with the highest estimated moments and a short position in the portfolio of securities with the lowest estimated moments. The three factors in the regressions are the return on the value-weighted market portfolio in excess of the risk-free rate ($r_{MRP,t}$), the return on a portfolio of small capitalization stocks in excess of the return on a portfolio of large capitalization stocks ($r_{SMB,t}$), and the return on a portfolio of firms with high book-to-market equity in excess of the return on a portfolio of firms with low book-to-market equity ($r_{HML,t}$). Since the dependent portfolios in our analysis are equally weighted, we construct factors on the basis of equally weighted portfolio returns as well. Firms are again grouped by maturity and sorted into portfolios on the basis of estimated moments (volatility, skewness, and kurtosis) using options closest to 3 and 12 months to maturity, respectively.[10] We report in Table IV intercepts, slope coefficients for the three factors, and adjusted R^2s. Test statistics for the null hypothesis that the coefficient is zero are presented below the point estimates. The first panel in Table IV contains the results for volatility-sorted portfolios. Consistent with the results in Table II for characteristic-adjusted returns, we observe negative alphas in our "High-Low" portfolio. The point estimates of the intercepts suggest that risk adjustment has little economic impact on the magnitude of the returns to the portfolios. Alphas for zero-cost portfolios sorted on the basis of 3-month

[10] The 1- and 6-month maturities reported in the Internet Appendix yield similar findings.

Table IV

Fama–French Factor Risk Adjustment: Risk-Neutral Moment-Sorted Portfolios

The table presents the results of time series regressions of excess return differentials (High-Low) between portfolios ranked on risk-neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a 1-month T-bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). The moment-sorted portfolios are equally weighted, formed on the basis of terciles, and reformed each quarter. The table presents point estimates of the coefficients and t-statistics. In Panel A, we use options closest to 3 months to maturity to calculate risk-neutral moments; 12 month options are used in Panel B. Data cover the period April 1996 through December 2005 for 117 monthly observations.

Rank	α	β_{MRP}	β_{SMB}	β_{HML}	Adj. R^2
Panel A: 3 Months to Maturity					
Volatility					
1	0.648	0.762	−0.448	−0.240	0.835
	3.337	17.817	−9.652	−4.340	
2	0.465	0.977	−0.383	−0.803	0.877
	1.652	15.750	−5.684	−10.010	
3	0.170	1.313	−0.230	−1.183	0.880
	0.414	14.551	−2.351	−10.149	
3−1	−0.478	0.550	0.218	−0.943	0.748
	−1.109	5.786	2.113	−7.674	
Skewness					
1	0.975	0.831	−0.240	−0.743	0.770
	2.630	10.173	−2.709	−7.042	
2	0.445	1.029	−0.368	−0.777	0.917
	1.911	20.057	−6.614	−11.716	
3	−0.126	1.175	−0.459	−0.716	0.896
	−0.455	19.314	−6.946	−9.110	
3−1	−1.100	0.344	−0.219	0.027	0.129
	−2.717	3.855	−2.255	0.235	
Kurtosis					
1	−0.223	1.237	−0.358	−0.709	0.905
	−0.804	20.211	−5.392	−8.965	
2	0.524	1.021	−0.395	−0.775	0.912
	2.210	19.554	−6.961	−11.477	
3	0.961	0.780	−0.304	−0.752	0.792
	2.916	10.741	−3.863	−8.018	
3−1	1.184	−0.457	0.054	−0.043	0.327
	3.677	−6.441	0.698	−0.472	
Panel B: 12 Months to Maturity					
Volatility					
1	0.666	0.734	−0.453	−0.246	0.829
	3.460	17.306	−9.828	−4.495	
2	0.500	0.971	−0.384	−0.806	0.878
	1.788	15.746	−5.737	−10.110	

(*Continued*)

Table IV—*Continued*

Rank	α	β_{MRP}	β_{SMB}	β_{HML}	Adj. R^2
Panel B: 12 Months to Maturity					
3	0.104	1.350	−0.224	−1.173	0.877
	0.247	14.551	−2.229	−9.785	
3−1	−0.562	0.615	0.228	−0.926	0.749
	−1.262	6.264	2.141	−7.298	
Skewness					
1	0.975	0.829	−2.232	−7.747	0.771
	2.630	10.146	−2.612	−7.076	
2	0.396	1.035	−0.380	−0.798	0.914
	1.657	19.642	−6.649	−11.720	
3	−0.059	1.169	−0.451	−0.684	0.901
	−0.224	20.065	−7.135	−9.087	
3−1	−1.034	0.341	−0.220	0.063	0.124
	−2.579	3.854	−2.292	0.549	
Kurtosis					
1	−0.246	1.231	−0.354	−0.684	0.906
	−0.908	20.585	−5.459	−8.853	
2	0.572	1.011	−0.382	−0.804	0.909
	2.347	18.830	−6.554	−11.582	
3	0.921	0.800	−0.326	−0.739	0.799
	2.853	11.246	−4.217	−8.037	
3−1	1.167	−0.431	0.029	−0.054	0.309
	3.699	−6.202	0.382	−0.605	

(12-month) options are –48 (–56) basis points. While the point estimates are not statistically distinguishable from zero, we conjecture that this result is again due to a relatively small sample size rather than the precision of the estimates. The magnitude and sign of these excess returns are consistent with those of Ang et al. (2006), who show that firms with high idiosyncratic volatility relative to the Fama and French (1993) model earn "abysmally low" returns.

The patterns in the intercepts for skewness-sorted portfolios are of the same sign as the volatility-sorted alphas but larger in magnitude and statistically significant at the 5% level. For 3-month (12-month) maturity options, the results indicate that a zero-cost portfolio earns –110 (–103) basis points per month relative to Fama and French (1993) risk adjustment. These findings are consistent with the summary statistics in Table II. The negative alphas still imply a "low skewness" premium, that is, securities with more negative skewness earn, on average, higher returns in the subsequent months, while securities with less negative or positive skewness earn lower returns in subsequent months.

We also report in Table IV the results for kurtosis-sorted portfolios. The economic magnitude and statistical significance of the zero-cost portfolio alphas are highest for this set of portfolios. For 3-month (12-month) options, we observe excess returns of 118 (117) basis points per month, statistically distinguishable from zero at the 1% level. Consistent with the results in Table II, we see positive intercepts in portfolios that are long kurtosis. The magnitude of the alphas with respect to kurtosis is comparable to that observed in the skewness- and volatility-sorted portfolios.

There is one other noteworthy feature of Table IV. The explanatory power of the Fama and French (1993) three factors is, on average, lower for the kurtosis-sorted High-Low portfolios, and much lower for the skewness-sorted High-Low portfolios, than the volatility-sorted portfolios. Some of this difference is likely due to the fact that, as Table II shows, skewness- and kurtosis-sorted portfolios exhibit much smaller differences in size and beta than do the volatility-sorted portfolios. However, it is also possible that there are features of the returns on moment-sorted portfolios that are not captured well by the usual firm characteristics. This evidence implies that there is potentially important variation in the returns of higher moment-sorted portfolios that is not captured by the Fama and French (1993) risk adjustment framework.

Recall that the evidence from the multivariate sorts in Section II.C indicate that, while moment estimates are correlated, the effects of volatility and skewness are robust to controlling for variation in other moments; kurtosis effects, in contrast, are much weaker after controlling for variation in volatility and skewness. The results in Table IV are also informative in this regard. The distinct pattern of pricing errors, and in particular the differences in explanatory power of Fama and French (1993) factors between volatility sorts on the one hand and skewness (and to a lesser extent kurtosis) sorts on the other, indicate that our results are not driven exclusively by confounding effects across moment classifications.

E. Robustness Checks

As noted above, one of our concerns following the findings of Battalio and Schultz (2006) is that results might be driven by stale or misleading prices. Consequently, as a robustness check, we perform other tests to examine the possibility that return differentials are driven by liquidity issues, either in the underlying equity returns or by stale or illiquid option prices. For example, we consider alternative minimum price criteria for the options included in our sample. We also risk-adjust returns relative to an aggregate liquidity factor, as in Pástor and Stambaugh (2003). The results presented in this section are robust to these additional requirements, and are discussed in more detail in the Internet Appendix.

III. Higher Moment Returns: Systematic and Idiosyncratic Components

In this section, we analyze the extent to which the cross-sectional relations between higher moments and returns presented in Tables II and IV are due to investors seeking compensation for higher co-moment risk, rather than idiosyncratic moments. We start in Section III.A with a characterization of co-skewness and co-kurtosis in the context of single-factor models, inspired by the analysis in Harvey and Siddique (2000) and Bakshi, Kapadia, and Madan (2003).

In Section III.B, we estimate the relation of risk-neutral co-moments to returns in our sample. In Section III.C, we decompose total moments into co-moments and idiosyncratic moments and examine the relationship of these components to subsequent returns. In a final subsection we report on various robustness checks using more general specifications.

A. Co-Skewness, Co-kurtosis, and a Single-Factor Model

Bakshi, Kapadia, and Madan (2003) suggest a procedure for computing the co-skewness of an asset with a factor. They assume a single-factor data generating process:

$$r_{i,t} = a_i + b_i r_{m,t} + e_{i,t}, \tag{5}$$

where $e_{i,t}$ is assumed to be independent of $r_{m,t}$. The authors note that, if the parameters a and b are "risk-neutralized," equation (5) is also well defined under the risk-neutral measure. With this single factor model, co-skewness can be calculated as:

$$\begin{aligned} COSKEW_t^Q(r_{i,t+\tau}, r_{m,t+\tau}) &= \frac{E_t^Q\left[(r_{i,t+\tau} - E_t^Q[r_{i,t+\tau}])(r_{m,t+\tau} - E_t^Q[r_{m,t+\tau}])^2\right]}{\sqrt{VAR_t^Q(r_{i,t+\tau})VAR_t^Q(r_{m,t+\tau})}} \\ &= b_i SKEW_{m,t}^Q(\tau)\frac{VAR_{i,t}^Q(\tau)}{\sqrt{VAR_{m,t}^Q(\tau)}}. \end{aligned} \tag{6}$$

In these expressions, $r_{i,t+\tau}$ is the τ-period return on the underlying security, $SKEW^Q$ is the risk-neutral skewness, and $COSKEW^Q$ is the risk-neutral co-skewness with the single factor m. Note that the formula in equation (6) is the risk-neutral equivalent of the co-skewness measure used by Harvey and Siddique (2000) (see their equation (11), and equations (26) and (27) in Bakshi, Kapadia, and Madan (2003)).

A similar argument can be invoked to derive co-kurtosis,

$$\begin{aligned} COKURT_t^Q(r_{i,t+\tau}, r_{m,t+\tau}) &= \frac{E_t^Q\left[\left(r_{i,t+\tau} - E_t^Q[r_{i,t+\tau}]\right)\left(r_{m,t+\tau} - E_t^Q[r_{m,t+\tau}]\right)^3\right]}{VAR_t^Q(r_{i,t+\tau})VAR_t^Q(r_{m,t+\tau})} \\ &= b_i \frac{KURT_{m,t}^Q(\tau)}{VAR_{i,t}^Q(\tau)VAR_{m,t}^Q(\tau)}. \end{aligned} \tag{7}$$

Since b_i is a risk-neutral parameter in equations (6) and (7), Bakshi, Kapadia, and Madan (2003) note that it can be estimated from options data. Accordingly, we estimate b_i using the procedure in Coval and Shumway (2001). Specifically, we compute the risk-neutral b_i as

$$b_i = \frac{S_{i,t}}{C_{i,t}}\mathcal{N}\left(\frac{\ln(S_{i,t}/K_i) + (r - \delta + 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}\right)\beta_i, \tag{8}$$

where $\mathcal{N}$ represents the normal distribution, δ is the stock's dividend yield, σ^2 is the volatility of the underlying stock return, and β_i is the slope coefficient from a projection of underlying stock returns on the single factor.[11] We estimate β_i using 1 year of past daily returns to the underlying equity, regressed on the S&P 500, ending on the day of the observed option prices. To avoid cross-sectional biases in β related to cross-sectional variation in liquidity, we use the procedure in Dimson (1979) that corrects for infrequent trading; our reported results use a 1-day lead and lag of the market return as additional regressors. We also compute Dimson β's with five leads and lags of the market return, and find very little difference in the results.

B. Relation of Risk-Neutral Co-moments to Returns

Given estimates of b_i, we compute co-skewness and co-kurtosis using equations (6) and (7); note that the estimate of b_i corresponds directly to a measure of risk-neutral covariance with the single factor m. Previous authors, such as Harvey and Siddique (2000) and Dittmar (2002), have reported significant cross-sectional relationships between physical co-moments and returns. We examine whether there are similar relationships between risk-neutral co-moments and expected returns. We form co-moment-sorted portfolios that are analogous to the total moment-sorted portfolios used in Table II. That is, we first calculate daily risk-neutral co-moments. We average these co-moments over the calendar quarter, rank into terciles, and then form equal-weight portfolios on the basis of these rankings over the next 3 months. Reported results in Table V have the same structure as the results for total moments reported in Table II. The portfolios sorted on risk-neutral covariance are associated with a positive premium of 41 basis points per month for 3-month options and 65 basis points for 12-month maturity options. This premium is reversed in sign, and roughly similar in magnitude, to the total volatility premium reported in Table II. The premium is substantially attenuated when we adjust for firm characteristics; it is reduced to 14 basis points (36 basis points) for 3-month (12-month) options. This indicates that, similar to the total volatility premium, differences in risk-neutral covariances are associated with significant differences in market capitalization and book-to-market equity ratios.

Differences in co-skewness are associated with significant negative differences in returns of 48 basis points for 3-month options, and 64 basis points for 12 month-options. The magnitude of this differential is of the same sign, and similar in magnitude to, the co-skewness premium reported in Harvey and Siddique (2000); they find a negative premium associated with co-skewness

[11] Coval and Shumway (2001) report that their estimates of b_i following this procedure are very similar to those calculated by directly regressing option returns on the market portfolio. We follow their lead and use the average ratio of $S_{i,t}/C_{i,t}$ across calls and the risk-neutral variance calculated for each security i to compute our estimates of this parameter. We also experimented with using the most and least out-of-the-money calls in our estimates and found little cross-sectional sensitivity to the choice of call options.

Table V

Descriptive Statistics: Risk-Neutral Co-Moment Portfolios

Panels A and B present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral covariance, co-skewness, and co-kurtosis within each calendar quarter into terciles based on 30th and 70th percentiles. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments and co-moments are calculated using the procedure in Bakshi, Kapadia, and Madan (2003). In Panel A we report results using options closest to 3 months to maturity, and in Panel B results with options closest to 12 months to maturity. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5×5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average risk-neutral volatility, skewness, and kurtosis of the stocks in the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
Panel A: 3 Months to Maturity								
Co-variance								
1	0.839	0.078	31.505	−0.879	10.309	1.511	14.457	0.313
2	0.913	0.155	31.159	−1.057	6.912	1.410	14.730	0.337
3	1.253	0.215	32.653	−1.845	9.728	1.224	15.642	0.382
3−1	0.414	0.137	1.148	−0.966	−0.581	−0.286	1.185	0.069
t(3−1)	0.812	0.323	1.337	−13.639	−0.590	−9.487	12.030	8.227
Co-skewness								
1	1.164	0.116	26.782	−1.780	10.119	1.068	15.790	0.366
2	1.093	0.327	32.797	−1.120	7.021	1.459	14.717	0.346
3	0.682	−0.040	35.171	−0.860	9.755	1.614	14.323	0.318
3−1	−0.482	−0.156	8.389	0.920	−0.365	0.545	−1.467	−0.048
t(3−1)	−0.791	−0.316	17.027	13.524	−0.376	22.463	−14.957	−6.440
Co-kurtosis								
1	0.732	0.006	39.428	−0.870	9.224	1.725	14.224	0.324
2	0.966	0.177	32.620	−1.146	7.139	1.445	14.730	0.342
3	1.284	0.260	22.779	−1.734	10.505	0.985	15.867	0.365
3−1	0.552	0.254	−16.650	−0.864	1.281	−0.740	1.643	0.042
t(3−1)	0.802	0.459	−49.652	−13.349	1.316	−27.949	17.534	5.260
Panel B: 12 Months to Maturity								
Co-variance								
1	0.682	−0.063	32.871	−0.792	9.584	1.640	14.265	0.303
2	0.972	0.206	30.316	−1.057	6.952	1.438	14.693	0.341
3	1.332	0.294	28.948	−1.850	10.276	1.077	15.875	0.387
3−1	0.649	0.357	−3.924	−1.058	0.692	−0.563	1.610	0.084
t(3−1)	0.950	0.668	−4.849	−15.490	0.708	−24.419	18.416	13.706
Co-skewness								
1	1.281	0.249	25.814	−1.834	10.563	0.992	15.955	0.378
2	1.039	0.220	31.240	−1.082	6.999	1.465	14.701	0.341
3	0.641	−0.043	34.754	−0.777	9.220	1.702	14.174	0.311

(*Continued*)

Table V—*Continued*

Panel B: 12 Months to Maturity								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
3-1	−0.640	−0.291	8.940	1.058	−1.343	0.710	−1.781	−0.068
$t(3-1)$	−0.867	−0.506	14.338	15.602	−1.391	33.332	−20.626	−10.138
Co-kurtosis								
1	0.753	0.027	36.878	−0.789	9.043	1.776	14.133	0.314
2	0.943	0.137	31.516	−1.104	6.950	1.449	14.687	0.344
3	1.296	0.281	23.339	−1.789	10.817	0.944	16.012	0.372
3-1	0.543	0.254	−13.538	−1.000	1.773	−0.832	1.878	0.058
$t(3-1)$	0.714	0.431	−35.948	−15.145	1.786	−30.623	22.235	7.796

of approximately 30 basis points per month. Adjusting for firm characteristics significantly reduces the co-skewness premium, to −16 basis points for 3-month options and −29 basis points for 12-month options. These results are also broadly consistent with Harvey and Siddique (2000), who link co-skewness to characteristics such as size and book-to-market equity.

Finally, portfolios sorted on co-kurtosis are associated with a positive return differential that is similar in sign and magnitude to portfolios sorted on total kurtosis. When measured using 3-month (12-month) maturity options, the difference in returns is 55 (54) basis points per month. Co-kurtosis is also associated with firm characteristics, and as a consequence these return differentials are also significantly reduced after characteristic adjustment, with characteristic-adjusted returns falling to 25 basis points per month for both option maturities. Although the economic magnitude of covariance, co-skewness, and co-kurtosis premia seem significant, the statistical evidence does not indicate that these premia are significantly different than zero at conventional levels of significance.

Overall, our estimates of risk-neutral co-moments appear to generate dispersion in returns that are consistent with the relation between physical co-moments and returns observed by other researchers. In addition, these risk-neutral co-moments are strongly associated with differences in firm characteristics. This association with characteristics may be due to the co-moments' representations of common factor exposures. Table VI reports the results of time series regressions of the co-moment-sorted High-Low portfolio returns on the three factors proposed in Fama and French (1993) as in Table IV. In sharp contrast to the results in Table IV on moment-sorted portfolios, all of the alphas in Table VI are insignificant, and the R^2s for portfolios sorted on co-skewness and co-kurtosis are substantially higher than the R^2s for skewness- and kurtosis-sorted portfolios. These results imply that a significant fraction of the returns differential in Table II is associated with the idiosyncratic component of higher moments, rather than co-moments with the market portfolio. We explore this further in the next subsection.

Table VI

Fama–French Factor Risk Adjustment: Risk-Neutral Co-Moment-Sorted Portfolios

The table presents the results of time series regressions of excess return differentials (High-Low) between portfolios ranked on risk-neutral covariance, co-skewness, and co-kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a 1-month T-bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). Risk-neutral moments and co-moments are calculated using the procedure in Bakshi, Kapadia, and Madan (2003). The moment-sorted portfolios are equally weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and t-statistics. In Panel A, we use options closest to 3 months to maturity to calculate risk-neutral moments; 12 month options are used in Panel B. Data cover the period April 1996 through December 2005 for 117 monthly observations.

Tercile	α	β_{MRP}	β_{SMB}	β_{HML}	Adj. R^2
		Panel A: 3 Months to Maturity			
Co-variance					
1	0.355	1.116	−0.366	−1.044	0.898
	1.142	16.301	−4.929	−11.807	
2	0.496	0.942	−0.366	−0.906	0.868
	1.634	14.088	−5.044	−10.480	
3	0.355	0.986	−0.380	−0.309	0.792
	1.196	15.084	−5.348	−3.655	
3−1	0.000	−0.129	−0.013	0.736	0.509
	0.001	−1.509	−0.143	6.632	
Co-skewness					
1	0.317	0.893	−0.416	−0.238	0.778
	1.178	15.042	−6.450	−3.096	
2	0.634	1.003	−0.409	−0.909	0.872
	2.069	14.838	−5.582	−10.405	
3	0.201	1.128	−0.272	−1.110	0.898
	0.613	15.590	−3.459	−11.877	
3−1	−0.116	0.235	0.144	−0.873	0.675
	−0.306	2.815	1.590	−8.094	
Co-kurtosis					
1	0.194	1.200	−0.215	−1.155	0.899
	0.556	15.577	−2.571	−11.605	
2	0.500	1.001	−0.459	−0.875	0.876
	1.704	15.467	−6.536	−10.458	
3	0.503	0.823	−0.406	−0.239	0.736
	1.793	13.307	−6.044	−2.985	
3−1	0.309	−0.377	−0.191	0.917	0.734
	0.797	−4.417	−2.060	8.313	
		Panel B: 12 Months to Maturity			
Co-variance					
1	0.332	1.127	−0.354	−1.246	0.880
	0.890	13.693	−3.968	−11.718	
2	0.500	0.964	−0.295	−0.895	0.861
	1.562	13.672	−3.859	−9.819	

(*Continued*)

Table VI—*Continued*

Panel B: 12 Months to Maturity					
Tercile	α	β_{MRP}	β_{SMB}	β_{HML}	Adj. R^2
3	0.372	0.946	−0.485	−0.121	0.852
	1.790	20.672	−9.766	−2.054	
3−1	0.039	−0.181	−0.131	1.125	0.729
	0.101	−2.114	−1.407	10.174	
Co-skewness					
1	0.402	0.863	−0.485	−0.122	0.834
	1.974	19.207	−9.940	−2.102	
2	0.561	1.003	−0.349	−0.913	0.869
	1.772	14.372	−4.610	−10.120	
3	0.218	1.157	−0.283	−1.221	0.877
	0.563	13.553	−3.052	−11.067	
3−1	−0.184	0.294	0.202	−1.099	0.744
	−0.453	3.272	2.072	−9.467	
Co-kurtosis					
1	0.274	1.219	−0.272	−1.233	0.900
	0.766	15.456	−3.173	−12.098	
2	0.465	0.984	−0.349	−0.889	0.864
	1.479	14.187	−4.630	−9.914	
3	0.472	0.827	−0.497	−0.143	0.798
	2.120	16.828	−9.320	−2.247	
3−1	0.198	−0.393	−0.225	1.091	0.799
	0.533	−4.789	−2.529	10.290	

C. Decomposing Total Moment Return Effects

We decompose the return differential observed for total moment-sorted portfolios in Table II into components related to dispersion in co-moments and dispersion in idiosyncratic moments. We begin by regressing the daily series of total moments for each firm on daily co-moments within the calendar quarter:

$$\mathcal{V}^{Q}_{i,t} = \kappa^{\mathcal{V}}_{0i} + \kappa^{\mathcal{V}}_{1i}\, COVAR^{Q}_{i,t} + \zeta^{\mathcal{V}}_{i,t}$$
$$\mathcal{S}^{Q}_{i,t} = \kappa^{\mathcal{S}}_{0i} + \kappa^{\mathcal{S}}_{1i}\, COSKEW^{Q}_{i,t} + \zeta^{\mathcal{S}}_{i,t}$$
$$\mathcal{K}^{Q}_{i,t} = \kappa^{\mathcal{K}}_{0i} + \kappa^{\mathcal{K}}_{1i}\, COKURT^{Q}_{i,t} + \zeta^{\mathcal{K}}_{i,t}.$$

In this specification, idiosyncratic moments are the intercepts, $\kappa^{\mathcal{V}}_{0i}$, $\kappa^{\mathcal{S}}_{0i}$, and $\kappa^{\mathcal{K}}_{0i}$, or the portion of the total moments that are not explained by co-moments.[12] While the relation between total moments and co-moments in the regressions above is significant, the explanatory power of the co-moments is not large.

Following the procedure for the total moments and co-moments above, we sort firms into tercile portfolios on the basis of the idiosyncratic moments.

[12] Note that within the quarter, the average errors must be equal to zero; as a consequence, including the residual in the average idiosyncratic moment would not change our results.

Summary statistics for these portfolios are presented in Table VII, and regressions adjusting for the contribution of Fama and French (1993) risk factors to returns are presented in Table VIII. These tables have the same structure as Tables II and IV in which we sort firms into portfolios on the basis of total moments. The results in Tables VII and VIII mimic to a large extent the total moment results reported in Tables II and IV. That is, while risk-neutral co-moments have significant relations with returns that are consistent with the relations found in physical co-moments by other authors, and while we do find dispersion in co-moments in portfolios sorted on total moments, the returns differentials associated with differences in idiosyncratic moments are not substantially different from those observed in total moments in Table II, and the alpha estimates obtained after Fama and French (1993) risk adjustment for idiosyncratic moments are not substantially different from those for total moments. In general, differences in idiosyncratic moments appear to drive most of the dispersion in total moments, and the returns differential associated with differences in idiosyncratic moments is both statistically and economically significant.

D. Robustness

The analysis in the previous subsections relied on the assumption of a single-factor data generating process. As a robustness check, we explore other approaches to the specification of systematic risks. We describe these results briefly here; they are available in the Internet Appendix.

We consider alternative specifications of the SDF, $M_t(\tau)$, where $M_t(\tau)$ satisfies the Euler equation

$$E_t[M_t(\tau)r_{i,t}(\tau)] = 0 \tag{9}$$

and $r_{i,t}$ is an excess return for asset i. In this setting, inferences about the importance of idiosyncratic moments are relative to a particular specification of the SDF. Failure of the Euler equation condition holding may represent the importance of idiosyncratic risk or misspecification of the SDFs.

We use several methods to estimate $M_t(\tau)$ that allow for higher co-moments to influence required returns. These methods differ in the details of specific factor proxies, the number of higher co-moments allowed, and the construction of the SDF. The goal in each case is to estimate the relation between idiosyncratic moments and residual returns, after adjusting for risk.

We begin by considering a parametric SDF that incorporates information about higher moments of the SDF, and consequently adjusts for securities' co-moment risk with the SDF. This approach is similar to that of Harvey and Siddique (2000) and Dittmar (2002), who examine polynomial SDFs that account for co-skewness and co-kurtosis risk, respectively. The evidence from these tests suggests that the payoffs to higher moment-sorted portfolios, particularly skewness-sorted portfolios, cannot be traced to higher co-moments with respect to a value-weighted market proxy. While the statistical magnitude of

Table VII

Descriptive Statistics: Risk-Neutral Idiosyncratic Moment Portfolios

Panels A and B present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral idiosyncratic volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30th and 70th percentiles. Idiosyncratic moments are calculated by regressing daily estimates of each firm's total moment on measures of the risk-neutral co-moment within a calendar quarter. The average unexplained portion of the moments is used as the measure of idiosyncratic moments. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia, and Madan (2003); in Panel A we report results using options closest to 3 months to maturity, and in Panel B results with options closest to 12 months to maturity. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5 × 5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average risk-neutral volatility, skewness, and kurtosis of the stocks in the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
				Panel A: 3 Months to Maturity				
Idiosyncratic Volatility								
1	1.295	0.358	18.307	−1.362	10.760	0.929	15.832	0.343
2	0.990	0.106	28.727	−1.226	9.756	1.375	14.754	0.337
3	0.691	0.010	49.090	−1.135	5.443	1.879	14.227	0.352
3−1	−0.603	−0.349	30.783	0.227	−5.317	0.950	−1.605	0.008
t(3−1)	−0.751	−0.542	43.943	5.258	−10.422	25.650	−20.252	1.091
Idiosyncratic Skewness								
1	1.331	0.474	31.172	−2.596	14.190	1.261	15.728	0.324
2	0.991	0.176	32.501	−1.030	5.745	1.448	14.780	0.344
3	0.658	−0.190	31.179	−0.166	7.394	1.403	14.306	0.365
3−1	−0.673	−0.664	0.008	2.430	−6.796	0.142	−1.422	0.041
t(3−1)	−1.741	−1.814	0.014	41.477	−6.808	5.687	−33.818	6.159
Idiosyncratic Kurtosis								
1	0.576	−0.257	37.530	−0.540	3.251	1.553	14.111	0.368
2	1.080	0.271	31.799	−0.997	5.770	1.388	14.839	0.344
3	1.290	0.415	25.778	−2.265	18.290	1.196	15.835	0.321
3−1	0.714	0.672	−11.752	−1.725	15.039	−0.357	1.724	−0.046
t(3−1)	1.899	2.098	−27.093	−28.392	14.862	−11.752	42.316	−6.348
				Panel B: 12 Months to Maturity				
Idiosyncratic Volatility								
1	1.282	0.348	17.692	−1.336	10.811	0.901	15.941	0.342
2	0.963	0.102	27.984	−1.146	9.212	1.378	14.755	0.335
3	0.740	0.019	47.234	−1.186	5.998	1.910	14.115	0.357
3−1	−0.542	−0.329	29.542	0.150	−4.813	1.009	−1.826	0.015
t(3−1)	−0.651	−0.499	50.832	3.095	−8.549	25.385	−24.935	1.695

(*Continued*)

Table VII—*Continued*

Panel B: 12 Months to Maturity								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
Idiosyncratic Skewness								
1	1.359	0.466	29.761	−2.618	14.620	1.284	15.672	0.326
2	0.962	0.203	31.108	−1.019	5.803	1.427	14.829	0.339
3	0.668	−0.215	30.988	−0.077	6.770	1.409	14.296	0.369
3−1	−0.692	−0.681	1.227	2.540	−7.850	0.125	−1.376	0.043
t(3−1)	−1.683	−1.699	2.471	37.366	−7.823	4.845	−33.172	5.927
Idiosyncratic Kurtosis								
1	0.657	−0.212	36.734	−0.521	3.243	1.540	14.100	0.371
2	1.012	0.256	30.332	−0.989	5.837	1.387	14.864	0.342
3	1.299	0.401	25.065	−2.213	18.093	1.210	15.814	0.320
3−1	0.642	0.613	−11.670	−1.693	14.850	−0.329	1.713	−0.050
t(3−1)	1.683	1.721	−27.075	−27.646	14.589	−11.455	37.402	−7.007

the pricing errors is not consistent across all specifications, the economic magnitude of the pricing errors is large. As a consequence, relative to the risks associated with returns on an S&P 500 tangency portfolio, the returns to the moment-sorted High-Low portfolios appear to be idiosyncratic.

In a second specification of $M_t(\tau)$, we estimate the parameters of the SDF polynomial using the returns on the tangency portfolio, where the basis assets used to construct the tangency portfolio consist of industry portfolios. This specification yields similar results to those obtained with the S&P 500. Finally, we consider a nonparametric estimate of $M_t(\tau)$, in which we construct the SDF by taking the ratio of the risk-neutral distribution of the market portfolio, constructed from index options data, to estimates of the physical distribution constructed from different samples of historical returns data. Although the precision of the estimates is poor, the results of this method are again similar, with idiosyncratic moments, particularly skewness, still significantly related to subsequent returns.

Overall, the results of our robustness analysis appear to corroborate the evidence we obtain using a simple single-factor model. This evidence indicates that the payoffs of moment-sorted portfolios are less related to systematic exposure to an SDF than they are to idiosyncratic components.

IV. Risk-Neutral and Physical Probability Distributions

Up to this point we have focused on the estimation of risk-neutral moments, and the relationship with subsequent returns. However, models such as Barberis and Huang (2008) and Brunnermeier, Gollier, and Parker (2007) that consider the effects of skewness and fat tails in individual securities' distributions on expected returns deal with investors' estimates of the physical

Table VIII

Fama–French Factor Risk Adjustment: Risk-Neutral Idiosyncratic Moment-Sorted Portfolios

The table presents the results of time series regressions of excess return differentials (High-Low) between portfolios ranked on idiosyncratic risk-neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a 1-month T-bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). Idiosyncratic moments are calculated by regressing daily estimates of each firm's total moment on measures of the risk-neutral co-moment within a calendar quarter. We take the average unexplained portion of the moments and use these as the measure of idiosyncratic moments. Moment-sorted portfolios are equally weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and t-statistics. In Panel A, we use options closest to 3 months to maturity to calculate risk-neutral moments; 12 month options are used in Panel B. Data cover the period April 1996 through December 2005 for 117 monthly observations.

		3 Months to Maturity			
Tercile	α	β_{MRP}	β_{SMB}	β_{HML}	Adj. R^2
Idiosyncratic Volatility					
1	0.576	0.788	−0.487	−0.238	0.835
	2.895	17.962	−10.236	−4.197	
2	0.524	0.956	−0.400	−0.842	0.872
	1.817	15.043	−5.803	−10.257	
3	0.092	1.295	−0.212	−1.199	0.873
	0.216	13.881	−2.089	−9.944	
3−1	−0.485	0.507	0.276	−0.961	0.732
	−1.068	5.073	2.541	−7.439	
Idiosyncratic Skewness					
1	0.891	0.824	−0.294	−0.754	0.762
	2.389	10.021	−3.298	−7.097	
2	0.441	1.009	−0.333	−0.831	0.910
	1.780	18.468	−5.621	−11.772	
3	−0.108	1.188	−0.495	−0.698	0.897
	−0.398	19.786	−7.589	−8.993	
3−1	−1.000	0.364	−0.200	0.056	0.146
	−2.569	4.244	−2.151	0.509	
Idiosyncratic Kurtosis					
1	−0.285	1.272	−0.394	−0.729	0.897
	−0.960	19.438	−5.550	−8.617	
2	0.571	0.973	−0.367	−0.823	0.897
	2.213	17.107	−5.946	−11.194	
3	0.892	0.788	−0.351	−0.736	0.809
	2.895	11.607	−4.758	−8.387	
3−1	1.177	−0.484	0.044	−0.007	0.377
	3.638	−6.793	0.565	−0.075	
		12 Months to Maturity			
Idiosyncratic Volatility					
1	0.595	0.743	−0.469	−0.230	0.818
	2.971	16.838	−9.792	−4.027	

(*Continued*)

Table VIII—*Continued*

12 Months to Maturity					
Tercile	α	β_{MRP}	β_{SMB}	β_{HML}	Adj. R^2
2	0.489	0.981	−0.415	−0.857	0.883
	1.741	15.857	−6.176	−10.727	
3	0.120	1.307	−0.211	−1.188	0.857
	0.264	13.067	−1.945	−9.188	
3−1	−0.475	0.565	0.258	−0.958	0.717
	−0.983	5.304	2.228	−6.965	
Idiosyncratic Skewness					
1	0.905	0.827	−0.261	−0.755	0.758
	2.369	9.822	−2.855	−6.932	
2	0.406	1.023	−0.334	−0.840	0.900
	1.522	17.399	−5.238	−11.058	
3	−0.075	1.166	−0.527	−0.686	0.897
	−0.284	19.938	−8.293	−9.071	
3−1	−0.981	0.339	−0.265	0.069	0.121
	−2.334	3.657	−2.640	0.577	
Idiosyncratic Kurtosis					
1	−0.217	1.276	−0.430	−0.699	0.897
	−0.744	19.815	−6.145	−8.403	
2	0.517	0.977	−0.374	−0.844	0.895
	1.968	16.858	−5.941	−11.263	
3	0.896	0.779	−0.307	−0.738	0.790
	2.732	10.771	−3.908	−7.900	
3−1	1.114	−0.497	0.123	−0.039	0.330
	3.272	−6.629	1.509	−0.400	

distribution. The purpose of this section is to analyze the relation between risk-neutral and physical moments, particularly skewness, in our sample, and whether the predictive power of risk-neutral skewness for subsequent returns is due to its relation to physical moments. In addition, we examine the relation between risk-neutral moments and forward-looking valuation ratios.

A. Risk-Neutral Moments, Physical Moments, and Subsequent Returns

Bakshi, Kapadia, and Madan (2003), who examine the relation between risk-neutral and physical distributions, note that under certain conditions the risk-neutral distribution can be obtained by simply exponentially "tilting" the physical density, with the tilt determined by the risk-aversion of investors. In a related study, Bliss and Panagirtzoglou (2004) assume a time-varying stationary risk aversion function and use estimates of the risk-neutral distribution taken from option prices, and a particular parametric form of the utility function, to estimate physical distributions. While time variation in risk premia, or equivalently risk aversion, may cause differences between risk-neutral and physical distributions over longer intervals, these papers suggest that, with a relatively constant pricing kernel over shorter periods, the cross-sectional variation in risk-neutral and physical moments will capture the same information.

We compute risk-neutral skewness using the standard measure based on the third moment of returns. It would therefore be natural to compute the physical measure third moments by using sample third moments to measure skewness. However, it is well known that skewness estimates based on sample averages are sensitive to outliers, even more so than are estimates of the first two moments. This fact has prompted researchers since Pearson (1895), Bowley (1920), and more recently Hinkley (1975) to look for robust measures of asymmetry that are not based on sample estimates of the third moment. Specifically, Hinkley's (1975) robust coefficient of asymmetry (skewness) is defined as

$$RA_{\theta}\left(r_{t}\right)=\frac{\left(q_{\theta}\left(r_{t}\right)-q_{0.50}\left(r_{t}\right)\right)-\left(q_{0.50}\left(r_{t}\right)-q_{1-\theta}\left(r_{t}\right)\right)}{q_{\theta}\left(r_{t}\right)-q_{1-\theta}\left(r_{t}\right)}, \tag{10}$$

where $q_{1-\theta}(r_t)$, $q_{0.50}(r_t)$, and $q_\theta(r_t)$ are the $1-\theta$, 0.5, and θ quantiles of r_t, and quantile θ is defined as $q_\theta(r_t) = F_n^{-1}(r_t)$, for $\theta \in (0, 1]$.[13] This skewness measure captures asymmetry of quantiles $q_{1-\theta}(r_t)$ and $q_\theta(r_t)$ with respect to the median (i.e., $q_{0.50}(r_t)$). In the specific case of $\theta = 0.75$, we are considering the interquartile range and, in that case, (10) is known as Bowley's (1920) statistic. The normalization in the denominator ensures that the measure is unit independent with values between negative and positive one. When $RA_\theta(r_t) = 0$ the distribution is symmetric, while values diverging to negative (positive) one indicate skewness to the left (right).

Two types of applications of the above measure have been proposed and used in the finance literature. Kim and White (2004), White, Kim, and Manganelli (2008), and Ghysels, Plazzi, and Valkanov (2011) adopt the above measure in a time series context, whereas Zhang (2006) and Green and Hwang (2009) apply the RA measure in a cross-sectional context. We pursue the cross-sectional approach and analyze the relationship between physical and risk-neutral skewness across securities.[14]

The approach of Zhang (2006) consists of pooling a cross-section of stocks along some common characteristic. In particular, Zhang (2006) and Green and Hwang (2009) group stocks by industry and compute the skewness of the distribution of pooled returns of all firms in the industry over the past few months. We are interested in studying the match between risk-neutral and physical measure skewness, so we use risk-neutral skewness as the characteristic, and pool firms within quantiles on the basis of risk-neutral skew. Specifically, we rank firms into three groups on the basis of risk-neutral skew.[15] Within each group, we calculate the quantile-based skewness measure appearing in

[13] The inverse of $F(r_t)$ is unique when it is assumed that $F(r_t)$ is strictly increasing. If $F(r_t)$ is not strictly increasing, then we can define the quantile as $q^*_{\theta_k}(r_t) \equiv \inf\{r : F_n(r_t) = \theta_k\}$.

[14] We are grateful to an anonymous referee for suggesting this method to us.

[15] We considered using the original industry classification as in Zhang (2006) and Green and Hwang (2009). Following this classification, we would first calculate the skew across firms within each industry. Then, one would group firms into skewness bins across industries. Unfortunately, our attempts at following this procedure were unsuccessful. Our industry groupings were very

equation (10) using monthly returns compounded over the 6 months ending in the calendar quarter. Each risk-neutral tercile k will then have a measure of average risk-neutral skew, $S^{Q}_{k,t}$, and a measure of physical skew, $S^{P}_{k,t}$. We use this information to compute frequency tables to assess the frequency with which the risk-neutral skew ranking is equal to the physical skew ranking. Under the hypothesis that the rankings assign firms randomly to each tercile, the frequency with which a first tercile risk-neutral skew firm is a first-tercile physical skew firm is 33.3%.

The results of this tabulation are presented in the left-most sections of Table IX, with results using 3-month options in Panel A and 12-month options in Panel B. In both sets of results, the assignment between P and Q measures of skewness appears to be nonrandom. Particularly in the first and third terciles, there is a propensity for low risk-neutral skew firms to have low physical skewness (48% and 48% for 3- and 12-month maturities), and for high risk-neutral skew firms to have high physical skewness (42% and 44% for 3- and 12-month maturities). While the propensity is somewhat lower for firms with intermediate skew, the ratio still exceeds the null of random assignment of 33.3%. A formal test of this null hypothesis is presented below each table; the χ^2 test of the null suggests strong rejection of the hypothesis of random assignment across terciles. Despite this evidence, while the diagonal elements are dominant, it is also clear that the mapping between Q and P is not perfect. We next ask whether the variation in average returns that we observe due to risk-neutral skewness is more closely associated with the risk-neutral skewness or physical skewness classification. Our approach to answering this question is similar to the contingency table results above. We begin with the risk-neutral terciles from our previous analysis. To generate dispersion in physical skewness within each tercile, we further subdivide each tercile into four groups on the basis of risk-neutral skew. We calculate the quantile-based physical skewness measure within each of these 12 groups, and then, conditional on the risk-neutral tercile ranking, sort on the basis of physical skewness into three groups.[16] As a result, we're left with nine classifications of securities, based on risk-neutral and physical skewness. We form equally weighted portfolio returns of these nine classifications.

Average returns for the portfolios appear in the second set of matrices in the panels of Table IX (titled "Mean Returns"). For ease of comparison, recall that the results in Table II generate a skewness "discount" of 0.82% (0.73%) per month for 3- (12-) month maturity options. In the mean returns panel of

sparsely populated, with half of the industry groupings having an average number of firms smaller than 14, and a third of the groupings having an average number of firms smaller than 10. In addition, six of the industries have months in which there are no data available. In our judgment, these groupings would not generate reliable results and, as a consequence, we pooled firms on the basis of risk-neutral skew.

[16] The three groups are (1) securities with "extreme" high physical skewness (2) securities with "extreme" low physical skewness, and (3) securities in the middle two groups of physical skewness. Using 12 groups helps us balance generating sufficient dispersion in physical skewness and having a sufficient number of firms in each group.

Table IX

Relation between Risk-Neutral and Physical Distribution Sorts

We calculate the quantile-based skewness measure appearing in equation (10) using monthly compounded returns over the past 6 months, ending in the quarter, using all the firms having the same risk-neutral moment classification. This yields for each of the three risk-neutral terciles a physical measure skewness estimate $S^{Q}_{k,t}$ for $k = 1, 2$, and 3, which are ranked from high to low yielding $S^{P}_{k,t}$, for $k = 1, 2$, and 3. The left panels are a contingency table for respectively 3- and 12-month maturity horizons. The percentage for the 1,1 element of the table represents the proportion of times that the lowest risk-neutral skew portfolio is the lowest physical skew portfolio, etc. Below each panel is χ^2 test for the null that all entries are randomly assigned, that is, all entries are 33.33%. Within each tercile of risk-neutral skewness, we calculate the asymmetry measure as in equation (10) as a physical skewness of the portfolio. The mean returns for the nine portfolios appear in the right panels. The numbers in square brackets reported below each of the nine entries are the skewness under Q and the physical measure skews for each of the nine portfolios.

Panel A: 3-Month Horizon															
	Contingency Table				Mean Returns				Risk-Neutral Skew				Physical Skew		
	P1	P2	P3		P1	P2	P3		P1	P2	P3		P1	P2	P3
Q1	47.86	29.06	23.08	Q1	1.250	1.293	1.743	Q1	−2.503	−7.543	−2.741	Q1	0.022	0.264	0.498
Q2	29.06	35.90	35.04	Q2	0.979	1.001	0.981	Q2	−0.943	−0.981	−1.031	Q2	0.053	0.279	0.486
Q3	23.08	35.04	41.88	Q3	1.026	0.461	0.722	Q2	0.055	0.139	0.000	Q3	0.028	0.280	0.488
$\chi^2 = 19.077(0.001)$															
Panel B: 12-Month Horizon															
Q1	47.86	29.91	22.22	Q1	1.089	1.464	1.523	Q1	−2.266	−7.599	−2.623	Q1	0.022	0.270	0.509
Q2	24.79	41.03	34.19	Q2	1.002	0.840	1.101	Q2	−0.964	−0.981	−0.987	Q2	0.023	0.279	0.499
Q3	27.35	29.06	43.59	Q3	0.867	0.708	0.574	Q3	−0.143	0.180	0.084	Q3	0.022	0.283	0.497
$\chi^2 = 22.410(0.000)$															

Table IX, holding physical skew constant, we continue to see average returns that decline as risk-neutral skewness increases (reading down each column). However, holding risk-neutral skewness constant (in each row), there is no clear pattern in average returns as physical skewness increases. For example, in the sample of 3-month options, for low risk-neutral portfolios average returns increase by 49 basis points per month as physical skew increases. For moderate risk-neutral portfolios, the returns are basically flat as physical skew increases, and for high risk-neutral portfolios returns decrease by 0.30% per month.

Overall, the results in Table IX tell us that there is an association between Q and P skewness measures, but that they are far from perfectly related. In addition, controlling for differences in physical skewness, the predictive power of differences in risk-neutral skewness for subsequent returns still holds, while, after controlling for differences in risk-neutral skewness, there is no clear pattern in returns related to physical skew measures. One advantage of risk-neutral skewness is that it is truly a market-based forward-looking prediction, while the skewness measure under P that we have used is historical in nature and therefore may not have the same predictive content. One could potentially invoke parametric prediction models for skewness under P, such as those proposed by Harvey and Siddique (1999). In general, however, it is a challenging task to estimate such time series models for individual firms. In such a setting, the use of option prices eliminates the need for a long time series of returns to estimate the moments of the return distribution.

B. Skewness, Valuation, and the Internet Bubble

We have presented evidence that risk-neutral higher moments are associated with cross-sectional variation in subsequent returns, and that a significant portion of this explanatory power is due to idiosyncratic moments. We have also shown in Table IX that estimates of risk-neutral and physical moments, where the latter are estimated from historical returns, are related, although the predictive power of physical measures, after controlling for risk-neutral moments, is relatively weak. In this subsection, we analyze the relation between higher risk-neutral moments and valuation ratios. Valuation ratios should, in equilibrium, be the (inverse of the) infinite sum of discounted expected future cash flows, where the expectations are taken under the physical distribution. As a consequence, they also represent beliefs at time t about *future* payoffs. Estimating the relation between higher moments and valuation ratios may be particularly interesting during the Internet bubble, which is included in our sample period. In addition, the association between risk-neutral skewness and valuation ratios relates to the analysis in the previous subsection pertaining to the relationship between risk-neutral and physical probability distributions.

We again use measures of risk-neutral higher moments constructed as in Section I. As a valuation measure, we use the earnings-to-price ratio (henceforth E/P) of individual securities.[17] We sort securities into decile portfolios based on risk-neutral higher-moment estimates, and then examine portfolio statistics. We present the results in Table X. In the table, we report the average firm's volatility (in Panel A), skewness (in Panel B), and kurtosis (in Panel C) across each decile portfolio. For each portfolio, we also report the mean of three valuation ratios: FY0, the historical E/P ratio, FY1, the next fiscal year forecast E/P ratio, and FY2, the 2-years-out forecasted E/P ratio.[18] As these results demonstrate, there are strong relations between risk-neutral moments and valuation ratios. Earnings-to-price ratios over all horizons decline as volatility and skewness increase and increase as kurtosis increases. Clearly, ex ante risk-neutral moments estimated from equity options are related to valuation ratios calculated for the underlying securities. In each case, the relation is sharpest for historical E/P measures, while the slope is most attenuated for FY2 forecasts, but the relations are there throughout. Intuitively, securities with higher volatility and higher skewness are more highly valued, that is have lower E/P ratios. For the same level of expected earnings, investors are more willing to pay a higher price for securities that are more volatile, and more right-skewed. In a discounted cash-flow model, this higher valuation should come from lower discount rates, or higher expected growth rates. Conversely, securities that have higher kurtosis have higher E/P ratios, or are valued less highly.

We further divide the sample into two subperiods, using March of 2000 as the dividing point. If the pricing of technology stocks in the 1998 to 2000 period was related to higher moments, we should observe substantial differences in higher moments, or a significant increase in the sensitivity of valuation ratios to higher moments, in the first subperiod. While estimates of higher moments tend to be more extreme during the first subperiod (with the exception of volatility in one case), these differences are relatively small. Moreover, the relations between higher moments and valuation ratios are not more steeply sloped during this subperiod; in fact, if we estimate a linear relation between individual higher moments and E/P ratios for each subperiod, we find that, for every E/P measure (historical, FY1 forecast, and FY2 forecast), the relation between moments and E/P ratios tends to be more pronounced, or steeper, in the second subperiod, with the single exception of FY1 forecasts and volatility. Thus, while the results in this subsection indicate that valuation ratios and option-based risk-neutral higher moments are related, we find little

[17] We use E/P ratios, rather than price-to-earnings, due to the prevalence of negative earnings for some firms in our sample. We do not exclude firms with negative earnings, since negative earnings are common during the Internet bubble, and it is precisely these firms for which higher moments may be more important in assessing valuation. Indeed, in our sample, firms with negative earnings per share tend to be smaller, more volatile, and have higher betas; they also exhibit substantial differences in volatility, skewness, and kurtosis.

[18] The historical E/P ratio is based on the month-end CRSP price per share and most recent Compustat earnings per share. Following Fama and French (1993), we assume that earnings per share data are available no more recently than 6 months after the fiscal year end. The forecast earnings per share are based on I/B/E/S mean estimates for the upcoming and following fiscal year ends.

Table X

Relation of Valuation Ratios to Risk-Neutral Moments

We sort securities into decile portfolios based on risk-neutral moment estimates, and then compute portfolio statistics. The statistics are the mean log market value, book-to-market ratio, beta, and risk-neutral moments of the portfolio. In addition, we calculate earnings to price (E/P) ratios for different measures of earnings. The E/P ratios are FY0, the E/P ratios based on closest monthly historical earnings per share figures (constructed as in Fama and French (1993)); FY1, the E/P ratio based on I/B/E/S consensus mean estimates of next fiscal-year-end's earnings per share; and FY2, the E/P ratio calculated based on I/B/E/S consensus mean earnings per share estimates for the fiscal year ending 2 years after the most recent fiscal year end.

Decile	MV	BM	Beta	Vol	Skew	Kurt	FY0	FY1	FY2
Panel A: Volatility									
1	16.088	0.350	0.817	0.141	−1.334	14.095	5.228	3.454	3.952
2	15.755	0.344	0.882	0.181	−1.253	8.924	4.656	3.510	4.124
3	15.385	0.354	1.050	0.212	−1.264	9.345	4.035	3.363	4.105
4	15.161	0.325	1.157	0.241	−1.372	9.202	4.069	3.116	3.937
5	14.701	0.347	1.288	0.268	−1.059	9.982	3.568	2.987	3.862
6	14.563	0.359	1.413	0.298	−1.101	11.891	2.858	2.442	3.561
7	14.377	0.338	1.568	0.333	−1.073	6.952	−0.094	1.951	3.140
8	14.086	0.362	1.710	0.380	−1.063	6.146	0.450	1.151	2.790
9	14.107	0.355	1.866	0.459	−1.146	5.498	−1.050	0.993	2.843
10	14.152	0.393	2.043	0.658	−1.611	7.706	−5.637	−1.263	1.455
Panel B: Skewness									
1	16.181	0.316	1.078	0.289	−4.279	26.083	3.757	3.116	3.816
2	15.546	0.344	1.266	0.322	−2.404	11.800	2.739	2.613	3.588
3	15.194	0.332	1.389	0.334	−1.793	8.669	2.757	2.004	3.042
4	14.918	0.367	1.449	0.334	−1.385	6.927	2.104	2.503	3.708
5	14.779	0.356	1.431	0.324	−1.094	5.657	1.502	2.029	3.334
6	14.692	0.333	1.416	0.320	−0.838	4.771	1.167	2.130	3.256
7	14.582	0.372	1.387	0.312	−0.608	4.227	0.643	1.475	2.833
8	14.326	0.357	1.405	0.310	−0.366	3.724	0.495	1.836	3.420
9	14.092	0.356	1.394	0.311	−0.100	3.411	0.901	1.632	3.261
10	14.092	0.398	1.425	0.313	0.556	15.066	2.170	2.012	3.449
Panel C: Kurtosis									
1	13.714	0.432	1.485	0.393	−0.182	1.571	−2.353	0.567	2.586
2	14.046	0.376	1.574	0.367	−0.370	2.402	−0.883	0.670	2.619
3	14.212	0.359	1.508	0.358	−0.552	3.120	1.174	1.204	2.666
4	14.509	0.364	1.394	0.331	−0.693	3.862	2.455	2.211	3.525
5	14.684	0.369	1.417	0.327	−0.839	4.722	2.269	2.535	3.664
6	14.846	0.344	1.360	0.309	−0.976	5.827	2.827	2.443	3.562
7	15.042	0.326	1.366	0.301	−1.311	7.371	2.130	2.814	3.837
8	15.352	0.341	1.249	0.279	−1.592	9.485	3.307	2.805	3.842
9	15.662	0.308	1.187	0.262	−2.140	13.111	3.450	2.819	3.662
10	16.293	0.316	1.094	0.242	−3.636	38.691	3.528	3.107	3.785

support for the hypothesis that higher moments of individual firms' payoff distribution contributed significantly to higher valuations during the Internet bubble.

V. Conclusions

We explore the possibility that higher moments of the returns distribution are important in explaining security returns. Using a sample of option prices from 1996 to 2005, we estimate the moments of the risk-neutral density function for individual securities using the methodology of Bakshi, Kapadia, and Madan (2003). We analyze the relation between volatility, skewness, and kurtosis and subsequent returns.

We find a strong relation between these moments and returns. Specifically, we find that high (low) volatility firms are associated with lower (higher) returns over the next month. This result is consistent with the results of Ang et al. (2006). We also find that skewness has a strong negative relation with subsequent returns; firms with less negative or positive skewness earn lower returns. That is, investors seem to prefer positive skewness, and the returns differential associated with skewness is both economically and statistically significant. We also find a positive relation between kurtosis and returns. These relations are robust to controls for differences in firm characteristics, such as firm size, book-to-market ratios, and betas, as well as liquidity and momentum. However, when we control for interactions between volatility, skewness, and kurtosis, we find that the evidence for an independent relation between kurtosis and returns is relatively weak.

We use several different methods to control for differences in higher co-moments, and their related compensation for risk, when estimating the relation between higher moments and returns. These methods range from a simple single-factor data-generating process, suggested in Bakshi, Kapadia, and Madan (2003) and, similar to the method (in physical returns) used in Harvey and Siddique (2000), to a nonparametric calculation of the SDF. After controlling for higher co-moments, we continue to find evidence that idiosyncratic moments matter.

Finally, we examine the relation between risk-neutral and physical skewness. Asset pricing models that consider the effects of skewness and fat tails in individual securities' distributions on expected returns deal with investors' estimates of the physical distribution. We therefore explore the relationship between the two probability measures to relate our empirical findings to the existing theoretical models. We find that P and Q skewness measures are strongly, but not perfectly, related. Risk-neutral skewness is truly a market-based forward-looking prediction, and the relation between risk-neutral moments and valuation ratios, for which we find evidence, is consistent with this interpretation. In contrast, the skewness under P is historical in nature and backward looking by construction, which may explain why the sorts under Q and P do not line up perfectly. However, the fact that there is an association between forward-looking, easy-to-compute skewness sorts under Q and historical sorts under P leads us to believe that the results we obtain relate to existing theoretical asset pricing models.

Initial submission: December 11, 2009; Final version received: September 17, 2012
Editor: Campbell Harvey

Appendix

A.1 Estimation of Risk-Neutral Moments—Details

To estimate the higher moments of the (risk-neutral) density function of individual securities, we use the results in Bakshi and Madan (2000) and Bakshi, Kapadia, and Madan (2003). Bakshi and Madan (2000) show that any payoff to a security can be constructed and priced using a set of option prices with different strike prices on that security. The assumptions used in Bakshi and Madan (2000) are standard for the no-arbitrage option pricing literature, and can cover various configurations of primitive uncertainty, ranging from discrete-time dynamics, continuous-time diffusion, pure-jump, to jump diffusion environments. Bakshi, Kapadia, and Madan (2003), assuming constant instantaneous interest rates, demonstrate how to express the risk-neutral density moments in terms of quadratic, cubic, and quartic payoffs. In particular, Bakshi, Kapadia, and Madan (2003) show that one can express the τ-maturity price of a security that pays the quadratic, cubic, and quartic return on the base security i as

$$V_{i,t}(\tau) = \int_{S_{i,t}}^{\infty} \frac{2(1 - \ln(K_i/S_{i,t}))}{K_i^2} C_{i,t}(\tau; K_i)\, dK_i + \int_0^{S_{i,t}} \frac{2(1 + \ln(K_i/S_{i,t}))}{K_i^2} P_{i,t}(\tau; K_i)\, dK_i \tag{A1}$$

$$W_{i,t}(\tau) = \int_{S_{i,t}}^{\infty} \frac{6(\ln(K_i/S_{i,t})) - 3(\ln(K_i/S_{i,t}))^2}{K_i^2} C_{i,t}(\tau; K_i)\, dK_i + \int_0^{S_{i,t}} \frac{6(\ln(K_i/S_{i,t})) + 3(\ln(K_i/S_{i,t}))^2}{K_i^2} P_{i,t}(\tau; K_i)\, dK_i \tag{A2}$$

$$X_{i,t}(\tau) = \int_{S_{i,t}}^{\infty} \frac{12(\ln(K_i/S_{i,t}))^2 - 4(\ln(K_i/S_{i,t})^3)}{K_i^2} C_{i,t}(\tau; K_i)\, dK_i + \int_0^{S_{i,t}} \frac{12(\ln(K_i/S_{i,t}))^2 + 4(\ln(K_i/S_{i,t})^3)}{K_i^2} P_{i,t}(\tau; K_i)\, dK_i, \tag{A3}$$

where $V_{i,t}(\tau)$, $W_{i,t}(\tau)$, and $X_{i,t}(\tau)$ are the time t prices of τ-maturity quadratic, cubic, and quartic contracts, respectively. $C_{i,t}(\tau; K)$ and $P_{i,t}(\tau; K)$ are the time t prices of European calls and puts written on the underlying stock with strike price K and expiration τ periods from time t. As equations (A1), (A2), and

(A3) show, the procedure involves using a weighted sum of (out-of-the-money) options across varying strike prices to construct the prices of payoffs related to the second, third, and fourth moments of returns.

A.2 Option Data Filters

We do not adjust for early exercise premia in our option prices. As Bakshi, Kapadia, and Madan (2003) note, the magnitude of such premia in OTM calls and puts is very small, and the implicit weight that options receive in our estimation of higher moments declines as they get closer to at-the-money. Using the same method, Bakshi, Kapadia, and Madan (2003) show in their empirical work that, for their sample of out-of-the-money options, the implied volatilities from the Black–Scholes model and a model of American option prices have negligible differences.

In estimating equations (A1) to (A3), we use equal numbers of out-of-the-money (OTM) calls and puts for each stock for each day. Thus, if there are n OTM puts with closing prices available on day t we require n OTM call prices. If there are $N > n$ OTM call prices available on day t, we use the n OTM calls that have the most similar distance from stock to strike as the OTM puts for which we have data. We require a minimum n of two. Dennis and Mayhew (2009) examine and estimate the magnitude of the bias induced in Bakshi–Kapadia–Madan estimates of skewness that is due to discreteness in strike prices. For \$5 (\$2.50) differences in strike prices, they estimate the bias in skewness to be approximately –0.07 (0.05). Since most stocks have the same differences across strike prices, however, the relative bias should be approximately the same across securities, and should not affect either the ranking of securities into portfolios based on skewness or the nature of the cross-sectional relation between skewness and returns that we examine. In our empirical implementation, the moneyness of the options in our sample ranges roughly from 0.8 to 1.2 with on average five equally spaced contracts. We eliminate options in which there is no trading volume in any option of the same maturity. We also eliminate options with prices less than \$0.50 to remove especially thinly traded options. In unreported results, we examine the sensitivity of our results to changing the requirement of options available and both increasing and decreasing the price filter. The results are qualitatively unchanged and are discussed in the Internet Appendix. The resulting set of data consists of 3,722,700 daily observations across firms and maturities over the 1996 to 2005 sample period.

REFERENCES

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2009, High idiosyncratic volatility and low returns: International and further U.S. evidence, *Journal of Financial Economics* 91, 1–23.

Bakshi, Gurdip, Nikunj Kapadia, and Dilip Madan, 2003, Stock return characteristics, skew laws, and the differential pricing of individual equity options, *Review of Financial Studies* 16, 101–143.

Bakshi, Gurdip, and Dilip Madan, 2000, Spanning and derivative-security valuation, *Journal of Financial Economics* 55, 205–238.

Barberis, Nicholas, and Ming Huang, 2008, Stocks as lotteries: The implications of probability weighting for security prices, *American Economic Review* 98, 2066–2100.

Bates, David S., 1991, The crash of 87: Was it expected? The evidence from options markets, *Journal of Finance* 46, 1009–1044.

Battalio, Robert, and Paul Schultz, 2006, Options and the bubble, *Journal of Finance* 61, 2071–2102.

Bliss, Robert R., and Nikolaos Panagirtzoglou, 2004, Option-implied risk aversion estimates, *Journal of Finance* 59, 411–446.

Bowley, Sir Anthony, 1920, *Elements of Statistics* (New York: Charles Scribners Sons).

Boyer, Brian, Todd Mitton, and Keith Vorkink, 2010, Expected idiosyncratic skewness, *Review of Financial Studies* 23, 169–202.

Brunnermeier, Markus K., Christian Gollier, and Jonathan A. Parker, 2007, Optimal beliefs, asset prices, and the preference for skewed returns, *American Economic Review* 97, 159–165.

Chabi-Yo, Fousseni, Eric Ghysels, and Eric Renault, 2010, Factor asset pricing models implied by heterogeneous beliefs and attitudes towards risk, Working paper, Ohio State University and University of North Carolina.

Chang, Bo Young, Peter Christoffersen, and Kris Jacobs, 2009, Market skewness risk and the cross-section of stock returns, Working paper, University of Toronto.

Coval, Joshua D., and Tyler Shumway, 2001, Expected option returns, *Journal of Finance* 56, 983–1009.

Daniel, Kent, Mark Grinblatt, Sheridan Titman, and Russ Wermers, 1997, Measuring mutual fund performance with characteristic-based benchmarks, *Journal of Finance* 52, 1035–1058.

Davis, James L., Eugene F. Fama, and Kenneth R. French, 2000, Characteristics, covariances, and average returns: 1929 to 1997, *Journal of Finance* 55, 389–406.

Dennis, Patrick, and Stewart Mayhew, 2002, Implied volatility skews: Evidence from options on individual securities, *Journal of Financial and Quantitative Analysis* 37, 471–493.

Dennis, Patrick, and Stewart Mayhew, 2009, Risk-neutral skewness: Evidence from stock options, *Journal of Financial and Quantitative Analysis* 37, 471–493.

Dimson, Elroy, 1979, Risk measurement when shares are subjected to infrequent trading, *Journal of Financial Economics* 7, 197–226.

Dittmar, Robert F., 2002, Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns, *Journal of Finance* 57, 369–403.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F. and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *The Journal of Political Economy*, 81, 607–636.

Ghysels, Eric, Alberto Plazzi, and Rossen Valkanov, 2011, Conditional skewness of stock market returns in developed and emerging markets and its economic fundamentals, Discussion paper available at http://papers.ssrn.com/so13/papers.cfm?abstract_id=1761446.

Green, T. Clifton, and Byoung-Hyoun Hwang, 2009, IPOs as lotteries: Expected skewness and first-day returns, Working paper, Emory University-Goizueta Business School.

Harvey, Campbell R., and Akhtar Siddique, 1999, Autoregressive conditional skewness, *Journal of Financial and Quantitative Analysis* 34, 465–487.

Harvey, Campbell R., and Akhtar Siddique, 2000, Conditional skewness in asset pricing tests, *Journal of Finance* 55, 1263–1295.

Hinkley, David V., 1975, On power transformations to symmetry, *Biometrika* 62, 101–111.

Jackwerth, Jens, and Mark Rubinstein, 1996, Recovering probability distributions from option prices, *Journal of Finance* 51, 1611–1631.

Kim, Tae-Hwan, and Halbert White, 2004, On more robust estimation of skewness and kurtosis, *Finance Research Letters* 1, 56–70.

Kraus, Allan, and Robert Litzenberger, 1976, Skewness preference and the valuation of risk assets, *Journal of Finance* 31, 1085–1100.

Kraus, Allan, and Robert Litzenberger, 1983, On the distributional conditions for a consumption oriented three moment CAPM, *Journal of Finance* 38, 1381–1391.

Mitton, Todd, and Keith Vorkink, 2007, Equilibrium underdiversification and the preference for skewness, *Review of Financial Studies* 20, 1255–1288.

Ofek, Eli, and Matthew Richardson, 2003, DotCom mania: The rise and fall of internet stock prices, *Journal of Finance* 58, 1113–1138.

Pástor, Ľuboš, and Robert F. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.

Pearson, Karl, 1895, Contributions to the mathematical theory of evolution. II. Skew variation in homogeneous material, *Philosophical Transactions of the Royal Society of London A* 186, 343–414.

Rubinstein, Mark E., 1973, The fundamental theorem of parameter-preference security valuation, *Journal of Financial and Quantitative Analysis* 8, 61–69.

Rubinstein, Mark E., 1994, Implied binomial trees, *Journal of Finance* 49, 771–818.

Shleifer, Andrei, and Robert W. Vishny, 1997, The limits of arbitrage, *Journal of Finance* 52, 35–55.

White, Halbert, Tae-Hwan Kim, and Simone Manganelli, 2008, Modeling autoregressive conditional skewness and kurtosis with multi-quantile CAViaR, in Robert F. Engle, and Halbert White, eds.: *A Festschrift in Honor of Clive W.J. Granger*. (Oxford University Press, New York).

Xing, Yuhang, Xiaoyan Zhang, and Rui Zhao, 2010, What does the individual option volatility smirk tell us about future equity returns?, *Journal of Financial and Quantitative Analysis* 45, 641–662.

Zhang, Yijie, 2006, Individual skewness and the cross-section of average stock returns, Discussion paper, Yale University.