International Journal of Financial Engineering
Vol. 2, No. 3 (2015) 1550028 (28 pages)

DOI: 10.1142/S2424786315500280

Scenario optimization technique for the assessment of downside-risk and investable portfolios in post-financial crisis

Mazin A. M. Al Janabi

Financial Engineering and Finance & Banking
Department of Economics and Finance
College of Business and Economics
United Arab Emirates University (UAEU), P. O. Box 15551
Al-Ain, Abu Dhabi, United Arab Emirates

Received: 10 May 2015; Revised: 18 August 2015; Accepted: 18 August 2015
Published: 22 September 2015

Abstract

The aim of this paper is to develop an optimization technique for the assessment of downside-risk limits and investable financial portfolios under crisis-driven outlooks subject to applying meaningful financial and operational constraints. The simulation and testing methods are based on the renowned concept of liquidity-adjusted value-at-risk (LVaR) along with the development of an optimization risk-algorithm utilizing matrix–algebra technique. With the purpose of demonstrating the effectiveness of LVaR and stress-testing techniques, real-world quantitative analysis of structured equity portfolios are depicted for the Gulf Cooperation Council (GCC) financial markets. To this end, several structural simulations studies are accomplished with the goal of establishing realistic financial modeling algorithm for the calculation of downside-risk parameters and to empirically assess portfolio managers' optimal and investable portfolios. The developed methodology and risk valuation algorithms can aid in advancing risk assessment and portfolio management practices in emerging markets, particularly in the wake of the most recent credit crunch and the subsequent financial turmoil.

Keywords: Emerging markets; financial engineering; financial risk management; GCC financial markets; liquidity-adjusted value-at-risk; optimization; portfolio management; stress testing.

JEL Classifications: C10; C13; G20; G28.

1. Introduction

The recent financial crisis has emphasized the necessity of accurate identification and assessment of embedded liquidity risk in financial investment portfolios. In

Email address: mazinaljanabi@gmail.com

essence, liquidity trading risk arises due to the incapability of financial entities to liquidate financial assets holdings, at rational prices, as time passes across the liquidation (close-out) period. Undeniably, certain collapses in some well-known financial entities and the consequential financial turmoil were caused, to some degree, by the impact of assets liquidity risk on structured portfolios. In this backdrop, the main objectives of this paper are threefold: (i) to develop and test a scenario optimization algorithm for the assessment of market price risk exposure for structured financial trading portfolios under crisis-driven circumstances; (ii) to propose a technique for the estimation of maximum downside-risk boundaries (maximum risk-threshold limits); (iii) to examine an optimization technique for the assessment of optimal portfolios and to empirically analyze the structuring of investable portfolios under crisis market prospects.[1]

To evaluate the risks included in their daily trading operations, major financial institutions are progressively developing and using value-at-risk (VaR) models. Given that financial institutions differ in their individual characteristics, tailor-made internal risk models are more suitable. Luckily, and in accordance with Basel II (and the forthcoming Basel III) regulatory accords on capital adequacy, financial entities are allowed to build and develop their own internal risk models with the determination of providing adequate risk measures and forecasting tools. In addition, internal risk models can be used in the determination of economic-capital cushion that financial institutions must hold to ratify their trading of financial assets and securities. The benefit of such an approach is that it takes into account the relationship between various asset types and can accurately assess the overall risk for a whole combination of trading assets (Al Janabi, 2011b).

In our time, VaR is by far the most widespread and most acknowledged risk measure among financial institutions. While VaR is very popular measure of market risk of financial trading portfolios, it is not a panacea for all risk assessments and has several drawbacks, limitations and undesirable properties (Sanders, 2002; Al Janabi, 2013). Indeed, if assets returns are not normally distributed, the computed VaR will play-down the true figure of VaR. In other words, if there are far more observations that is numerically distant from the rest of the data (i.e., outliers) in the actual return distribution of underlying assets than would be expected given the normality assumption, the actual VaR will be much higher than the computed VaR. An interrelated hurdle occurs when the variances and covariances matrices across the underlying assets change over time. This nonstationarity

[1]In this paper, the concept of investable market portfolios refers to rational portfolios that are contingent on meaningful financial and operational constraints. In this sense, investable market portfolios are not located on the efficient frontiers as defined by Markowitz (1952), and instead have logical and well-structured long/short asset allocation proportions.

in values is not unusual because the ground-rules driving these numbers amend over time. As such, it is not surprising that much of the work that has been done to invigorate the methodology has been focused on dealing with these critiques. To this end, a cluster of researchers have questioned how best to compute VaR with assumptions other than the standardized normal distribution of underlying assets returns. As such, Hull and White (1998) suggest ways of estimating VaR when variables are not normally distributed by permitting users to postulate any probability distribution for variables but require that transformations of the distribution still fall within a multivariate normal distribution. These and other related papers develop interesting variations but have to overwhelm several practical implementation problems. For instance, estimating inputs for non-normal models can be very problematic, especially when working with historical data. Moreover, the probabilities of losses and VaR are easiest to handle with the normal distribution and can get increasingly much more difficult with asymmetric and fat-tailed distributions. Furthermore, since the returns on many financial assets are heavy-tailed and skewed, one can obtain a better estimate for VaR by accounting for the deviations from normality assumption. For this purpose, Zangari (1996) provides a modified VaR calculation that takes the higher moments of non-normal distributions (skewness, kurtosis) into account.

One other critique that can be leveled against the variance–covariance estimate of VaR is that it does not explicitly consider asset liquidity risk. Typical VaR models assess the worst change in the mark-to-market portfolio value over a given time horizon, but do not account for the actual trading risk of liquidation. Indeed, neglecting asset liquidity risk can lead to an underestimation of the overall market risk and misapplication of capital cushion for the safety and soundness of financial institutions (Al Janabi, 2011a). In effect, the conventional VaR approach to computing market (or trading) risk of a portfolio does not explicitly consider liquidity risk. Typical VaR models are based on modern portfolio management theory and assess the worst change in mark-to-market portfolio value over a given time horizon, but do not account for the actual trading risk of liquidation. In general, customary fine-tunings are made on an ad hoc basis. At most, the holding period (or liquidation horizon) over which the VaR number is calculated is adjusted to ensure the inclusion of liquidity risk. As a result, liquidity trading risk can be imprecisely factored into VaR assessments by assuring that the liquidation horizon is as a minimum larger than an orderly liquidation interval. Moreover, the same liquidation horizon is employed to all trading asset classes, albeit some assets may be more liquid than others (Al Janabi, 2011a).

Effectively, this technique not only differentiate between the various kinds of asset market/liquidity risks, but also employs the square-root of time multiplier, in which it is assumed that no autocorrelation exists between the rates of return on asset from one measurement period to another (Al Janabi, 2011a).[2] The presumption of a lack of autocorrelation permits for simple addition of individual variances to produce the overall variance of the holding period. This postulation has been disputed over the past two decades by several authors (see for example, Danielsson and Zigrand, 2006). In addition, Al Janabi (2011a, 2011c, 2011d) recently reveals that the square-root of time rule leads to a systematic overestimation of market risk. The author concludes that despite the widespread application and implementation of the square-root of time multiplier in Basel II regulatory accord, it nevertheless fails short to address the aim of Basel II Accord on capital adequacy. It is clear that a comprehensive investigation into the nature of asset liquidity risk and its effect on distinct structured portfolios of both long and short-sales illiquid assets is necessary (Al Janabi, 2011a).[3]

In this backdrop and despite some research into asset liquidity risk measurement, the financial services industry still finds it difficult to quantify and predict measurable attributes of market liquidity risk.[4] In fact, most financial institutions have committed or are preparing to commit significant resources for managing liquidity risk. But there are no clear standards regarding the definition of the problem these entities' efforts are meant to solve, let alone the risk measures themselves. In the absence of accepted industry standards, risk professionals are faced with the challenge of defining an organization-specific internal approach that addresses market liquidity risk not only for a single trading security, but rather on an aggregate portfolio level (Al Janabi, 2011a). This is where this research paper comes in, as we strive to clarify the essence of asset liquidity risk, offer a clear-cut delineation of the topic and suggest a scenario optimization technique for the assessment of downside-risk boundary limits. Further, we empirically test for optimal and investable portfolios and provide some practical recommendations for

[2]A multiplier that is frequently used by researchers and practitioners is the square root of time. However, it overstates the overall impact of market risk since assets liquidation is not permitted during the close-out (unwinding) period.

[3]In another relevant study, Dowd *et al.* (2004) tackle the problem of estimating VaR for long-term horizon. In their paper they offer a different; however a rather straightforward, approach that avoids the inherited problems associated with the square-root of time rule, as well as those associated with attempting to extrapolate day-to-day volatility forecasts over long horizons.

[4]Within the VaR framework, Jarrow and Subramanian (1997) provide a market impact model of liquidity by considering the optimal liquidation of an investment portfolio over a fixed horizon. Bangia *et al.* (2002) approach the liquidity risk from another angle and provide a model of VaR adjusted for what they call exogenous liquidity — defined as common to all market players and unaffected by the actions of any one participant. It comprises such execution costs as order processing costs and adverse selection costs resulting in a given bid-ask spread faced by investors in the market. In a different vein, Almgren and Chriss (1999) present a concrete framework for deriving the optimal execution strategy using a mean–variance approach, and show a specific calculation method.

financial institutions. Effectively, the developed methodology and risk assessment algorithms can aid in evolving risk management practices in emerging markets, predominantly in light of the aftermaths of the recent sub-prime credit crunch and the resultant financial turmoil.

The remainder of the paper is organized as follows. The following section defines the mathematical procedure of a closed-form LVaR method and a robust model that integrates the effects of illiquidity of financial assets. The results of empirical tests and structural simulations of investable portfolios are drawn in the final section along with concluding remarks. Furthermore, Appendix A includes detailed derivation of the mathematical approach of LVaR during the close-out period. Full set of empirical testing and simulation analysis of maximum downside boundary limits and investable portfolios are included within Appendix B.

2. Closed-Form Parametric Process for the Assessment of Liquidity Adjusted Value-at-Risk

To estimate VaR employing the parametric method, the volatility of each risk factor is extracted from a pre-defined historical observation period. The potential effect of each component of the portfolio on the overall portfolio value is then worked out. These effects are then aggregated across the whole portfolio using the correlations between the risk factors (which are, again, extracted from the historical observation period) to give the overall VaR value of the portfolio with a given confidence level (Al Janabi, 2011c, 2013). A streamlined calculation route for the estimation of VaR risk factors (using a closed-form parametric process) for single and multiple assets positions are clarified as follows:

From fundamental statistics it is well acknowledged that for a normal distribution, 68% of the observations lie within 1σ (standard deviation) from the expected value, 95% within 2σ and 99% within 3σ from the expected value. As a result for single trading positions the absolute value of VaR in monetary terms can be defined as follows:

$$\mathrm{VaR}_i = |(\mu_i - \alpha * \sigma_i)[\mathrm{Asset}_i * Fx_i]| \approx |\alpha * \sigma_i[\mathrm{Asset}_i * Fx_i]| \tag{1}$$

where μ_i is the expected return of the asset, α is the confidence level (or in other words, the standard normal variant at confidence level α) and σ_i is the forecasted standard deviation (or conditional volatility) of the return of the security that constitutes the single position. The Asset_i is the mark-to-market value of the trading asset and indicates the monetary amount of equity position in asset i and Fx_i denotes the unit foreign exchange rate of asset i. Without a loss of generality, we can assume that the expected value of daily returns μ_i is close to zero. As such, though Eq. (1) includes some simplifying assumptions, yet it is routinely used by researchers and practitioners in the financial markets for the estimation of VaR for a single trading position (Al Janabi, 2013).

Portfolio trading risk in the existence of multiple risk factors is determined and assessed by the mutual influence of individual risk factors. The extent of total risk is determined not only by the magnitudes of the individual risk factors but also by their correlations parameters. Portfolio effects are crucial in risk management not only for large diversified portfolios but also for individual instruments that depends on several risk factors (Al Janabi, 2013). For multiple assets or portfolio of assets, VaR is a function of each individual security's risk and the correlation factor $[\rho_{i,j}]$ between the returns on the individual securities, detailed as follows:

$$\mathrm{VaR}_P = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{VaR}_i \mathrm{VaR}_j \rho_{i,j}} = \sqrt{|\mathrm{VaR}|^T |\rho| |\mathrm{VaR}|} \tag{2}$$

The above formula is a common one for the calculation of VaR for every financial portfolio irrespective of the number of securities that constitutes the portfolio. It should be stressed that the second term of the above equation is represented in terms of matrix–algebra techniques — a very convenient procedure to circumvent mathematical intricacy, as more and more financial securities are added to the trading portfolio. This tactic can streamline the algorithmic programming process and can enable straightforward assimilation of short-sales positions in the market risk assessment process. As such, in order to estimate VaR (of a portfolio of any number of financial assets), one needs first to generate a vector $|\mathrm{VaR}|$ of individual VaR positions — evidently n rows and one column (*nx1*) matrix — a transpose vector $|\mathrm{VaR}|^T$ of the distinct VaR positions — an (*1xn*) vector, and for this reason the superscript "T" donates transpose of the vector — and finally a correlation matrix, $|\rho|$, of all correlation factors (ρ) — explicitly an (*nxn*) matrix. Therefore, as one multiplies the two vectors and the correlation matrix and then examines the square root of the result, one ends up with the VaR_P of any portfolio with any n-number of securities (Al Janabi, 2013). As a result, this simple number summarizes the portfolio's exposure to market risk. Portfolio managers can then conclude whether they feel satisfied with this level of risk. If the answer is no, then the procedure that led to the assessment of VaR can be exerted to deduce where to reduce and curtail redundant risk.

Illiquid financial assets such as equities are very common in emerging markets. Typically these securities are traded infrequently (at very low volume) and particularly during financial crisis periods. Their quoted prices should not be regarded as a representative of the traders' consensus vis-à-vis their real value, but rather as the transaction price that arrived at by two counterparties under special market conditions. This of course represents a real dilemma to anybody who seeks to measure the market risk of these securities with a methodology which is based on volatilities and correlation matrices. The main problem arises when the historical price time-series are not obtainable for some assets or, when they are available, they are not fully reliable due to the lack of liquidity.

In fact, if assets returns are independent and they can have any elliptical multivariate distribution, then it is possible to convert the VaR horizon parameter from daily to any t-day horizon. The variance of a t-day return should be t times the variance of a 1–day return or $\sigma^2 = f(t)$. Thus, in terms of standard deviation (or volatility), $\sigma = f\ (\sqrt{t})$ and the daily VaR number [VaR (1-day)] can be adjusted for any t horizon to yield an LVaR estimate, such as:

$$\mathrm{LVaR}(t - day) = \mathrm{VaR}(1 - day)\sqrt{t} \tag{3}$$

The above formula was proposed and used by J. P. Morgan in their earlier *RiskMetrics™* method (Morgan Guaranty Trust Company, 1994). This methodology implicitly assumes that liquidation occurs in one block sale at the end of the holding period and that there is one holding period for all assets, regardless of their inherent trading liquidity structure. Unfortunately, the latter approach does not consider real-life-trading situations, where traders can liquidate (or re-balance) small portions of their trading portfolios on a daily basis. Moreover, this could generate unreliable risk assessments and can lead to considerable overestimates of LVaR figures, especially for the purposes of economic-capital allocation between trading and/or investment units (Al Janabi, 2011a; 2013).

The assumption of a given holding period for orderly liquidation inevitably implies that assets' liquidation occurs during the holding period. Accordingly, scaling the holding period to account for orderly liquidation can be justified if one allows the assets to be liquidated throughout the holding (close-out) period. In order to perform the calculation of LVaR under more realistic illiquid market conditions, we can define the following throughout the close-out period[5]:

$$L\mathrm{VaR}_{\mathrm{adj}} = \mathrm{VaR}\sqrt{\frac{(2t+1)(t+1)}{6t}} \tag{4}$$

where t is the number of liquidation days (*t-day* to liquidate the entire asset fully), VaR $=$ Value-at-Risk under liquid market conditions (as presented formerly in Eq. (1)) and $\mathrm{LVaR}_{\mathrm{adj}}$ $=$ Value-at-Risk under illiquid market conditions. The latter equation indicates that $\mathrm{LVaR}_{\mathrm{adj}} > \mathrm{VaR}$, and for the special case when the number of days to liquidate the entire assets is one trading day, then $\mathrm{LVaR}_{\mathrm{adj}} = \mathrm{VaR}$.

[5]For further details on the mathematical derivation and rational usefulness of this formula one can refer to Appendix A.

Consequently, the difference between $\text{LVaR}_{\text{adj}} - \text{VaR}$ should be equal to the residual market risk due to the illiquidity of any asset under illiquid markets conditions. As a matter of fact, the number of liquidation days (t) necessary to liquidate the entire assets fully is related to the choice of the liquidity threshold; however the size of this threshold is likely to change under adverse market conditions. Indeed, the choice of the liquidation horizon can be estimated from the total trading position size and the daily trading volume that can be unwound into the market without significantly disrupting market prices. Effectively, a linear liquidation procedure of assets is assumed in the above formula, i.e., selling equal parts of each asset every day till the last trading day (t), where the entire asset is sold. The above model is more appropriate for daily trading circumstances where traders can unwind part of their positions on a daily basis.[6]

3. Modeling of Downside-Risk Exposures and Investable Portfolios Structural Simulation Study of GCC Financial Markets

In this paper, databases of daily price returns of the six GCC stock markets' main indicators (indices) are implemented. The total number of indices that are considered in this work are nine; seven local indices for the six GCC stock markets (including two indices for the UAE markets) and two benchmark indices, detailed as follow: DFM General Index (Dubai Financial Market General Index, Country: United Arab Emirates); ADSM Index (Abu Dhabi Stock Market Index, Country: United Arab Emirates); BA All Share Index (All Share Stock Market Index, Country: Bahrain); KSE General Index (Stock Exchange General Index, Country: Kuwait); MSM30 Index (Muscat Stock Market Index, Country: Oman); DSM20 Index (Doha Stock Market General Index, Country: Qatar); SE All Share Index (All Share Stock Market Index, Country: Saudi Arabia); Shuaa GCC Index (GCC Stock Markets Benchmark Index, Provider: Shuaa Capital in UAE); and Shuaa Arab Index (Arab Stock Markets Benchmark Index, Provider: Shuaa Capital in UAE).

For this particular study we have chosen a confidence interval of 95% (or 97.5% with "one-tailed" loss side) and several liquidation time horizons (close-out periods) to calculate LVaR under crisis market situations. Historical database (of more than six years) of daily closing index levels, for the period 17/10/2004–22/05/2009, are collected for the purpose of implementing this research study and further for the construction of market risk management parameters, downside-risk

[6]It is important to note that Eq. (4) can be used to calculate LVaR for any time horizon subject to the constraint that the overall LVaR figure should not exceed at any setting the nominal exposure, in other words the total trading volume.

limits, and in assessing optimal and investable portfolios.[7] In fact, the selected time-series datasets fall within the period of the most severe part of the latest subprime financial crunch. The analysis of data and discussions of relevant findings and results of this research are organized and explained in the following sections

3.1. Testing for non-normality patterns and statistical analysis of volatility structures

To examine the statistical traits of the dataset, we have calculated the log returns of each time-series. As such, Table B.1 shows the daily volatility of each of the sample stock market indices under normal market and adverse (crisis-driven) market circumstances.[8] Crisis market volatilities are determined by applying the empirical distributions of past asset returns for all stock market indices' time-series. Therefore, the maximum negative assets returns (or downside losses), which are perceived in the historical time-series, are chosen for this end. This tactic can help in overcoming certain restrictions of normality supposition and can offer a superior analysis of LVaR particularly under adverse and illiquid market situations. As such, Table B.1 represents the maximum daily positive gains and negative losses and their respective dates of occurrences during the crisis period. To this end, downside-risk under adverse market state of affairs is simulated as the conditional volatility of the maximum daily assets losses. From Table B.1 we can note that the stock market index with the uppermost volatility is the Saudi SE All Share Index (under normal market stipulation) whereas the Dubai DFM General Index has confirmed the uppermost volatility under unfavorable market environments.

A noted result of the empirical testing of sensitivity factors (beta factors for systematic market risk) is the way in which the outcomes are differed across the sample stock market indices as indicated in Table B.1. The Saudi SE All Share Index seems to have the highest sensitivity factor (0.98) vis-à-vis the Shuaa Arab Index (that is the highest systematic market risk) and the Bahraini BA All Share Index seems to have the slightest beta factor (0.06). Likewise, and in line with general opinions, Shuaa GCC Index is the top nominee of the full stock market sample indices that acts to vary closely with regard to the benchmark Shuaa Arab Index.

In alternative testing, assessments of kurtosis and skewness are realized on the sample stock market indices. The results of these empirical testing are also described in Table B.1. It is perceived, as a general rule, that all stock market indices

[7]The historical database of daily indices levels is drawn from Reuters 3000 Xtra Hosted Terminal Platform and Thomson Reuters Datastream datasets.

[8]In this paper, crisis or severe market conditions refer to unexpected extreme adverse market situations at which losses could be several-fold larger than losses under normal market situation. Stress-testing technique is usually used to estimate the impact of unusual and severe events.

have displayed asymmetric characteristics. In addition, kurtosis testing studies have revealed parallel forms of abnormality. At the higher edge, the Omani MSM 30 Index has presented a negative skewness (−0.57) and a very high Kurtosis pattern — peakedness of (18.40). Some stock market indices, such as in the case of Qatari DSM30 Index, has presented the nearest association to normality — kurtosis of 5.59 and skewness of −0.11. Nevertheless, the Jarque–Bera (JB) normality test indicates noticeable common deviations from normality and, in consequence, discards the hypothesis that GCC stock markets daily returns are normally distributed.

3.2. Maximum downside-risk parameters and threshold LVaR limit-setting

Downside-risk limits parameters (or maximum risk budgeting thresholds) are imperative components for any asset management firm and it should be well-defined and exercised judiciously to guarantee control on the maximum risk exposures under normal settings and/or as a result of a financial turmoil under unfavorable market conditions.

How should we establish downside-risk limits to protect against maximum loss amounts? These are certain fundamental questions risk managers and/or portfolio managers must consider. In this research paper a simplified hands-on methodology is described for the calculation of maximum downside LVaR boundary limits. In order to ascertain coherent maximum LVaR trading limits, the mathematical formulation of an algorithm for the optimization process is set as follows:

From Eq. (4), we can express the liquidation horizon factor (LHF_i) for each trading asset as:

$$\mathrm{LHF}_i = \sqrt{\frac{(2t_i+1)(t_i+1)}{6t_i}} \geq 1.0; \quad \forall_i \quad i = 1, 2, \ldots, n \tag{5}$$

After substituting Eq. (5) into Eq. (2), we can calculate the maximum portfolio downside-risk $L\mathrm{VaR}_{P_{\mathrm{adj}}}$ limits by resolving the following quadratic programming formulation (objective function) under illiquid and adverse (crisis) market state of affairs[9]:

$$\begin{aligned} \text{Maximize} : L\mathrm{VaR}_{P_{\mathrm{adj}}} &= \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} L\mathrm{VaR}_{i_{\mathrm{adj}}} L\mathrm{VaR}_{j_{\mathrm{adj}}} \rho_{i,j}} \\ &= \sqrt{\left|L\mathrm{VaR}_{\mathrm{adj}}\right|^{T} |\rho| \left|L\mathrm{VaR}_{\mathrm{adj}}\right|} \end{aligned} \tag{6}$$

[9]For more details on the mathematical derivation of $L\mathrm{VaR}_{P_{\mathrm{adj}}}$, one can refer to Eq. (A.8) in Appendix A.

The above objective function can be maximized conditional on the following operational and financial budget constraints as specified by the portfolios' trading risk manager and/or portfolio manager

$$\sum_{i=1}^{n} R_i x_i = R_P; \quad l_i \leq x_i \leq u_i \quad i = 1, 2, \ldots, n \tag{7}$$

$$\sum_{i=1}^{n} x_i = 1.0; \quad l_i \leq x_i \leq u_i \quad i = 1, 2, \ldots, n \tag{8}$$

$$\sum_{i=1}^{n} V_i = V_P \quad i = 1, 2, \ldots, n \tag{9}$$

$$|\mathrm{LHF}| \geq 1.0; \quad \forall_i \quad i = 1, 2, \ldots, n \tag{10}$$

Here R_P and V_P denote the target portfolio mean expected return and total portfolio volume, respectively, and x_i is the weight or percentage asset allocation for each asset. The values l_i and u_i, $i = 1, 2, \ldots, n,$ denote the lower and upper constraints for the portfolio weights x_i. If we choose $l_i = 0$, $i = 1, 2, \ldots, n$, then we have the situation where no short-sales are allowed (Al Janabi, 2013). Moreover, |LHF| indicates an *(nx1)* liquidity risk factor vector for all $i = 1, 2, \ldots, n$.

For this purpose, Tables B.2 and B.3 present four structural simulation case studies for setting maximum downside LVaR boundary trading limits. In all four case studies, the effects of various asset allocations (with or without short-sales) are investigated for the purpose of deciding on acceptable LVaR threshold limits. In all case studies, the optimization algorithm is based on the definition of LVaR as the maximum possible loss over a specified time horizon within a given confidence level. The optimization technique solves the problem by locating market positions that maximize downside loss, subject to that all imposed constraints are fulfilled within their boundary values. For the sake of restraining the optimization process and thereafter its analysis, a volume trading limit of AED 200,000,000 (UAE Dirham) is assumed as a constraint — that is the financial entity must keep a maximum overall market value of stocks of no more than AED 200,000,000 (between long-only and short-sales positions).[10] Further, in all cases a liquidation horizon (or a close-out period) of 10 trading days is assumed constant during the optimization procedure for all trading assets. This indeed is in line with Basel rules

[10]Albeit current operational platform in GCC financial markets does not permit short-sales of assets, and in anticipation that this restriction might be relaxed in the future, we perform an analysis for long/short-sales positions and determine potential downside asset liquidity risk exposure and structured investable portfolios accordingly.

on capital adequacy which stipulates that market risk usually has to be measured using longer horizons for regulatory purposes.

While in Case Study (1) diverse asset allocation percentages are postulated, in Case Study (2) all equity assets position is centered in one market index that has, under adverse market conditions, the highest daily return volatility — that is, the Dubai Financial Market (DFM) General Index. Finally, in Case Study (3) and Case Study (4) the influence of short-sales of the sample stocks (or stock market indices) is also contemplated by randomly short-selling certain stocks.

The principal effect of diversification on LVaR downside limits setting seems to be through the long-only Case Study (1); that is with uneven asset allocation. On the other hand, using empirical correlations only we can stress that the highest downside LVaR figures, which are obtained from the optimization engine, are for Case Study (3), i.e., when the trading budget is allocated between long and short-sales equity positions. Nonetheless, for correlation factors other than empirical (that is, $\rho = +1$ and 0) the optimization results are quite mixed and spread between case studies 2, 3 and 4, respectively as presented in the grey-shaded areas of Tables B.2 and B.3.

Furthermore, it is interesting to note here that under special long/short-sales trading volume circumstances [see for instance, the results of Case-Study (3) in Tables B.2 and B.3], our optimization algorithm indicates that it is possible to obtain optimally maximized LVaR figures for a diversified portfolio in comparison to a fully non-diversified portfolio (in other words, in relation to the case of unity correlation factors ($\rho = +1$) among equity assets in the GCC financial markets). Albeit this observation might sound rather odd and contradicts general norms in modern portfolio management practices, it has its own merits and logical explanation. The rationality behind these results is basically due to the fact that our proposed portfolio LVaR — which is a combination of a closed-form parametric LVaR along with a stress-testing approach that is based on historical simulations of real severe upheavals in the GCC financial markets — can reveal new interesting phenomena (which has not been observed otherwise) of the impacts of different correlation factors (under special trading volume settings) on the total downside LVaR exposure of the portfolio.

In fact, and in accordance with some earlier studies on other financial markets (see for instance, Al Janabi, 2008), the empirical results indicate, by and large, that short-sales decreases LVaR figures and, hence, the downside LVaR trading limits. In the case of the GCC stock markets, however, the above phenomena (of high LVaR figures) can be explained by the nature of the diminutive correlation factors that we can observe in the entire GCC stock markets (see Al Janabi, 2013 for further insights). As a result, these tiny correlation factors have led to grand diversification benefits for long-only equity positions and visa-versa for short-sales positions.

As a conclusion of this empirical study, the asset management entity can set the maximum daily downside LVaR threshold limits for their structured equity portfolios as follows:

- Maximum daily downside LVaR threshold limit under normal market conditions = AED 21,553,941.
- Maximum daily downside LVaR threshold limit under crisis market conditions = AED 125,538,018.
- Maximum daily volume limit = AED 200,000,000 (between long-only and short-sales trading positions).
- The maximum liquidity horizons (unwinding periods) of all equity indices are set for 10 trading days and in line with Basel accord on capital adequacy regulations.

It should be noted here that the above maximum downside LVaR trading threshold limits are in their equivalent UAE dirham (AED) values at the prevailing foreign exchange rates of the other GCC countries against the UAE dirham.[11]

3.3. Empirical analysis of investable portfolios under crisis market prospects

For more than five decades a wide body of knowledge has been accumulated about the performance, strengths, and weaknesses of the Markowitz's (1952) classical mean–variance approach when applied to equity portfolios. However, much less is known about portfolio optimization techniques in emerging equity markets and particularly under austere and crisis-driven market conditions (Al Janabi, 2013).

Likewise, and given the fact that mean–variance optimizers have serious financial deficiencies (which could often lead to financially meaningless "*optimal*" portfolios) and since asset-allocation decision is the most fundamental issue facing portfolio managers who invest across multiple asset classes, the aim in this research paper is to look at the scenario optimization problem from a different realistic operational angle.[12] In view of that, the optimization-algorithms are

[11] After running several maximization simulation studies, by and large our empirical results indicate that under crisis market conditions the actual obtained structural portfolios can expect to realize greater downside-risk of approximately 6 times normal market conditions simulation case.

[12] It is well documented (Michaud, 1989) that mean–variance optimizers, if left to their own devices, can sometimes lead to unintuitive portfolios with extreme positions in asset classes. Consequently, these "optimized" portfolios are not necessarily well diversified and exposed to unnecessary ex-post risk (Michaud, 1989).

formulated by finding the set of structured investible portfolios that maximize risk-adjusted-returns, with risk-tolerance, and regulatory restrictions are constrained according to the requirements of institutional investors (i.e., asset management firms in this case). Moreover, a unique feature of this study that warrant particular emphasis is the superiority of the robust risk-adjusted-returns benchmark for "non-normal" returns that will be employed. As such, the focus in this work is on the forecast of risk-adjusted-returns measure, rather than on expected returns for two reasons: first, several studies have analyzed the forecasts of expected returns in the context of mean–variance optimization (see for instance, Best and Grauer, 1991). The common opinion is that expected returns are not easy to forecast, and that the optimization process is very sensitive to these variations. Second, there exists a general notion that risk-adjusted-returns, in a wide sense, are simpler to assess than expected returns from historical data (Al Janabi, 2013).

Set against this background, in this work we develop a model for optimizing portfolio risk-return with LVaR constraints using realistic operational and financial scenarios and conduct a simulation study on optimizing equity portfolios of the six GCC stock markets. The simulation study shows that the optimization algorithm, which is based on quadratic programming techniques, is very stable and efficient in handling different liquidity horizons and correlation factors. Moreover, the approach can tackle large number of equity securities and rational asset management scenarios. Indeed, the LVaR's risk management constraints (reduced to linear constraints) can be used in various portfolio management applications to bound percentiles of loss distributions.

Essentially, our approach is a robust improvement to the classic Markowitz mean–variance approach, where the original risk measure, variance, is replaced by LVaR algorithms. The task is attained here by minimizing LVaR, while requiring a minimum expected return subject to several financially meaningful operational constraints. Thus, by considering different expected returns, we can generate an optimal LVaR frontier. Alternatively, we can also maximize expected returns while not allowing for large risks. For the purpose of this study, the optimization problem is formulated as follow:

It is possible to compute from Eqs. (4), (2) and (A.8) the minimum LVaR necessary to support current asset management operation by solving for the following quadratic programming objective function:

$$\begin{aligned}\text{Minimize} : L\text{VaR}_{P_{\text{adj}}} &= \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} L\text{VaR}_{i_{\text{adj}}} L\text{VaR}_{j_{\text{adj}}} \rho_{i,j}} \\ &= \sqrt{\left|L\text{VaR}_{\text{adj}}\right|^{T} |\rho| \left|L\text{VaR}_{\text{adj}}\right|} \qquad (11)\end{aligned}$$

The above objective function can be minimized subject to operational and financial budget constraints as specified by the portfolio manager and as indicated in Eqs. (7) to (10).

Now the portfolio manager can identify different liquidity horizon and correlation factors and compute the required amount of LVaR to sustain trading operations of the financial unit without subjecting the entity to insolvency matters. The rationality behind imposing the above constraints is to comply with contemporary regulations which enforce capital requirements on investment companies, proportional to their LVaR and economic-capital, besides other operational limits (for instance, volume trading limits and the choice of long/short-sales positions).

In this backdrop, the empirical optimization process is based on the definition of LVaR as the minimum possible loss over a specified time horizon within a given confidence level. The iterative-optimization modeling algorithm solves the problem by finding the market positions that minimize the loss, subject to the fact that all constraints are satisfied within their boundary values.[13] Further, in all cases the liquidation horizons (unwinding periods) as indicated in Tables B.4–B.7 are assumed constant throughout the optimization process. For the sake of restraining the optimization algorithm and thereafter its analysis, a volume trading position limit of 10 million AED is assumed as a constraint — that is the equity trading entity must keep a maximum overall market value of diverse equities of no more than 10 million AED (between long and short-sales positions). As such, Fig. 1 provides evidence of the empirical LVaR optimal frontier (under 10-days liquidation horizons and for crisis market conditions) defined using a 97.5% confidence level. As mentioned above, the optimal portfolio selection is performed by

[13]In his research paper, Al Janabi (2013) has investigated and applied similar optimization algorithm for GCC financial markets under normal market conditions along with some assets liquidity stress testing analysis. Indeed, his empirical analysis was directed to investigate an optimization case study for merely long/short-sales trading positions under normal market perceptions combined with stressing the liquidity horizons of trading assets. While some of his general optimization elements and parameters are adapted herein, this work builds upon Al Janabi (2011, 2013) research papers and differs entirely in the sense that the optimization algorithm was carried out under severe crisis market circumstances as a result of a financial crunch (e.g., the most recent financial turmoil) and for investable portfolios that have both long and short-sales trading positions (that is, disallowing both pure long positions and borrowing constraints). Indeed, our optimization algorithm and the newly obtained empirical results have shown that portfolio managers can obtain financially meaningful investable portfolios (under crisis market settings) and demonstrated new interesting market-microstructures' patterns (e.g., the impact of close-out periods, overall trading volume, expected returns, etc., on the optimization-algorithm process). In fact, these patterns cannot be attained by using the classical Markowitz's mean–variance approach. In addition, the new market-microstructures' patterns and the drawn conclusions were not evident in Al Janabi (2013) research paper.

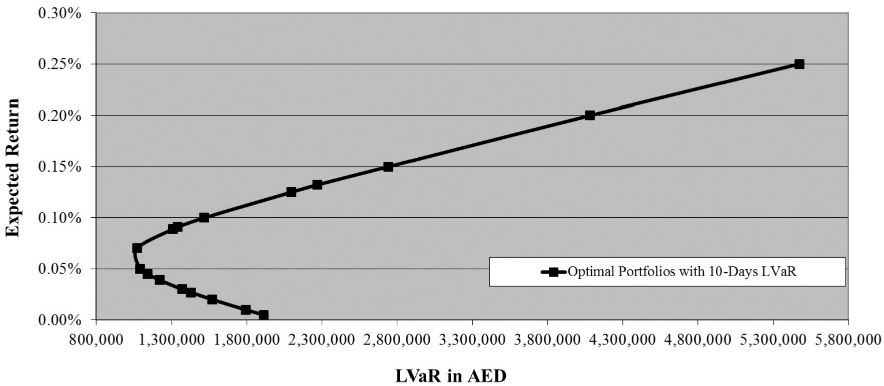

Fig. 1. Optimal portfolios with 10-days LVaR technique (case of long and short-sales positions under crisis market conditions).

relaxing the short-sales constraint, for the different equity assets. On the other hand, optimal portfolios cannot always be attained (e.g., short-sales without realistic lower boundaries on x_i) in the day-to-day real-world portfolio management operations and, hence, the portfolio manager should establish proactive investable portfolios under realistic and restricted dynamic budget constraints, detailed as follows:

- The total trading volume (between long and short-sales equity trading positions) of any investable portfolio is 10 million AED.
- For any investable portfolio, the asset allocation for long equity trading position varies from 10% to 100%.
- For any investable portfolio, the asset allocation for short-sales equity trading position varies from 10–60%.
- All liquidity horizons (close-out periods) for all equities are kept constant and in accordance with the specified values indicated in Tables B.4–B.7.
- Volatilities under crisis market notions are estimated as the maximum historical-simulation events with the highest downside-risk. These conditional volatilities are kept constant throughout the optimization process and in accordance with the quantified values indicated in Table B.1.

Now the asset allocation weights are allowed to take negative or positive values, however, since arbitrarily high or low percentages have no financial logic, we determined to introduce lower and upper boundaries for the weights and in accordance with reasonable trading practices. Furthermore, for comparison purposes and since the endeavor in this work is to minimize LVaR under crisis market prospects subject to specific expected returns, we decide to plot LVaR versus expected returns and not the reverse, as is commonly done in the various modern

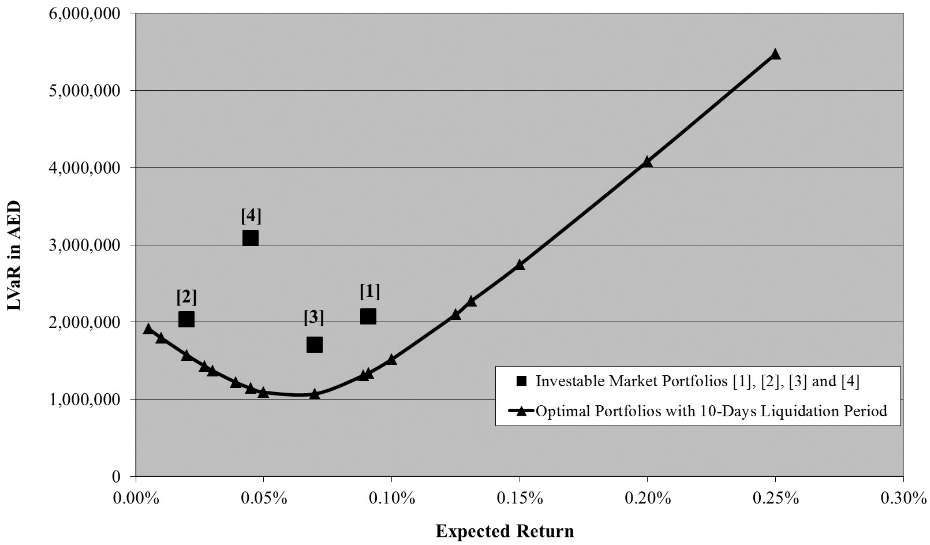

Fig. 2. Optimal and investable portfolios with LVaR technique (case of long and short-sales positions under crisis market conditions).

portfolio management literature. As a result, it is worth noting that the four benchmark investable portfolios (investable portfolios [1], [2], [3] and [4]) are not positioned near the optimal frontier as indicated in Fig. 2. This is because financially and operationally real-world investment considerations make it unlikely that an investment portfolio will behave exactly as theory predicts under crisis-driven market's perspectives. Imperfections such as restriction on long and short-sales trading positions, total trading volume and liquidation horizons make it unlikely to create an optimal equity trading portfolio (Al Janabi, 2013). Thus, the portfolio manager should apply active strategies in order to earn excess returns and particularly under adverse market perceptions. These considerations are especially relevant for individual portfolio managers who may spread their trading positions across a few securities.

In order to clarify the structural composition of investable portfolios [1], [2], [3] and [4], Tables B.4–B.7 point out the asset allocation weights for all equity assets in all the liquidation periods under consideration in addition to their expected return and sensitivity factor. Similarly, the four tables show the minimum needed LVaR to maintain the operations of these precise portfolios under the notion of three different correlation factors as well as the LVaR/volume ratio under crisis market settings. In this way, portfolio managers should employ appropriate downside-risk measures which allow them to take prompt decisions which would produce a risk budget lower than a specific target. Thus, this analysis is substantially a generalized enhancement to the classical Markowitz's (1952) analysis that permits one to determine and verify the asymmetric aspects of risk and investable portfolios under crisis market notions.

4. Concluding Remarks

The empirical testing outcomes for the GCC zone imply that in almost all tests, there are clear and strong asymmetric behaviors in the distribution of assets return of the sample stock market indices. The appealing outcome of this study suggests the inevitability of uniting LVaR computations with other techniques such as stress-testing and scenario analysis to grasp a systematic view of other residual risks (such as, fat-tails in the probability distribution) that cannot be revealed, under crisis market circumstances, with the plain assumption of normality. As such, by and large our empirical results indicate that under crisis market conditions the actual obtained investable portfolios can expect to realize greater downside-risk of approximately 6 times the case under normal market conditions. The outcomes of the results of this study on the GCC stock markets advocate that while there is a strong deviation from normality in the distribution of daily assets return, this matter can be addressed without the necessity of complex mathematical and analytical processes. In effect, it is feasible to handle these issues for equity cash assets with the clear-cut use of a closed-form parametric method within matrix–algebra arrangement; along with the integration of a credible stress-testing tactic (under adverse crisis market conditions); as well as by complementing the risk analysis with a rational illiquidity risk factor that considers real-world trading situations all the way through the close-out period.

For this purpose, in this paper a robust optimization technique for the assessment of illiquidity of both short-sales and long trading position is incorporated. In contrast to other commonly used liquidity models, the liquidity technique that is applied in this work is more appropriate for real-world trading practices since it considers selling small fractions of the long/short trading securities on a daily basis. Further, this liquidity model can be implemented for the entire portfolio or for each individual security within the equity trading portfolio. Indeed, the developed methodology and risk assessment algorithms can aid in progressing risk management practices in emerging markets and above all in the wake of the latest credit crunch and the ensuing financial upheavals.

Indeed, our suggested modeling technique is in-line with the recommendation of previous studies (see, for example, Neftci, 2000). In essence, a number of authors have argued that many assets distributions have "fat tails" and that

RiskMetrics (1994), by assuming the normal distribution, underestimates the risk of extreme losses. As such, our proposed portfolio LVaR is a combination of a closed-form parametric LVaR along with a stress-testing approach that is based on historical simulation of real severe upheavals in the GCC markets. Furthermore, our empirical results reveal new interesting phenomena in the sense that under the traditional variance/covariance method, one expects to obtain lower LVaR figure when the correlation is lower than unity, nonetheless our empirical results indicate otherwise for certain long/short trading positions under different correlation factors.

Similarly, the empirically obtained investable benchmark portfolios, under the notion of a crisis-driven event, are noticeably located away from the optimal frontier. This is due to the fact that financially and operationally real-world investment considerations make it unlikely that an investable portfolio will behave exactly as theory predicts. Imperfections such as restriction on long and short-sales trading positions, total trading volume and liquidation horizons make it unlikely to create an optimal equity portfolio. As such, the portfolio manager should apply active strategies in order to earn excess returns and particularly under adverse market perceptions.[14] In a nutshell, the obtained empirical results can have several practical applications and could aid in overcoming some of the shortcomings of conventional VaR and the classical mean–variance approach, especially in light of the aftermaths of the latest financial crunch.

In conclusion, the presented methodology and empirical results of this paper have important practical uses and applications for financial institutions, risk managers, portfolio managers, financial regulators and policymakers operating in the GCC and other emerging markets, and particularly in the wake of the most recent financial crisis. For instance, the proposed modeling technique and simulation algorithms can be used by risk managers and portfolio managers for the assessment of appropriate asset allocations for different structured investable portfolios. Further, the technique can be used in optimizing realistic asset management portfolios and in setting maximum downside LVaR limits under normal and adverse market circumstances.

[14] As a matter of fact, our optimization algorithm and the newly obtained empirical results have shown that portfolio managers can obtain financially meaningful investable portfolios (under crisis market settings) and demonstrated new interesting market-microstructures' patterns (e.g., the impact of close-out periods, overall trading volume, expected returns, etc., on the optimization-algorithm process). In fact, these newly observed market-microstructures' patterns cannot be attained by using the classical Markowitz's mean–variance approach. Furthermore, the new market-microstructures' patterns and the drawn conclusions were not evident in Al Janabi (2013) research paper.

Appendix A. Derivation of Liquidity-Adjusted Value-at-Risk (LVaR) Mathematical Structure During the Close-Out Period

In this section, we present a simple re-engineered modeling approach for calculating a closed-form parametric LVaR.[15] The proposed model and liquidity scaling factor is more realistic and less conservative than the conventional root-t multiplier. In essence, the suggested multiplier is a function of a predetermined liquidity threshold defined as the maximum position which can be unwound without disturbing market prices during one trading day. The essence of the model relies on the assumption of a stochastic stationary process and some rules of thumb, which can be of crucial value for more accurate overall trading risk assessment during market stress periods when liquidity dries up. To this end, a simplified mathematical approach is described below with the purpose of incorporating and calculating illiquid assets' daily LVaR, detailed as follows:

The market risk of an illiquid trading position is larger than the risk of an otherwise identical liquid position. This is because unwinding the illiquid position takes longer than unwinding the liquid position, and, as a result, the illiquid position is more exposed to the volatility of the market for a longer period of time. In this approach, a trading position will be considered illiquid if its size surpasses a certain liquidity threshold. The threshold (which is determined by each trader and/or portfolio manager) and defined as the maximum position which can be unwound, without disrupting market prices, in normal market conditions and during one trading day. Consequently, the size of the trading position relative to the threshold plays an important role in determining the number of days that are required to close the entire position (Al Janabi, 2013, 2011a, 2011c). This effect can be translated into a liquidity increment (or an additional liquidity risk factor) that can be incorporated into VaR analysis. If for instance, the par value of a position is $10,000 and the liquidity threshold is $5000, then it will take two days to sell out the entire trading position. Therefore, the initial position will be exposed to market variation for one day, and the rest of the position (that is $5000) is subject to market variation for an additional day. If it assumed that daily changes of market values follow a stationary stochastic process, the risk exposure due to illiquidity effects is given by the following illustration, detailed along these lines:

In order to take into account the full illiquidity of assets (that is, the required unwinding period to liquidate an asset) we define the following:

t = number of liquidation days (t–days to liquidate the entire asset fully)

σ^2_{adj} = overnight (daily) variance of the illiquid position; and

[15] The mathematical approach and optimization algorithms of this appendix are largely drawn from Al Janabi (2013) research paper.

σ_{adj} = liquidity risk factor or overnight (daily) standard deviation of the illiquid position.

The proposed approach assumes that the trading position is closed out linearly over t-days and hence it uses the logical assumption that the losses due to illiquid trading positions over t-days are the sum of losses over the individual trading days. Moreover, we can assume with reasonable accuracy that asset returns and losses due to illiquid trading positions are independent and identically distributed (iid) and serially uncorrelated day-to-day along the liquidation horizon and that the variance of losses due to liquidity risk over t-days is the sum of the variance (σ_i^2, for all $i = 1, 2 \ldots, t$) of losses on the individual days, thus

$$\sigma_{\text{adj}}^2 = \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots + \sigma_{t-2}^2 + \sigma_{t-1}^2 + \sigma_t^2\right) \tag{A.1}$$

In fact, the square root-t approach (Eq. (2)) is a simplified special case of Eq. (A.1) under the assumption that the daily variances of losses throughout the holding period are all the same as first day variance, σ_1^2, thus, $\sigma_{\text{adj}}^2 = (\sigma_1^2 + \sigma_1^2 + \sigma_1^2 + \cdots + \sigma_1^2) = t\sigma_1^2$. Basically, the square root-$t$equation overestimates asset liquidity risk since it does not consider that traders can liquidate small portions of their trading portfolios on a daily basis and then the whole trading position can be sold completely on the last trading day. Undeniably, in real financial markets operations, liquidation occurs during the holding period and thus scaling the holding period to account for orderly liquidation can be justified if one allows the assets to be liquidated throughout the holding period. As such, for this special linear liquidation case and under the assumption that the variance of losses of the first trading day decreases linearly each day (as a function of t) we can derive from Eq. (A.1) the following:

$$\sigma_{\text{adj}}^2 = \left(\left(\frac{t}{t}\right)^2 \sigma_1^2 + \left(\frac{t-1}{t}\right)^2 \sigma_1^2 + \left(\frac{t-2}{t}\right)^2 \sigma_1^2 + \cdots + \left(\frac{3}{t}\right)^2 \sigma_1^2 \right.$$
$$\left. + \left(\frac{2}{t}\right)^2 \sigma_1^2 + \left(\frac{1}{t}\right)^2 \sigma_1^2\right) \tag{A.2}$$

Evidently, the additional liquidity risk factor depends only on the number of days needed to sell an illiquid equity position linearly. In the general case of t-days, the variance of the liquidity risk factor is given by the following mathematical functional expression of t

$$\sigma_{\text{adj}}^2 = \sigma_1^2\left(\left(\frac{t}{t}\right)^2 + \left(\frac{t-1}{t}\right)^2 + \left(\frac{t-2}{t}\right)^2 + \cdots + \left(\frac{3}{t}\right)^2 + \left(\frac{2}{t}\right)^2 + \left(\frac{1}{t}\right)^2\right) \tag{A.3}$$

To calculate the sum of the squares, it is convenient to use a short-cut approach. From mathematical series the following relationship can be obtained:

$$(t)^2 + (t-1)^2 + (t-2)^2 + \cdots + (3)^2 + (2)^2 + (1)^2 = \frac{t(t+1)(2t+1)}{6} \tag{A.4}$$

Accordingly, from Eqs. (A.3) and (A.4) the liquidity risk factor can be expressed in terms of volatility (or standard deviation) as

$$\sigma_{\text{adj}} = \sigma_1 \left(\sqrt{\frac{1}{t^2}[(t)^2 + (t-1)^2 + (t-2)^2 + \cdots + (3)^2 + (2)^2 + (1)^2]} \right)$$

$$\text{or } \sigma_{\text{adj}} = \sigma_1 \left(\sqrt{\frac{(2t+1)(t+1)}{6t}} \right) \tag{A.5}$$

The final result is of course a function of time and not the square root of time as employed by some financial market's participants based on the *RiskMetrics™* methodologies. The above approach can also be used to calculate the *L*VaR for any time horizon. In order to perform the calculation of *L*VaR under illiquid market conditions, the liquidity risk factor of Eq. (A.5) can be implemented in LVaR modeling, hence, one can define the following:

$$L\text{VaR}_{\text{adj}} = \text{VaR}\sqrt{\frac{(2t+1)(t+1)}{6t}} \tag{A.6}$$

where, VaR = Value-at-Risk under liquid market conditions (as presented formerly in Eq. (1)] and $L\text{VaR}_{\text{adj}}$ = Value-at-Risk under illiquid market conditions. The latter equation indicates that $L\text{VaR}_{\text{adj}} > \text{VaR}$, and for the special case when the number of days to liquidate the entire assets is one trading day, then $L\text{VaR}_{\text{adj}} = \text{VaR}$. Consequently, the difference between $L\text{VaR}_{\text{adj}} - \text{VaR}$ should be equal to the residual market risk due to the illiquidity of any asset under illiquid markets conditions. As a matter of fact, the number of liquidation days (t) necessary to liquidate the entire assets fully is related to the choice of the liquidity threshold; however the size of this threshold is likely to change under severe market conditions. Indeed, the choice of the liquidation horizon can be estimated from the total trading position size and the daily trading volume that can be unwound into the market without significantly disrupting market prices; and in actual practices it is generally estimated as:

$$t = \text{Total Trading Position Size of Equity Asset}_i / \text{Daily Trading Volume of Equity Asset}_i \tag{A.7}$$

As such, the close out time (t) is the time required to bring the positions to a state where the financial entity can make no further loss from the trading positions. It is the time taken to either sell the long positions or alternatively the time required to buy securities in case of short-sales positions. In real practice, the daily trading volume of any asset is estimated as the average volume over some period of time, generally a month of trading activities. In effect, the daily trading volume of assets can be regarded as the average daily volume or the volume that can be unwound under a severe market period. Thus, if trading volume is low because of a "one-way market," in that most people are seeking to sell rather than to buy, then (t) can rise substantially (Saunders and Cornett, 2008). The trading volume in a severe market period can be roughly approximated as the average daily trading volume less a number of standard deviations. Albeit this alternative approach is quite simple, it is still relatively objective. Moreover, it is reasonably easy to gather the required data to perform the necessary liquidation scenarios. In essence, the above liquidity scaling factor (or multiplier) is more realistic and less conservative than the conventional root-t multiplier and can aid financial entities in allocating reasonable and liquidity market-driven regulatory and economic-capital requirements.

Furthermore, the above mathematical formulas can be applied for the calculation of LVaR for each trading position and for the entire portfolio. In order to calculate the LVaR for the full trading portfolio under illiquid market conditions ($\mathrm{LVaR}_{P_{\mathrm{adj}}}$), the above mathematical formulation can be extended, with the aid of Eq. (2), into a matrix–algebra form to yield the following:

$$\mathrm{LVaR}_{P_{\mathrm{adj}}} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} L\mathrm{VaR}_{i_{\mathrm{adj}}} L\mathrm{VaR}_{j_{\mathrm{adj}}} \rho_{i,j}} = \sqrt{\left|L\mathrm{VaR}_{\mathrm{adj}}\right|^{T} |\rho| \left|L\mathrm{VaR}_{\mathrm{adj}}\right|} \quad (\mathrm{A.8})$$

The above mathematical structure (in the form of two vectors, $|L\mathrm{VaR}_{\mathrm{adj}}|$, $|L\mathrm{VaR}_{\mathrm{adj}}|^{T}$ and a correlation matrix, $|\rho|$) can facilitate the programming process so that the portfolio trading risk manager and/or portfolio manager can specify different liquidation days for the whole portfolio and/or for each individual trading security according to the necessary number of days to liquidate the entire asset fully. The latter can be achieved by specifying an overall benchmark liquidation to liquidate the entire constituents of the portfolio fully.

Appendix B. Results of Empirical Testing and Simulation Analysis of Investable Portfolios

Table B.1. Risk assessment dataset, basic statistics and test for non-normality.

Stock market indices	Volatility (normal market) (%)	Volatility (crisis market) (%)	Expected return (%)	Maximum positive return (gain) (%)	Date of occurrence	Maximum negative return (loss) (%)	Date of occurrence	Sensitivity (Beta) factor	Skewness	Kurtosis	Jarque-Bera (JB) test
DFM General Index	1.93	12.16	0.12	9.94	23/1/2008	−12.16	14/3/2006	0.58	0.01	7.86	955**
ADSM Index	1.42	7.08	0.07	6.57	09/5/2005	−7.08	22/1/2008	0.40	0.12	7.26	734**
BA All Share Index	0.59	3.77	0.05	3.61	24/1/2006	−3.77	13/8/2007	0.06	0.43	10.24	2142**
KSE GeneralIndex	0.76	3.74	0.09	5.05	16/3/2006	−3.74	14/3/2006	0.14	−0.18	8.38	1173**
MSM30 Index	0.84	8.70	0.12	5.22	16/10/2007	−8.70	22/1/2008	0.10	−0.57	18.40	9617**
DSM20 Index	1.53	8.07	0.06	6.22	04/2/2008	−8.07	22/1/2008	0.31	−0.11	5.59	273**
SE All AU Share Index	2.08	11.03	0.03	9.39	13/5/2006	−11.03	21/1/2008	0.98	−0.97	8.47	1361**
Shuaa GCC Index	1.45	8.10	0.06	11.14	13/5/2006	−8.10	21/1/2008	1.05	−0.66	14.00	4949**
Shuaa Arab Index	1.28	7.57	0.07	9.43	13/5/2006	−7.57	21/1/2008	1.00	−0.61	13.79	4758**

Notes: (1) Asterisk ** denotes statistical significance at the 0.01 level.
(2) Downside risk under crisis market conditions is simulated as the conditional volatility of the maximum negative daily return (loss).

Table B.2. Downside-risk parameters in AED and with different correlation factors (ρ) (simulation under the assumption of normal market conditions).

Downside LVaR limit-setting	$\rho =$ empirical	$\rho = +1$	$\rho = 0$
Long-only Case study 1	6,401,699	10,777,025	4,908,240
Long-only Case study 2	15,111,606	15,111,606	15,111,606
Long/short Case study 3	21,553,941	7,832,512	22,772,680
Long/short Case study 4	15,414,438	19,433,231	12,775,513

Note: In all the above optimization case studies for the assessment of downside risk-budgeting threshold, the liquidity horizons for all equity indices are assumed 10 trading days and in line with Basel accord on capital adequacy regulations.

Table B.3. Downside-risk parameters in AED and with different correlation factors (ρ) (simulation under the assumption of crisis market conditions).

Downside LVaR limit-setting	$\rho =$ empirical	$\rho = +1$	$\rho = 0$
Long-only Case study 1	37,457,121	63,967,200	28,728,113
Long-only Case study 2	95,417,112	95,417,112	95,417,112
Long/short Case study 3	125,538,018	60,978,300	131,830,223
Long/short Case study 4	87,355,735	92,984,460	76,297,610

Note: In all the above optimization case studies for the assessment of downside risk-budgeting threshold, the liquidity horizons for all equity indices are assumed 10 trading days and in line with Basel accord on capital adequacy regulations.

Table B.4. Composition of investable market portfolio [1] (case analysis of long and short-sales positions under crisis market conditions).

Market index	Liquidation period (in days)	Market value in AED	Asset allocation per market
DFM General Index	2.0	(2,000,000)	−20%
ADSM Index	3.0	7,000,000	70%
BA All Share Index	4.0	6,000,000	60%
KSE General Index	3.0	(2,000,000)	−20%
MSM30 Index	4.0	6,000,000	60%
DSM20 Index	3.0	(1,000,000)	−10%
SE All Share Index	2.0	(4,000,000)	−40%
Correlation factor	LVaR (crisis market)	LVaR/volume	Expected return
$\rho =$ *Empirical*	2,066,833	20.7%	0.090%
$\rho = 1$	1,365,712	13.7%	Sensitivity Factor
$\rho = 0$	2,301,653	23.0%	−0.192

Table B.5. Composition of investable market portfolio [2] (case analysis of long and short-sales positions under crisis market conditions).

Market index	Liquidation Period (in Days)	Market Value in AED	Asset Allocation per Market
DFM General Index	2.0	(2,000,000)	−20%
ADSM Index	3.0	3,000,000	30%
BA All Share Index	4.0	5,000,000	50%
KSE General Index	3.0	3,000,000	30%
MSM30 Index	4.0	(3,000,000)	−30%
DSM20 Index	3.0	(3,000,000)	−30%
SE All Share Index	2.0	7,000,000	70%
Correlation factor	LVaR (crisis market)	LVaR/volume	Expected return
$\rho = Empirical$	2,033,085	20.3%	0.021%
$\rho = 1$	1,189,397	11.9%	Sensitivity Factor
$\rho = 0$	2,185,975	21.9%	0.633

Table B.6. Composition of investable market portfolio [3] (case analysis of long and short-sales positions under crisis market conditions).

Market index	Liquidation period (in days)	Market value in AED	Asset allocation per market
DFM General Index	2.0	4,000,000	40%
ADSM Index	3.0	2,000,000	20%
BA All Share Index	4.0	3,000,000	30%
KSE General Index	3.0	(2,000,000)	−20%
MSM30 Index	4.0	1,000,000	10%
DSM20 Index	3.0	(1,000,000)	−10%
SE All Share Index	2.0	3,000,000	30%
Correlation factor	LVaR (crisis market)	LVaR/volume	Expected return
$\rho = Empirical$	1,702,833	17.0%	0.076%
$\rho = 1$	2,340,483	23.4%	Sensitivity Factor
$\rho = 0$	1,443,144	14.4%	0.572

Table B.7. Composition of investable market portfolio [4] (case analysis of long and short-sales positions under crisis market conditions).

Market index	Liquidation period (in days)	Market value in AED	Asset allocation per market
DFM General Index	2.0	9,000,000	90%
ADSM Index	3.0	(6,000,000)	−60%

(*Continued*)

Table B.7. (*Continued*)

Market index	Liquidation period (in days)	Market value in AED	Asset allocation per market
BA All Share Index	4.0	5,000,000	50%
KSE General Index	3.0	(2,000,000)	−20%
MSM30 Index	4.0	(6,000,000)	−60%
DSM20 Index	3.0	4,000,000	40%
SE All Share Index	2.0	6,000,000	60%
Correlation factor	LVaR (crisis market)	LVaR/volume	Expected return
$\rho = Empirical$	3,085,371	30.9%	0.044%
$\rho = 1$	2,573,648	25.7%	Sensitivity Factor
$\rho = 0$	3,505,921	35.1%	0.933

References

Al Janabi, MAM (2013). Optimal and coherent economic-capital structures: Evidence from long and short-sales trading positions under illiquid market perspectives, *Annals of Operations Research*, 205(1), 109–139.

Al Janabi, MAM (2011a). A generalized theoretical modeling approach for the assessment of economic capital under asset market liquidity risk constraints, *Service Industries Journal*, 31(13–14), 2193–2221.

Al Janabi, MAM (2011b). Economic capital allocation under coherent market liquidity constraints, *Journal of Economic Research*, 16(2), 147–186.

Al Janabi, MAM (2011c). Modeling coherent trading risk parameters under illiquid market perspective, *Studies in Economics and Finance*, 28(4), 301–320.

Al Janabi, MAM (2011d). Dynamic equity asset allocation with liquidity-adjusted market risk criterion: Appraisal of efficient and coherent portfolios, *Journal of Asset Management*, 12(6), 378–394.

Al Janabi, MAM (2008). A practical approach to market risk analysis and control: Empirical test of the Mexican foreign exchange and stock markets, *International Journal of Risk Assessment and Management*, 9(1–2), 70–103.

Almgren, R and N Chriss (1999). Optimal Execution of Portfolio Transaction. Working Paper, Department of Mathematics, The University of Chicago.

Bangia, A, F Diebold, T Schuermann and J Stroughair (2002). Modeling liquidity risk with implications for traditional market risk measurement and management, *Risk Management: The State of the Art*, S Figlewski and RM Levich (eds.), The New York University Salomon Center Series on Financial Markets and Institutions, Vol. 8, pp. 3–13. Springer.

Best, MJ and RR Grauer (1991). On the sensitivity of mean-variance-efficient portfolios to changes in asset means: Some analytical and computational results, *Review of Financial Studies*, 4, 315–342.

Danielsson, J and J-P Zigrand (2006). On time-scaling of risk and the square-root-of-time rule, *Journal of Banking & Finance*, 30(10), 2701–2713.

Dowd, K, D Blake and A Cairns (2004). Long-term value at risk, *Journal of Risk Finance*, 5, 52–57.

Hull, J and A White (1998). Value at risk when daily changes are not normally distributed, *Journal of Derivatives*, 5(3), 9–19.

Jarrow, R and A Subramanian (1997). Mopping up liquidity, *Risk*, 10(12), 170–173.

Markowitz, H (1952). Portfolio selection, *Journal of Finance*, 7(1), 77–91.

Michaud, RO (1989). The Markowitz optimization enigma: Is "optimized" optimal? *Financial Analysts Journal*, 45(1), 31–42.

Morgan Guaranty Trust Company (1994). *RiskMetrics: Technical Document*, New York: Morgan Guaranty Trust Company, Global Research.

Neftci, SF (2000). Value at risk calculations, extreme events and tail estimations, *Journal of Derivatives*, 23–37.

Sanders, DR (2002). The role of value-at-risk in purchasing: An application to the foodservice industry, *The Journal of Supply Chain Management*, 38–45.

Saunders, A and M Cornett (2008). *Financial Institutions Management*, 6th edn. Singapore: McGraw Hill International Edition.

Zangari, P (1996). A VaR methodology for portfolios that include options, *RiskMetrics Monitor*, 4–12.