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Market skewness risk and the cross section of stock returns ☆

Bo Young Chang [a], Peter Christoffersen [b,c,*], Kris Jacobs [d,e]

[a] Bank of Canada, Canada
[b] Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario, Canada M5P 3E6
[c] Copenhagen Business School and CREATES, Denmark
[d] C.T. Bauer College of Business, University of Houston, United States
[e] Tilburg University, The Netherlands

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ABSTRACT

The cross section of stock returns has substantial exposure to risk captured by higher moments of market returns. We estimate these moments from daily Standard & Poor's 500 index option data. The resulting time series of factors are genuinely conditional and forward-looking. Stocks with high exposure to innovations in implied market skewness exhibit low returns on average. The results are robust to various permutations of the empirical setup. The market skewness risk premium is statistically and economically significant and cannot be explained by other common risk factors such as the market excess return or the size, book-to-market, momentum, and market volatility factors, or by firm characteristics.

1. Introduction

Stock market index volatility is an indicator of market-wide risk, and the capital asset pricing model (CAPM) predicts that it is a determinant of the market equity premium. Recent studies by Ang, Hodrick, Xing, and Zhang (2006) and Adrian and Rosenberg (2008) demonstrate that, contrary to the CAPM intuition, market-wide volatility risk is also priced in the cross section of stock returns. Given these findings, and given the overwhelming evidence in the literature that market-wide skewness and kurtosis are important indicators of market-wide risk and that those risks do not co-vary perfectly with volatility risk, an investigation of higher moments of the market return as pricing factors in the cross section of stock returns seems worthwhile. We extend the investigation of Ang, Hodrick, Xing, and Zhang (2006) and examine if market skewness and kurtosis risks affect the cross section of stock returns.

Our empirical tests use moments of market returns implied by Standard and Poor's (S&P) 500 index options. Moment estimates are extracted from option prices using the model-free methodology of Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003). These moments are forward-looking. They are computed every day using just one day of option price data, so that the resulting estimates are genuinely

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* Corresponding author at: Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario, Canada M5P 3E6. Tel.: +1 416 946 5511; fax: +1 416 971 3048.
E-mail address: peter.christoffersen@rotman.utoronto.ca (P. Christoffersen).

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conditional. Our approach avoids the traditional trade-off problem with estimates of higher moments from historical returns data that need to use long windows to increase precision, but short windows to obtain conditional instead of unconditional estimates.

We conduct two types of empirical exercises. First, by sorting all stocks on the NYSE, Amex, and Nasdaq from 1996 through 2007 in quintiles based on the exposures of their returns to innovations in market moments, we find evidence that market skewness is priced in the cross section of stocks and that stock returns have substantial exposure to higher moment risk. We find that stocks with high exposures to innovations in market skewness exhibit low returns on average. Stocks with high exposure to volatility and kurtosis exhibit somewhat higher returns on average in our sample. We extensively investigate the robustness of our empirical results and find that the effect of market skewness risk is robust, whereas that of market volatility and kurtosis risk is sensitive to variations in test methodology and data sample.

Skewness and kurtosis innovations are highly negatively correlated (-0.83), and so we rely on kurtosis innovations that have been orthogonalized by skewness. Nevertheless, the large negative correlation suggests that it might be difficult to fully separate the effects of skewness and kurtosis. We, therefore, construct factor portfolios for market skewness and kurtosis risk using a quadruple independent sort of stocks with respect to their exposures to market excess returns, innovations in market volatility, innovations in market skewness, and innovations in market kurtosis.

We find that the average return on the market skewness risk factor portfolio is -0.78% per month, or -9.36% per year, and this return cannot be explained by market beta, the size factor, the book-to-market factor, or the momentum factor. We also estimate the price of market skewness risk by running Fama and MacBeth regressions. We find that estimates of the premium for market skewness risk are consistently negative and significant with values of approximately -7% per year.

Our empirical findings contribute to an existing literature that emphasizes the importance of higher-moment risk in asset pricing. Part of this literature investigates three- and four-factor CAPMs, building on the seminal contribution by Kraus and Litzenberger (1976). For theory and empirical results on the three- and four-factor CAPM, see, for example, Dittmar (2002), Friend and Westerfield (1980), Harvey and Siddique (1999, 2000a, 2000b), Hwang and Satchell (1999), Lim (1989), Sears and Wei (1985, 1988), and Chabi-Yo (2008). The higher-moment CAPMs are static models, and as a result, risk in time-varying market moments is not priced in these models. We conjecture that once moments are allowed to vary over time, their innovations will also be priced. Chabi-Yo (2012) proposes an intertemporal asset pricing model whose pricing kernel includes higher moments of market returns as well as their innovations.

Also, the higher moments of the market returns could simply be valuable indicators of the underlying economic conditions. This is the idea behind Ang, Hodrick, Xing, and Zhang (2006), who study the performance of volatility as a risk factor. Ang, Hodrick, Xing, and Zhang (2006) motivate their study using the Merton (1973) intertemporal asset pricing model (ICAPM), and because it has been established empirically that increased volatility is associated with a deterioration of the investment opportunity set, this suggests a negative price of risk. Unfortunately, theoretically determining the price of skewness and kurtosis risk using this approach is less straightforward.

Our results are related to several other studies. Kapadia (2006) finds a negative price of market skewness risk, but he uses the cross-sectional skewness across stocks at a particular time, which is very different from our approach. Adrian and Rosenberg (2008) find a positive price of market skewness risk when the market skewness is estimated from a historical time series of daily stock returns. Our experimentation with historical higher moments did not produce robust results and so we prefer working with option-implied moments, as in Ang, Hodrick, Xing, and Zhang (2006). Conrad, Dittmar, and Ghysels (2012) and Xing, Zhang, and Zhao (2010) study the cross-sectional differences in stock returns as a function of the risk-neutral skewness of individual stocks.

Vanden (2004, 2006) shows that, in an equilibrium setup with non-negative wealth constraints, the pricing kernel is a function of the return on a stock market index and the return on market index options as well as higher powers of these two returns. Vanden's models are related to our approach in that his pricing kernel includes information from the index option market.

Our study is also related to Agarwal, Bakshi, and Huij (2009). They examine a linear multifactor model in which the factors include innovations in implied market volatility, innovations in implied market skewness, and innovations in implied market kurtosis, using the same methodology that we use to extract implied moments of the market return. They find that hedge funds are systematically exposed to volatility, skewness, and kurtosis risk. Our work differs from theirs in that we explain the expected returns of individual stocks, not hedge funds.

The paper proceeds as follows. In Section 2 we introduce the empirical model and we discuss the relation between our findings and the existing literature. In Section 3, we discuss the data, as well as the methods used to extract higher moments from option data, and the models used to extract innovations from option-implied moments. Section 4 presents empirical results obtained by sorting the cross section of stocks into quintiles based on exposures to market moments. Section 5 constructs factor portfolios, and Section 6 estimates the price of market skewness risk using cross-sectional regressions. Section 7 concludes.

2. The analytical framework

We use two strategies to empirically investigate a conditional multifactor representation of equilibrium expected returns, with the moments of the stock market return, namely, volatility, skewness, and kurtosis, as state variables.

The first strategy is based on univariate sorting. We use a sample of returns and moments for a time period $t=1,\ldots,T$ to estimate the risky assets' loadings on innovations in market moments through time series regressions of the form

$$R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta VOL}^i\Delta VOL_t \\ +\beta_{\Delta SKEW}^i\Delta SKEW_t+\beta_{\Delta KURT}^i\Delta KURT_t+\varepsilon_{i,t}, \quad (1)$$

where $R_{i,t}$, $R_{m,t}$, and $R_{f,t}$ are the rates of return on the ith risky asset, the market portfolio, and the risk-free asset, respectively. Further $\Delta VOL_t=VOL_t-E_{t-1}[VOL_t]$, $\Delta SKEW_t=SKEW_t-E_{t-1}[SKEW_t]$, and $\Delta KURT_t=KURT_t-E_{t-1}[KURT_t]$. The regression coefficients, β_{MKT}^i, $\beta_{\Delta VOL}^i$, $\beta_{\Delta SKEW}^i$, and $\beta_{\Delta KURT}^i$, are the measures of the ith risky asset's exposures to market excess return, market volatility, market skewness, and market kurtosis risks, respectively. We assume that $\varepsilon_{i,t}$ is homoskedastic and independent of R_m-R_f, ΔVOL, $\Delta SKEW$, or $\Delta KURT$.

Estimates of the regression coefficients, β_{MKT}^i, $\beta_{\Delta VOL}^i$, $\beta_{\Delta SKEW}^i$, and $\beta_{\Delta KURT}^i$, are used to sort the available assets into different portfolios, and the cross-sectional performance of these portfolios is indicative of the price of risk associated with the different factors.

The second set of results is based on multivariate sorts and cross-sectional regressions. We use regression coefficients for assets $i=1,\ldots,N$ obtained from time series regressions to estimate the prices of the market moment risks λ from the cross-sectional relation:

$$E[R_i]-R_f=\lambda_0+\lambda_{MKT}\beta_{MKT}^i+\lambda_{\Delta VOL}\beta_{\Delta VOL}^i+\lambda_{\Delta SKEW}\beta_{\Delta SKEW}^i \\ +\lambda_{\Delta KURT}\beta_{\Delta KURT}^i. \quad (2)$$

Economic theory provides little guidance on the signs of the prices of the market skewness and kurtosis risks. It is very important in this regard to note the differences between our setup and available results on the three-moment and four-moment CAPM.

The main insight of the four-moment CAPM is that only systematic volatility, skewness, and kurtosis matter in pricing. Investors with reasonable utility functions (Kimball, 1993) prefer wealth portfolios with low volatility, high positive skewness, and low kurtosis, and so they need to be compensated with higher expected wealth when their portfolios exhibit returns with high volatility, low skewness, or high kurtosis. However, it is not straightforward to relate the prices of risk from the four-factor CAPM to the prices of risk in Eq. (2).

The ICAPM, following Merton (1973), Campbell (1996), Ang, Hodrick, Xing, and Zhang (2006), and Chen (2003), allows us to formulate a prior for the sign of the price of volatility risk. Volatility matters in the cross section of returns because it allows investors to hedge against changes in future investment opportunities. The prices of risk of the factors, therefore, depend on whether they reflect improvements or deteriorations in the economy's (future) opportunity set. For instance, if high market volatility today is related to an unfavorable investment opportunity set tomorrow, then an asset whose return is positively related to the innovation in market volatility provides a hedge against a deterioration in the investment opportunity set. When investors are risk averse, the hedge provided by this asset is desirable, resulting in a lower expected return for such asset. The price of market volatility risk is then negative. In the opposite scenario in which high market volatility is related to a favorable future investment opportunity set, the price of market volatility risk is positive. Previous studies have found that market volatility is high when market returns are low, a phenomenon sometimes termed the leverage effect. Therefore, we can expect the sign of the price of market volatility risk, $\lambda_{\Delta VOL}$, to be negative.

Following the reasoning regarding the price of market volatility risk $\lambda_{\Delta VOL}$, empirical findings on the correlation between higher market moments and market returns could be used to provide guidance regarding the price of market skewness and kurtosis risk. However, this reasoning ignores the investors' preferences for skewness and is, therefore, incomplete. In Chabi-Yo (2012), which can be seen as an intertemporal extension of the three-moment and four-moment CAPM, market volatility, skewness, and kurtosis show up as cross-sectional pricing factors. The price of market volatility risk $\lambda_{\Delta VOL}$ is negative if agents' preference for skewness is smaller than one, which depends on the third derivative of the utility function. Similarly, the price of market skewness risk $\lambda_{\Delta SKEW}$ and the price of market kurtosis risk $\lambda_{\Delta KURT}$ depend, respectively, on the fourth and the fifth derivative of the utility function, which are hard to sign. Chabi-Yo (2012) estimates the preference parameters characterizing his model and finds that the price of volatility risk and skewness risk are both negative.

Option-implied skewness has an alternative interpretation as a measure of jump risk or downside risk (Bates, 2000; Pan, 2002; Doran, Peterson, and Tarrant, 2007). Under this interpretation of option-implied skewness, a positive innovation in option-implied market skewness indicates decreased jump risk in the stock market, which is likely to be related to an improved investment opportunity set. The expected relation between $\Delta SKEW$ and the market excess return is, therefore, positive, and stocks with low exposures to innovations in market skewness provide a valuable hedge against downside risk of the stock market. Investors require lower returns on these stocks, and we would expect the price of market skewness risk to be positive.

Recent studies by Bollen and Whaley (2004) and Gârleanu, Pedersen, and Poteshman (2009) suggest that the skewness of the implied volatility curve is mainly caused by the imbalance in supply and demand for options with different strike prices. Unfortunately, because these studies take the imbalance in supply and demand in options as exogenous, it is not straightforward to interpret the relation between innovations in option-implied market skewness and changes in the investment opportunity set using their models.

In summary, when using skewness and kurtosis as cross-sectional pricing factors, theory does not provide much guidance regarding the a priori expected sign of the prices of risk. Determining the sign is, therefore, largely an empirical exercise.

3. Market moment innovations

In this section, we first introduce the option-implied moments of market returns. We then model the conditional moment expectations that allow us to define moment innovations. Finally, we describe the stock return sample.

3.1. Estimating higher moments of market returns

Several methods are available to estimate moments of the market return. The most widely used estimator is the sample moment from historical returns. However, the computation of sample moments necessitates a choice of time window. It is well known that it is difficult to estimate higher moments precisely (Kim and White, 2004), which would suggest the use of long windows. However, it is preferable to use short windows to capture the conditional nature of the factor loadings. To obtain more reliable estimates of conditional higher moments, a time series model can be used, but then the question arises whether the empirical results are robust to the choice of time series model.

The availability of tick-by-tick price data provides an interesting alternative. We can estimate the return variance by summing the squares of high frequency returns (e.g., 5 minutes) as described in Andersen, Bollerslev, Diebold, and Labys (2003) and Barndorff-Nielsen and Shephard (2002). However, the estimation of skewness or kurtosis from high frequency data is much less customary, perhaps because the sampling properties of these estimators are as yet unknown.

In this study, we instead estimate the higher moments of market returns by extracting moments implied by daily index option prices. We use the model-free methodology proposed by Carr and Madan (2001), Bakshi and Madan (2000), Bakshi, Kapadia, and Madan (2003), and Jiang and Tian (2005) to estimate the moments of market returns implied by S&P 500 index option prices and call them *VOL*, *SKEW*, and *KURT*. We use implied moments over a 30-day horizon for our tests, based on options with one-month maturity. The details of the methodology and implementation are provided in the Appendix. Data on S&P 500 index options from 1996 through 2007 are obtained from OptionMetrics. Because *VOL* has a correlation of 0.99 with the Chicago Board Options Exchange market volatility index (*VIX*), we use *VIX* for *VOL* in our analysis to facilitate replication of our results and maximum comparability with the results of Ang, Hodrick, Xing, and Zhang (2006).

Three remarks are in order at this point. First, in contrast with traditional historical moment estimates, option implied moments are obtained using a single day of option data and are, therefore, conceptually more suited for the testing of conditional asset pricing models. Second, option implied moments have the advantage of being forward-looking, which is more consistent with underlying theories of expected, i.e., future, returns. Third, option implied moments are risk-neutral moments. Following Ang, Hodrick, Xing, and Zhang (2006), we do not try to specify the risk premiums required to convert the risk-neutral moments to their physical counterparts. Doing so would require choosing one from many possible specifications of the volatility risk and jump risk premiums. See Bates (2000), Broadie, Chernov, and Johannes (2007), Pan (2002), Jones (2003), and Eraker (2004) for specifications of volatility and jump risk premia in option valuation. While it is a potential disadvantage that changes in risk-neutral moments reflect both changes in the physical moment and changes in the risk premium, this has to be traded off against their advantages, notably, that they can be obtained using a single day of data using the rich information available in option prices. The usefulness of these moments as pricing factors is thus largely an empirical question, and our empirical results show that risk-neutral skewness implied from options is an important risk factor.

The daily measures of *VOL*, *SKEW*, and *KURT* are shown in Fig. 1. All three time series vary significantly through time. Consistent with available empirical evidence, the implied market skewness measures are always negative. Furthermore, kurtosis is always larger than three.

3.2. Measuring innovations in market moments

To obtain estimates of innovations in market moments, we fit an appropriate autoregressive moving average (ARMA) model to the time series for each moment. The results are reported in Table 1. We report the autocorrelation functions (ACF) of the original time series, the ACF of the first differences, and the ACF of the ARMA(1,1) residuals for *VOL*, *SKEW*, and *KURT* in Fig. 2 to demonstrate our choice of time series model for each market moment. For *VOL*, taking the first difference removes most of the autocorrelation in the data, whereas for *SKEW* and *KURT*, ARMA(1,1) models are needed to remove the autocorrelation. Using the first difference of *VOL* has an additional advantage that it makes it easier to compare our empirical results with those of other related studies such as Ang, Hodrick, Xing, and Zhang (2006), because many of these studies employ the change in *VOL* in their analyses. To check for robustness, we repeat our tests using the ARMA(1,1) residuals of *VOL* as our measures of ΔVOL and find that the results are very similar. The results of this robustness test are not reported in the paper but are available upon request.

Our main results use the entire time series to calibrate the ARMA parameters and then use these parameters to compute $\Delta SKEW$ and $\Delta KURT$. The use of the entire time series can seem somewhat unrealistic because investors can observe only past moments when forming their expectations on future moments. We also compute innovations recursively using only past moments in ARMA estimation, and this yields a very similar time series.

The resulting measures of innovations in market moments are obtained using the following models:

$$\Delta VOL_t = VOL_t - VOL_{t-1}, \tag{3}$$

$$\Delta SKEW_t = 100 \times (SKEW_t - 0.9962 \times SKEW_{t-1} + 0.3618 \times \Delta SKEW_{t-1}) \tag{4}$$

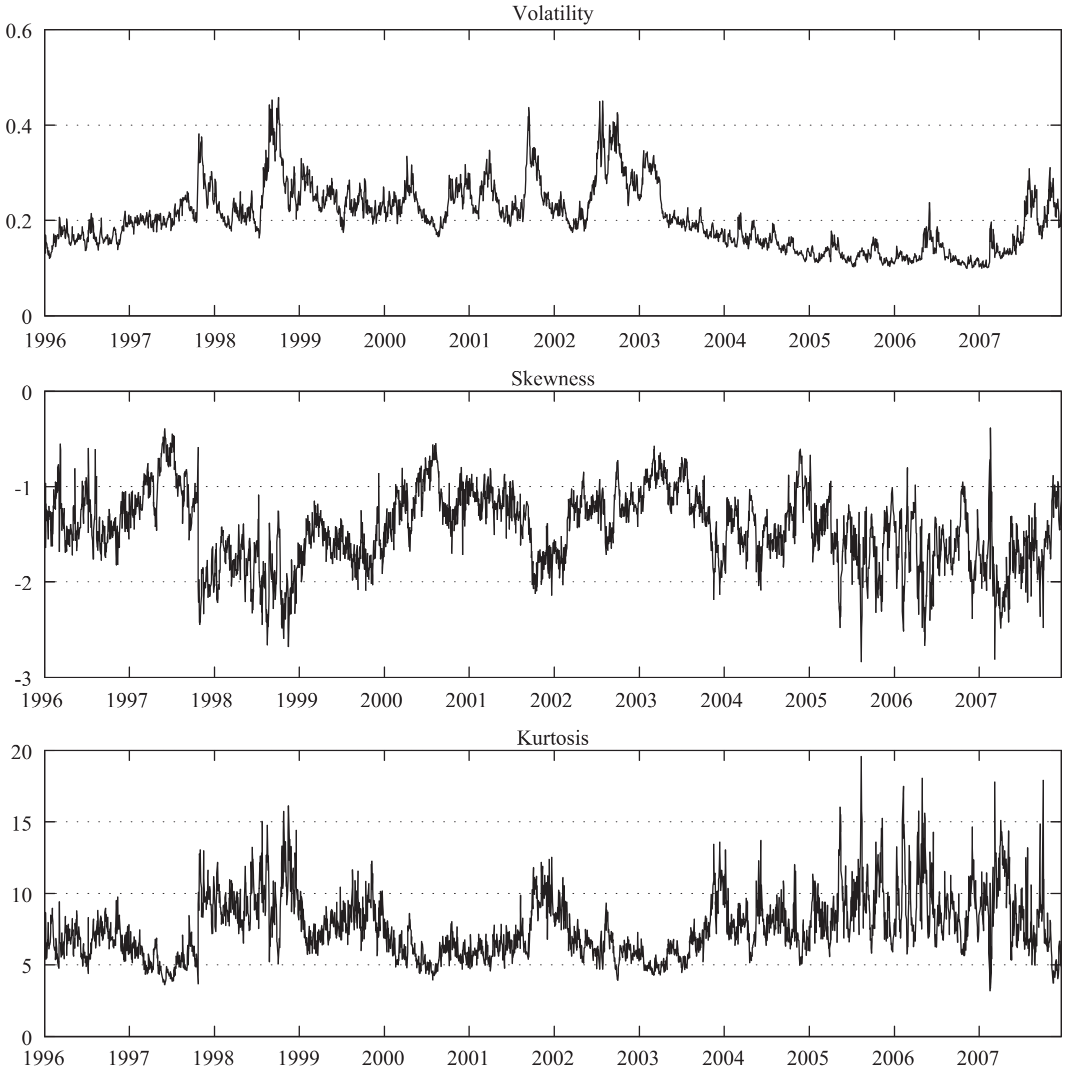

Fig. 1. Daily option implied moments of Standard & Poor's 500 index returns. We plot daily option implied volatility, skewness, and kurtosis of the S&P 500 index return from 1996 through 2007. The model-free methodology developed in Carr and Madan (2001), Bakshi and Madan (2000), and Bakshi, Kapadia, and Madan (2003) is applied to extract the option implied moments using option data available from OptionMetrics Ivy DB. See the Appendix for details of the methodology and implementation. The time series of moments can be downloaded from jfe.rochester.edu/data.htm.

Table 1
Daily risk factors: dynamics and moments.

We report the correlations between daily innovations in implied moments, ΔVOL, $\Delta SKEW$, and $\Delta KURT$ and the standard pricing factors, $R_m - R_f$, *SMB*, *HML*, and *UMD*. $\Delta VOL_t = VOL_t - VOL_{t-1}$, and $\Delta SKEW$ and $\Delta KURT$ are the residuals from fitting an ARMA(1,1) to the time series of corresponding moments using the entire sample. *SMB*, *HML*, and *UMD* are the returns on the factor portfolios for size, book-to-market, and momentum risks. We also report the average of each factor as well as the AR(1) and MA(1) parameters used to construct the $\Delta SKEW$ and $\Delta KURT$ residuals.

Risk factor	AR(1) Parameter	MA(1) Parameter	Average	Correlation ΔVOL	Correlation $\Delta SKEW$	Correlation $\Delta KURT$
ΔVOL	−1.0000	0.0000	2.9E−05		0.17	−0.25
$\Delta SKEW$	−0.9962	0.3618	−8.4E−05			−0.83
$\Delta KURT$	−0.9936	0.4032	8.0E−04			
$R_m - R_f$			2.7E−04	−0.79	−0.20	0.28
SMB			3.9E−05	0.10	0.01	−0.04
HML			1.8E−04	0.42	0.09	−0.13
UMD			4.5E−04	0.01	−0.01	0.01

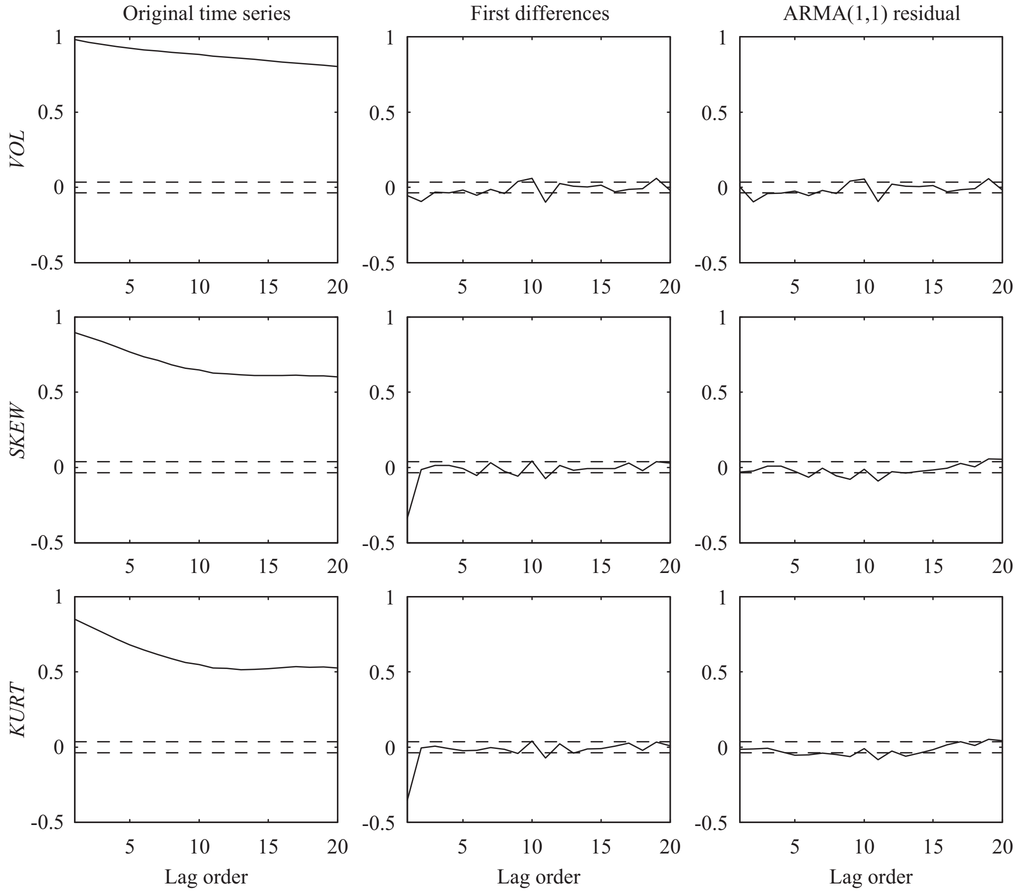

Fig. 2. Autocorrelation functions of option implied market moments. We plot sample autocorrelations of the original time series, their first differences, and their autoregressive moving average ARMA(1,1) residuals for volatility (*VOL*), skewness (*SKEW*), and kurtosis (*KURT*). The horizontal dashes around zero indicate 95% confidence intervals.

and

$$\Delta KURT_t = 100 \times (KURT_t - 0.9936 \times KURT_{t-1} + 0.4032 \times \Delta KURT_{t-1}). \quad (5)$$

The AR(1) parameters for both *SKEW* and *KURT* are very close to -1. This indicates that we can use an MA(1) model on the first differences to obtain the innovations in *SKEW* and *KURT*. The results from using this alternative assumption on the innovations are very similar to the results reported and are available upon request.

Table 1 reports the correlations between ΔVOL, $\Delta SKEW$, and $\Delta KURT$, as well as correlations with known pricing factors, namely, the excess market return $R_m - R_f$, the Fama and French (1993) pricing factors *SMB* and *HML*, and the momentum factor *UMD*. ΔVOL is highly negatively correlated (-0.79) with the market excess return and highly positively correlated (0.42) with *HML*. Also, $\Delta SKEW$ and $\Delta KURT$ are highly negatively correlated (-0.83).

A negative relation between $\Delta SKEW$ and $\Delta KURT$ is to be expected. The option implied distribution has a fat left tail and, so, negative skewness on average. A negative shock to skewness increases the fat left tail further and, thus, increases kurtosis. That is, the relation between $\Delta SKEW$ and $\Delta KURT$ is negative. To separate the effect from skewness on kurtosis from pure kurtosis dynamics, we orthogonalize $\Delta KURT$ by regressing it on the contemporaneous $\Delta SKEW$. Throughout the paper we use the residuals from this regression as $\Delta KURT$.

Fig. 3 plots the daily innovations for the three option implied moments. ΔVOL are the first differences in *VOL*. $\Delta SKEW$ are the ARMA residuals from Eq. (4), and $\Delta KURT$ are the ARMA residuals from Eq. (5) orthogonalized by $\Delta SKEW$.

3.3. Return data

We use returns on all stocks included in the Center for Research in Securities Prices NYSE/Amex/Nasdaq daily stock file. The stock index return, risk-free rate, and the factor mimicking portfolio returns for size, book-to-market, and momentum factors, as well as the returns of the 25 Fama and French portfolios formed on size and book-to-market, are obtained from the online data library of Ken French which can be found at mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Because OptionMetrics option data start in 1996, all our tests are focused on the 12-year period from 1996 through 2007.

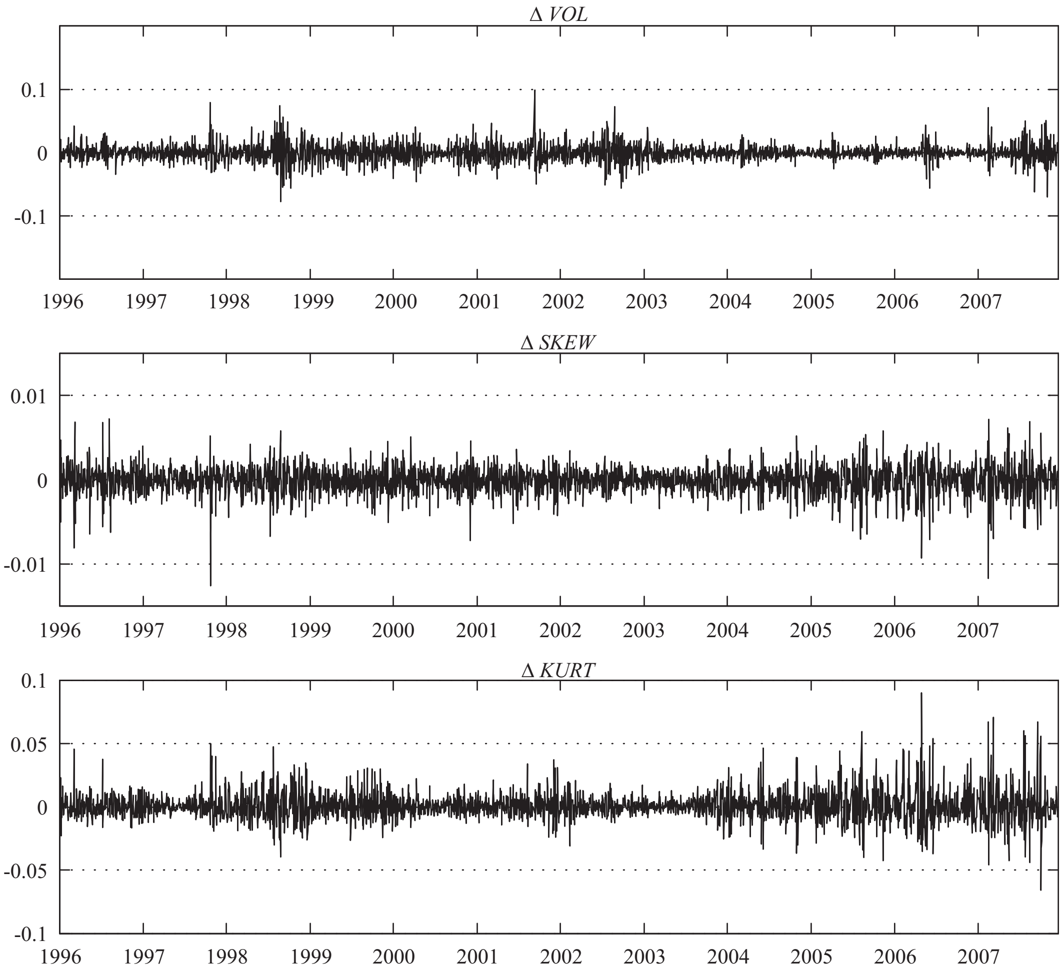

Fig. 3. Daily innovations in option implied moments of Standard and Poor's (S&P) 500 index returns. We plot daily innovations in option implied volatility, skewness, and kurtosis of the S&P 500 index from 1996 through 2007. For $\Delta SKEW$ and $\Delta KURT$ the innovations are the residuals obtained after fitting the entire time series of the moments to the appropriate autoregressive moving average model. $\Delta KURT$ is further orthogonalized by $\Delta SKEW$. For ΔVOL we simply use first differences of the *VOL* series.

4. Portfolio sorts on exposure to market moments

This section reports on three empirical exercises that all follow a similar approach. In each case, we sort the cross section of stock returns into quintiles based on the stock's exposure to innovations in one of the market moments. We first sort based on the exposure to innovations in market volatility, and subsequently we sort based on exposure to market skewness and kurtosis. Finally, we briefly report on various robustness exercises.

4.1. Portfolios sorted on market volatility exposure

The main implication of the model in Eq. (2) is that stocks with different exposures to innovations in market volatility, skewness, or kurtosis exhibit different returns on average. In this subsection, we test if this implication holds for market volatility risk by first constructing portfolios based on their exposures to ΔVOL and subsequently comparing the average returns and alphas of these portfolios. To avoid spurious effects we consider out-of-sample returns and alphas, following the procedures in Ang, Hodrick, Xing, and Zhang (2006), Agarwal, Bakshi, and Huij (2009), and Harvey and Siddique (2000a), among others.

Ang, Hodrick, Xing, and Zhang (2006) have demonstrated that stocks with high exposure to ΔVOL exhibit lower average returns compared with stocks with low exposure to ΔVOL, using all stocks in the NYSE, Amex, and Nasdaq between 1986 and 2000. See Adrian and Rosenberg (2008), Goyal and Santa-Clara (2003), Fu (2009), and Bali and Cakici (2008) for additional work on volatility and the cross section of returns. They do not consider market skewness and kurtosis risks. We investigate whether the effect of market volatility risk persists even after taking market skewness and kurtosis risks into account.

To capture the conditional nature of the factor exposures, we use daily return data with fairly short windows. Following Pástor and Stambaugh (2003), Ang, Hodrick, Xing, and Zhang (2006), and Lewellen and Nagel (2006), we start by using a one-month window that appears to strike a good balance between getting reasonably precise estimates while allowing for time-varying factor loadings. At the end of each month, we run one of the following time series regressions on daily returns of each stock during that month to estimate its exposure to ΔVOL. The first specification measures stocks' exposure to ΔVOL after controlling for exposure to market excess return.

$$R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta VOL}^i\Delta VOL_t+\epsilon_{i,t}. \tag{6}$$

The second specification controls for exposure to market excess returns, $\Delta SKEW$, and $\Delta KURT$.

$$R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta VOL}^i\Delta VOL_t + \beta_{\Delta SKEW}^i\Delta SKEW_t+\beta_{\Delta KURT}^i\Delta KURT_t+\epsilon_{i,t}. \tag{7}$$

We then sort the stocks into quintiles based on their regression coefficients, $\beta_{\Delta VOL}$, so that Quintile 1 contains stocks with the lowest $\beta_{\Delta VOL}$ and Quintile 5 contains stocks with the highest $\beta_{\Delta VOL}$.

We form value-weighted portfolios by weighing each stock in the quintile by its relative market value within the quintile at the end of the beta estimation period. After portfolio formation, we record the daily returns of each quintile portfolio during the one-month period following the estimation period and refer to these returns as post-ranking returns. We repeat the procedure by rolling the beta estimation window forward one month at a time. At the end of the procedure, we have time series of daily post-ranking returns as well as time series of monthly pre-ranking $\beta_{\Delta VOL}$ for each quintile portfolio.

To assess if the effect of ΔVOL persists after controlling for other well known factors including market excess return, size, book-to-market, and momentum, we also compute the Jensen alpha of each quintile portfolio with respect to the Carhart four-factor model (Carhart, 1997) using post-ranking daily returns over the whole sample.

For each quintile portfolio, Panel A of Table 2 reports the average pre-ranking $\beta_{\Delta VOL}$, the average post-ranking monthly returns, and the Carhart four-factor alphas. We also report the average return and Carhart four-factor alpha of a portfolio that is long the highest quintile portfolio and short the lowest quintile portfolio, denoted as 5-1. The results for Eq. (6) are referred to as univariate results, and the results for Eq. (7) are referred to as multivariate results.

If the innovation in market volatility is a priced risk factor, we would ideally like to see a monotonic pattern in average returns for portfolios sorted on their exposure to innovations in market volatility. Because high volatility is generally associated with a deterioration in the investment opportunity set, we would expect to see a decreasing pattern in average returns and alphas from Quintile 1 (lowest exposure) to Quintile 5 (highest exposure). We would also expect to see a negative average return and alpha for the high-low portfolio. In Panel A of Table 2, the univariate Carhart four-factor alphas have a dispersion between Quintiles 5 and 1 of -0.71% per month, or -8.52% per year. In the multivariate case the dispersion is -0.53% per month, or -6.36% per year, but the pattern is not entirely monotonic across quintiles. The average raw returns show smaller negative dispersions of -0.46% and -0.29% per month, respectively.

Panel B of Table 2 repeats the analysis of Panel A, but now the betas are obtained using six months of daily data. The longer estimation period makes the estimate less genuinely conditional but can lead to more precise estimates because of the increased sample size. The six-month betas yield small and statistically insignificant dispersion estimates. Results for three-month betas are not reported to save space, but they also yield small and statistically insignificant dispersion estimates.

Overall, we conclude that some evidence exists that ΔVOL is a priced risk factor with a negative price of risk, which confirms the finding of Ang, Hodrick, Xing, and Zhang (2006), but the estimates are statistically significant only in the case of one-month betas, and when $\Delta SKEW$ and $\Delta KURT$ are added as factors the significance decreases. The difference in significance could well be driven by the difference in sample periods. Ang, Hodrick, Xing, and Zhang (2006) use data for 1986–2000, whereas our sample covers the period from 1996 through 2007.

4.2. Portfolios sorted on market skewness exposure

The portfolio formation procedure and the empirical strategy for market skewness risk are identical to that for market volatility risk, except that stocks are sorted on $\beta_{\Delta SKEW}$. Regression (6) is replaced by

$$R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta SKEW}^i\Delta SKEW+\epsilon_{i,t}. \tag{8}$$

The results of this procedure are reported in Table 3. We again report results for one-month and six-month betas. Results for three-month betas are similar but are not reported to save space.

The results show that portfolios sorted on $\beta_{\Delta SKEW}$ typically exhibit a monotonically decreasing pattern in average alphas and returns. The estimate of dispersion is negative in all cases. For one-month betas, the dispersion of the alphas between Quintiles 1 and 5 in Panel A is -0.80% per month in the univariate case and -1.26% per month in the multivariate case, or -9.60% and -15.12% per year, respectively. Both estimates are statistically significant. The spread in average return between the fifth and the first quantile is also significantly negative in both the univariate and multivariate cases.

The estimates in Panel B, obtained with six-month betas, are also large and the alphas are statistically significant.

Overall, strong evidence exists that $\Delta SKEW$ is a priced risk factor with a negative price of risk. Comparing the results in Tables 2 and 3, the exposure to skewness risk seems quantitatively larger than the exposure to volatility risk, and statistically more significant.

4.3. Portfolios sorted on market kurtosis exposure

The portfolio formation procedure for market kurtosis risk is identical to that for market volatility risk, except that stocks are sorted on $\beta_{\Delta KURT}$ and that Eq. (6) is

Table 2
Sorting on ΔVOL loadings.

At the end of each rolling one-month (Panel A) and six-month (Panel B) period, we run the following regressions on the daily returns of each stock:

$$\text{(Univariate)} \quad R_{i,t}-R_{f,t} = \beta_0^i + \beta_{MKT}^i(R_{m,t}-R_{f,t}) + \beta_{\Delta VOL}^i \Delta VOL_t + \epsilon_{i,t},$$

and

$$\text{(Multivariate)} \quad R_{i,t}-R_{f,t} = \beta_0^i + \beta_{MKT}^i(R_{m,t}-R_{f,t}) + \beta_{\Delta VOL}^i \Delta VOL_t + \beta_{\Delta SKEW}^i \Delta SKEW_t + \beta_{\Delta KURT}^i \Delta KURT_t + \epsilon_{i,t}.$$

We then sort the stocks into quintiles based on their regression coefficients, $\beta_{\Delta VOL}$, so that Quintile 1 contains stocks with the lowest $\beta_{\Delta VOL}$ and Quintile 5, those with the highest $\beta_{\Delta VOL}$. We form value-weighted portfolios by weighing each stock in the quintile by its relative market value within the quintile at the end of the beta estimation period. After portfolio formation, we record the daily returns of each quintile portfolio during the one-month period following the estimation period and refer to these returns as post-ranking returns. We repeat the procedure by rolling the beta estimation window forward by one month at a time. At the end of the procedure, we have time series of daily post-ranking returns as well as time series of monthly pre-ranking $\beta_{\Delta VOL}$ for each quintile portfolio. This table reports the average pre-ranking beta and post-ranking return (monthly return in percent) for each quintile portfolio. We also compute the Jensen alpha of each quintile portfolio with respect to the Carhart four-factor model by running a time series regression of the post-ranking daily returns on daily R_m-R_f, *SMB*, *HML*, and *UMD*. We multiply daily alphas by 21 to obtain monthly alphas and report the monthly alphas in percent. The *t*-statistics for the Carhart four-factor alpha estimates are reported in parentheses. The *t*-statistics that are significant with 90% confidence are boldfaced.

	Quintile portfolio					
Sorting statistic	1	2	3	4	5	5-1
Panel A: One-month beta estimation period						
Volatility beta (univariate)	−1.30	−0.44	0.00	0.44	1.40	
	(−3.79)	**(−2.08)**	(−0.01)	**(2.02)**	**(3.70)**	
Average return	1.05	0.90	0.98	1.08	0.59	−0.46
	(2.08)	**(2.29)**	**(2.84)**	**(2.76)**	(0.99)	(−1.13)
Carhart four-factor alpha	0.34	0.07	0.08	0.12	−0.37	−0.71
	(1.58)	(0.68)	(0.85)	(1.05)	(−1.60)	**(−1.92)**
Volatility beta (multivariate)	−1.44	−0.48	0.00	0.49	1.54	
	(−3.72)	**(−2.03)**	(0.01)	**(3.00)**	**(3.65)**	
Average return	0.95	0.86	1.12	1.01	0.66	−0.29
	(1.84)	**(2.24)**	**(3.18)**	**(2.55)**	(1.11)	(−0.72)
Carhart four-factor alpha	0.22	0.02	0.27	0.03	−0.31	−0.53
	(1.03)	(0.22)	**(2.80)**	(0.26)	(−1.37)	(−1.49)
Panel B: Six-month beta estimation period						
Volatility beta (univariate)	−0.48	−0.17	0.00	0.18	0.56	
	(−2.12)	(−1.08)	(0.02)	(1.04)	**(1.97)**	
Average return	0.95	0.80	0.83	1.01	1.06	0.11
	(1.88)	**(2.05)**	**(2.22)**	**(2.40)**	(1.64)	(0.22)
Carhart four-factor alpha	0.25	−0.10	−0.09	0.06	0.40	0.15
	(1.20)	(−0.92)	(−0.79)	(0.49)	(1.47)	(0.37)
Volatility beta (multivariate)	−0.49	−0.17	0.00	0.18	0.57	
	(−2.11)	(−1.08)	(0.02)	(1.04)	**(1.97)**	
Average return	1.01	0.82	0.79	0.95	1.00	−0.01
	(2.00)	**(2.17)**	**(2.11)**	**(2.22)**	(1.53)	(−0.03)
Carhart four-factor alpha	0.32	−0.10	−0.18	0.05	0.37	0.05
	(1.55)	(−0.89)	(−1.60)	(0.37)	(1.37)	(0.13)

replaced by

$$R_{i,t}-R_{f,t} = \beta_0^i + \beta_{MKT}^i(R_{m,t}-R_{f,t}) + \beta_{\Delta KURT}^i \Delta KURT + \epsilon_{i,t}. \quad (9)$$

The results of this procedure are reported in Table 4.

The results in Table 4 show that portfolios sorted on $\beta_{\Delta KURT}$ do not exhibit a monotonically increasing pattern in average returns and alphas. The average returns and alphas are never significant and exhibit different signs when using one-month (positive) versus six-month (negative) beta estimation periods. Overall, little evidence exists that $\Delta KURT$ is a priced risk factor.

The results from Tables 2–4 are summarized in Fig. 4. For each quintile we plot the average pre-ranking beta on the horizontal axis and the average monthly return (circle marker) and average monthly alpha (asterisk marker) on the vertical axis. The top panel shows the betas based on ΔVOL, the middle panel show the betas based on $\Delta SKEW$, and the bottom panel shows the betas based on $\Delta KURT$. The figure uses one-month betas and the multivariate version of the exposure regression. Fig. 4 highlights that the cross-sectional results for market skewness are much stronger than those for volatility and kurtosis.

4.4. Results on subperiods

The period from 1996 through 2007 is characterized by an initial bubble-like stock market boom (until August 2000), followed by a sharp stock market decline and then recovery. To verify that our results are not driven by the peculiar circumstances in this sample period, we repeat our tests on two subperiods: 1996–2000 and 2001–2007. We report results obtained using the one-month beta estimation period and betas from Eq. (7), which includes innovations in all market moments.

Table 3
Sorting on $\Delta SKEW$ loadings.

At the end of each rolling one-month (Panel A) and six-month (Panel B) period, we run the following regressions on the daily returns of each stock:

$$\text{(Univariate)} \quad R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta VOL}^i \Delta SKEW_t+\epsilon_{i,t},$$

and

$$\text{(Multivariate)} \quad R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta VOL}^i \Delta VOL_t+\beta_{\Delta SKEW}^i \Delta SKEW_t+\beta_{\Delta KURT}^i \Delta KURT_t+\epsilon_{i,t}.$$

We then sort the stocks into quintiles based on their regression coefficients, $\beta_{\Delta SKEW}$, so that Quintile 1 contains stocks with the lowest $\beta_{\Delta SKEW}$ and Quintile 5, those with the highest $\beta_{\Delta SKEW}$. We form value-weighted portfolios by weighing each stock in the quintile by its relative market value within the quintile at the end of the beta estimation period. After portfolio formation, we record the daily returns of each quintile portfolio during the one-month period following the estimation period and refer to these returns as post-ranking returns. We repeat the procedure by rolling the beta estimation window forward by one month at a time. At the end of the procedure, we have time series of daily post-ranking returns as well as time series of monthly pre-ranking $\beta_{\Delta SKEW}$ for each quintile portfolio. This table reports the average pre-ranking beta and post-ranking return (monthly return in percent) for each quintile portfolio. We also compute the Jensen alpha of each quintile portfolio with respect to the Carhart four-factor model by running a time series regression of the post-ranking daily returns on daily R_m-R_f, *SMB*, *HML*, and *UMD*. We multiply daily alphas by 21 to obtain monthly alphas and report the monthly alphas in percent. The *t*-statistics for the Carhart four-factor alpha estimates are reported in parentheses. The *t*-statistics that are significant with 90% confidence are boldfaced.

	Quintile portfolio					
Sorting statistic	1	2	3	4	5	5-1
Panel A: One-month beta estimation period						
Skewness beta (univariate)	−6.27	−2.03	0.08	2.16	6.48	
	(−3.76)	**(−1.95)**	(0.14)	**(2.09)**	**(3.77)**	
Average return	1.22	1.12	0.89	0.84	0.63	−0.59
	(2.40)	**(2.97)**	**(2.47)**	**(2.08)**	(1.12)	**(−1.88)**
Carhart four-factor alpha	0.52	0.27	0.02	−0.13	−0.28	−0.80
	(2.53)	**(2.58)**	(0.16)	(−1.18)	(−1.37)	**(−2.42)**
Skewness beta (multivariate)	−8.44	−2.65	0.15	2.95	8.74	
	(−3.68)	**(−1.90)**	(0.14)	**(2.05)**	**(3.73)**	
Average return	1.44	1.02	0.96	0.91	0.53	−0.91
	(2.77)	**(2.62)**	**(2.76)**	**(2.29)**	(0.97)	**(−2.57)**
Carhart four-factor alpha	0.84	0.19	0.04	−0.05	−0.43	−1.26
	(3.89)	**(1.83)**	(0.47)	(−0.47)	**(−2.03)**	**(−3.66)**
Panel B: Six-month beta estimation period						
Skewness beta (univariate)	−2.30	−0.78	0.05	0.88	2.54	
	(−1.93)	(−0.95)	(0.11)	(1.09)	**(1.91)**	
Average return	1.08	0.98	0.89	0.91	0.81	−0.27
	(2.05)	**(2.62)**	**(2.36)**	**(2.15)**	(1.36)	(−0.73)
Carhart four-factor alpha	0.52	0.15	0.05	−0.04	−0.12	−0.64
	(2.45)	(1.44)	(0.57)	(−0.32)	(−0.58)	**(−1.88)**
Skewness beta (multivariate)	−2.66	−0.87	0.11	1.08	2.97	
	(−1.93)	(−0.92)	(0.15)	(1.13)	**(1.95)**	
Average return	1.08	0.95	0.85	0.96	0.85	−0.23
	(1.96)	**(2.43)**	**(2.26)**	**(2.31)**	(1.47)	(−0.57)
Carhart four-factor alpha	0.62	0.21	−0.06	0.03	−0.15	−0.77
	(2.56)	**(1.82)**	(−0.65)	(0.24)	(−0.67)	**(−1.98)**

The results are reported in Table 5. When sorting on skewness beta, we find that the patterns in average returns and in Carhart four-factor model alphas observed in Table 3 for the period 1996–2007 are robustly present in the subperiods, although the pattern is not always strictly monotonic. The estimates of the dispersion of the alphas are relatively robust, and they are statistically significant.

The estimate of the volatility dispersion for the Carhart four-factor model alphas is negative in both subperiods, but it is statistically significant for the 1996–2000 period only. The estimate for this period is much larger than the corresponding one in Table 2, even though the dispersion estimate based on returns is comparable. This result indicates that the effect of ΔVOL was significant during 1996–2000 but weakened during 2001–2007. When sorting on kurtosis, the 1996–2000 period does not yield conclusive evidence. We find a positive but insignificant dispersion in both alphas and returns for the 2001–2007 period.

4.5. Alternative measurements of innovations

The results in Tables 2–4 are obtained using innovations in market moments resulting from fitting ARMA models to the entire time series of moments. We obtain a very similar time series of innovations when estimating innovations recursively through time using only past data.

We repeat our tests using the ARMA(1,1) residuals of *VOL* as our measure of ΔVOL and also using the model-free *VOL* series constructed according to Bakshi, Kapadia, and Madan (2003) instead of the VIX. Finally, we repeat our tests using ARIMA(0,1,1) instead of ARMA(1,1) models. In all cases, results are very similar. We do not report these

Table 4
Sorting on $\Delta KURT$ loadings.

At the end of each rolling one-month (Panel A) and six-month (Panel B) period, we run the following regressions on the daily returns of each stock:

$$\text{(Univariate)} \quad R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta VOL}^i\Delta KURT_t+\epsilon_{i,t},$$

and

$$\text{(Multivariate)} \quad R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta VOL}^i\Delta VOL_t+\beta_{\Delta SKEW}^i\Delta SKEW_t+\beta_{\Delta KURT}^i\Delta KURT_t+\epsilon_{i,t}.$$

We then sort the stocks into quintiles based on their regression coefficients, $\beta_{\Delta KURT}$, so that Quintile 1 contains stocks with the lowest $\beta_{\Delta KURT}$ and Quintile 5, those with the highest $\beta_{\Delta KURT}$. We form value-weighted portfolios by weighing each stock in the quintile by its relative market value within the quintile at the end of the beta estimation period. After portfolio formation, we record the daily returns of each quintile portfolio during the one-month period following the estimation period and refer to these returns as post-ranking returns. We repeat the procedure by rolling the beta estimation window forward by one month at a time. At the end of the procedure, we have time series of daily post-ranking returns as well as time series of monthly pre-ranking $\beta_{\Delta KURT}$ for each quintile portfolio. This table reports the average pre-ranking beta and post-ranking return (monthly return in percent) for each quintile portfolio. We also compute the Jensen alpha of each quintile portfolio with respect to the Carhart four-factor model by running a time series regression of the post-ranking daily returns on daily R_m-R_f, *SMB*, *HML*, and *UMD*. We multiply daily alphas by 21 to obtain monthly alphas and report the monthly alphas in percent. The *t*-statistics for the Carhart four-factor alpha estimates are reported in parentheses. The *t*-statistics that are significant with 90% confidence are boldfaced.

	Quintile portfolio					
Sorting statistic	1	2	3	4	5	5-1
Panel A: One-month beta estimation period						
Kurtosis beta (univariate)	−1.91	−0.64	−0.03	0.58	1.81	
	(−3.96)	**(−2.20)**	(−0.10)	**(2.06)**	**(3.95)**	
Average return	0.80	0.87	0.99	0.95	1.10	0.30
	(1.54)	**(2.23)**	**(2.73)**	**(2.47)**	**(1.98)**	(0.92)
Carhart four-factor alpha	−0.01	0.02	0.07	0.00	0.31	0.32
	(−0.06)	(0.14)	(0.75)	(0.04)	(1.47)	(0.93)
Kurtosis beta (multivariate)	−2.47	−0.84	−0.04	0.75	2.37	
	(−3.68)	**(−1.90)**	(0.14)	**(2.05)**	**(3.73)**	
Average return	0.79	0.94	0.97	1.06	0.96	0.17
	(1.51)	**(2.45)**	**(2.71)**	**(2.71)**	**(1.71)**	(0.47)
Carhart four-factor alpha	−0.15	0.01	0.08	0.19	0.28	0.43
	(−0.71)	(0.09)	(0.85)	**(1.76)**	(1.28)	(1.22)
Panel B: Six-month beta estimation period						
Kurtosis beta (univariate)	−0.74	−0.27	−0.04	0.19	0.62	
	(−2.04)	(−1.19)	(−0.16)	(0.95)	**(1.89)**	
Average return	1.04	0.89	0.98	1.02	0.62	−0.42
	(1.86)	**(2.22)**	**(2.68)**	**(2.58)**	(1.15)	(−1.29)
Carhart four-factor alpha	0.31	0.03	0.10	0.10	−0.10	−0.41
	(1.53)	(0.27)	(1.02)	(1.01)	(−0.56)	(−1.39)
Kurtosis beta (multivariate)	−0.84	−0.32	−0.05	0.22	0.73	
	(−2.06)	(−1.19)	(−0.16)	(0.93)	**(1.86)**	
Average return	1.11	0.97	0.94	0.96	0.61	−0.50
	(2.09)	**(2.45)**	**(2.57)**	**(2.40)**	(1.04)	(−1.32)
Carhart four-factor alpha	0.19	0.10	0.02	0.10	0.09	−0.10
	(0.97)	(0.94)	(0.21)	(1.01)	(0.39)	(−0.29)

results because of space constraints. They are available upon request.

4.6. Varying the option maturity

The methodology of Bakshi, Kapadia, and Madan (2003) allows us to estimate option-implied moments using different option maturities. For instance, one-month option-implied moments of the S&P 500 index reflect the investors' expectation of risk in the stock market over the next one month. We focus our analysis in this paper on the one-month option-implied moments and their effects on the stock returns in the next month. But for other applications analyzing asset returns over a period longer than one month, the use of option-implied moments over a longer horizon would be more appropriate.

We investigate (not reported) longer horizons using three-month and six-month options. For three-month options we find again that portfolios sorted on $\beta_{\Delta SKEW}$ exhibit a decreasing pattern in average alphas and returns, but the results for the volatility and kurtosis factors are not robust. We do not find significant results when using six-month options to estimate ΔVOL, $\Delta SKEW$, and $\Delta KURT$. One possible reason is that six-month options are much less liquid, and, therefore, estimates of six-month option-implied moments are more noisy.

4.7. Using historical moments

We repeat our procedure (not reported) using ΔVOL, $\Delta SKEW$, and $\Delta KURT$ estimated from moving windows of daily historical returns instead of option implied moments. In general, we were not able to arrive at robust conclusions. The signs of volatility, skewness, and kurtosis risk typically depend on the window used when estimating the moments. Moreover, the patterns across the

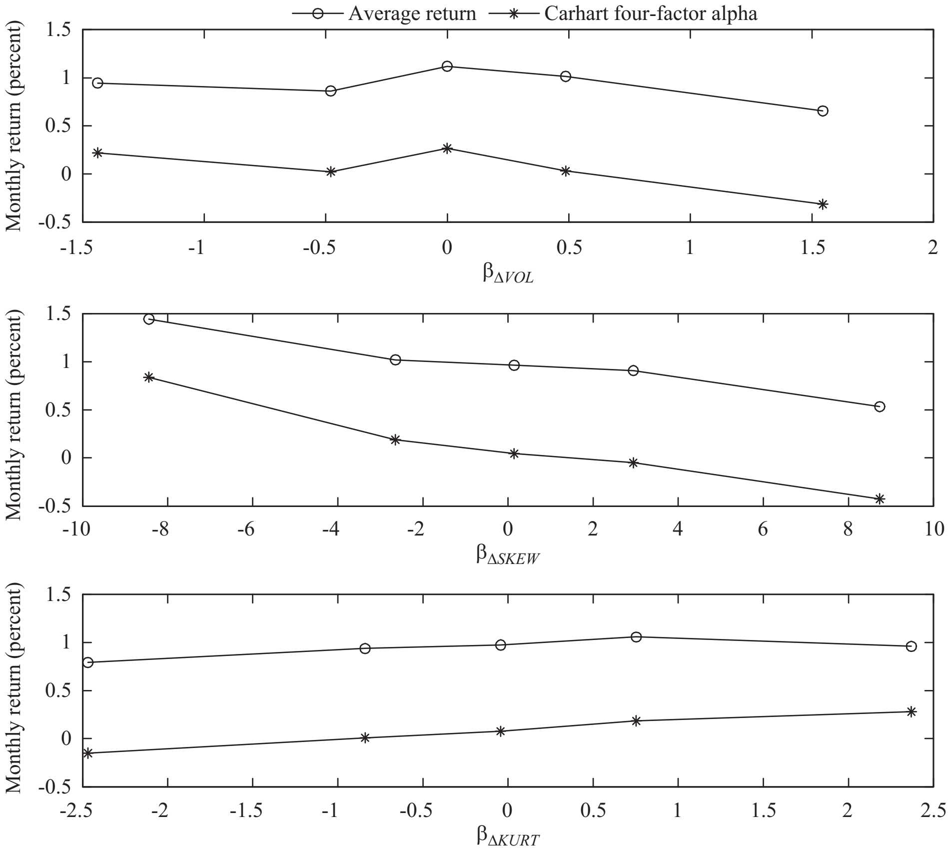

Fig. 4. Average return and alpha versus moment betas. For each quintile we plot the average pre-ranking beta on the horizontal axis and the average monthly return (circle marker) and average monthly alpha (asterisk marker) on the vertical axis. The top panel shows the betas based on ΔVOL, the middle panel show the betas based on $\Delta SKEW$, and the bottom panel shows the betas based on $\Delta KURT$. The numbers are from Panel A of Tables 2–4. The figure uses the multivariate version of the exposure regression.

quintile portfolios are typically not monotonic. Whereas some empirical results are consistent with the findings in Tables 2–4, such findings are hardly meaningful given the lack of robustness, and we do not tabulate them here.

These findings are not necessarily surprising, and they do not invalidate the findings in Tables 2–4. They merely indicate that the option implied moments used in our analysis are different from historical moments. They are estimated using one day of options data with many strike prices, and they incorporate expectations about the future distribution of returns.

5. Returns on factor portfolios

The analysis in Section 4 shows that the effect of $\Delta SKEW$ on the cross section of stock returns is robust to variations in the empirical setup and across sample periods. The results on the effect of ΔVOL and $\Delta KURT$, however, change when the betas are estimated using longer windows or for certain subperiods.

One problem in interpreting the results in Section 4 is the correlation between different market moments. If exposures to different factors are correlated, then it is important to separate the pricing effects of different factors to identify the implication of each market moment separately. To this end we use a four-way sort on β_{MKT}, $\beta_{\Delta VOL}$, $\beta_{\Delta SKEW}$, and $\beta_{\Delta KURT}$ following the sorting approach used in Fama and French (1993), Cochrane (2005), and Liew and Vassalou (2000), among others. Agarwal, Bakshi, and Huij (2009) use a three-way sort on $\beta_{\Delta VOL}$, $\beta_{\Delta SKEW}$, and $\beta_{\Delta KURT}$ when analyzing higher moments. We use a four-way sort because the large negative correlation between the market excess return and innovation in market volatility reported in Table 1 suggests that controlling for β_{MKT} is also important when separating out the pricing effects of different market moments.

At the end of each month, we run a regression with market excess return, ΔVOL, $\Delta SKEW$, and $\Delta KURT$ as factors for each stock, as in Eq. (7). We construct tercile portfolios based on β_{MKT} (lowest in tercile 1 and highest in tercile 3) and also based on $\beta_{\Delta VOL}$, $\beta_{\Delta SKEW}$, and $\beta_{\Delta KURT}$. We then construct 81 portfolios using the intersection of these four sorting criteria.

We repeat the procedure from Section 4 to obtain a time series of pre-ranking betas and post-ranking returns as well as the Carhart four-factor alphas for each of the 81 portfolios. We do not report all the details of these results, but they are available upon request. The dispersion in alphas after controlling for the factors in the Carhart four-factor model is substantial. The impact of each risk factor

Table 5
Sorting on moment risk loadings in subperiods.

At the end of each month, we run the following regression on daily returns of each stock:

$$R_{i,t}-R_{f,t}=\beta_0^i+\beta_{MKT}^i(R_{m,t}-R_{f,t})+\beta_{\Delta VOL}^i\Delta VOL_t+\beta_{\Delta SKEW}^i\Delta SKEW_t+\beta_{\Delta KURT}^i\Delta KURT_t+\epsilon_{i,t}.$$

The remainder of the procedure is the same as in Tables 2–4. The *t*-statistics are reported in parentheses. The *t*-statistics that are significant with 90% confidence are boldfaced.

	Quintile portfolio					5-1 portfolio	
Sorting statistic	1	2	3	4	5	Estimate	*t*-statistic
Panel A: 1996–2000							
Volatility beta	−1.51	−0.52	−0.02	0.50	1.59		
Average return	1.47	1.36	1.75	1.62	1.06	−0.41	(−0.54)
Carhart four-factor alpha	0.50	0.00	0.48	0.09	−0.64	−1.13	**(−1.92)**
Skewness beta	−9.93	−3.17	0.10	3.40	9.93		
Average return	1.94	1.57	1.41	1.54	0.99	−0.94	**(−1.98)**
Carhart four-factor alpha	0.87	0.35	−0.01	0.05	−0.59	−1.46	**(−2.84)**
Kurtosis beta	−2.99	−1.02	−0.07	0.88	2.76		
Average return	1.38	1.61	1.48	1.55	1.08	−0.30	(−0.56)
Carhart four-factor alpha	−0.08	0.23	0.01	0.21	−0.11	−0.03	(−0.05)
Panel B: 2001–2007							
Volatility beta	−1.38	−0.45	0.01	0.48	1.51		
Average return	0.49	0.51	0.60	0.58	0.24	−0.26	(−0.61)
Carhart four-factor alpha	0.12	0.10	0.11	0.01	−0.27	−0.39	(−0.92)
Skewness beta	−7.18	−2.22	0.20	2.61	7.84		
Average return	0.79	0.54	0.60	0.48	0.18	−0.61	(−1.35)
Carhart four-factor alpha	0.56	0.05	0.07	−0.03	−0.30	−0.86	**(−1.91)**
Kurtosis beta	−2.10	−0.71	−0.03	0.65	2.06		
Average return	0.36	0.47	0.60	0.60	0.56	0.20	(0.50)
Carhart four-factor alpha	−0.14	−0.04	0.13	0.09	0.24	0.38	(0.96)

can be traced by looking at the variation in returns when varying that factor and keeping the others constant.

Table 6 summarizes the results for the 81 portfolios by grouping them according to high, medium, or low exposure to each of the factors one at a time and averaging over the 27 portfolios in each group. This grouping procedure allows us to obtain portfolios that differ in exposure to one factor but are neutral in the other factors. The row H–L reports the average returns and alphas of the high-low portfolios that are long 27 high exposure portfolios and short 27 low exposure portfolios with respect to a given factor. The high-low portfolios capture the risk premium of being exposed to the corresponding factors and are equivalent to the *SMB* and *HML* factor portfolios for size and book-to-market. We find that the average return of the $\beta_{\Delta VOL}$ high-low portfolio is 0.30% per month, that of the $\beta_{\Delta SKEW}$ high-low portfolio is -0.78% per month, and that of the $\beta_{\Delta KURT}$ high-low portfolio is 0.03% per month, but only the estimate for the $\beta_{\Delta SKEW}$ high-low portfolio is statistically significant with a *t*-statistic of -3.37.

We also report the Carhart four-factor alpha of the high-low portfolios to see if the return spread is captured by the Carhart four factors. We find that, for the $\Delta SKEW$ exposure portfolios, not only do the alphas decrease from low to high exposure groups as in average returns, but also the magnitudes of the dispersions are even wider for the alphas. The difference in alphas is -1.00 with a *t*-statistic of -2.85. This result shows that the difference between the high and low skewness exposure portfolios cannot be explained by the market excess return, size, book-to-market, or momentum effects.

Thus, in summary, $\Delta SKEW$ exposure portfolios show decreasing patterns in average returns and in Carhart four-factor alphas, consistent with our earlier results. Results for ΔVOL and $\Delta KURT$ are not necessarily robust to the design of the empirical setup.

The returns on the volatility, skewness, and kurtosis factor portfolios constructed above can be used as proxies for risk factors, ΔVOL, $\Delta SKEW$, and $\Delta KURT$, in the same vein as *SMB* and *HML*. We refer to the volatility factor portfolio as *FVOL*, the skewness factor portfolio as *FSKEW*, and the kurtosis factor portfolio as *FKURT*. They correspond to the high-low portfolios in Table 6 and are explicitly defined as

$$FVOL=(1/27)(R\beta_{\Delta VOL,H}-R\beta_{\Delta VOL,L}),$$

$$FSKEW=(1/27)(R\beta_{\Delta SKEW,H}-R\beta_{\Delta SKEW,L}) \tag{10}$$

and

$$FKURT=(1/27)(R\beta_{\Delta KURT,H}-R\beta_{\Delta KURT,L}),$$

where $R\beta_{\Delta VOL,H}$ and $R\beta_{\Delta VOL,L}$ denote the sum of the returns on the 27 portfolios with highest and lowest exposure to ΔVOL, respectively. $R\beta_{\Delta SKEW,H}$, $R\beta_{\Delta SKEW,L}$, $R\beta_{\Delta KURT,H}$, and $R\beta_{\Delta KURT,L}$ are defined in a similar fashion.

The *FVOL*, *FSKEW*, and *FKURT* factors represent the compensation for taking on the risk of time-varying implied market volatility, skewness, and kurtosis, respectively. In Table 7 we report again the means and corresponding

Table 6
Portfolios sorted on exposure to $R_m - R_f$, ΔVOL, $\Delta SKEW$, and $\Delta KURT$.

At the end of each month, we run the following regression on one month's worth of daily returns of each stock:

$$R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i (R_{m,t} - R_{f,t}) + \beta_{\Delta VOL}^i \Delta VOL_t + \beta_{\Delta SKEW}^i \Delta SKEW_t + \beta_{\Delta KURT}^i \Delta KURT_t + \epsilon_{i,t}.$$

We assign each stock into three different groups-high (H), medium (M), and low (L)-based on their exposures (βs) to each of the four market moment factors. Then, the stocks that belong to the same groups based on all four factors are put together to form a market-weighted portfolio, and the portfolio's return in the following month is recorded. The procedure yields the post-ranking returns of 81 portfolios with different exposures to the market moment factors. The table reports the average monthly returns, Carhart four-factor alphas, and pre-ranking betas of the 81 portfolios by group. We also report the Newey and West (1987) t-statistics with 12 lags for the difference in average returns and alphas between the high and low exposure groups. The t-statistics that are significant with 90% confidence are boldfaced.

Portfolio	Average return (% month)	Carhart four-factor alpha (% month)	Pre-ranking exposure			
			β_{MKT}	$\beta_{\Delta VOL}$	$\beta_{\Delta SKEW}$	$\beta_{\Delta KURT}$
β_{MKT}						
L	1.23	0.47	−0.47	−0.14	−0.09	0.04
M	0.98	0.11	0.66	−0.04	0.14	−0.02
H	0.91	0.02	2.00	0.12	0.25	−0.09
H-L	−0.32	−0.45				
t-statistic	(−0.57)	(−1.34)				
$\beta_{\Delta VOL}$						
L	0.92	0.07	0.56	−0.95	0.16	−0.09
M	0.98	0.12	0.72	−0.02	0.11	−0.02
H	1.22	0.41	0.92	0.90	0.03	0.03
H-L	0.30	0.34				
t-statistic	(0.72)	(1.01)				
$\beta_{\Delta SKEW}$						
L	1.40	0.68	0.72	0.00	−5.52	0.04
M	1.11	0.23	0.72	−0.02	0.12	−0.04
H	0.62	−0.31	0.76	−0.04	5.69	−0.07
H-L	−0.78	−1.00				
t-statistic	**(−3.37)**	**(−2.85)**				
$\beta_{\Delta KURT}$						
L	1.09	0.32	0.78	−0.04	0.34	−1.64
M	0.91	0.06	0.72	−0.02	0.15	−0.03
H	1.12	0.22	0.70	0.00	−0.20	1.60
H-L	0.03	−0.09				
t-statistic	(0.14)	(−0.41)				

t-statistics of the factor portfolio returns from Table 6. The risk premium on these factors are simply their average returns, which were estimated to be 0.30%, −0.78%, and 0.03% per month, respectively. To check if these estimates are economically significant, we compare them with the average returns of $R_m - R_f$, *SMB*, *HML*, and *UMD* in the same period. *SMB* and *HML* are Fama and French (1993) size and book-to-market factors and *UMD* is the momentum factor constructed by Ken French.

Interestingly, we find that the average returns on only the skewness and momentum factor portfolios are significantly different from zero during 1996–2007. The average return on *UMD* is the largest in magnitude, 0.83% per month, but the average return on *FSKEW* is almost as large, at −0.78% per month, or −9.36% per year. *FSKEW* has relatively low correlations with other risk factors. The new *FSKEW* factor portfolio thus earns a substantial (negative) return while being relatively orthogonal to other portfolios.

The quadruple sort using terciles based on β_{MKT}, $\beta_{\Delta VOL}$, $\beta_{\Delta SKEW}$, and $\beta_{\Delta KURT}$ yields considerable cross-sectional dispersion, but some of the 81 portfolios consist of relatively few stocks. It is worth investigating if this affects the results.

In Panel B of Table 7, we report estimates of *FVOL* and *FSKEW* obtained using a trivariate sort using terciles based on β_{MKT}, $\beta_{\Delta VOL}$, and $\beta_{\Delta SKEW}$ only. This yields an estimate of −0.30% per month for the average return on the $\beta_{\Delta VOL}$ high-low portfolio, which is consistent with the results in Table 2. The estimate of the average return of the $\beta_{\Delta SKEW}$ high-low portfolio is once again negative at −0.49% per month and statistically significant. The Carhart four-factor returns are not shown in Panel B of Table 7 because they are identical to the Carhart four-factor returns reported in Panel A.

Fig. 5 shows the cumulative returns of the seven factors from Panel A of Table 7. The top panel in Fig. 5 shows the three moment-based factor portfolios and the bottom panel shows the four Carhart factor portfolios. For ease of comparison we show the negative of the return on the *FSKEW* portfolio. The *FSKEW* portfolio not only has the largest total return of the seven factors, but it also appears to be one of the most stable over time. The much lower

Table 7
Factor portfolio returns.

In Panel A, we report the average monthly returns of the factor portfolios *FVOL*, *FSKEW*, and *FKURT*, corresponding to the factors ΔVOL, $\Delta SKEW$, and $\Delta KURT$. The factor portfolios are constructed as follows. At the end of each month, we regress the daily returns of each stock on $R_m - R_f$, ΔVOL, $\Delta SKEW$, and $\Delta KURT$. We then assign each stock into three different groups-high (H), medium (M), and low (L)-based on their exposures (βs) to each of the four market moment factors. Finally, the stocks that belong to the same groups based on all four factors are put together to form a market-weighted portfolio, and the portfolio's return in the following month is recorded. The procedure yields 81 portfolios with different exposures to the market moment factors. Given these portfolios, we form factor portfolios by going long stocks in the group with the highest exposure to the corresponding risk factor and taking short positions in stocks in the group with the lowest exposure to the same risk factor. We report the average monthly returns of the factor portfolios as well as their correlations with $R_m - R_f$, *SMB*, *HML*, and *UMD*. *SMB* and *HML* are Fama and French (1993) size and book-to-market factors, respectively, and UMD is the momentum factor constructed by Kenneth French. The period covered is 1996 to 2007. We report the Newey and West *t*-statistics with 12 lags for the average returns. Significant *t*-statistics at the 90% level are boldfaced. Panel B reports the results from the same procedure using only the first three factors, $R_m - R_f$, ΔVOL, and $\Delta SKEW$, which gives 27 portfolios.

Factors	Mean (percent per month)	*t*-statistic	Correlation						
			FVOL	*FSKEW*	*FKURT*	$R_m - R_f$	*SMB*	*HML*	*UMD*
Panel A: Quadruple sort									
FVOL	0.30	(0.72)	1.00	−0.25	−0.37	−0.28	0.05	0.24	−0.02
FSKEW	−0.78	**(−3.37)**		1.00	0.28	−0.15	−0.07	0.21	0.26
FKURT	0.03	(0.14)			1.00	0.12	0.06	−0.14	0.20
$R_m - R_f$	0.55	(1.42)				1.00	0.22	−0.53	−0.21
SMB	0.22	(0.72)					1.00	−0.49	0.17
HML	0.40	(1.00)						1.00	−0.06
UMD	0.83	**(2.24)**							1.00
Panel B: Triple sort									
FVOL	−0.30	(−1.44)	1.00	0.13		−0.39	0.11	0.27	0.37
FSKEW	−0.49	**(−1.96)**		1.00		−0.01	0.22	−0.06	0.15

volatility in *FSKEW* compared with *UMD* implies that the total return during the 1996–2007 period in Fig. 5 is larger for *FSKEW* than *UMD* in spite of *UMD* having a slightly higher average simple return than *FSKEW* in Table 7.

6. Exploring the risk premiums

In Section 5, we construct a factor for market skewness risk, *FSKEW*, and estimate its premium to be −0.78% per month by simply computing the average monthly return of *FSKEW*. The premium on market volatility risk and market kurtosis risk are much smaller in magnitude, at 0.30% and 0.03% per month, respectively.

In this section, we compute the price of risk by running a series of cross-sectional regressions using different test portfolios and different regression procedures. The 81 portfolios constructed in Section 5 have the largest spread in their exposures to ΔVOL, $\Delta SKEW$, and $\Delta KURT$ by construction, so that the cross-sectional regression tests based on these portfolios yield a good estimate of the prices of market volatility, market skewness, and market kurtosis risk. We check the robustness of the cross-sectional regression test using two other sets of test portfolios: 25 Fama and French portfolios sorted on size and book-to-market ratio and 49 industry portfolios.

6.1. Fama and MacBeth regressions on the 81 factor portfolios

We apply the two-pass regressions of Fama and MacBeth (1973) to the 81 portfolios from Section 5. In the first stage, we estimate betas by running a time series regression of six months of daily returns. In the second stage, we regress the cross section of excess returns of the 81 portfolios on their estimated factor betas to obtain the estimate of the price of risk the next month. The monthly estimates of the price of risk are then averaged to yield the final estimate.

To run the Fama and MacBeth regression, we need to make an assumption on the form of the factor model. We consider several different specifications that control for one or more of the known pricing factors, including $R_m - R_f$, *SMB*, *HML*, *UMD*, ΔVOL, $\Delta SKEW$, and $\Delta KURT$. We include several benchmark models that do not include $\Delta SKEW$ and $\Delta KURT$ so that we can compare the pricing performance of the models that include these factors with those that do not. The factor models considered are (i) *CAPM*, (ii) $CAPM+\Delta VOL$, (iii) $CAPM+\Delta SKEW$, (iv) $CAPM+\Delta KURT$, (v) $CAPM+\Delta VOL+\Delta SKEW$, (vi) $CAPM+\Delta VOL+\Delta KURT$, (vii) $CAPM+\Delta SKEW+\Delta KURT$, (viii) $CAPM+\Delta VOL+\Delta SKEW+\Delta KURT$, (ix) Carhart four-factor, (x) Carhart four-factor$+\Delta VOL$, (xi) Carhart four-factor$+\Delta SKEW$, (xii) Carhart four-factor$+\Delta KURT$, (xiii) Carhart four-factor$+\Delta VOL+\Delta SKEW$, (xiv) Carhart four-factor$+\Delta VOL+\Delta KURT$, (xv) Carhart four-factor$+\Delta SKEW+\Delta KURT$, and (xvi) Carhart four-factor$+\Delta VOL+\Delta SKEW+\Delta KURT$.

The testable prediction of Model (xvi) that includes all the risk factors is

$$E[R_i]-R_f = \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{\Delta VOL}\beta^i_{\Delta VOL} + \lambda_{\Delta SKEW}\beta^i_{\Delta SKEW} + \lambda_{\Delta KURT}\beta^i_{\Delta KURT} + \lambda_{SMB}\beta^i_{SMB} + \lambda_{HML}\beta^i_{HML} + \lambda_{UMD}\beta^i_{UMD}. \quad (11)$$

The predictions of all the other factor models can be formulated in a similar way. The results of the Fama and MacBeth regressions are reported in Table 8.

The estimate of the price of market skewness risk, $\lambda_{\Delta SKEW}$, is negative in all cases. The prices of risk of the

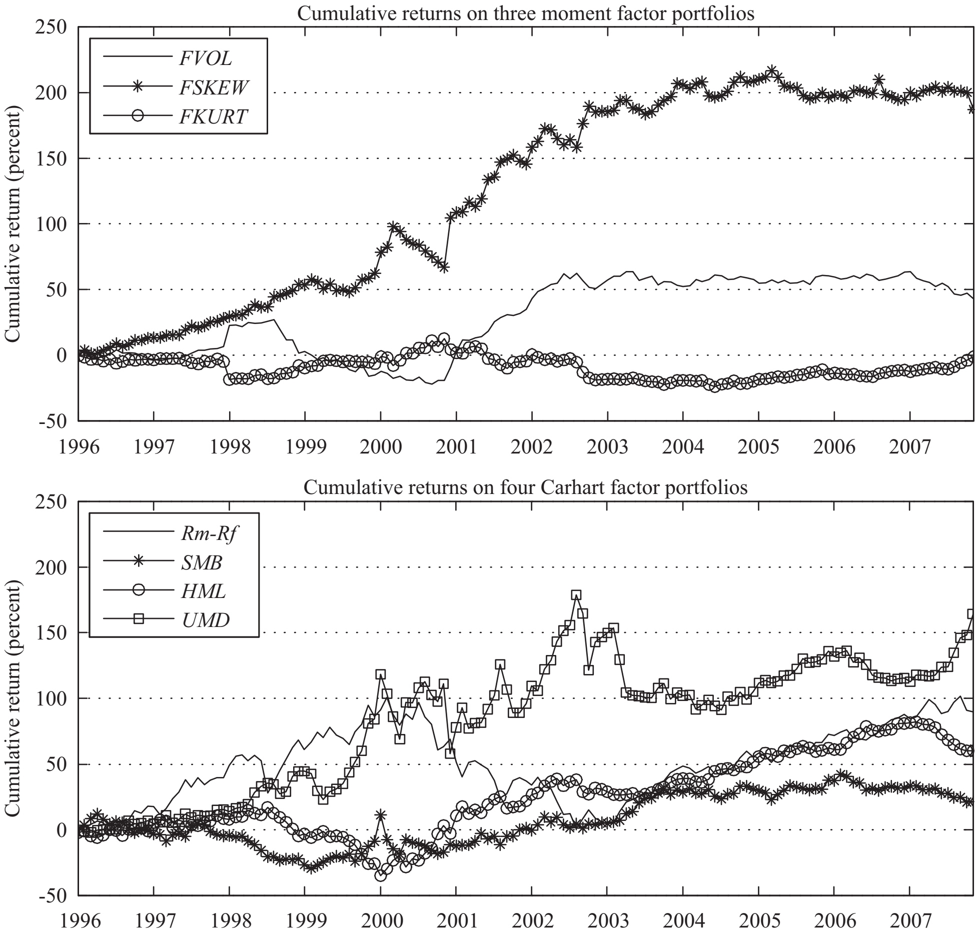

Fig. 5. Cumulative return on factor portfolios. In the top panel, we plot the cumulative return on the three moment-based factor portfolios, *FVOL*, *FSKEW*, and *FKURT*. In the bottom panel, we plot the cumulative returns on the standard market (R_m-R_f), size (*SMB*), value (*HML*), and momentum (*UMD*) portfolios. We plot the negative of the *FSKEW* return to facilitate comparison with the other factors. The times series of cumulative factor returns in the top panel can be downloaded from jfe.rochester.edu/data.htm.

factors commonly used in the literature, *SMB*, *HML*, and *UMD*, are only rarely significantly estimated in our relatively short sample. The estimates of λ_{FSKEW} are robustly estimated with values between -0.48 and -0.62.

The economic significance of a risk factor can be gauged from the magnitude of its price of risk and the spread of betas among different assets. Using the $\beta_{\Delta SKEW}$ for the 81 portfolios estimated from the first stage time series regression in the Fama and MacBeth procedure, for Model (xvi) the highest $\beta_{\Delta SKEW}$ is 0.60 and the lowest, -0.53 (not reported). The market skewness risk premium, therefore, results in the average return on the portfolio with the lowest $\beta_{\Delta SKEW}$ to be higher than the average return on the portfolio with the highest $\beta_{\Delta SKEW}$ by $(-0.53-0.60)\times(-0.53)=0.60\%$ per month, or 7.19% per year. Note that we have used the price of skewness risk of -0.53 in the last column of Table 7. An annual risk premium of 7.19% is clearly economically significant.

Regarding the pricing performance of the different models, the adjusted R^2 statistics range from 6.0% for the CAPM to 14.2% for Model (xvi) with all factors. In terms of adjusted R^2, Model (viii) with three market moment factors performs similarly to the Carhart four-factor Model (ix), with a slightly lower R^2 but also a lower root mean squared error (*RMSE*).

6.2. Fama and MacBeth regressions on other test portfolios

We check the robustness of the estimate of $\lambda_{\Delta SKEW}$ in Table 8 by running the same Fama and MacBeth regression on other test portfolios, namely, the 25 Fama and French portfolios sorted on size and book-to-market and 49 industry portfolios. The latter test portfolios are useful because some of the 81 portfolios are composed of few stocks, which sometimes yield rather large returns and alphas. It is, therefore, worthwhile to investigate if the negative sign of $\lambda_{\Delta SKEW}$ also obtains for less dispersed portfolios. We use a specification that includes all seven factors in all our robustness tests in this subsection. The results of the regressions are reported in Table 9. Panel A uses six-month betas, Panel B uses nine-month betas, and Panel C uses 12-month betas.

In the case of the 49 industry portfolios, the price of risk estimates are insignificant for all risk factors. The

Table 8
Price of market volatility, skewness, and kurtosis risk.

We report the estimated prices of risk for various multifactor models with $R_m - R_f$, ΔVOL, $\Delta SKEW$, $\Delta KURT$, *SMB*, *HML*, and *UMD* as factors. For each model considered, we estimate the prices of risk λ by applying the two-pass regression procedure of Fama and MacBeth (1973) to the post-ranking returns of the 81 portfolios sorted on exposures to $R_m - R_f$, ΔVOL, $\Delta SKEW$, and $\Delta KURT$. In the first stage, we estimate betas by running a time series regression of six months of daily returns on $R_m - R_f$ and the factor of interest. We then run the cross-sectional regression on next month's returns. We repeat the procedure by rolling the beta estimation window by one month. Newey and West *t*-statistics with 12 lags are reported in parentheses. Significant *t*-statistics at the 90% confidence level are boldfaced.

	Benchmark: capital asset pricing model								Benchmark: Carhart four-factor model							
Factor	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)	(xiv)	(xv)	(xvi)
Constant	1.09	0.90	1.21	1.13	1.08	0.94	1.20	1.08	1.22	1.29	1.35	1.25	1.38	1.34	1.23	1.32
	(1.98)	**(1.85)**	**(2.16)**	**(2.12)**	**(2.25)**	**(1.97)**	**(2.25)**	**(2.34)**	**(1.94)**	**(2.39)**	**(2.11)**	**(1.96)**	**(2.49)**	**(2.44)**	**(1.93)**	**(2.41)**
$R_m - R_f$	−0.30	−0.14	−0.41	−0.35	−0.31	−0.20	−0.42	−0.33	−0.51	−0.62	−0.64	−0.52	−0.70	−0.66	−0.53	−0.65
	(−0.47)	(−0.24)	(−0.62)	(−0.56)	(−0.52)	(−0.34)	(−0.67)	(−0.57)	(−0.67)	(−0.92)	(−0.82)	(−0.67)	(−0.99)	(−0.96)	(−0.69)	(−0.93)
ΔVOL		−3.10			−2.70	−3.00		−2.75		−3.49			−3.00	−3.38		−2.81
		(−2.38)			**(−2.07)**	**(−2.34)**		**(−2.13)**		**(−2.57)**			**(−2.26)**	**(−2.58)**		**(−2.13)**
$\Delta SKEW$			−0.59		−0.57		−0.62	−0.58			−0.48		−0.48		−0.53	−0.53
			(−3.16)		**(−3.22)**		**(−2.87)**	**(−2.71)**			**(−2.40)**		**(−2.66)**		**(−2.15)**	**(−2.10)**
$\Delta KURT$				−1.56		−1.69	−0.89	−1.07				−1.70		−1.78	−0.85	−0.92
				(−2.42)		**(−2.35)**	(−1.42)	(−1.49)				**(−3.03)**		**(−2.97)**	(−1.14)	(−1.10)
SMB									0.18	0.06	0.11	−0.04	−0.05	−0.14	0.11	−0.05
									(0.33)	(0.11)	(0.24)	(−0.09)	(−0.10)	(−0.32)	(0.23)	(−0.11)
HML									−0.11	−0.17	−0.32	−0.30	−0.32	−0.31	−0.23	−0.21
									(−0.25)	(−0.40)	(−0.95)	(−0.85)	(−0.95)	(−0.88)	(−0.59)	(−0.51)
UMD									0.97	1.07	0.87	1.07	0.83	1.16	0.86	0.82
									(2.46)	**(1.88)**	**(2.29)**	**(2.53)**	**(1.75)**	**(1.97)**	**(2.37)**	**(1.91)**
Average adj. R^2 (percent)	6.0	7.3	7.3	7.2	8.5	8.4	8.3	9.5	11.2	12.1	12.4	12.2	13.2	13.0	13.4	14.2
RMSE (percent per month)	1.00	0.91	0.92	0.95	0.83	0.85	0.90	0.81	0.93	0.86	0.88	0.90	0.80	0.82	0.86	0.79

Table 9
Price of market volatility, skewness, and kurtosis risk with different test portfolios and different beta estimation periods.
We report the estimated prices of risk in a multifactor model of the form

$$E[R_i]-R_f=\lambda_0+\lambda_{MKT}\beta^i_{MKT}+\lambda_{\Delta VOL}\beta^i_{\Delta VOL}+\lambda_{\Delta SKEW}\beta^i_{\Delta SKEW}+\lambda_{\Delta KURT}\beta^i_{\Delta KURT}+\lambda_{SMB}\beta^i_{SMB}+\lambda_{HML}\beta^i_{HML}+\lambda_{UMD}\beta^i_{UMD}.$$

We estimate the prices of risk λ by applying the two-pass regression procedure of Fama and MacBeth (1973) to the post-ranking returns of 81 portfolios sorted on exposures to R_m-R_f, ΔVOL, $\Delta SKEW$, and $\Delta KURT$, the 25 Fama and French portfolios sorted on size and book-to-market as well as 49 industry portfolios. In Panel A, we estimate the λs by running a time series regression of six months of daily returns on R_m-R_f and the factor of interest. We then estimate λs by running cross-sectional regressions over the next month. In Panels B and C, the procedure is the same as in Panel A but uses rolling nine-month and 12-month beta estimation periods, respectively. Newey and West t-statistics with twelve lags are reported in parentheses. Significant t-statistics at the 90% confidence level are boldfaced.

Test portfolio	Price of risk (percent per month)							Average adj. R^2 (percent)	RMSE (percent per month)
	λ_{MKT}	$\lambda_{\Delta VOL}$	$\lambda_{\Delta SKEW}$	$\lambda_{\Delta KURT}$	λ_{SMB}	λ_{HML}	λ_{UMD}		
Panel A: Six-month beta									
81 portfolios	−0.65	−2.81	−0.53	−0.92	−0.05	−0.21	0.82	14.2	0.79
	(−0.93)	**(−2.13)**	**(−2.10)**	(−1.10)	(−0.11)	(−0.51)	**(1.91)**		
25 size and book-to-market portfolios	−0.59	0.68	−0.28	−0.02	−0.57	−0.58	0.58	54.4	0.11
	(−0.94)	(1.00)	(−0.94)	(−0.02)	(−1.42)	(−1.30)	(0.83)		
49 industry portfolios	0.71	0.21	0.23	0.29	0.03	0.33	0.29	27.6	0.35
	(1.22)	(0.25)	(1.56)	(0.81)	(0.08)	(1.02)	(0.67)		
Panel B: Nine-month beta									
81 portfolios	−0.56	−2.12	−0.56	−0.80	−0.16	−0.44	0.68	15.1	0.81
	(−0.70)	(−1.37)	**(−1.89)**	(−1.49)	(−0.28)	(−0.82)	(1.34)		
25 size and book-to-market portfolios	−0.78	2.10	−0.58	−0.77	−0.44	−0.38	0.45	54.4	0.14
	(−1.10)	**(2.09)**	**(−1.86)**	(−0.53)	(−0.78)	(−0.87)	(0.62)		
49 industry portfolios	0.84	0.48	0.18	0.46	0.09	0.44	0.07	26.9	0.34
	(1.59)	(0.43)	(1.07)	(0.91)	(0.20)	(1.08)	(0.19)		
Panel C: 12-month beta									
81 portfolios	−0.71	−3.24	−0.28	−1.33	−0.62	−0.30	0.96	15.3	0.86
	(−0.82)	**(−2.28)**	(−1.49)	**(−2.05)**	(−0.93)	(−0.56)	(0.92)		
25 size and book-to-market portfolios	−1.15	2.90	−0.88	−0.77	−0.61	−1.21	0.11	3.2	0.55
	(−1.60)	**(2.23)**	**(−2.09)**	(−0.48)	(−1.21)	**(−1.86)**	(0.11)		
49 industry portfolios	0.70	0.47	0.15	0.29	0.06	0.63	−0.32	27.6	0.34
	(1.25)	(0.38)	(0.87)	(0.38)	(0.09)	(1.13)	(−0.61)		

industry portfolios do not appear to contain sufficient dispersion in exposure to the risk factors in our sample.

We find that the estimate of $\lambda_{\Delta SKEW}$ is significantly negative in four out of the remaining six cases considered. The estimates vary from −0.53% to −0.88% per month when significant.

The estimates of $\lambda_{\Delta VOL}$ in Table 9 are significantly negative in two cases and significantly positive in two cases. The estimate of the price of kurtosis risk is significant in only one case.

The fact that the prices of risk for the moment factors are not statistically significantly estimated in some cases must be interpreted carefully in light of the fact that traditional pricing factors such as *SMB*, *HML*, and *UMD* are also rarely estimated significantly, even when using the 25 size and book-to-market portfolios, which should give an advantage to the *SMB* and *HML* factors. The typically negative price of risk for the skewness factor, therefore, suggests that the skewness factor could constitute a valuable alternative to the Carhart factors. These factors are, moreover, all the more attractive because they can be clearly motivated from an economic perspective.

6.3. Fama and MacBeth regressions using size and book-to-market characteristics

The prices of risk on the *SMB* and *HML* factors are not statistically significant in Tables 8 and 9. This could favor the performance of the moment factors or, more seriously, the inclusion of *SMB* and *HML* could lead to problems associated with useless factors (see Kan and Zhang, 1999). We investigate the robustness of the estimates of $\lambda_{\Delta SKEW}$, $\lambda_{\Delta VOL}$, and $\lambda_{\Delta KURT}$ to the use of characteristics instead of size and book-to-market factor exposure in the cross-sectional Eq. (11), as in Daniel and Titman (1997). Specifically, we replace the *SMB* and *HML* factors by firm size, measured by market equity value *ME*, and book-to-market ratio *BM*, respectively, and the resulting specification is

$$E[R_i]-R_f=\lambda_0+\lambda_{MKT}\beta^i_{MKT}+\lambda_{\Delta VOL}\beta^i_{\Delta VOL}+\lambda_{\Delta SKEW}\beta^i_{\Delta SKEW}+\lambda_{\Delta KURT}\beta^i_{\Delta KURT}+\lambda_{ME}ME+\lambda_{BM}BM+\lambda_{UMD}\beta^i_{UMD}. \tag{12}$$

The market equity value of each stock is computed by multiplying the daily stock price by the number of shares outstanding. The book value of a stock for each quarter is computed as the Compustat book value of stockholders' equity, plus deferred taxes and investment tax credit, minus the book value of preferred stock, following Fama and French (1993). The same book value is used for the entire quarter to compute daily book-to-equity ratios. *ME* and *BM* are the averages of the stocks in each portfolio.

Table 10 presents results using the 81 portfolios from Table 9. Table 10 indicates that the estimates of $\lambda_{\Delta SKEW}$ are negative in all three cases and significant when the betas are estimated on six and nine months of daily data.

6.4. Interpreting the negative price of market skewness risk

Using ICAPM reasoning, the negative price of market volatility risk found in Ang, Hodrick, Xing, and Zhang (2006) suggests that increased stock market volatility is an indication of a deteriorating investment opportunity set. Because investors want to hedge themselves against the risk of a deteriorating investment opportunity set, they want to hold stocks that have high returns when the market volatility is higher than expected. High demand for stocks whose returns are highly correlated with innovations in market volatility leads to lower required returns for these stocks according to the ICAPM. In other words, the price of market volatility risk must be negative, consistent with the empirical results.

This argument is based on the assumption that increased stock market volatility is an indication of a deteriorating investment opportunity set. It is important to point out that what makes this assumption reasonable is the well-documented empirical fact that the innovation in stock market volatility is negatively correlated with the stock market return, referred to as the leverage effect (Black, 1976). To see why the leverage effect is consistent with the fact that increased stock market volatility is related to a deteriorating investment opportunity set, consider the following scenario. Suppose that there is an upward surprise (or positive innovation) in stock market volatility. If high stock market volatility is considered bad news for the economy, then a positive innovation in volatility leads to an immediate drop in the stock market index and vice versa. Therefore, the negative relation between volatility and the return of the stock market index implies that increased stock market volatility is considered bad news for the economy, i.e., an indication of a deteriorating investment opportunity set.

Interpreting skewness as a pricing factor in the ICAPM similar to the case of volatility is less obvious, because the continuous-time CAPM is based on local normality and, therefore, investors care only about mean and variance. In a discrete-time CAPM, skewness matters but then the investors' preference for skewness should be considered more explicitly. Ignoring this theoretical inconsistency, an exploratory investigation of the relation between skewness and the market return, in the vein of the results described above for volatility, could be worthwhile. The correlation between $\Delta SKEW$ and the market excess return is −0.20, as reported in Table 1. Using an argument similar to the one used to explain the negative price of market volatility risk, this negative correlation is consistent with a negative price of market skewness.

To check that the negative relation between $\Delta SKEW$ and the market excess return is significant, we regress the market excess return on $\Delta SKEW$ and compute the *t*-statistics of the slope coefficients. The results are reported in Table 11. We include the lagged market excess return for robustness, but this does not affect the results much. The slope coefficient is −1.33 with a significant *t*-statistic of −6.33. We also add ΔVOL and $\Delta KURT$ as regressors. The point estimates and *t*-statistics change somewhat when ΔVOL is included, but the estimate stays negative and statistically significant.

The negative relation between $\Delta SKEW$ and market excess returns is consistent with our finding of a negative price of market skewness risk. Also, all point estimates for ΔVOL are negative and statistically significant, and all point estimates for $\Delta KURT$ are positive and statistically significant. These findings are consistent with Table 1, as well as with our negative respective positive estimates of the price of volatility and kurtosis risk in Tables 2–9, although these estimates are not always robust.

Furthermore, Campbell (1996), Chen (2003), and Ang, Hodrick, Xing, and Zhang (2006) argue that candidate pricing factors in the ICAPM ought to forecast the future investment opportunity set. Chen (2003) extends the Campbell (1996) model to allow for stochastic second moments and shows that risk-averse investors want to hedge directly against changes in future market volatility in this environment. We, therefore, investigate the forecasting power of ΔVOL, $\Delta SKEW$, and $\Delta KURT$ in Table 12.

Table 10
Using size and book-to-market ratio instead of *SMB* and *HML* betas.

We report the estimated prices of risk in a multifactor model of the form

$$E[R_i]-R_f = \lambda_0+\lambda_{MKT}\beta^i_{MKT}+\lambda_{\Delta VOL}\beta^i_{\Delta VOL}+\lambda_{\Delta SKEW}\beta^i_{\Delta SKEW}+\lambda_{\Delta KURT}\beta^i_{\Delta KURT}+\lambda_{SME}ME^i+\lambda_{BM}BM^i+\lambda_{UMD}\beta^i_{UMD}.$$

The procedure follows Table 9 except that the market equity value (ME) and the book-to-market ratio (BM) of the portfolios are used instead of β_{SMB} and β_{HML} in the second-stage cross-sectional regression. We use the 81 portfolios sorted on exposures to R_m-R_f, ΔVOL, $\Delta SKEW$, and $\Delta KURT$ as test assets. ME and BM are the averages for the companies in each portfolio. The market equity value of each stock is computed by multiplying the daily stock price by the number of shares outstanding. The book value of a stock for each quarter is computed as the Compustat book value of stockholders' equity, plus deferred taxes and investment tax credit, minus the book value of preferred stock, following Fama and French (1993). The same book value is used for the entire quarter to compute daily book-to-equity ratios.

Price of risk (percent per month)						
λ_{MKT}	$\lambda_{\Delta VOL}$	$\lambda_{\Delta SKEW}$	$\lambda_{\Delta KURT}$	λ_{ME}	λ_{BM}	λ_{UMD}
Panel A: Six-month beta						
−0.25	−0.69	−0.23	0.54	0.10	−0.05	−0.27
(−0.43)	**(−1.74)**	**(−1.68)**	(0.86)	(1.37)	(−1.36)	(−0.94)
Panel B: Nine-month beta						
−0.09	0.04	−0.27	0.90	0.04	−0.04	−0.20
(−0.13)	(0.07)	**(−1.90)**	**(1.75)**	(0.65)	(−1.47)	(−0.61)
Panel C: Twelve-month beta						
0.47	−0.29	−0.14	0.97	−0.01	−0.04	−0.12
(0.68)	(−0.33)	(−0.91)	(1.54)	(−0.13)	(−0.97)	(−0.28)

Table 11
Time series relationship between ΔVOL, $\Delta SKEW$, $\Delta KURT$, and $R_m - R_f$.

This table reports the estimates and Newey and West (1987) t-statistics of the slope coefficients from regressing the daily excess returns of the Standard & Poor's 500 index on the lagged excess returns, ΔVOL, $\Delta SKEW$, and $\Delta KURT$. Significant t-statistics at the 90% confidence level are boldfaced.

	Dependent variable: $R_m - R_f$							
Regressors	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
$\Delta VOL(t)$		−0.661			−0.651	−0.654		−0.644
		(−36.50)			**(−35.05)**	**(−35.24)**		**(−34.57)**
$\Delta SKEW(t)$			−1.334		−0.471		−1.330	−0.480
			(−6.33)		**(−3.67)**		**(−6.02)**	**(−3.69)**
$\Delta KURT(t)$				0.353		0.066	0.352	0.071
				(7.65)		**(3.00)**	**(7.59)**	**(3.26)**
$R_m - R_f(t-1)$	−0.003	0.057	0.005	−0.014	0.059	0.055	−0.007	0.056
	(−0.12)	**(3.62)**	(0.26)	(−0.72)	**(3.87)**	**(3.48)**	(−0.37)	**(3.71)**
Adj. R^2 (percent)	−0.03	62.51	4.05	3.92	63.00	62.63	7.98	63.14

Table 12
Forecast regressions for market return and volatility.

We report the estimates and Newey and West (1987) adjusted t-statistics of the slope coefficients from regressing the monthly excess return of the Standard & Poor's 500 index (Panel A) and the monthly $MVOL$ (Panel B), which is defined as the volatility of the daily market excess return, on innovations in VOL, $SKEW$, and $KURT$ in the previous month. Significant t-statistics at the 90% confidence level are boldfaced.

Regression	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
Panel A: Dependent variable: $R_m - R_f(t)$								
$\Delta VOL(t-1)$		−0.073			−0.045	−0.012		0.021
		(−0.74)			(−0.43)	(−0.11)		(0.20)
$\Delta SKEW(t-1)$			1.133		1.103		1.135	1.149
			(1.57)		(1.56)		(1.53)	(1.55)
$\Delta KURT(t-1)$				0.393		0.385	0.394	0.407
				(1.95)		**(1.75)**	**(2.06)**	**(1.99)**
$R_m - R_f(t-1)$	0.022	−0.026	0.034	−0.011	0.004	−0.018	0.001	0.013
	(0.40)	(−0.26)	(0.64)	(−0.19)	(0.04)	(−0.19)	(0.01)	(0.14)
Adj. R^2 (percent)	−0.66	−1.17	0.50	0.07	−0.13	−0.65	1.25	0.55
Panel B: Dependent variable: MVOL(t)								
$\Delta VOL(t-1)$		0.371			0.373	0.339		0.344
		(3.07)			**(3.34)**	**(2.64)**		**(2.84)**
$\Delta SKEW(t-1)$			1.208		1.251		1.158	1.230
			(1.93)		**(2.05)**		**(1.73)**	**(1.95)**
$\Delta KURT(t-1)$				−0.619		−0.228	−0.604	−0.207
				(−2.43)		(−0.95)	**(−2.45)**	(−0.87)
$MVOL(t-1)$	0.619	0.589	0.624	0.607	0.594	0.588	0.613	0.593
	(12.41)	**(11.13)**	**(12.50)**	**(12.03)**	**(11.25)**	**(11.19)**	**(12.09)**	**(11.27)**
Adj. R^2 (percent)	37.90	41.64	38.26	38.88	42.08	41.39	39.18	41.80

To benchmark our results to the available literature on equity premium forecasting (see, for instance, Bollerslev, Tauchen, and Zhou, 2009; Goyal and Welch, 2008), we use monthly data. In the equity premium literature, successful candidate forecasts such as the price-earnings ratio, price-dividend ratio, market volatility, or the volatility risk premium typically yield adjusted R^2s in the 1%–2% range.

Table 12 reports results on forecasting the excess market return and market volatility using lagged moments of the market return for our 1996–2007 sample. Monthly market volatility, $MVOL$, is estimated using the daily excess market returns within the month. Panel B indicates that changes in skewness are positively related to higher future volatility and, therefore, to a deterioration of the investment opportunity set, and the estimate is statistically significant. We obtain similar results (not reported) when using daily option-implied skewness to forecast daily realized volatility estimated from intraday returns.

Panel A indicates that lagged market skewness is not a significant predictor of future market returns. Moreover, the estimated sign is positive. Interestingly, though, the t-statistics and adjusted R^2s are higher than for market volatility. Lagged kurtosis is estimated with the expected positive sign, and the estimates are statistically significant. However, forecasting market returns is notoriously difficult and sensitive to outliers, and given the limited size of our sample these results ought to be interpreted cautiously.

As mentioned in Section 2, the model in Chabi-Yo (2012) can be seen as an intertemporal extension of the higher-moment CAPM in which market volatility, skewness, and kurtosis as well as their innovations are cross-sectional pricing factors. In that model the price of market skewness risk depends on the fourth derivative of the utility function, which is difficult to sign from theory alone. Chabi-Yo (2012) estimates the preference parameters characterizing his model and finds that the price of skewness risk is negative, thus confirming our empirical result.

7. Conclusion

We investigate the pricing implications of market volatility, skewness, and kurtosis for the cross section of stock returns, using estimates of the moments of the market return extracted from index options. We find that stocks with high exposure to innovations in implied market skewness exhibit low returns on average. The results are weaker for volatility and kurtosis: Stocks with higher exposure to innovations in implied market volatility exhibit somewhat lower returns on average, whereas stocks with high exposure to innovations in implied market kurtosis exhibit somewhat higher returns on average.

While the results on market skewness risk indicate a robustly negative risk premium, the results on market volatility and kurtosis risk are sensitive to variations in the empirical setup and across sample periods. The estimated premium for bearing market skewness risk is economically significant and cannot be explained by known risk factors such as the market excess return, the size, book-to-market, momentum, and market volatility factors, or by firm characteristics such as firm and book-to-market.

A number of extensions of our investigation could prove worthwhile.

First, we use the cross section of stock returns for 1996–2007 because we are constrained by the availability of index option data from OptionMetrics. Using other sources of option data, we could obtain a longer time series of option-implied moments, which would allow us to enlarge the sample by going further back in time.

Second, the three- and four-factor CAPM differ from our ICAPM approach in the sense that the signs on some of the factors can be motivated by utility theory. Investigating the three-factor CAPM using option-implied moments could be worthwhile.

Finally, Albuquerque (2012) theoretically establishes a relation between firm-level returns and negatively skewed market returns, and it could prove interesting to explore the implications of his framework for our empirical findings.

Appendix A. Extracting option implied moments

Let $R(t,\tau) = \ln S(t+\tau) - \ln S(t)$. We want to extract the following moments from option prices:

$$VOL(t,\tau) = \{E_t^Q[(R(t,\tau) - E_t^Q[R(t,\tau)])^2]\}^{1/2}, \tag{13}$$

$$SKEW(t,\tau) = \frac{E_t^Q[(R(t,\tau) - E_t^Q[R(t,\tau)])^3]}{\{E_t^Q[(R(t,\tau) - E_t^Q[R(t,\tau)])^2]\}^{3/2}} \tag{14}$$

and

$$KURT(t,\tau) = \frac{E_t^Q[(R(t,\tau) - E_t^Q[R(t,\tau)])^4]}{\{E_t^Q[(R(t,\tau) - E_t^Q[R(t,\tau)])^2]\}^2}, \tag{15}$$

where $E_t^Q[\cdot]$ is the expected value under the risk-neutral measure. If we expand the powers inside the expectations, we can see that these moments are functions of

$$E_t^Q[R(t,\tau)], \quad E_t^Q[R^2(t,\tau)], \quad E_t^Q[R^3(t,\tau)], \quad E_t^Q[R^4(t,\tau)] \tag{16}$$

or

$$E_t^Q[e^{-r\tau}R(t,\tau)], \quad E_t^Q[e^{-r\tau}R^2(t,\tau)], \quad E_t^Q[e^{-r\tau}R^3(t,\tau)],$$
$$E_t^Q[e^{-r\tau}R^4(t,\tau)], \tag{17}$$

where r is the constant risk-free rate. The quantities in Eq. (17) can be interpreted as prices of the contracts whose payoffs, $H[S]$ ($S = S(t+\tau)$ for notational convenience), are

$$H[S] = \begin{cases} R(t,\tau), \\ R^2(t,\tau), \\ R^3(t,\tau), \\ R^4(t,\tau). \end{cases} \tag{18}$$

Bakshi and Madan (2000) have shown that any twice-continuously differentiable payoff function, $H[S]$, can be spanned by a portfolio of risk-free bonds, the underlying asset, and out-of-the-money (OTM) calls and puts

$$H[S] = H[\bar{S}] + (S - \bar{S})H_S[\bar{S}] + \int_{\bar{S}}^{\infty} H_{SS}[K](S-K)^+ \, dK + \int_0^{\bar{S}} H_{SS}[K](K-S)^+ \, dK. \tag{19}$$

The prices of these contracts are

$$E_t^Q\{e^{-r\tau}H[S]\} = (H[\bar{S}] - \bar{S}H_S[\bar{S}])e^{-r\tau} + H_S[\bar{S}]S(t) + \int_{\bar{S}}^{\infty} H_{SS}[K]C(t,\tau;K)\, dK + \int_0^{\bar{S}} H_{SS}[K]P(t,\tau;K)\, dK, \tag{20}$$

where $C(t,\tau;K)$ and $P(t,\tau;K)$ are prices of European call and put options with maturity τ and strike price K. As a result, we can calculate the prices of derivatives whose payoffs depend only on S, given the prices of a risk-free zero coupon bond, the underlying stock, and a series of OTM calls and puts. We can use this methodology to first calculate the quantities in Eq. (17) and then use these quantities to calculate the option implied moments in Eqs. (13), (14), and (15).

We use the data on S&P 500 index options from 1996 through 2007 available through OptionMetrics Ivy DB. We use the average of the bid and ask quotes for each option contract and filter out options with zeros bids as well as those whose average quotes are less than \$3/8. We also filter out quotes that do not satisfy standard no-arbitrage conditions. Finally, we eliminate in-the-money options because they are less liquid than out-of-the-money and at-the-money options. We eliminate put options with strike prices of more than 103% of the underlying asset price ($K/S>1.03$) and call options with strike prices of less than 97% of the underlying asset price ($K/S<0.97$). We estimate only the moments for days that have at least two OTM call prices and two OTM put prices available.

Because we do not have a continuum of strike prices, we calculate the integrals using cubic splines. For each maturity, we interpolate implied volatilities using a cubic spline across moneyness levels (K/S) to obtain a continuum of implied volatilities. For moneyness levels below or above the available moneyness level in the market, we use the implied volatility of the lowest or highest available strike price. After implementing this interpolation-extrapolation technique, we obtain a fine grid of one thousand implied volatilities for moneyness levels between 0.01% and 300%. We then convert these implied volatilities into call and put prices using the following rule: Moneyness levels smaller than 100% ($K/S<1$) are used to generate put prices and moneyness levels larger than 100% ($K/S>1$) are used to generate call prices using trapezoidal numerical integration. Linear interpolation between maturities is used to calculate the moments at a fixed 30-day horizon.

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