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Jump risk, stock returns, and slope of implied volatility smile ☆

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ABSTRACT

In the presence of jump risk, expected stock return is a function of the average jump size, which can be proxied by the slope of option implied volatility smile. This implies a negative predictive relation between the slope of implied volatility smile and stock return. For more than four thousand stocks ranked by slope during 1996–2005, the difference between the risk-adjusted average returns of the lowest and highest quintile portfolios is 1.9% per month. Although both the systematic and idiosyncratic components of slope are priced, the idiosyncratic component dominates the systematic component in explaining the return predictability of slope. The findings are robust after controlling for stock characteristics such as size, book-to-market, leverage, volatility, skewness, and volume. Furthermore, the results cannot be explained by alternative measures of steepness of implied volatility smile in previous studies.

1. Introduction

The finance literature shows extensively that distributions of stock returns are leptokurtic or "fat-tailed." Fat-tailed distributions can be caused by jumps, that is, sudden but infrequent movements of large magnitude. Modeling dynamics of jumps in stock prices dates back to Press (1967) and Merton (1976a). Subsequent studies such as Ball and Torous (1983), Jarrow and Rosenfeld (1984), and Jorion (1989) provide convincing support for the presence of jumps in stock prices. Another strand of papers, following the approach of Cox and Ross (1976) and Merton (1976b), examines the effects of jumps to option pricing beyond the classical diffusion model of Black and Scholes (1973). Articles such as Ball and Torous (1985), Naik and Lee (1990), Amin and Ng (1993), Bakshi, Cao, and Chen (1997), Bates (2000), Duffe, Pan, and Singleton (2000), Anderson, Benzoni, and Lund (2002), Pan (2002), and Eraker, Johannes, and Polson (2003) demonstrate that incorporating jumps is essential in explaining observed option prices. Despite the overwhelming evidence for jumps, a lack of understanding exists on the relation between jump risk and cross-sectional expected stock returns. In this paper, I try to shed some light on the subject by examining two questions: (1) How is the expected return of a stock dependent on jump risk? (2) How can jump risk be measured?

To address the first question, I adopt the stochastic discount factor (SDF) framework and present a general and yet parsimonious continuous-time model in which the SDF and stock prices follow correlated jump-diffusion processes. In the absence of arbitrage, there exists an SDF

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that prices all assets. (See, for example, Rubinstein, 1976; Ross, 1978; Harrison and Kreps, 1979; and Cochrane, 2005.) The model contains two types of risk: diffusive risk and jump risk, driven by a Brownian motion and a Poisson process, respectively. The expected excess stock return is dependent on both sources of risk. The diffusive component of the return is determined by the covariance between the Brownian motions driving the SDF and stock processes, a well-known continuous-time analogue of the discrete-time $\beta-$representation. The jump component of the return is captured by the covariance between the Poisson processes driving the SDF and stock processes, the covariance between the jump distributions of the SDF and stock when a systematic jump occurs, and the product of the average jump sizes of the SDF and stock. This decomposition highlights the sources affecting the component of expected stock return that compensates for the jump risk.

Applying the jump-diffusion model empirically leads to the second question, that is, how to measure or estimate jump risk. There are a couple of major challenges. First, the SDF is not identified due to market incompleteness in the presence of jumps. (See, for example, Naik and Lee, 1990.) Nonetheless, I argue that, based on existing asset pricing models such as the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) and the consumption based capital asset pricing model (CCAPM) of Breeden (1979), the average jump size of the SDF is positive. This seems to be a small step toward understanding the jump risk, but it generates some strong implications. Specifically, I demonstrate that, for reasonable model parameters, the expected excess stock return is monotonically decreasing in the average stock jump size.

Identifying the average stock jump size empirically is the second challenge in implementing the jump-diffusion model. Jumps are rare events and estimating average jump size precisely requires long time series samples, which are often unavailable.[1] Even with large samples, jumps could fail to realize due to the peso problem. Exacerbating the problem, jump distributions could be time-varying, causing model misspecification and estimation bias. I finesse these difficulties by using information from the option market. The intuition arises from the groundbreaking work of Merton (1976a), who demonstrates the impact of jumps on option prices. Conversely, from observed option prices, I extract information about the underlying jump distribution. The main advantage of this approach is that options are forward-looking contracts and can provide ex ante measures of jump risk. This mitigates the peso problem and reduces the bias caused by in-sample fitting.

To proxy jump risk using option data, I propose the slope of implied volatility smile, defined to be the difference between the fitted implied volatilities of one-month-to-expiration put and call options with deltas equal to -0.5 and 0.5, respectively. Theoretically, I demonstrate that the slope measures the local steepness of the smile for near-the-money near-expiration options. In addition, I prove that the slope is approximately proportional to the average stock jump size. Combining these results with the relation between stock return and average stock jump size, I obtain the main hypothesis of the paper: *If stock portfolios are formed by ranking on the slope, then the future returns of low slope portfolios are higher than those of high slope portfolios*.

My empirical analysis is conducted using the option data on 4,048 stocks from January 1996 to June 2005. At first, I employ two tests, one indirect and one direct, to establish the link between the slope of implied volatility smile and average stock jump size. The indirect test is based on the well-known positive relation between jump and skewness. I also propose a new way of computing skewness by taking into account time-varying jump risk. The direct test is based on the jump identification algorithm of Jiang and Yao (2009), which provides estimates of realized jump sizes. This allows for an examination of the predictive power of slope on future jump sizes using time series regressions. The evidence from both tests strongly support the slope being a proxy of average jump size.

Next, I examine the relation between slope and future stock returns by considering five equally weighted quintile portfolios formed by sorting stocks on slope at the end of each month. Confirming my hypothesis, the average portfolio returns in the following month exhibit a monotonic decreasing pattern in slope. The pattern does not change even after I adjust portfolio returns using some popular factor models such as the CAPM, the Fama-French three-factor model, and the four-factor model of Carhart (1997). The difference between the risk-adjusted (using the four-factor model) average monthly returns of the lowest and highest quintile portfolios is 1.9%. The evidence supports the notion that the jump risk embedded in the slope is priced. However, an important question arises: Which component of the jump risk is priced by the market—the systematic component or the idiosyncratic component?

To address this issue empirically, I proxy the market jump risk by the slope of Standard & Poor's (S&P) 500 index options and decompose the slope of a stock into the systematic and idiosyncratic components. Both components are found to be priced as they can predict stock returns in the same way as the slope. Although neither component is able to explain the slope fully, the idiosyncratic component dominates the systematic component as it captures most variation and return predictability in the slope. Consistent with my findings, Jiang and Yao (2009) estimate realized jumps from stock returns and find stock jumps tend to be idiosyncratic. They also find that stock jumps tend to be positive, consistent with my data of positive average slope.

It is a puzzle that the idiosyncratic jump risk is priced and even dominates the systematic jump risk. This could be caused by my specific decomposition of the slope, where some systematic factors other than the market

[1] Significant progress has been made in estimating jumps in asset prices. Recent papers include Bates (1996), Bakshi, Cao, and Chen (1997), Anderson, Benzoni, and Lund (2002), Pan (2002), Carr and Wu (2003), Chernov, Gallant, Ghysels, and Tauchen (2003), Eraker, Johannes, and Polson (2003), Ait-Sahalia (2004), and Jiang and Yao (2009).

slope are missing. But identifying these missing factors posts a challenge as the risk models considered above and the stock characteristics that I control for in robustness checks do not capture these missing factors. An alternative point of view is that the stock market is inefficient as investors mistakenly undervalue (overvalue) stocks with expected negative (positive) idiosyncratic surprises. But this contradicts my model, which assumes efficient stock and option markets. One possible rational explanation of the puzzle could lie in investors' ability of identifying and aggregating firm-specific information. An idiosyncratic jump in the price of a stock should be totally driven by firm-specific information shocks. But investors are able to forecast precisely the expected idiosyncratic jump size for the stock. When well-diversified portfolios of stocks of similar expected idiosyncratic jump sizes are formed, a low jump-size portfolio has more bad firm-specific surprises on average than a high jump-size portfolio. A utility-maximizing investor, who is averse to bad surprises, should demand higher rate of return for holding the low jump-size portfolio. In the meantime, the total information shock to the market can be negligible if the idiosyncratic jumps cancel each other. According to this explanation, as long as investors do not like adverse jumps, the idiosyncratic jump risk becomes systematic when it is identified and aggregated. Therefore, the idiosyncratic jump risk is nondiversifiable, in contrast to the fact that the idiosyncratic diffusive risk is diversifiable. This is not surprising because jumps are rare and extreme events.

For robustness checks of my findings, I control for a number of stock characteristics such as past return, size, book-to-market, leverage, volatility, idiosyncratic volatility, skewness, co-skewness, option trading volume, stock trading volume, and stock turnover rate.[2] None of the control variables is found to explain the return predictability of slope. Although the return predictability is persistent up to six months, it does not show any obvious seasonality. My findings are also robust to various data filter rules.

In the literature, jump risk is often argued to be reflected by the over pricing of deep out-of-the-money (OTM) put options. In fact, various measures for steepness of implied volatility smile proposed previously use implied volatilities of deep OTM puts. (See, for example, Toft and Prucyk, 1997; Bollen and Whaley, 2004; and Xing et al., 2010.) One problem of using deep OTM puts is that measurement errors can be significant. In contrast, the slope in this paper uses at-the-money options. Furthermore, my model relates the slope to jump risk while previous studies offer different interpretations.[3] To differentiate my paper from earlier papers, I compare the return predictability of slope against the slope measures that use OTM puts. The evidence suggests that the OTM slope measures are unable to capture the return predictability in the slope, while the slope can explain most return predictability in the OTM slope measures.

The paper proceeds as follows. In Section 2, I present the jump-diffusion model and all the theoretical results. Section 3 contains the main empirical analysis. In Section 4, I conduct robustness checks. Section 5 concludes. Technical results are provided in the Appendix.

2. Jump-diffusions and asset pricing

In this section, we first present the model of stock returns and then demonstrate the relation between jump size and slope of implied volatility smile.

2.1. Stochastic discount factor and stock returns

It is natural to formulate jumps using the continuous-time approach. A stochastic discount factor, $M(t)$, is a positive stochastic process so that MS is a martingale for any stock price process $S(t)$. Specifically, I model $M(t)$ as a jump-diffusion process:

$$\frac{dM}{M} = (-r_f - \lambda_M \mu_{J_M})\,dt + \sigma_M dW_M + J_M dN_M, \tag{1}$$

where W_M is a standard Brownian motion and N_M is a Poisson process with intensity $\lambda_M (\geq 0)$, that is, $\mathrm{Prob}(dN_M = 1) = \lambda_M dt$. J_M is the jump size with a displaced lognormal distribution independent over time:

$$\ln(1+J_M) \sim \mathcal{N}(\ln(1+\mu_{J_M}) - \tfrac{1}{2}\sigma^2_{J_M}, \sigma^2_{J_M}). \tag{2}$$

The lognormal specification of J_M ensures positivity of M, which guarantees no arbitrage. W_M, N_M, and J_M are independent of each other. r_f is the risk-free interest rate. The term $\lambda_M \mu_{J_M}$ adjusts the drift for the average jump size. σ_M is the instantaneous diffusive standard deviation. This type of model for stock prices was introduced by Merton (1976a). I use one-dimensional Brownian motion and Poisson process for simplicity. The model can be extended to incorporate multi-dimensional Brownian motions and Poisson processes. Similarly, I let the price of the ith stock follow a jump-diffusion process:

$$\frac{dS_i}{S_i} = (\mu_i - \lambda_i \mu_{J_i})\,dt + \sigma_i dW_i + J_i dN_i, \tag{3}$$

where W_i is a standard Brownian motion and N_i is a Poisson process with intensity λ_i. Like J_M, J_i has a displaced lognormal distribution independent over time:

$$\ln(1+J_i) \sim \mathcal{N}(\ln(1+\mu_{J_i}) - \tfrac{1}{2}\sigma^2_{J_i}, \sigma^2_{J_i}). \tag{4}$$

[2] These variables are motivated by a long list of papers including Banz (1981), Basu (1983), Rosenberg, Reid, and Lanstein (1985), Fama and French (1992), Jegadeesh and Titman (1993), Lakonishok, Shleifer, and Vishny (1994), Harvey and Siddique (2000), Ang, Hodrick, Xing, and Zhang (2006), and Pan and Poteshman (2006).

[3] For example, Toft and Prucyk (1997) relate slope to firm leverage; Dennis and Mayhew (2002) and Bakshi, Kapadia, and Madan (2003) draw connection between slope and risk-neutral skewness; Cremers, Driessen, Maenhout, and Weinbaum (2008) examine the relation between slope and credit spread; Bollen and Whaley (2004) show slope to be affected by the net buying pressure from public order flow; Xing et al. (2010) argue, based on the model of Easley, O'Hara, and Srinivas (1998), that slope reflects informed investors' demand of OTM puts in anticipating bad news; and Duan and Wei (2009) find slope to be dependent on the systematic risk proportion in the total risk.

Again, W_i, N_i, and J_i are independent of each other, but they are related to the corresponding components in the SDF. Specifically, I assume that W_M and W_i, N_M and N_i, and J_M and J_i are pairwise correlated with correlation coefficients $\mathrm{Corr}(dW_M,dW_i)=\rho_i$, $\mathrm{Corr}(N_M,N_i)=\eta_i$, and $\mathrm{Corr}(\ln(1+J_M),\ln(1+J_i))=\psi_i$, respectively. Notice that η_i is non-negative, while ρ_i and ψ_i can be negative. Ito's lemma for jump-diffusions implies the following result.

Proposition 1. *Given the dynamics of the SDF and stock price in Eqs.* (1)–(4), *the expected excess stock return can be expressed as*

$$\mu_i - r_f = -\rho_i\sigma_M\sigma_i - \eta_i\sqrt{\lambda_M\lambda_i}[(1+\mu_{J_M})(1+\mu_{J_i})e^{\psi_i\sigma_{J_M}\sigma_{J_i}} - \mu_{J_M} - \mu_{J_i} - 1]. \quad (5)$$

Moreover, the expected excess stock return is

(i) *decreasing in* ρ_i *and* ψ_i;
(ii) *decreasing* (*increasing*) *in* η_i *if* $\Theta_i \equiv (1+\mu_{J_M})(1+\mu_{J_i})e^{\psi_i\sigma_{J_M}\sigma_{J_i}} - \mu_{J_M} - \mu_{J_i} - 1 > 0$ (<0); *and*
(iii) *decreasing* (*increasing*) *in* μ_{J_i} *if* $\Phi_i \equiv (1+\mu_{J_M})e^{\psi_i\sigma_{J_M}\sigma_{J_i}} - 1 > 0(<0)$.

Although various forms of Proposition 1 exist in the literature, it is worthwhile to make several observations. First, in the absence of jumps, Eq. (5) is the well-known continuous-time analogue of the discrete-time $\beta-$representation of expected stock return. Second, when jumps are present but nonsystematic ($\eta_i=0$), Eq. (5) is the same as that in the case of no jumps. This is exactly what Merton (1976a) argues—that idiosyncratic (diversifiable) jumps do not affect expected stock return. In the presence of systematic jump risk ($\eta_i>0$), the expected stock return depends on the jump distributions. (i) of Proposition 1 says that stocks whose systematic jumps are more negatively correlated with jumps of the SDF ($\psi_i<0$) earn higher returns ceteris paribus. However, the relation between η_i and expected return and the relation between μ_{J_i} and expected return depend on the signs of quantities Θ_i and Φ_i as defined in (ii) and (iii) of Proposition 1, respectively. For the rest of the paper, I focus on the latter because I can infer μ_{J_i} from the option data.

To explore the effect of μ_{J_i} on expected stock return, I first consider the special case of uncorrelated jump distributions of the stock and SDF, i.e., $\psi_i=0$. The determining quantity Φ_i in (iii) of Proposition 1 simplifies to μ_{J_M}. To draw inference, I have to know the sign of μ_{J_M}, the average jump size of M, which is not explicitly specified in the model. The main problem is the nonuniqueness of the SDF because of market incompleteness. I can, however, resort to some well-known asset pricing models to argue that $\mu_{J_M}>0$. In the CAPM, M is inversely proportional to the market portfolio. Then the empirical evidence that the average jump size of the market portfolio is negative implies $\mu_{J_M}>0$. As a second example, the SDF in the consumption-based CAPM is proportional to the intertemporal marginal rate of substitution. For a representative investor with a time-separable power utility function, jumps in the SDF are negatively related to jumps in the consumption growth. (See, for example, Cochrane, 2005.) Therefore, $\mu_{J_M}>0$ holds if the average jump in consumption is negative. Barro (2006) shows strong evidence supporting this assumption. Given $\mu_{J_M}>0$, Proposition 1 indicates that expected stock return is monotonically decreasing in the average stock jump size.

In the case of $\psi_i \neq 0$, the value of Φ_i depends on the jump parameters μ_{J_M}, ψ_i, σ_{J_M}, and σ_{J_i}. To get some sense on the sign of Φ_i, I start with a case in which the CAPM holds, and $\mu_{J_M}=10\%$ and $\sigma_{J_M}=15\%$.[4] As a worst scenario against $\Phi_i>0$, I let $\psi_i=-1$ and further let $\sigma_{J_i}=40\%$, which is very generous as it is more than two-thirds of the average standard deviation of realized stock returns in the sample. Even for these extreme values of ψ_i and σ_{J_i}, $\Phi_i>0$. In general, $\Phi_i>0$ as long as μ_{J_M} and σ_{J_M} are of similar magnitude and the product $\psi_i\sigma_{J_i}$ is not too negative, which can be due to either small ψ_i or reasonable magnitude of σ_{J_i}. It is possible that $\Phi_i<0$ for some stocks. But these stocks should be outnumbered by stocks with $\Phi_i>0$ in well-diversified portfolios. It is important to note that whether the expected stock return is monotonically decreasing in μ_{J_i} is ultimately an empirical issue. What I estimate from the data is basically the empirical SDF, which could well be different from the theoretical SDFs in models such as the CAPM. I thank the referee for this point.

2.2. Jump size and slope of implied volatility

Testing the relation between stock return and average jump size requires estimating μ_{J_i}. As argued by Merton (1980), the parameters related to the diffusive risk such as σ_i can be accurately estimated by quadratic variation of realized stock returns. But the parameters related to the jump risk such as μ_{J_i} are difficult to pin down because jumps are rare events and could fail to materialize in the sample. Moreover, the parameter could change over time and historical estimate can be biased. In this paper, I propose a rather simple method to proxy μ_{J_i} that uses information from the option market.

Consider a European call option on the ith stock with strike price K and maturity T. Let q_i be the dividend yield and let $\sigma_i^{\mathrm{imp}}(K,T)$ denote the Black-Scholes implied volatility. I define log moneyness of the option to be $X \equiv \ln(Ke^{-(r_f-q_i)T}/S_i(0))$, which is more convenient to work with than K. The log-transformed definition takes into account time value and leads to cleaner formulae than the conventional definition of moneyness K/S. Without ambiguity, I write the implied volatility as $\sigma_i^{\mathrm{imp}}(X,T)$, which is referred as the implied volatility smile for fixed T.

[4] The values used are consistent with those in the literature. For the same sample period, the estimates of the average market jump size and standard deviation in Santa-Clara and Yan (2010) are −9.8% and 16%, respectively. There are other estimates for different sample periods and using different methods. For example, the estimates in Bakshi, Cao, and Chen (1997) are −5% and 7%, and the estimates in Eraker, Johannes, and Polson (2003) are about −3% and 4%. Despite the differences in estimates, $\Phi_i>0$ holds when these alternative parameter values are used.

Proposition 2 summarizes some local properties of the smile at $X=0$.

Proposition 2. *For T small, the Black-Scholes implied volatility of the at-the-money European call option satisfies*

$$\sigma_i^{\text{imp}}(X,T)|_{X=0} = \sigma_i + O(T) \tag{6}$$

and

$$\left.\frac{\partial \sigma_i^{\text{imp}}(X,T)}{\partial X}\right|_{X=0} = \frac{\lambda_i \mu_{J_i}}{\sigma_i} + O(T), \tag{7}$$

where $O(T)$ means in the same order as T.

According to Eq. (6), the at-the-money implied volatility converges to the instantaneous diffusive volatility of stock returns as the maturity approaches zero. This extends the similar result of Ledoit, Santa-Clara, and Yan (2003) for diffusions. The jump risk has no impact on the level of the at-the-money implied volatility. But it affects the local steepness of implied volatility smile near-the-money to the extent, as seen in Eq. (7), that the slope, defined to be the partial derivative of implied volatility in terms of moneyness, is proportional to the average jump size. Technically, the parameters such as λ_i and μ_{J_i} should be specified under the risk-neutral probability measure. In the Appendix, we discuss the transformation between the objective and risk-neutral probability measures. The proposition also holds for put options.

I implicitly assume the model parameters to be constant. It is important to note that Proposition 2 can be extended to general settings in which parameters such as the diffusive volatility, average jump size, and jump intensity are time-varying. The findings of Bakshi, Cao, and Chen (1997), Bates (2000), Pan (2002), and Santa-Clara and Yan (2010), among others, strongly support these more general specifications. In the Appendix, I present evidence that Proposition 2 holds when the diffusive volatility σ_i follows the square-root process of Heston (1993).

To implement Proposition 2, I fix time-to-maturity to be small and consider implied volatility $\sigma_{i,\text{put}}^{\text{imp}}$ ($\sigma_{i,\text{call}}^{\text{imp}}$) of the put (call) option on the ith stock with $\Delta = -0.5$ (0.5). These options are not exactly at-the-money but very close to being at-the-money. Define proxies of volatility (ν_i) and slope of implied volatility smile (s_i) by

$$\nu_i \equiv 0.5(\sigma_{i,\text{put}}^{\text{imp}}(-0.5) + \sigma_{i,\text{call}}^{\text{imp}}(0.5)) \tag{8}$$

and

$$s_i \equiv \sigma_{i,\text{put}}^{\text{imp}}(-0.5) - \sigma_{i,\text{call}}^{\text{imp}}(0.5). \tag{9}$$

One practical problem is that individual equity options are American style and their implied volatilities are not obtained by inverting the Black-Scholes formula. Nonetheless, because the options that I use are short-term and near-the-money contracts, their prices are close to the prices of similar European options because early exercise value is low. For example, Bakshi, Kapadia, and Madan (2003) examine a sample of 30 largest stocks in the S&P 100 index and find the difference between Black-Scholes and American option implied volatilities is small enough to be ignored. In the Appendix, I prove Proposition 3.

Proposition 3. *ν_i is approximately equal to the diffusive volatility σ_i, and s_i is approximately proportional to the product of jump intensity and average stock jump size. For constant $L_i > 0$,*

$$\nu_i \approx \sigma_i \tag{10}$$

and

$$s_i \approx L_i \lambda_i \mu_{J_i}. \tag{11}$$

Comparing Eq. (11) with Eq. (7), s_i is approximately proportional to the local steepness of the implied volatility smile.[5] Combining this observation with the discussion following Proposition 1, I can argue that the expected stock return is decreasing in s. To reduce noises in individual stock returns and increase the power of statistical analysis, I consider stock portfolios and formulate my main empirical hypothesis: *For stock portfolios formed by ranking on the slope, the returns of low slope portfolios are higher than the returns of high slope portfolios.*

One could be concerned about the precision of the approximations of Eqs. (6) and (7) and Eqs. (10) and (11). To examine the impact of errors in these approximations, I conduct Monte-Carlo simulations (see the Appendix). Several interesting results are worth commenting upon. First, the errors in the implied volatility level are small even for maturities beyond one month. However, the errors in the slope are relatively large even for maturities less than a month. This is not surprising given that the slope is the derivative of implied volatility. Second, the errors in the slope tend to be negative and are increasing in T, μ_{J_i}, and λ_i. Third, the slope is a monotonic increasing function of μ_{J_i} despite approximation errors. This point is critical and provides the foundation for my empirical analysis in which I rank stocks by the slope. The positive relation between the slope and μ_{J_i} implies that the errors in the slope should not bias the cross-sectional ranking of stocks in μ_{J_i}.

3. Empirical analysis

In this section, I first discuss the data used in the paper. Then I present evidence that the slope does forecast future stock jump size. Next, the main hypothesis is tested. I further investigate the return predictability of the systematic and idiosyncratic components of slope.

3.1. Data

At the end (last trading day) of each month during January 1996–June 2005, the option data from the OptionMetrics are matched with stock return data from the Center for Research in Security Prices (CRSP) and accounting data from the Compustat. Monthly frequency is chosen for two reasons. First, it is the frequency considered by most studies on cross-sectional stock

[5] To be exact, I should use s_i/ν_i as the definition of the slope. But I choose the current version for simplicity. My later robustness checks show qualitatively and quantitatively similar results using this alternative definition.

Table 1
Stock summary statistics.

This table reports, for January 1996–June 2005, the summary statistics (mean and standard deviation) of the firm accounting and stock return data obtained from the Compustat and the Center for Research in Security Prices, respectively. At the end of each month, I use the firm market capitalization, book-to-market ratio, and leverage observed two quarters ago to define the variables ME (in billions of dollars), BM, and LV, respectively. A stock's β is estimated by regressing its monthly returns on the returns of the Standard & Poor's (S&P) 500 index. The second last column shows the sample length (in months) of match stock and option data. The last column reports the total number of stocks in the data set.

	ME	BM	LV	β	Monthly returns: Mean	Monthly returns: Standard deviation	Monthly returns: Skewness	Monthly returns: Kurtosis	Sample length	Number of stocks
Mean	3.252	1.036	2.024	1.339	0.010	0.162	0.408	4.367	47	4,048
Standard deviation	13.108	5.704	16.617	1.003	0.060	0.083	0.783	3.083	34	

Table 2
Option implied volatilities.

This table reports the mean and standard deviation of fitted implied volatilities of the individual equity options with one month to expiration and fixed deltas obtained from OptionMetrics. For each fitted implied volatility, OptionMetrics calculates a dispersion value, which is essentially a weighted average of standard deviations measuring the accuracy of the fitting procedure at that point. DS is the average dispersion over time and across stocks.

	Calls												
Δ_{call}	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
Mean	0.584	0.572	0.565	0.560	0.559	0.558	0.559	0.562	0.566	0.571	0.576	0.583	0.591
Standard deviation	0.237	0.239	0.240	0.241	0.242	0.240	0.240	0.241	0.241	0.241	0.241	0.241	0.240
DS	0.028	0.028	0.027	0.024	0.023	0.022	0.014	0.014	0.014	0.014	0.014	0.016	0.020
	Puts												
Δ_{put}	−0.80	−0.75	−0.70	−0.65	−0.60	−0.55	−0.50	−0.45	−0.40	−0.35	−0.30	−0.25	−0.20
Mean	0.593	0.584	0.576	0.571	0.569	0.569	0.569	0.572	0.576	0.582	0.590	0.600	0.613
Standard deviation	0.248	0.248	0.246	0.245	0.245	0.244	0.242	0.242	0.241	0.241	0.240	0.237	0.232
DS	0.026	0.023	0.020	0.017	0.014	0.015	0.013	0.012	0.012	0.013	0.015	0.019	0.026

returns. Second, it has the benefit of homogeneity, as the options for estimating implied volatility surface in different months have similar maturities. A stock's β is estimated by regressing its monthly returns on the returns of the S&P 500 index. I also use stock returns of last four years and use the CRSP value-weighted index as the proxy for the market portfolio and obtain similar results. A stock is excluded if it does not have at least two previous years of return data to estimate market beta. Following the convention of the literature, I use the market capitalization, book-to-market ratio, and leverage of each stock observed two quarters ago to define the variables ME, BM, and LV, respectively. I consider three liquidity measures: OV is the total option trading volume; SV is the total stock trading volume; and TO is the stock turnover rate. The data of the risk-free interest rate, Fama-French factors [R_M–R_f, small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)], and the momentum factor (MOM) are downloaded from Kenneth French's website. The summary statistics of the stocks are reported in Table 1. The sample contains 4,048 stocks with an average time series length of 47 months. The mean market capitalization is over \$3 billion and the mean book-to-market ratio is a bit higher than one. On average, the stock returns are positively skewed and fat-tailed.

As individual equity options are American style, OptionMetrics employs an algorithm based on the binomial tree model of Cox, Ross, and Rubinstein (1979) to compute option implied volatilities. The implied volatility surface is then constructed from estimated implied volatilities with a kernel smoothing technique, which is described in detail in the OptionMetrics data manual. OptionMetrics reports the fitted implied volatilities (of both calls and puts) on a grid of fixed maturities and option deltas. The maturities are one month, two months, three months, six months, and one year, and option deltas are 0.2, 0.25, …, 0.8 for calls and −0.8, −0.75,…,−0.2 for puts. For each fitted implied volatility, OptionMetrics also calculates a dispersion value, which is essentially a weighted average of standard deviations that measures the accuracy of the fitting procedure at that point. Table 2 presents the sample statistics of end-of-month fitted implied volatilities with one month to expiration. Clearly, there is a smile, as the at-the-money implied volatility (with $\Delta=0.5$ and -0.5 for call and put, respectively) is on average lower than in-the-money and out-of-the-money implied volatilities. The row for average dispersion (DS) shows increasing estimation errors for options deep in-the-money or out-of-the-money.

I use $\upsilon^{imp}_{put}(\Delta_{put})$ and $\upsilon^{imp}_{call}(\Delta_{call})$ to denote, respectively, the fitted implied volatilities of put and call options with one month to expiration and deltas equal to Δ_{put} and Δ_{call}. Following Eqs. (8) and (9), I define $\upsilon\equiv 0.5(\upsilon^{imp}_{put}(-0.5)+\upsilon^{imp}_{call}(0.5))$ and $s\equiv \upsilon^{imp}_{put}(-0.5)-\upsilon^{imp}_{call}(0.5)$, and report the

Table 3
ν and various measures of slope.

This table reports the mean and standard deviation of ν and various measures of slope of implied volatility smile. Let $\nu^{imp}_{put}(\Delta_{put})$ and $\nu^{imp}_{call}(\Delta_{call})$ denote the fitted implied volatilities with one month to expiration and option deltas equal to Δ_{put} and Δ_{call}, respectively. ν is defined by $\nu \equiv 0.5(\nu^{imp}_{put}(-0.5)+\nu^{imp}_{call}(0.5))$. s is defined by $s \equiv \nu^{imp}_{put}(-0.5)-\nu^{imp}_{call}(0.5)$. The systematic and idiosyncratic components of s (s^{sys} and s^{idio}) are defined to be, respectively, the fitted value and residual of the time series regression of s on the slope of the S&P 500 index options for the last 12 months. The slope measures using OTM puts are defined as $s(\Delta) \equiv \nu^{imp}_{put}(\Delta)-\nu^{imp}_{call}(0.5)$, for $-0.45 \le \Delta \le -0.20$.

					Slope measures using OTM puts					
	ν	s	s^{sys}	s^{idio}	$s(-0.45)$	$s(-0.40)$	$s(-0.35)$	$s(-0.30)$	$s(-0.25)$	$s(-0.20)$
Mean	0.567	0.010	0.010	0.000	0.013	0.017	0.023	0.030	0.040	0.054
Standard deviation	0.243	0.048	0.033	0.072	0.047	0.048	0.049	0.050	0.052	0.054

summary statistics in Table 3. The average implied volatility ν is 56.7%, more than twice of the average implied volatility of the S&P 500 index options (about 20%) for the same period. The slope s is positive on average but shows significant variation as the standard deviation of s across stocks is almost five times of the average slope. Because the slope is a proxy of jump risk with measurement error, a wide range of cross-sectional differences in slope alleviates the concern that my subsequent portfolio sorting analysis is affected by measurement errors. Furthermore, s varies significantly over time in terms of (unreported) high standard deviation of change of s, implying time-varying jump risk. Almost all correlations among return, ν, and s or changes of these variables are insignificant and not reported for brevity. The exception is the negative correlation between return and change of ν, which is consistent with the leverage effect suggested by Black (1976) and Christie (1982).

I further decompose s into the systematic and idiosyncratic components, using the slope for the S&P 500 index options, $s_{S\&P500}$, to proxy the market jump risk. Specifically, for the ith stock at the end of month t, I estimate the time series regression of the stock slope on the market slope for the last 12 months: $s_{i,k} = a_i + b_i s_{S\&P500,k} + \varepsilon_{i,k}, k = t-11,\dots,t$.[6] I define the systematic and idiosyncratic slopes, $s^{sys}_{i,t}$ and $s^{idio}_{i,t}$, to be the fitted value and residual of the regression, respectively.[7] Clearly, most variation in s is captured by the idiosyncratic component. In addition to s, I examine some other slope measures. Particularly, I consider the measures that use out-of-the-money puts, defined as $s(\Delta) \equiv \nu^{imp}_{put}(\Delta)- \nu^{imp}_{call}(0.5)$ for $-0.45 \le \Delta \le -0.20$. The summary statistics of the alternative slope measures using OTM puts are also presented in Table 3.

3.2. Slope predicting jump size

One implication of the theoretical results in Section 2 is that the realized jump size is monotonically increasing in the slope of implied volatility smile. Testing this implication empirically has two difficulties. First, identifying realized jumps generally requires long time series of stock returns, which are unavailable. The second difficulty, closely related to the first one, is that jump distributions could change over time, making identification of jumps even harder. In this section, I employ two different tests, one indirect and one direct, to demonstrate that the slope predicts average jump size.

Table 4
Average skewness of stock returns in slope quintiles.

I consider the 585 stocks that have the slope data during the entire period of January 1996–June 2005. For the ith stock, let $\{r^i_t\}^T_{t=1}$ denote its monthly return series. Define a ranking series $\{I^i_t\}$ so that $I^i_t=n$ if the slope of the stock in month $t-1$ is ranked in the nth quintile, where $n \in (1,\dots,5)$. Fixing a number $n \in (1,\dots,5)$, I collect observations in $\{r^i_t\}^T_{t=1}$ with slope ranking equal to n, that is, $\{r^i_{t_j} : I^i_{t_j} = n\}$. I then calculate the skewness of the subseries $\{r^i_{t_j} : I^i_{t_j} = n\}$. I consider only subseries of at least 10 observations. So I have (at most) five skewnesses for each stock corresponding to five slope rankings. This table reports the statistics of the skewnesses for the five quintile rankings. The last row shows the number of subseries of stock returns in each quintile ranking.

	Q_1	Q_2	Q_3	Q_4	Q_5
Mean	0.075	0.109	0.187	0.181	0.327
Standard deviation	0.062	0.121	0.144	0.157	0.276
Maximum	2.802	2.923	2.804	3.557	5.614
Minimum	−2.980	−2.627	−2.528	−2.619	−2.095
Number of observations	516	541	527	549	491

The indirect test is based on the well-known fact that jumps are positively related to skewness. Because the slope is a proxy of average jumps size, high (low) slope should predict high (low) future return skewness. To ensure enough observations in computing sample moments, I consider only the 585 stocks that have the slope data for the whole period. To take into account of time variation in slope and skewness, I propose a new way to compute skewness. Let $\{r^i_t\}^T_{t=1}$ denote the monthly return series of the ith stock. Define an auxiliary ranking series $\{I^i_t\}$ so that $I^i_t=n$ if the slope of the stock at the end of month $t-1$ is ranked in the nth quintile, where $n \in (1,\dots,5)$. Fixing a number $n \in (1,\dots,5)$, I collect observations in $\{r^i_t\}^T_{t=1}$ with slope ranking equal to n, that is, the subseries $\{r^i_{t_j} : I^i_{t_j} = n\}$. I then calculate the skewness of the subseries $\{r^i_{t_j} : I^i_{t_j} = n\}$. For accurate estimation, I consider only subseries with at least 10 observations. So I have (at most) five skewnesses for each

[6] One year of data is lost to estimating the regression. The regression is not defined if there are not enough (or 12) observations. I also use two years of data to estimate the regression and find similar results.

[7] The intercept of the regression is part of the systematic slope in this definition. Alternatively, I can incorporate the intercept into the idiosyncratic slope. Another definition that I consider uses the historical estimate of market β for the decomposition. The results for these alternative approaches are similar to those presented in the paper.

Table 5
Returns of portfolios formed on slope.

Panels A–C of this table report, respectively, the statistics for monthly returns of equally weighted quintile portfolios as well as the long-short portfolio Q_1–Q_5 by long the lowest quintile portfolio and short the highest quintile portfolio, formed on slope and its systematic and idiosyncratic components (s, s^{sys}, and s^{idio}) during January 1996–June 2005. In addition to the unadjusted raw returns, I consider the risk-adjusted returns, obtained from three models: the capital asset pricing model (CAPM), the Fama-French three-factor [R_M–R_f, small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)] model, and the four-factor model that extends the Fama-French three-factor model by incorporating the momentum factor (MOM). The t-statistics for the average (unadjusted and risk-adjusted) returns of Q_1–Q_5 are reported in brackets. The standard deviation, Sharpe ratio, skewness, kurtosis, and autocorrelation coefficient are calculated for the unadjusted returns.

Quintile	Unadjusted mean	Risk-adjusted mean: CAPM	Risk-adjusted mean: Three-factor	Risk-adjusted mean: Four-factor	Standard deviation	Sharpe ratio	Skewness	Kurtosis	Autocorrelation coefficient
Panel A: Quintile portfolios formed on s									
Q_1	0.021	0.013	0.008	0.012	0.080	0.225	0.003	3.978	0.115
Q_2	0.013	0.007	0.004	0.005	0.059	0.175	−0.608	3.878	0.092
Q_3	0.010	0.004	0.002	0.001	0.055	0.131	−0.665	3.475	0.123
Q_4	0.008	0.002	−0.001	−0.001	0.059	0.089	−0.586	3.357	0.112
Q_5	0.002	−0.005	−0.009	−0.008	0.072	−0.008	−0.499	3.358	0.132
Q_1–Q_5	0.018	0.018	0.017	0.019	0.024	0.642	2.256	13.267	0.053
	[8.168]	[8.128]	[8.158]	[9.638]					
Panel B: Quintile portfolios formed on s^{sys}									
Q_1	0.013	0.008	0.003	0.006	0.073	0.140	−0.076	4.733	0.117
Q_2	0.012	0.008	0.005	0.006	0.058	0.161	−0.355	3.839	0.072
Q_3	0.009	0.004	0.001	0.002	0.057	0.102	−0.631	3.899	0.103
Q_4	0.010	0.005	0.001	0.002	0.061	0.109	−0.519	3.969	0.145
Q_5	0.006	0.000	−0.004	−0.002	0.072	0.045	−0.232	3.473	0.177
Q_1–Q_5	0.007	0.008	0.007	0.009	0.023	0.174	1.860	11.369	−0.028
	[3.248]	[3.269]	[3.220]	[3.829]					
Panel C: Quintile portfolios formed on s^{idio}									
Q_1	0.018	0.013	0.007	0.012	0.077	0.194	0.068	4.170	0.132
Q_2	0.011	0.007	0.004	0.004	0.059	0.138	−0.567	3.961	0.133
Q_3	0.009	0.005	0.002	0.002	0.054	0.113	−0.688	4.073	0.130
Q_4	0.007	0.002	−0.001	−0.001	0.055	0.067	−0.557	3.775	0.101
Q_5	0.003	−0.002	−0.007	−0.005	0.069	0.006	−0.516	3.767	0.151
Q_1–Q_5	0.014	0.015	0.015	0.017	0.022	0.527	2.038	10.453	0.050
	[7.083]	[7.322]	[7.238]	[8.415]					

stock corresponding to the five slope rankings, respectively. Table 4 presents the summary statistics of the skewnesses. As expected, the average skewness increases from 0.075 for the lowest quintile (n=1) to 0.327 for the highest quintile (n=5). A direct t-test confirms that the skewness of quintile one is larger than the skewness of quintile five. So the evidence on future skewness supports that a stock with higher slope is more likely to have larger-size jumps.

The second test is based on the jump-identification methodology of Jiang and Oomen (2008) and Jiang and Yao (2009). I follow Jiang and Yao (2009) to estimate realized jump sizes. For the 12-month period ended in month t, I use daily returns to construct their jump test statistic, which asymptotically follows a standard normal distribution. If the null of no jumps is rejected at the 5% critical level, an estimate of the annual jump size for that year is derived, which I call JR_t. If the null is not rejected at the 5% critical level, I let the annual jump size in that period be a missing observation. I repeat these steps for the next 12-month period ending in month t+1, and so on. Because the time series JR_t is constructed with rolling windows, it is the change of JR_t that measures the average realized jump size in month t. To test the predictability of slope on future average jump size, I run the following time series regression:

$$\Delta JR_{t+1} = a + bs_t + \varepsilon_{t+1}. \tag{12}$$

I expect the estimated b to be positive. For precise estimation, I exclude stocks with time series shorter than 24 observations, and I end up with 806 stocks. The average estimate of b is 0.037, the t-statistic for all estimates of b is 2.536, and the average R^2 is 0.036. The evidence from the predictive regression again supports the positive relation between the slope and average stock jump size. Having established the slope as a proxy of the average jump size, I are ready to test the main hypothesis that the slope predicts stock returns.

3.3. Predicting returns

Stocks are ranked, on the last trading day of a month, in ascending order according to s into quintiles, and five portfolios are formed by equally weighing the stocks within each quintile. On average, a quintile portfolio contains 402 stocks. I then record the realized returns of the portfolios in the next month. Repeating these steps for every month in the sample period generates the time series of monthly returns for the five quintiles. Panel A of Table 5 reports the statistics of the quintile portfolio returns.[8] As shown in the first column, the average monthly portfolio return decreases from 2.1% for quintile one to 0.2% for quintile five, which is consistent with the main hypothesis. The average monthly return of the long-short portfolio Q_1–Q_5, formed by long quintile one and short quintile five, is 1.8% with t-statistic of 8.168. The return of Q_1–Q_5 is also economically significant even in the presence of transaction costs. On average, the quintile portfolios have a turn-over rate of 73.1% per month. Assuming a 0.5% one-way transaction cost as in Jegadeesh and Titman (1993), the long-short portfolio still generates 1.1% profit per month.

The quintile portfolios could have different risk profiles and thus have different returns. I use three different models to adjust for variations in risk: the CAPM, the three-factor model of Fama and French (1993), and the four-factor model of Carhart (1997) that extends the Fama-French three-factor model by incorporating the momentum factor. The results for the three models are similar. For example, for the four-factor model, the risk-adjusted quintile portfolio returns are lower than the unadjusted returns but the decreasing pattern of returns in slope is the same. The risk-adjusted return for the long-short portfolio Q_1–Q_5 is 1.9%, even a bit higher than the unadjusted return. Therefore, the factor models cannot explain the returns of quintile portfolios formed on slope. Without ambiguity, I use the four-factor model to estimate risk-adjusted returns for the rest of the paper.

As another measure of performance, the Sharpe ratios of the quintile portfolios also decrease in slope. The Sharpe ratio of Q_1–Q_5 is almost three times that of quintile one. There seems no obvious patterns in other return characteristics such as skewness, kurtosis, and autocorrelation coefficient of the portfolio returns except that the skewness is positive and close to zero for quintile one but negative for other quintiles. Panel A of Fig. 1 plots monthly average slopes of the quintile portfolios. Panel B plots risk-adjusted monthly returns of the quintiles, while Panel C plots risk-adjusted monthly returns of the long-short portfolio Q_1–Q_5. The risk-adjusted return of Q_1–Q_5 is positive in 95 of 114 months and achieves the maximum in January 2001. For robustness check, I also consider forming equally weighted decile portfolios and find results similar to those for the quintile portfolios. As expected, the average unadjusted and risk-adjusted returns of Q_1–Q_{10} for the decile portfolios are even higher than those of Q_1–Q_5 for the quintile portfolios. These results are not presented for brevity.

3.4. Systematic versus idiosyncratic jump risks

As the slope of implied volatility smile is a measure of total jump risk, it is interesting to ask whether the relation between slope and return is driven by systematic or idiosyncratic jump risk.[9] Theoretically, Merton (1976a) assumes stock jump risk diversifiable, while papers such as Bates (1996) and Santa-Clara and Yan (2010) assume market jump risk priced. However, the empirical evidence on this issue is sparse, mainly because of the difficulty of disentangling market and idiosyncratic jump risks. Fortunately, the slope of implied volatility smile allows a natural decomposition into the systematic and idiosyncratic components: s^{sys} and s^{idio}.

Panels B and C of Table 5 report the statistics of returns of quintile portfolios formed by sorting stocks on s^{sys} and s^{idio}, respectively. Both components predict (unadjusted and risk-adjusted) portfolio returns, indicating that both components are priced. But the decreasing pattern of portfolio returns for the idiosyncratic component is more pronounced and closer to that for s, while the portfolio returns for the systematic component are much flatter. The average unadjusted return of Q_1–Q_5 is 1.4% for s^{idio} but only 0.7% for s^{sys} albeit statistically significant. Similar patterns are found when performance is measured in terms of Sharpe ratio.

To further examine the contributions of the systematic and idiosyncratic components to the slope, I conduct a double-sort exercise, following the methodology of Fama and French (1992). I initially divide stocks into five quintiles by ranking on one of the two components (s^{sys} or s^{idio}) and then within each component quintile I further divide stocks into five quintiles by ranking on s. If the decreasing pattern of portfolio returns in s becomes less significant within a component quintile, it is evidence that the component explains the return predictability of s. The risk-adjusted returns of $25(=5\times5)$ double-sorted quintile portfolios and long-short portfolio Q_1^s–Q_5^s are reported in Panels A and B of Table 6 for s^{sys} and s^{idio}, respectively. When stocks are sorted on s^{sys} first and then on s, the returns of s quintile portfolios are still decreasing in s in all s^{sys} quintiles. The return of Q_1^s–Q_5^s remains large (1.4% on average) and highly significant. However, the decreasing pattern of returns in s becomes much less pronounced when stocks are first sorted on s^{idio} and then on s. The returns of Q_1^s–Q_5^s for the s^{idio} quintiles are still positive but much smaller (0.9% on average) in magnitude. The results seem intuitive given that s^{sys} accounts for most variation in s. In sum, neither component can explain all the return predictability of s. Between the two components, s^{idio} dominates s^{sys} as it captures more variation and predictability in the slope.

4. Robustness checks

In this section, robustness checks are conducted on the findings that the slope predicts stock returns. In particular, I control for a number of variables that have been found to explain cross-sectional stock returns. I further examine persistence, seasonality, and the impact of data filter rules on the results. I also consider alternative definitions of slope and differentiate my findings from those in some previous studies.

[8] The holding period of the portfolios starts on the first business day in the next month. As a robustness check, I also allow a one-day delay in starting the portfolio holding period and find essentially the same results.

[9] I thank the referee for raising the issue and pointing out the direction of the analysis.

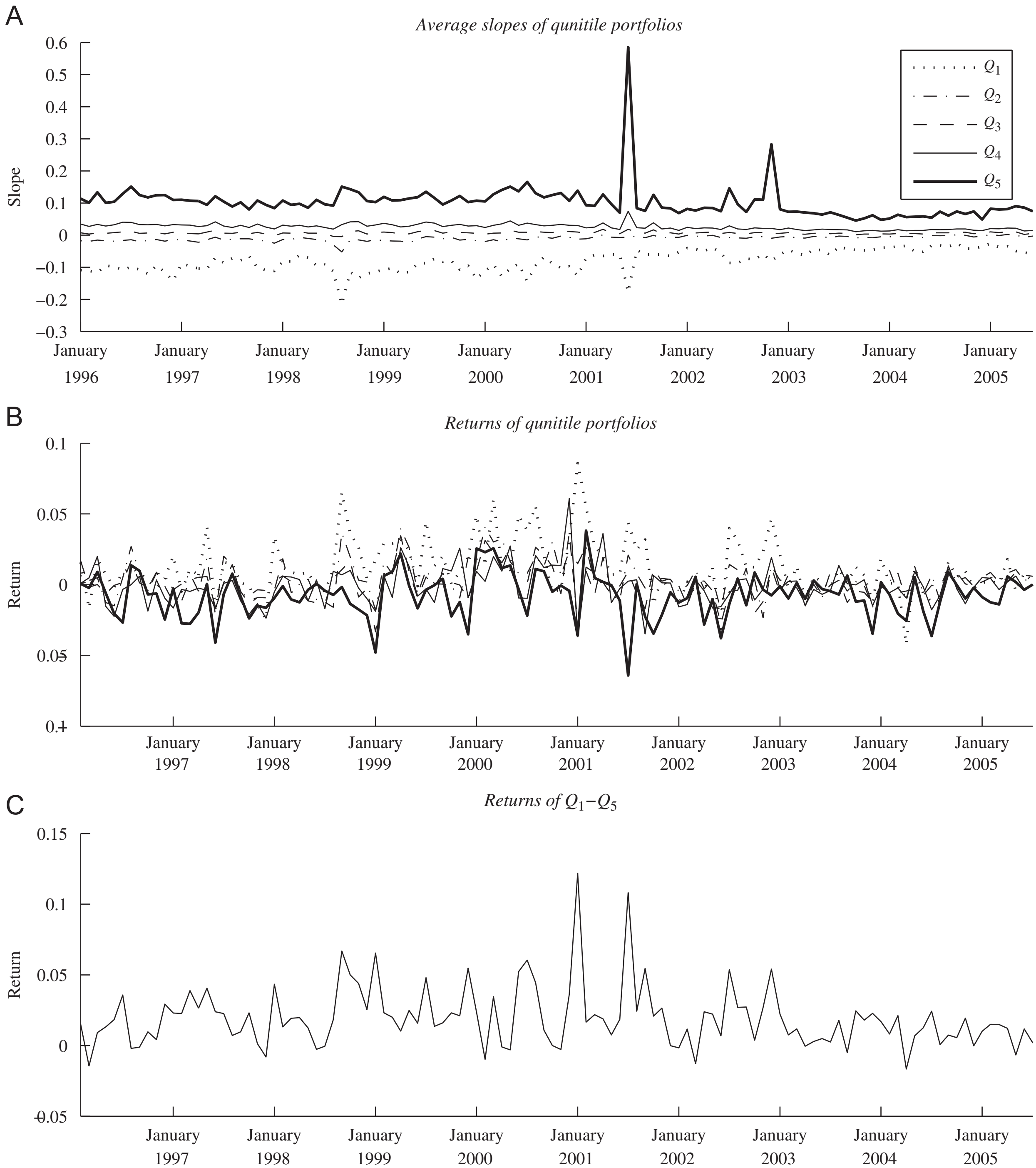

Fig. 1. Average slopes and returns of quintile portfolios. Panel A plots the monthly average slopes of the quintile portfolios formed on s during January 1996–June 2005. Panel B plots the risk-adjusted (using the four-factor model) monthly returns of these portfolios during February 1996–July 2005. Panel C plots the risk-adjusted returns of the long-short portfolio Q_1–Q_5 for the same period.

4.1. Control for other explanatory variables

The factor models cannot explain the return predictability of slope. But s still could be a proxy of some stock characteristics that are related to stock returns. This paper considers market β, past stock return, past idiosyncratic stock return, size, book-to-market ratio, leverage, implied volatility, idiosyncratic implied volatility, historic idiosyncratic volatility, skewness, co-skewness, systematic volatility, option volume, stock volume, and stock turnover rate.

Past return r is the stock return in the month when stocks are ranked and portfolios are formed, and past

Table 6
Double sorts on s, s^{sys}, and s^{idio}.

This table reports the average risk-adjusted (using the four-factor model) monthly returns of double-sorted quintile portfolios formed on s, s^{sys}, and s^{idio}. The last column of each panel reports the average risk-adjusted monthly returns (and t-statistics in brackets) of the long-short portfolio Q_1^s–Q_5^s. The last row of each panel reports the averages across the quintiles in each column. In Panel A, stocks are sorted on s^{sys} first and then on s. In Panel B, stocks are sorted on s^{idio} first and then on s.

	Q_1^s	Q_2^s	Q_3^s	Q_4^s	Q_5^s	$Q_1^s - Q_5^s$
Panel A: Sort on s^{sys} first and then on s						
$Q_1^{s^{sys}}$	0.011	0.008	0.003	0.000	−0.005	0.016
						[4.484]
$Q_2^{s^{sys}}$	0.012	0.004	0.001	0.001	−0.001	0.013
						[4.552]
$Q_3^{s^{sys}}$	0.008	0.003	−0.001	−0.000	−0.007	0.015
						[5.843]
$Q_4^{s^{sys}}$	0.004	−0.000	−0.000	−0.000	−0.005	0.009
						[3.966]
$Q_5^{s^{sys}}$	0.006	−0.002	−0.004	−0.006	−0.014	0.020
						[6.789]
Average	0.008	0.002	−0.000	−0.001	−0.006	0.014
Panel B: Sort on s^{idio} first and then on s						
$Q_1^{s^{idio}}$	0.014	0.012	0.011	0.006	0.000	0.013
						[3.474]
$Q_2^{s^{idio}}$	0.005	0.004	0.002	0.000	−0.002	0.008
						[3.454]
$Q_3^{s^{idio}}$	0.002	0.001	0.001	0.000	−0.002	0.004
						[2.164]
$Q_4^{s^{idio}}$	−0.002	0.002	−0.001	−0.002	−0.007	0.005
						[2.191]
$Q_5^{s^{idio}}$	0.002	−0.004	−0.006	−0.008	−0.015	0.017
						[5.485]
Average	0.004	0.003	0.001	−0.001	−0.005	0.009

idiosyncratic return is defined as $r_{idio} \equiv r - \beta R_M$, where R_M is the return of the S&P 500 index during the month. Because slope is constructed from option implied volatilities, it is natural to examine if the results are driven by v. Recent studies such as Goyal and Santa-Clara (2003) and Ang, Hodrick, Xing, and Zhang (2006) show that idiosyncratic volatilities have explanatory power on cross-sectional stock returns. Following Dennis, Mayhew, and Stivers (2006), I define the idiosyncratic implied variance as $v_{idio}^2 \equiv v^2 - \beta^2 v_M^2$, where v_M is the implied volatility of the S&P 500 index option. I also look at the historic idiosyncratic volatility v_{idio}^{hist}, defined to be the standard deviation of the residuals of the aforementioned market regression. Harvey and Siddique (2000) find that conditional (co-)skewness helps explain cross-sectional stock returns. I follow their method to examine two measures of conditional skewness: SK, defined as the total skewness of stock returns during the last two years; and CSK, defined as the coefficient of regressing last two years stock returns on the squares of market returns. Duan and Wei (2009) find that the systematic risk proportion in the total risk determines the risk-neutral skewness, which in turn affects the implied volatility smile as shown by Bakshi, Kapadia, and Madan (2003). Following Duan and Wei (2009), I define the systematic risk proportion to be $v_{sys}^2 \equiv \beta^2 v_M^2 / v^2$ and refer to it as *systematic volatility* without ambiguity. The liquidity variables are motivated by studies such as Bollen and Whaley (2004), Ofek, Richardson, and Whitelaw (2004), Pan and Poteshman (2006), and Cremers and Weinbaum (2010) that document evidence of market microstructure effects on option prices and stock returns.

I adopt the cross-sectional regression approach of Fama and MacBeth (1973) as it can examine multiple explanatory variables simultaneously.[10] For each month during the sample period, I run the cross-sectional regression of the unadjusted stock returns in the subsequent month on certain explanatory variables. Table 7 reports the time series averages of estimated regression coefficients and t-statistics.

First, consider univariate regressions that include either s or one control variable. The coefficient for s is negative and highly significant, confirming the earlier results based on the portfolio sorting approach. Among the control variables, only ln(ME) is significant and the negative coefficient is consistent with the size effect shown in the literature. For bivariate regressions that include s and one control variable, the coefficient on s remains negative and significant, while none of the control variables is significant. Next, consider incorporating multiple control variables. Given the large number of control variables, there are numerous possible

[10] I also use the double-sorting methodology to analyze the effectiveness of control variables in explaining the return predictability of slope. The results are similar to those based on the Fama and MacBeth regressions and are not reported for brevity. This approach, however, can consider only one control variable at a time.

Table 7
Slope and control variables.
This table reports the averages of estimated coefficients (and t-statistics in brackets) of Fama and MacBeth regressions for monthly stock returns on slope and control variables. The control variables include β, lagged return (r), lagged idiosyncratic return (r_{idio}), log size [ln(ME)], book-to-market ratio (BM), leverage (LV), implied volatility (ν), idiosyncratic variance (ν^2_{idio}), historic idiosyncratic volatility (ν^{hist}_{idio}), skewness (SK), co-skewness (CSK), systematic risk (ν^2_{sys}), option trading volume (OV), stock trading volume (SV), and stock turnover rate (TO). In univariate regressions, either s or one control variable is used. In bivariate regressions, s and one control variable are used. In multivariate regressions, s and multiple control variables are used.

		Bivariate		Multivariate					
	Univariate	s	Control	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
s	−0.057			−0.059	−0.057	−0.058	−0.061	−0.057	−0.057
	[−9.804]			[−10.837]	[−9.500]	[−10.172]	[−10.701]	[−10.090]	[−9.469]
β	0.001	−0.061	0.001	0.002					0.000
	[0.560]	[−10.552]	[0.414]	[0.647]					[0.236]
r	−0.009	−0.056	−0.007	−0.013					−0.025
	[−0.613]	[−9.847]	[−0.491]	[−1.076]					[−2.979]
r_{idio}	−0.014	−0.060	−0.011						
	[−1.007]	[−10.480]	[−0.814]						
ln(ME)	−0.005	−0.055	−0.000		−0.000				−0.001
	[−3.348]	[−10.036]	[−0.129]		[−0.186]				[−1.701]
BM	−0.004	−0.059	−0.133		−0.075				0.335
	[−0.051]	[−9.479]	[−0.712]		[−0.331]				[0.854]
LV	−0.000	−0.060	−0.000		−0.000				−0.000
	[−1.455]	[−9.645]	[−0.506]		[−0.671]				[−1.367]
ν	−0.009	−0.054	−0.009			−0.003			−0.012
	[−0.594]	[−9.580]	[−0.582]			[−0.199]			[−1.012]
ν^2_{idio}	−0.011	−0.060	−0.011						
	[−1.507]	[−10.037]	[−1.444]						
ν^{hist}_{idio}	0.021	−0.059	−0.048			−0.041			−0.019
	[0.385]	[−10.503]	[−0.657]			[−1.232]			[−0.681]
SK	0.002	−0.057	−0.002				−0.002		−0.001
	[0.952]	[−10.039]	[−1.007]				[−1.144]		[−0.670]
CSK	0.001	−0.061	0.001				0.001		
	[0.560]	[−10.552]	[0.414]				[0.423]		
ν^2_{sys}	−0.004	−0.062	−0.004						
	[−1.404]	[−10.222]	[−1.384]						
OV	−0.000	−0.057	−0.000					−0.000	−0.000
	[−1.560]	[−9.840]	[−0.156]					[−0.559]	[−0.606]
SV	−0.000	−0.057	0.000					0.000	0.000
	[−1.345]	[−9.859]	[0.784]					[1.102]	[1.434]
TO	0.000	−0.057	0.000					0.000	0.000
	[0.484]	[−9.983]	[0.564]					[0.371]	[0.683]

multi-variate regressions. I show only six representative models for brevity. The first five models include either two or three control variables, and the last model contains most of the control variables. Due to collinearity among the control variables, I drop several variables in Model 6. Again, the coefficient on s is significant in all multi-variate regressions. Among the control variables only r is significant and ln(ME) is marginally significant in Model 6. Overall, there is no evidence that any of the control variables can explain the return predictability of slope.

4.2. Persistence, seasonality, and filters

Next, I investigate the performance persistence of the quintile portfolios formed on s by considering holding horizons up to six months and report average risk-adjusted monthly portfolio returns in Table 8. Because of overlapping samples, the holding period returns are serially correlated for horizons beyond one month, and I calculate the t-statistics using the Newy and West procedure. The decreasing pattern of portfolio returns in s is still present but becomes less pronounced as the holding period increases. The return of the long-short portfolio Q_1–Q_5 goes down to 1% at two-month horizon and becomes as low as 0.5% at six-month horizon, albeit statistically significant. In spite of some degree of persistence, most of the profit generated by the long-short portfolio comes in the first month immediately after the portfolios being formed. It implies that jumps are short-lived and average jump sizes are time-varying. This is exactly what is observed in the data: The slope of a stock changes over time.

One could be interested in whether there is any seasonality in the return predictability of s. I conduct the portfolio sorting exercises and Fama and MacBeth regressions for 12 calender months and find no apparent differences across different months. Another concern is that the findings could be driven by the choice of data. To address the issue, I employ a number of different filters to the data and repeat the analysis. First, stocks for which s is too high or too low are excluded to make sure the findings not dominated by extreme values of s. Second, financial firms are excluded. Third, I use only the 585 stocks that have the slope data for the whole period. Finally, I look at

Table 8
Different holding period returns of portfolios formed on slope.

This table reports the average risk-adjusted (using the four-factor model) monthly returns of the quintile portfolios formed on *s* for holding periods of one month to six months. The last column reports the average risk-adjusted monthly returns (and *t*-statistics in brackets) of the long-short portfolio Q_1–Q_5. For horizons longer than one month, I follow the Newy and West procedure to compute the *t*-statistics because the returns are serially correlated due to overlapping samples.

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_1–Q_5
One month	0.012	0.005	0.001	−0.001	−0.008	0.019 [9.638]
Two months	0.008	0.004	0.002	0.001	−0.003	0.010 [7.560]
Three months	0.008	0.004	0.003	0.002	0.001	0.007 [6.073]
Four months	0.008	0.005	0.003	0.003	0.002	0.006 [4.765]
Five months	0.008	0.005	0.004	0.004	0.002	0.006 [4.601]
Six months	0.009	0.005	0.003	0.004	0.004	0.005 [3.837]

the subsamples of stocks that either paid dividends (2,821 stocks) or did not pay dividends (1,227 stocks) during the sample period. In sum, the results for different subsamples are similar to those for the full sample. These results are not shown for brevity but are available upon request.

4.3. Alternative definitions of slope

In the literature, jump risk is often argued to be reflected by the implied volatilities of deep OTM put options. However, as seen in the data, implied volatilities of deep OTM put options can be noisy and therefore might not provide accurate estimates of jump risk.[11] To examine the extent moneyness affects measurement of jump risk, I consider alternative slope measures that use OTM put options: $s(\Delta)=\sigma^{imp}_{put}(\Delta)-\sigma^{imp}_{call}(0.5)$, for $\Delta=-0.45,\ldots,-0.2$. Panel A of Table 9 reports the average risk-adjusted monthly returns of the quintile portfolios formed on $s(\Delta)$ s next to those for *s*. It is interesting to observe similar decreasing portfolio returns for all slope measures. The return of the long-short portfolio Q_1–Q_5 is always positive and significant, but it becomes relatively lower as Δ increases. This indicates potential larger measurement errors for slope measures using deeper OTM puts.

I further examine the issue by running the Fama and MacBeth regressions and report the results in Panel B of Table 9. For the univariate regressions with either *s* or one of $s(\Delta)$s as the explanatory variable, the coefficient is negative and statistically significant for all slope measures. But the magnitude of the coefficient, together with the *t*-statistic, decreases as Δ increases, consistent with the findings in Panel A. For the bivariate regressions with *s* and one of $s(\Delta)$s the explanatory variables, the coefficient on *s* is on average more than two times the coefficient on $s(\Delta)$. Moreover, the coefficient on *s* is always significant, and the coefficient on $s(\Delta)$ is only significant in two of six cases. These results suggest that $s(\Delta)$s, the slope measures that use OTM puts, cannot explain the return predictability of *s*, while *s* can explain most of the return predictability of $s(\Delta)$s. I also use the double-sorting method to examine the explanatory power of *s* and $s(\Delta)$s and obtain similar findings. However, OTM put options could contain information beyond that in the at-the-money option. A future research direction is to extract all the information embedded in the implied volatility smile.

It is important to note that for a fixed value of Δ, say −0.2, $s(\Delta)$ resembles the *skew* measure in Xing et al. (2010). Using the ratio of strike price to stock price as moneyness, they define skew as the difference between implied volatilities of out-of-the-money put and at-the-money call options. Xing et al. (2010) find that low skew stocks outperform high skew stocks, similar to the results for $s(\Delta)$. They argue that the skew reflects informed investors' demand of OTM puts in anticipating bad news about future stock prices. The implication is that the option market leads the stock market and is more efficient in incorporating information. In contrast, I assume efficient stock and option markets, and my slope of implied volatility smile proxies the jump risk. The put option used in defining *s* is slightly in-the-money. So a high value of *s* cannot be interpreted as anticipation of bad news.

I next look at another measure of slope defined as $sl\equiv\sigma^{imp}_{put}(\Delta)-\sigma^{imp}_{put}(-0.5)$, for $-0.45\leq\Delta\leq-0.2$. This is similar to the measure in Bollen and Whaley (2004), which is basically the percentage difference between implied volatilities of out-of-the-money put and at-the-money put with $\Delta=-0.25$ and −0.5, respectively. It is also similar to the slope variable of Xing et al. (2010), although they use put options with different moneynesses instead of different deltas. For my sample, I do not find return predictability of *sl*. It is interesting to realize that all the alternative measures of slope considered above capture the global steepness of implied volatility smile because the two options used for the definitions have distinct strike prices. In contrast, my slope *s* is a local steepness measure as the put and call options that I use are both close to being at-the-money.

[11] I thank the referee for suggesting this robustness analysis. I also consider using implied volatilities of deep in-the-money puts and obtain similar results.

Table 9
Slope measures using out-of-the-money (OTM) put options.
This table examines s and $s(\Delta)$s, the slope measures that use OTM put options. Panel A reports the average risk-adjusted (using the four-factor model) monthly returns of the quintile portfolios as well as the long-short portfolio Q_1–Q_5 formed on s and $s(\Delta)$s. The t-statistics for the average returns of Q_1–Q_5 are reported in brackets. Panel B reports the averages of estimated coefficients (and t-statistics in brackets) of the Fama and MacBeth regressions. The univariate regressions use either s or one $s(\Delta)$, while the bivariate regressions use s and one $s(\Delta)$.

Panel A: Portfolio returns

		Slope measures using OTM puts					
	s	$s(-0.45)$	$s(-0.40)$	$s(-0.35)$	$s(-0.30)$	$s(-0.25)$	$s(-0.20)$
Q_1	0.012	0.012	0.012	0.012	0.012	0.011	0.011
Q_2	0.005	0.003	0.003	0.004	0.003	0.003	0.003
Q_3	0.001	0.002	0.002	0.001	0.001	0.001	0.001
Q_4	−0.001	−0.001	−0.000	−0.001	−0.001	−0.001	−0.001
Q_5	−0.008	−0.008	−0.007	−0.007	−0.005	−0.005	−0.004
Q_1–Q_5	0.019	0.019	0.019	0.018	0.017	0.016	0.015
	[9.638]	[9.654]	[9.371]	[8.960]	[7.716]	[7.390]	[6.602]

Panel B: Fama and MacBeth regressions

	Univariate	Bivariate	
		s	$s(\Delta)$
s	−0.057		
	[−9.804]		
$s(-0.45)$	−0.057	−0.048	−0.010
	[−9.586]	[−1.983]	[−0.411]
$s(-0.4)$	−0.054	−0.038	−0.020
	[−9.445]	[−2.567]	[−1.400]
$s(-0.35)$	−0.053	−0.036	−0.024
	[−9.397]	[−3.133]	[−2.057]
$s(-0.3)$	−0.049	−0.039	−0.021
	[−8.987]	[−3.798]	[−2.033]
$s(-0.25)$	−0.044	−0.042	−0.018
	[−7.825]	[−4.380]	[−1.764]
$s(-0.2)$	−0.039	−0.045	−0.014
	[−6.118]	[−4.887]	[−1.353]

The last alternative measure of slope that I consider is essentially s normalized by ν, that is, $\hat{s} \equiv s/\nu$. This is similar to the normalization in Bollen and Whaley (2004). Toft and Prucyk (1997) also use the percentage difference between implied volatilities of call (put) options with strike prices 10% below and 10% above the stock price, respectively. The results for $\hat{s}$ are very similar to those for s and are not reported for brevity.

5. Conclusion

Overwhelming empirical evidence exists for jumps in stock prices. Based on a stylized jump-diffusion model for the SDF and stock price processes, I demonstrate that expected stock return should be monotonically decreasing in average stock jump size. Overcoming the difficulties of estimating jump distributions, I show that the average stock jump size can be proxied by the slope of option implied volatility smile.

After empirically establishing the relation between the slope and average future jump size, I test the hypothesis that the slope predicts future stock returns and find strong supporting evidence. Low slope portfolios earn higher returns than high slope portfolios. The trading strategy that long the lowest slope quintile portfolio and short the highest slope quintile portfolio generates monthly profit of 1.9% on a risk-adjusted basis. Interestingly, it is the idiosyncratic component of slope that accounts for most of the return predictability of slope. My findings are robust to a number of stock characteristics that have been found to explain stock returns. The results cannot be explained by other slope measures in the literature.

Appendix A

I first prove the propositions and then present simulation results.

A.1. Proofs

Proof of Proposition 1. I first decompose the Poisson processes into independent components: $N_M = N_C + \tilde{N}_M$ and $N_i = N_C + \tilde{N}_i$, where N_C, $\tilde{N}_M$, and $\tilde{N}_i$ are independent Poisson processes with intensities λ_C, $\tilde{\lambda}_M$, and $\tilde{\lambda}_i$, respectively. Direct calculation shows that $\mathrm{Corr}(N_M, N_i) = \lambda_C/\sqrt{\lambda_M \lambda_i}$. Hence $\lambda_C = \eta_i \sqrt{\lambda_M \lambda_i}$, $\tilde{\lambda}_M = \lambda_M - \lambda_C$, and $\tilde{\lambda}_i = \lambda_i - \lambda_C$. Next, I apply the Itô's formula for jump-diffusions (see, for example, Protter, 2004) to MS_i:

$$\frac{d(MS_i)}{MS_i} = (\mu_i - r_f + \rho_i\sigma_M\sigma_i - \lambda_M\mu_{J_M} - \lambda_i\mu_{J_i})\,dt + \sigma_M dW_M + \sigma_i dW_i + J_M dN_M + J_i dN_i + J_M J_i dN_C. \tag{13}$$

MS_i being a martingale implies $\mu_i - r_f + \rho_i\sigma_M\sigma_i + \eta_i\sqrt{\lambda_M\lambda_i}\mathrm{E}[J_M J_i] = 0$. I rewrite $J_M J_i = (1+J_M)(1+J_i) - J_M - J_i - 1$. Then direct computation of the above expectation leads to Eq. (5). The monotonicity of excess stock return in (i)–(iii) can be derived by differentiating the right-hand side of Eq. (5) with respect to the corresponding parameters. □

Proof of Proposition 2. Under the risk-neutral probability measure, the stock price follows

$$dS_i/S_i = (r_f - q_i - \lambda_i\mu_{J_i})\,dt + \sigma_i dW_i + J_i dN_i.\text{[12]} \tag{14}$$

The call price (C) is equal to the discounted expected payoff: $C = e^{-r_f T}\mathrm{E}_0[(S_i(T) - K)^+]$, where $\mathrm{E}_0(.)$ denotes the expectation. For small T, the probability that one jump occurs before T is λT, and the probability of multiple jumps is of order $O(T^2)$. So up to the order of T^2, the log terminal stock price can be approximated by the mixture of normal distributions:

$$\ln S_i(T) = \begin{cases} \ln S_i(0) + \left(r_f - q_i - \frac{1}{2}\sigma_i^2 - \lambda_i\mu_{J_i}\right)T + \sigma_i\sqrt{T}\varepsilon & \text{w/Prob.}1-\lambda_i T \\ \ln S_i(0) + \left(r_f - q_i - \frac{1}{2}\sigma_i^2 - \lambda_i\mu_{J_i}\right)T + \sigma_i\sqrt{T}\varepsilon + \mu_{J_i} + \sigma_{J_i}\zeta & \text{w/Prob.}\lambda_i T \end{cases}, \tag{15}$$

where ε and ζ are independent standard normally distributed variables. The option price can be written as

$$C = I_1 + I_2, \tag{16}$$

where I_1 and I_2 correspond to the components without and with the jump, respectively. I use the Black-Scholes formula to compute I_1 and I_2 to get

$$C = S_i(0)\left[\Phi\left(\frac{-X + \left(-\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right) - e^X\Phi\left(\frac{-X + \left(-\lambda_i\mu_{J_i} - \frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)\right] + O(T), \tag{17}$$

where $\Phi(.)$ is the standard normal distribution function. Letting X=0 and applying the Taylor expansion of Φ around zero ($\Phi(z) = \frac{1}{2} + z/\sqrt{2\pi} + O(z^2)$) results in

$$C|_{X=0} = \frac{1}{\sqrt{2\pi}}S_i(0)\sigma_i\sqrt{T} + O(T). \tag{18}$$

For the derivative, I differentiate Eq. (16) with respect to X and evaluate at X=0:

$$\left.\frac{\partial C}{\partial X}\right|_{X=0} = S_i(0)\left\{-\frac{e^{-\lambda_i\mu_{J_i}T}}{\sigma_i\sqrt{T}}\phi\left(\frac{\left(-\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)\sqrt{T}}{\sigma_i}\right) - \left[\Phi\left(\frac{-\left(\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)\sqrt{T}}{\sigma_i}\right) - \frac{1}{\sigma_i\sqrt{T}}\phi\left(\frac{-\left(\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)\sqrt{T}}{\sigma_i}\right)\right]\right\} + O(T), \tag{19}$$

where $\phi(.)$ is the standard normal density. Applying Taylor approximations for e^z, ϕ, and Φ around zero (e^z=1+z+$O(z^2)$, $\phi(z) = 1/\sqrt{2\pi}(1 - z^2/2) + O(z^4)$) leads to

$$\left.\frac{\partial C}{\partial X}\right|_{X=0} = S_i(0)\left[-\frac{1}{2} + \frac{1}{\sqrt{8\pi}}\left(\sigma_i + \frac{2\lambda_i\mu_{J_i}}{\sigma_i}\right)\sqrt{T}\right] + O(T). \tag{20}$$

Next, I compute the option price and the derivative of the option price in terms of moneyness using an alternative method. Let C^{BS} denote the option value derived from the Black-Scholes formula using some implied volatility function, $\sigma_i^{imp}(X,T)$, so that $C = C^{BS} = S_i(0)e^{-q_i T}[\Phi(d_1) - e^X\Phi(d_2)]$, where $d_1 = (-X + \frac{1}{2}(\sigma_i^{imp})^2 T)/\sigma_i^{imp}\sqrt{T}$ and $d_2 = (-X - \frac{1}{2}(\sigma_i^{imp})^2 T)/\sigma_i^{imp}\sqrt{T}$. Letting X=0 (so that $d_1 = \frac{1}{2}\sigma_i^{imp}\sqrt{T}$ and $d_2 = -\frac{1}{2}\sigma_i^{imp}\sqrt{T}$) and using the Taylor expansion of Φ results in

$$C|_{X=0} = \frac{1}{\sqrt{2\pi}}S_i(0)\sigma_i^{imp}\sqrt{T} + O(T). \tag{21}$$

For the derivative, $\partial C/\partial X = (\partial C^{BS}/\partial X) + (\partial C^{BS}/\partial\sigma_i^{imp})(\partial\sigma_i^{imp}/\partial X)$. Setting X=0 and applying Taylor approximations for Φ and ϕ results in

$$\left.\frac{\partial C}{\partial X}\right|_{X=0} = S_i(0)\left[-\frac{1}{2} + \frac{1}{\sqrt{8\pi}}\left(\sigma_i^{imp} + \frac{2\partial\sigma_i^{imp}}{\partial X}\right)\sqrt{T}\right] + O(T). \tag{22}$$

Comparing Eq. (18) with Eqs. (21) and (20) with Eq. (22), respectively, I derive Eqs. (6) and (7).

Proof of Proposition 3. Let X_{put} and X_{call} be the log moneyness of the put and call options. From the Black-Scholes formula, $\Delta_{put} = e^{-q_i T}[\Phi(d_{1,put}) - 1]$ and $\Delta_{call} = e^{-q_i T}\Phi(d_{1,call})$, where $d_{1,put} = (-X_{put} + \frac{1}{2}(\sigma_{i,put}^{imp})^2 T)/\sigma_{i,put}^{imp}\sqrt{T}$ and $d_{1,call} = (-X_{call} + \frac{1}{2}(\sigma_{i,call}^{imp})^2 T)/\sigma_{i,call}^{imp}\sqrt{T}$. By the fact that $\Delta_{put} = -0.5$ and $\Delta_{call} = 0.5$, then $\Phi(d_{1,put}) = 1 - 0.5e^{q_i T}$ and $\Phi(d_{1,call}) = 0.5e^{q_i T}$. Using the Taylor approximations of Φ and $e^{q_i T}$, I get X_{put}=$O(T)$ and X_{call}=$O(T)$. The implied volatilities of the put and call options are therefore close to the instantaneous stock volatility: $\sigma_{i,put}^{imp} = \sigma_i + O(T)$ and $\sigma_{i,call}^{imp} = \sigma_i + O(T)$. Combining these two equations proves Eq. (10). To prove Eq. (11), further computations show

[12] To be rigorous, the jump intensity and jump size distribution have to be modified when I switch the probability measure. Technically, I should use λ_i^*, $\mu_{J_i}^*$, and $\sigma_{J_i}^*$ to denote the jump intensity, average jump size, and jump volatility, respectively, under the risk-neutral probability measure. Because the market is incomplete in the presence of jumps, the transformation between the two probability measures is not unique. Santa-Clara and Yan, 2010, for example, find a transformation for their equilibrium model, which depends on the risk aversion of the representative investor. I abuse the notation here by using the same parameters for two different probability measures. However, ignoring the change of probability measure might not be a serious problem because the same transformation is applied to all stocks. As I consider cross-sectional stock returns, the probability transformation would not change the inference much.

$X_{\text{put}} = \frac{1}{2}\sigma_i^2 T + O(T^{3/2})$ and $X_{\text{call}} = \frac{1}{2}\sigma_i^2 T + O(T^{3/2})$. Take the difference between $d_{1,\text{put}}$ and $d_{1,\text{call}}$ to get

$$\frac{-X_{\text{put}} + \frac{1}{2}(\sigma_{i,\text{put}}^{\text{imp}})^2 T}{\sigma_{i,\text{put}}^{\text{imp}}\sqrt{T}} - \frac{-X_{\text{call}} + \frac{1}{2}(\sigma_{i,\text{call}}^{\text{imp}})^2 T}{\sigma_{i,\text{call}}^{\text{imp}}\sqrt{T}} \approx \sqrt{2\pi}(1-e^{q_i T}). \tag{23}$$

Rewrite the above equation as

$$X_{\text{put}} - X_{\text{call}} \approx \sqrt{2\pi}(e^{q_i T}-1)\sigma_{i,\text{put}}^{\text{imp}}\sqrt{T} + \frac{1}{2}(\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}})\sigma_{i,\text{put}}^{\text{imp}} T + \frac{(\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}})X_{\text{call}}}{\sigma_{i,\text{call}}^{\text{imp}}}. \tag{24}$$

By earlier results, $\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}}$ is of order $O(T^{3/2})$ and X_{call} is of order $O(T)$. $\sigma_{i,\text{put}}^{\text{imp}}$ can be approximated by $v_i = 0.5(\sigma_{i,\text{call}}^{\text{imp}} + \sigma_{i,\text{put}}^{\text{imp}})$ up to order $O(T)$. So, I can drop the last two terms in Eq. (24), which are of order $O(T^2)$, and have the following approximation:

$$X_{\text{put}} - X_{\text{call}} \approx \sqrt{2\pi}(e^{q_i T}-1)v_i\sqrt{T}. \tag{25}$$

The value of Eq. (25) is nonzero only when the dividend yield q_i is nonzero. If that is the case, I can approximate the slope of the implied volatility smile by

$$\left.\frac{\partial \sigma_i^{\text{imp}}(X,T)}{\partial X}\right|_{X=0} \approx \frac{\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}}}{X_{\text{put}} - X_{\text{call}}} = \frac{\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}}}{\sqrt{2\pi}}(e^{q_i T}-1)v_i\sqrt{T}. \tag{26}$$

Using the approximation $v_i \approx \sigma_i$ and comparing Eq. (26) with Eq. (7), s_i is proportional to $\lambda_i \mu_{J_i}$ up to the constant $L_i = 2\sqrt{2\pi T}(e^{q_i T}-1)$. And this proves Eq. (11).

The results depend on the assumption of nonzero dividend yield. However, the traded stock options are American style. Even for non-dividend-paying stocks, the put and call options with $\Delta = -0.5$ and 0.5 can have different strikes because of early exercise opportunities. I leave generalization to American options for future research. In fact, my empirical results for non-dividend-paying stocks are similar to those for dividend-paying stocks.

A.2. Monte-Carlo simulations

Monte-Carlo simulations are conducted to examine the approximation errors in Proposition 2. I extend the model to incorporate stochastic volatility because of overwhelming empirical evidence of time-varying volatility. In particular, the return volatility follows the square-root process of Heston (1993):

$$d\sigma_i^2 = \kappa_i(\theta_i - \sigma_i^2)\,dt + \phi_i\sqrt{\sigma_i^2}\,dZ_i, \tag{27}$$

where Z_i is a standard Brownian motion correlated with W_i and the correlation coefficient is $\text{Corr}(dW_i, dZ_i) = \zeta_i$. Although semi-analytical option pricing formula is available for jump-diffusion model of Eqs. (3), (4), and (27) (see, for example, Pan, 2002), I adopt the simulation approach here to compute option prices because of its simplicity. To simulate paths of stock prices, the Euler scheme is used to discretize the continuous-time model and the time interval Δt is set to one-fifth of a day. I approximate the Poisson process by a Bernoulli process, that is, there is at most one jump during an interval.

For the benchmark case, I use the following parameter values: r_f=0.06, q_i=0.02, $\lambda_i = 0.5$, $\mu_{J_i} = 0.1$, $\sigma_{J_i} = 0.1$, $\kappa_i = 0.02$, $\theta_i = 0.025$, $\phi_i = 0.025$, and $\zeta_i = -0.25$. The initial stock price is $S(0)$=\$40 and the initial volatility is $\sigma_i(0) = 0.5$. One million paths of stock prices are generated and the price of an option is calculated by discounting the average payoff. I then invert the Black-Scholes formula to get the option implied volatility. For a particular maturity T, I compute the at-the-money implied volatility for which the moneyness X is zero. I also compute the implied volatility for the option with the same maturity but with strike price \$0.001 higher than the strike price of the at-the-money option. I approximate the slope by the ratio between the difference of the two implied volatilities and the difference between the two moneyness values. Four different maturities are considered: one day, one week, one month, and two months. I also consider the effects of changing certain parameter values and report the at-the-money implied volatility and slope in Table A.1.

In Panel A, I use different values of average jump size, μ_{J_i}. The left half of the panel reports the implied volatility. For a fixed value of T, a U-shaped pattern of implied volatility is seen as a function of μ_{J_i}. The implied volatility is biased upward, that is, higher than the instantaneous diffusive volatility, which is equal to 0.5. The bias is very small for maturity of one day but becomes larger for long maturities. For example, at the two-month horizon and when $\mu_{J_i} = 0.2$, the implied volatility error is 0.04. As expected, the estimated slope of implied volatility smile shows an increasing pattern in terms of μ_{J_i} when T is fixed. The rate of increase is highest when T is one day, and it gets smaller as T becomes larger. To get a sense of the accuracy of the approximation, I compare slope with μ_{J_i} as Eq. (7) suggests that these two quantities should be close because of the choice of $\lambda_i = \sigma_i(0) = 0.5$. When $\mu_{J_i} = 0.2$ and T is one day, slope is 0.143, so the error is -0.057. For $\mu_{J_i} = 0.1$, the error is -0.045. The magnitude of error is smaller for negative jump sizes. For example, for $\mu_{J_i} = -0.1$, the bias is only 0.002. Fixing a value of μ_{J_i}, the error is increasing with T and becomes significant, particularly for positive values of μ_{J_i}. Overall, the approximation error is significant when average jump size is positive or when maturity is long, or both. However, it is important to notice that the slope of implied volatility maintains an increasing pattern in terms of μ_{J_i}. The implication is that high slope stocks have more positive jumps than low slope stocks. This is exactly what is needed to formulate the main hypothesis of the paper.

Panel B examines the effect of jump intensity, λ_i. As λ_i increases, the error in implied volatility becomes larger but still relatively small in magnitude. For values of λ_i equal to one and two, compare μ_{J_i} with half of and quarter of slope. As λ_i increases, the approximation error decreases.

Panel C examines the effect of correlation between the stock and volatility processes, ζ_i. The error in implied volatility is not affected by ζ_i, while the error in slope

Table A.1
Implied volatility and slope from Monte-Carlo simulations. GB=Black and Scholes's Geometric Brownian Motion model, SV=Heston's stochastic volatility model, GB-J=Merton's jump-diffusion model, SV-J=model with stochastic volatility and jump.

	$\sigma_i^{imp}(X,T)\|_{X=0}$				$\left.\frac{\partial\sigma_i^{imp}(X,T)}{\partial X}\right\|_{X=0}$			
T	One day	One week	One month	Two months	One day	One week	One month	Two months
Panel A: μ_{J_i}								
0.2	0.508	0.519	0.532	0.540	0.143	0.094	0.025	−0.014
0.1	0.504	0.509	0.515	0.519	0.055	0.027	−0.011	−0.029
0.05	0.503	0.506	0.510	0.513	0.014	0.002	−0.021	−0.032
−0.05	0.502	0.505	0.508	0.511	−0.059	−0.042	−0.042	−0.042
−0.1	0.503	0.507	0.511	0.515	−0.098	−0.068	−0.054	−0.052
−0.2	0.506	0.514	0.523	0.529	−0.188	−0.146	−0.100	−0.088
Panel B: λ_i								
0.5	0.504	0.509	0.515	0.519	0.055	0.027	−0.011	−0.029
1	0.508	0.517	0.528	0.533	0.129	0.073	0.007	−0.024
2	0.516	0.534	0.552	0.562	0.280	0.160	0.035	−0.022
Panel C: ζ_i								
−0.5	0.504	0.509	0.515	0.519	0.053	0.024	−0.014	−0.032
−0.25	0.504	0.509	0.515	0.519	0.055	0.027	−0.011	−0.029
0	0.504	0.509	0.515	0.519	0.058	0.030	−0.008	−0.026
0.25	0.504	0.509	0.515	0.519	0.060	0.033	−0.004	−0.023
0.5	0.504	0.509	0.515	0.519	0.063	0.036	−0.001	−0.020
Panel D: Model								
GB	0.500	0.501	0.502	0.505	−0.018	−0.017	−0.024	−0.029
SV	0.500	0.501	0.502	0.505	−0.020	−0.020	−0.027	−0.032
GB-J	0.504	0.509	0.515	0.520	0.058	0.029	−0.007	−0.026
SV-J	0.504	0.509	0.515	0.519	0.055	0.027	−0.011	−0.029

becomes smaller for higher values of ζ_i although the improvements are small.

My most general model includes Poisson jump and stochastic volatility, and I call it the SV-J model. When there is no jump and volatility is constant ($\lambda_i = 0, \phi_i = 0$), it becomes Black and Scholes's Geometric Brownian motion (GB) model. When volatility is stochastic but there are no jumps ($\lambda_i = 0$), the model becomes Heston's (SV) model. In the case of constant volatility ($\phi_i = 0$), it becomes Merton's jump-diffusion (GB-J) model. Panel D of Table A.1 reports the implied volatility and slope for these different models. For implied volatility, the approximation error is larger for the models with jumps. But the magnitude of errors is small. When jumps are absent, slope is negative and small. In contrast, for the SV-J and GB-J models, slope is positive at least for short maturities. To summarize, slope is related to jumps and not affected much by stochastic volatility.

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