What is the Value of Knowing Uninformed Trades?†

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Abstract

Extending the multiperiod Kyle (1985) model, I study strategic trading when an informed trader receives a noisy signal of uninformed trades. I show that the value of this signal is positive when it has perfect precision, but can turn negative when it becomes less precise.

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Key Words: value of information, uninformed trades, informed trading.

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1 Introduction

The seminal work of Kyle (1985) describes how an insider uses his private information to take advantage of noise traders in a security market. A large body of literature has since then been devoted to the issue of strategic trading of informed traders[1]. However, all of these models assume that the informed trader is not aware of the volume of uninformed trades submitted by noise traders. A question that arises naturally is, can the informed increase his profits by acquiring information on uninformed trades?

When the number of trading rounds, N, is either 1 or ∞, the answer is known. In the appendix of Rochet and Vila (1994), a single period model is solved when the insider knows uninformed trades. Their result shows that the insider's expected profits are the same as in the original Kyle model. On the other end of the scale, Back (1992) solves Kyle's model in continuous time. In this case, whether the insider sees the uninformed trades or not should not matter, since this information can be inferred from the continuous sample path of the Brownian motion which represents uninformed trades. Therefore in both cases, information on uninformed trades brings no extra profits.

This paper addresses the remaining part of the question, that is, the cases when $1 < N < \infty$. To better understand the nature of information on uninformed trades, I also assume that this information takes the form of a signal, whose precision, ρ, ranges from 0 to 1. The equilibrium of the model is compared with that of the case with no signal, so that the insider's expected gain from information can be extracted and characterized as a function of N and ρ. The way I show existence of a unique linear equilibrium is similar to Kyle (1985), and the way I solve the equilibrium parallels Holden and Subrahmanyam (1992).

The main findings are that the expected gain is positive when the signal has perfect precision, but can turn negative when the signal becomes less precise. That information has negative value here is not a contradiction to common wisdom, since the price-setting agent, the market maker, is assumed to know that the insider knows the signal. It is observed, however, that irrespective of the value of N and ρ, the expected gain is rather small. But conditioning on the terminal value of the traded security, expected gain shows large variation.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 shows the existence of a unique linear equilibrium and provides a characterization of this equilibrium by a set of difference equations. Section 4 presents findings concerning expected gains from information. Section 5 concludes.

2 The Model

A security is traded in N rounds of sequential batch trading which take place at $0 < t_1 < \cdots < t_N = 1$. At each round, market orders are submitted by noise traders. Let Δu_n denote the order placed by noise traders in the nth round, where Δu_n is normally distributed with mean 0 and variance $\sigma_u^2 \Delta t_n$, with $\Delta t_n = t_n - t_{n-1}$.

There is a risk-neutral insider who knows the terminal value of the security v, which is assumed to be normally distributed with mean 0 and variance normalized to 1 for simplicity. Before submitting his order, the insider receives in each round a noisy signal of the order placed by the noise traders. Let $\Delta z_n = \Delta u_n + \Delta \epsilon_n$ be a noisy version of Δu_n, where $\Delta \epsilon_n$ is normally distributed with mean 0 and variance $\sigma_e^2 \Delta t_n$. Define $\rho = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$. Note that ρ is equal to the square of the correlation between Δz and Δu. Thus it measures the precision of the insider's signal. It is assumed that $\{v, \Delta u_1, \ldots, \Delta u_N, \Delta \epsilon_1, \ldots, \Delta \epsilon_N\}$ are mutually independent. Let Δx_n denote the market order placed by the insider in the nth round of trading. Let π_n denote the profits of the insider from the nth round to the end of trading, that is,

$$\pi_n = \sum_{i=n}^{N} (v - p_n) \Delta x_n \tag{1}$$

There is a risk-neutral market maker who facilitates trading by absorbing the order flow submitted by both the insider and the noise traders. He observes the total order flow, $\Delta x_n + \Delta u_n$, but not its individual components. He sets a price, p_n, conditional on his information set in round n, so that he earns zero expected profits. Let J_n denote the market maker's information set at the nth round, where $J_n = \{\Delta x_1 + \Delta u_1, \ldots, \Delta x_n + \Delta u_n\}$. Let I_n denote the information set of the insider at the nth round, where $I_n = \{p_1, \ldots, p_{n-1}, \Delta z_1, \ldots, \Delta z_n, v\}$.

3 The Equilibrium

Following Kyle (1985), equilibrium is defined by three conditions: (i) The insider maximizes his expected profits $E[\pi_n|I_n]$ at each round, taking as given the market maker's pricing rule; (ii) The price set by the market maker is equal to the expected value of the security conditional on his information set; (iii) The market maker's conjecture of the insider's trading strategy and the insider's conjecture of the market maker's pricing rule are both correct.

A *linear equilibrium* is an equilibrium in which the insider's strategy and the market maker's pricing function are linear. A *recursive linear equilibrium* is a linear equilibrium in which there are constants $\lambda_1, \ldots, \lambda_N$ such that $p_n = p_{n-1} + \lambda_n(\Delta x_n + \Delta u_n)$, for $n = 1, \ldots, N$. The following proposition characterizes the unique linear equilibrium of the model:

Proposition 1 *There exists a unique linear equilibrium and it is a recursive linear equilibrium. In this equilibrium there are constants β_n, λ_n, α_n, δ_n and Σ_n such that for $n = 1, \ldots, N$,*

$$\Delta x_n = \beta_n \Delta t_n (v - p_{n-1} - \rho\lambda_n \Delta z_n) \tag{2}$$

$$\Delta p_n = \lambda_n(\Delta x_n + \Delta u_n) \tag{3}$$

$$E[\pi_n|I_n] = \alpha_{n-1}(v - p_{n-1} - \rho\lambda_n\Delta z_n)^2 + \delta_{n-1} \tag{4}$$

$$\Sigma_n = var[v|J_n] \tag{5}$$

where

$$\beta_n \Delta t_n = \frac{1 - 2\alpha_n\lambda_n}{2\lambda_n(1 - \alpha_n\lambda_n)} \tag{6}$$

$$\alpha_{n-1} = \frac{1}{4\lambda_n(1 - \alpha_n\lambda_n)} \tag{7}$$

$$\begin{aligned} \lambda_n &= \frac{\beta_n \Sigma_{n-1}}{\beta_n^2 \Sigma_{n-1}\Delta t_n + (1 - \rho\lambda_n\beta_n\Delta t_n)^2\sigma_u^2 + \rho^2\lambda_n^2\beta_n^2\Delta t_n^2\sigma_e^2} \\ &= \frac{\beta_n}{\sigma_u^2(1 - 2\rho\lambda_n\beta_n\Delta t_n + \rho\lambda_n^2\beta_n^2\Delta t_n^2)}\Sigma_n \end{aligned} \tag{8}$$

$$\begin{aligned} \Sigma_n &= \frac{(1 - \rho\lambda_n\beta_n\Delta t_n)^2\sigma_u^2 + \rho^2\lambda_n^2\beta_n^2\Delta t_n^2\sigma_e^2}{\beta_n^2\Sigma_{n-1}\Delta t_n + (1 - \rho\lambda_n\beta_n\Delta t_n)^2\sigma_u^2 + \rho^2\lambda_n^2\beta_n^2\Delta t_n^2\sigma_e^2}\Sigma_{n-1} \\ &= (1 - \lambda_n\beta_n\Delta t_n)\Sigma_{n-1} \end{aligned} \tag{9}$$

$$\delta_{n-1} = \delta_n + \rho\alpha_n\lambda_{n+1}^2\sigma_u^2\Delta t_{n+1} + \rho\alpha_n\lambda_n^2\sigma_e^2\Delta t_n \tag{10}$$

with $\alpha_N = 0$ *and* $\delta_N = 0$ *being the boundary conditions and* $\lambda_n(1-\alpha_n\lambda_n) > 0$, $\forall n$ *resulting from second order conditions for the profit maximization.*

Proof: See Appendix A.

Here β describes the trading aggressiveness of the insider in response to a mispricing. $\frac{1}{\lambda}$ is commonly referred to as "market depth" since λ is the price response to a unit-sized order; smaller λ implies a more "liquid" or "deeper" market. Σ is called the error variance of price and the sequence of Σ_n tells the speed at which private information on v is revealed. Note the negative dependence of informed trade Δx on the signal Δz in (2). This is indicative of insider's effort to hide his own trade behind uninformed trades, in order to reduce market maker's price response.

Next, I characterize the solutions to the difference equation system given in equations (6)-(10) in the following proposition.

Proposition 2 *Define* $q_n \equiv \lambda_n\alpha_n$. *To solve the difference equations, start from* $q_N = 0$ *and iterate backwards for* $q_{N-1}, \ldots, q_1$ *by solving for the unique root of the quartic equation*

$$4a_n(1-\rho)q_{n-1}^4 - 8a_n(1-\rho)q_{n-1}^3 + [a_n(4-3\rho)-2]q_{n-1}^2 + 3q_{n-1} - 1 = 0 \tag{11}$$

which lies inside $(0, \frac{1}{2})$, *where*

$$a_n = \frac{8(1-2q_n)(1-q_n)^2\frac{\Delta t_{n-1}}{\Delta t_n}}{1 + 2(1-\rho)(1-2q_n) + (1-\rho)(1-2q_n)^2}$$

Then, start from the given $\Sigma_0 = var(v) = 1$, *iterate forward for each of the following:*

$$\Sigma_n = \frac{1}{2(1-q_n)}\Sigma_{n-1} \tag{12}$$

$$\lambda_n = \{\frac{2(1-2q_n)(1-q_n)\Sigma_n}{\sigma_u^2\Delta t_n[1 + 2(1-\rho)(1-2q_n) + (1-\rho)(1-2q_n)^2]}\}^{\frac{1}{2}} \tag{13}$$

$$\beta_n = \{\frac{\sigma_u^2(1-2q_n)[1 + 2(1-\rho)(1-2q_n) + (1-\rho)(1-2q_n)^2]}{8(1-q_n)^3\Sigma_n\Delta t_n}\}^{\frac{1}{2}} \tag{14}$$

Finally, α_n *and* δ_n *can be iterated backwards using equations (7) and (10).*

Proof: See Appendix B.

4 Expected Gain from Information

The ex ante expected profits of the insider is obtained by taking the expectation with respect to v and Δz_1 of $E[\pi_1|I_1]$ in (4).

$$E[\pi_1] = \alpha_0(1 + \rho\lambda_1^2\sigma_u^2\Delta t_1) + \delta_0 \tag{15}$$

One can then use Proposition 2 to iterate backwards to obtain the necessary coefficients. The ex ante expected gain is the difference between $E\left[\pi_1\right]$ and the expected profits in the original Kyle model.

When the insider's signal of uninformed trades is exact, that is, when $\rho = 1$, Figure 1[2] shows that the insider's ex ante expected gain is strictly positive except when $N = 1$ or $N \to \infty$, where it is zero. Percentage expected gain is defined as expected gain divided by expected profits in the Kyle benchmark model. What is particularly interesting is that percentage gain is largest when N is approximately equal to 9, although its economic interpretation is not clear. An inspection of Figure 1 also reveals that the maximum of percentage gain is on the order of one percent. Thus although a gain from information exists, it is rather small.

For various values of N, Figure 2 shows the ex ante expected gain as a function of ρ. It is noted that when N is less than approximately 50, more precise signal does not necessarily mean the insider is better off. In fact, unless his signal attains a certain precision, he will be hurt by his information. The insider seems to be losing out to a decreasing market depth as the precision of signal increases (see Figure 4).

To the extent that the expected gain conditional on v is significantly positive (negative), the insider will want to adopt (disadopt) the signal after seeing v. The following equation provides a formula for the conditional expected profits. It can be obtained by taking the expectation with respect to Δz_1 of $E[\pi_1|I_1]$ in (4).

$$E[\pi_1|v] = \alpha_0(v^2 + \rho\lambda_1^2\sigma_u^2\Delta t_1) + \delta_0 \tag{16}$$

The conditional expected profits in Kyle's model can be calculated using (16), specializing to the case where $\sigma_e^2 \to \infty$. The two variables are compared to yield the conditional expected gain.

Figure 3 demonstrates that percentage conditional expected gain is negative when v is far from $E[v] = 0$, and positive when v is close to 0. This percentage also decreases monotonically when v increases. In fact, this is typical for all possible values of σ_e^2 and N. To interpret this result, note that when v is very far from 0, profits of the insider come from consistent buying or selling. He does this mostly in disregard of the direction of uninformed trades; knowing uninformed trades is unimportant to him. In fact, his information reduces profits because it causes market depth to decrease and the value of the security to be revealed faster (see Figure 5). When v is close to 0, the insider derives most of his profits by taking the opposite side of uninformed traders, in order to systematically mislead the market maker; knowing uninformed trades is now quite important to the insider.

A glance of Figure 3 reveals that the maximum percentage conditional expected gain is roughly 13%. In fact, with other parameter values, this number could be even larger. Thus although ex ante expected gain is negligible, ex post incentive of adopting (disadopting) the signal can be quite significant. This suggests a certain degree of inappropriateness of the Kyle equilibrium concept when it is used to address our problem[3].

5 Conclusion

This paper shows that the knowledge of uninformed trades can be valuable when it is sufficiently precise, and useless otherwise. Nevertheless, these interesting results are subsumed by the fact that the ex ante value of information is on the order of one percent, a magnitude perhaps too small to motivate informed traders to collect information on uninformed trades.

Though ex ante incentive is small, I show that this is the result of taking expected value of non-trivial ex post incentive with respect to terminal value of the security. How to find an equilibrium that is compatible with ex post incentive remains unresolved.

Appendix

A Proof of Proposition 1

Proof: Using backward induction, assuming expected profits of the insider have the form in (4), we get

$$\begin{aligned} E[\pi_n|I_n] &= \max_{\Delta x} E[(v-p_n)\Delta x + E[\pi_{n+1}|I_{n+1}]|I_n] \\ &= \max_{\Delta x} E[(v-p_n)\Delta x + \alpha_n(v-p_n-\rho\lambda_{n+1}\Delta z_{n+1})^2 + \delta_n|I_n] \end{aligned} \tag{17}$$

Above also applies to the case when $n = N$ since it is assumed that $\alpha_N = 0$ and $\delta_N = 0$.

In a linear equilibrium, prices obey

$$p_n = p_{n-1} + \lambda_n(\Delta x_n + \Delta u_n) + h \tag{18}$$

where h is some linear function of $\Delta x_1 + \Delta u_1, \ldots, \Delta x_{n-1} + \Delta u_{n-1}$. Substitute (18) into (17), we obtain

$$\begin{aligned} E[\pi_n|I_n] &= \max_{\Delta x} E[(v-p_{n-1}-\lambda_n(\Delta x+\Delta u_n)-h)\Delta x + \\ &\quad \alpha_n(v-p_{n-1}-\lambda_n(\Delta x+\Delta u_n)-h-\rho\lambda_{n+1}\Delta z_{n+1})^2 + \delta_n|I_n] \\ &= \max_{\Delta x}(v-p_{n-1}-\lambda_n(\Delta x+\rho\Delta z_n)-h)\Delta x + \\ &\quad \alpha_n(v-p_{n-1}-\lambda_n(\Delta x+\rho\Delta z_n)-h)^2 + \\ &\quad \rho\alpha_n\lambda_{n+1}^2\sigma_u^2\Delta t_{n+1} + \rho\alpha_n\lambda_n^2\sigma_e^2\Delta t_n + \delta_n \end{aligned} \tag{19}$$

where we use

$$\begin{aligned} E[\Delta u_n|I_n] &= \rho\Delta z_n \\ E[\Delta u_n\Delta z_{n+1}|I_n] &= E[\Delta z_{n+1}]E[\Delta u_n|I_n] = 0 \\ E[\Delta u_n^2|I_n] &= var[\Delta u_n|I_n] + E[\Delta u_n|I_n]^2 = \rho^2\Delta z_n^2 + \rho\sigma_e^2\Delta t_n \\ E[\Delta z_{n+1}^2|I_n] &= (\sigma_u^2+\sigma_e^2)\Delta t_{n+1} \\ E[\Delta z_{n+1}|I_n] &= 0 \end{aligned}$$

Solving the first order condition gives

$$\Delta x_n = \frac{1 - 2\alpha_n\lambda_n}{2\lambda_n(1 - \alpha_n\lambda_n)}(v - p_{n-1} - \rho\lambda_n\Delta z_n - h) \tag{20}$$

To show that $h = 0$, note that

$$\begin{aligned} E[\Delta p_n | J_{n-1}] &= h + \lambda_n E[\Delta x_n + \Delta u_n | J_{n-1}] \\ &= \frac{h}{2(1 - \alpha_n\lambda_n)} \\ &= 0 \end{aligned}$$

where we use (18), (20) and the market efficiency condition. Since the coefficient of h is positive by the second order condition, we have $h = 0$. Upon comparing (20) and (2) we see that (6) is obtained.

We then look at the profits by substituting (2) into (19):

$$\begin{aligned} E[\pi_n | I_n] &= (1 - \lambda_n\beta_n\Delta t_n)(\beta_n\Delta t_n + \alpha_n(1 - \lambda_n\beta_n\Delta t_n))(v - p_{n-1} - \rho\lambda_n\Delta z_n)^2 + \\ &\quad \delta_n + \rho\alpha_n\lambda_{n+1}^2\sigma_u^2\Delta t_{n+1} + \rho\alpha_n\lambda_n^2\sigma_e^2\Delta t_n \end{aligned} \tag{21}$$

where we obtain (10). To obtain (7), note that from (6) we have

$$1 - \lambda_n\beta_n\Delta t_n = \frac{1}{2(1 - \alpha_n\lambda_n)} \tag{22}$$

Substitute this into (21) and comparing with (4) we get (7).

To obtain (8) and (9), use the market efficiency condition

$$\Delta p_n = p_n - p_{n-1} = E[v - p_{n-1} | J_n] \tag{23}$$

However, market efficiency also implies that $v - p_{n-1}$ is independent of $\Delta x_1 + \Delta u_1, \ldots, \Delta x_{n-1} + \Delta u_{n-1}$ and that these volumes themselves are mutually independent. With this, we can simplify the above to

$$E[v - p_{n-1} | J_n] = E[v - p_{n-1} | \Delta x_n + \Delta u_n] \tag{24}$$

Next, we use the standard formulae on conditional means and variances to derive (8) and (9).

$$\begin{aligned} \Delta p_n &= E[v - p_{n-1}] + \frac{cov[v - p_{n-1}, \Delta x_n + \Delta u_n]}{var[\Delta x_n + \Delta u_n]}(\Delta x_n + \Delta u_n - E[\Delta x_n + \Delta u_n]) \\ &= \frac{cov[v - p_{n-1}, \Delta x_n + \Delta u_n]}{var[\Delta x_n + \Delta u_n]}(\Delta x_n + \Delta u_n) \end{aligned} \tag{25}$$

The first part of (8) is obtained after substituting into (25) the covariance and the variance, which are

$$
\begin{aligned}
cov[v - p_{n-1}, \Delta x_n + \Delta u_n] &= \beta_n \Delta t_n \Sigma_{n-1} && (26)\\
var[\Delta x_n + \Delta u_n] &= \beta_n^2 \Delta t_n^2 \Sigma_{n-1} + (1 - \rho\lambda_n\beta_n\Delta t_n)^2 \sigma_u^2 \Delta t_n + \\
&\quad \rho^2 \lambda_n^2 \beta_n^2 \Delta t_n^2 \sigma_e^2 \Delta t_n && (27)
\end{aligned}
$$

To get (9), substitute (26) and (27) into the expression below and note that $var[v - p_{n-1}] = \Sigma_{n-1}$.

$$
\begin{aligned}
\Sigma_n &= var[v - p_{n-1} | \Delta x_n + \Delta u_n] \\
&= var[v - p_{n-1}] - \frac{cov^2[v - p_{n-1}, \Delta x_n + \Delta u_n]}{var[\Delta x_n + \Delta u_n]} && (28)
\end{aligned}
$$

The second parts of (8) and (9) are easily obtained by cross substitution.

This demonstrates the conditions that characterize linear equilibria. To establish uniqueness, we have to show that the set of difference equations (6)-(10) has a unique solution. This we defer to Proposition 2.

Q.E.D.

B Proof of Proposition 2

Proof: Multiply both sides of (7) by λ_{n-1}, we obtain

$$
\left(\frac{\lambda_n}{\lambda_{n-1}}\right)^2 = \frac{1}{16 q_{n-1}^2 (1 - q_n)^2} \tag{29}
$$

Using (8) we obtain

$$
\frac{\lambda_n}{\lambda_{n-1}} = \frac{\beta_n}{\beta_{n-1}} \frac{(1 - \rho\lambda_{n-1}\beta_{n-1}\Delta t_{n-1})^2 \sigma_u^2 + \rho^2 \lambda_{n-1}^2 \beta_{n-1}^2 \Delta t_{n-1}^2 \sigma_e^2}{(1 - \rho\lambda_n\beta_n\Delta t_n)^2 \sigma_u^2 + \rho^2 \lambda_n^2 \beta_n^2 \Delta t_n^2 \sigma_e^2} (1 - \lambda_n \beta_n \Delta t_n) \tag{30}
$$

where we use (9). Now replace β_n and β_{n-1} with (6) and using (22), we get

$$
\left(\frac{\lambda_n}{\lambda_{n-1}}\right)^2 = \frac{(1 - 2q_n)\Delta t_{n-1}}{2(1 - 2q_{n-1})(1 - q_{n-1})\Delta t_n} \frac{[2 - \rho - 2(1 - \rho)q_{n-1}]^2 + \rho^2 (1 - 2q_{n-1})^2 \frac{\sigma_e^2}{\sigma_u^2}}{[2 - \rho - 2(1 - \rho)q_n]^2 + \rho^2 (1 - 2q_n)^2 \frac{\sigma_e^2}{\sigma_u^2}} \tag{31}
$$

Equating (31) and (29) and simplifying, we obtain (11).

To show that equation (11) produces a unique root inside $(0, \frac{1}{2})$, first note that the RHS of (11) evaluated at $q_{n-1} = 0$ is equal to $-1 < 0$ and it evaluated at $q_{n-1} = \frac{1}{2}$ is equal to $\frac{a_n}{4} > 0$, if it is assumed that $q_n \in [0, \frac{1}{2})$. This establishes the existence of a root in the interval $(0, \frac{1}{2})$. To prove uniqueness, we look at the first derivative of (11), given below:

$$f'(q_{n-1}) = a_n q_{n-1}[16(1-\rho)q_{n-1}^2 - 24(1-\rho)q_{n-1} + 2(4-3\rho)] + 3 - 4q_{n-1} \tag{32}$$

Let $L(q_{n-1})$ denote what is inside the curly brackets of (32). If it is possible to show that $L(q_{n-1})$ is always non-negative when $q_{n-1} \in (0, \frac{1}{2})$, then we are done, since the last two terms are always positive. It is easy to check that both $L(0)$ and $L(\frac{1}{2})$ are non-negative. It suffices to show that the minimum of L is attained outside $(0, \frac{1}{2})$. In fact, the following holds when $0 \leq \rho < 1$.

$$q_{n-1}^* = \frac{3}{4} > \frac{1}{2} \tag{33}$$

where q_{n-1}^* denotes the value of q_{n-1} that minimizes L. When $\rho = 1$, $L(q_{n-1}) = 2 \geq 0$ trivially.

Next, we need to show that only a root from the interval $(0, \frac{1}{2})$ can be used in the backward iteration. First, note that the second order condition and (7) imply that $\alpha_n > 0$. If it can be shown that $\lambda_n > 0$, then $q_n > 0$. Furthermore, (8) implies that $\beta_n > 0$ and then (6) suggests that $q_n < \frac{1}{2}$. Suppose otherwise, that $\lambda_n \leq 0$. Then since $\alpha_n > 0$, the second order condition is violated: $\lambda_n(1 - \alpha_n\lambda_n) \leq 0$.

Thus the backward iteration produces a unique sequence of q_n. To get (12), substitute (6) into (9). To get (13), substitute (6) into (8). (14) is obtained from (8) and (13).

The proof of Proposition 2 also establishes the uniqueness of linear equilibrium in Proposition 1.

Q.E.D.

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Footnotes

1. In this paper, the words "insider" and "informed trader" are used interchangeably.

2. In all figures, Σ_0 is normalized to 1. σ_u^2 is also set to 1 without loss of generality because percentage gain is independent of σ_u and only depends on the ratio σ_e/σ_u.

3. We need to make the equilibrium consistent with ex post incentives. Figure 3 suggests a decision rule for the insider in which there is a critical level of v above which he disadopts the signal. Market maker's pricing rule must then be made consistent with this conjecture. However, a straightforward exercise involving a one-period model with perfect signals demonstrates that no linear equilibrium can exist with such a decision rule. The reason is that the distribution of a normal random variable conditional on a correlated normal random variable being greater than a certain value is no longer normal. It seems that normality, which is what drives the linear equilibrium, can no longer be preserved. I leave this to future research.

Figure Legends

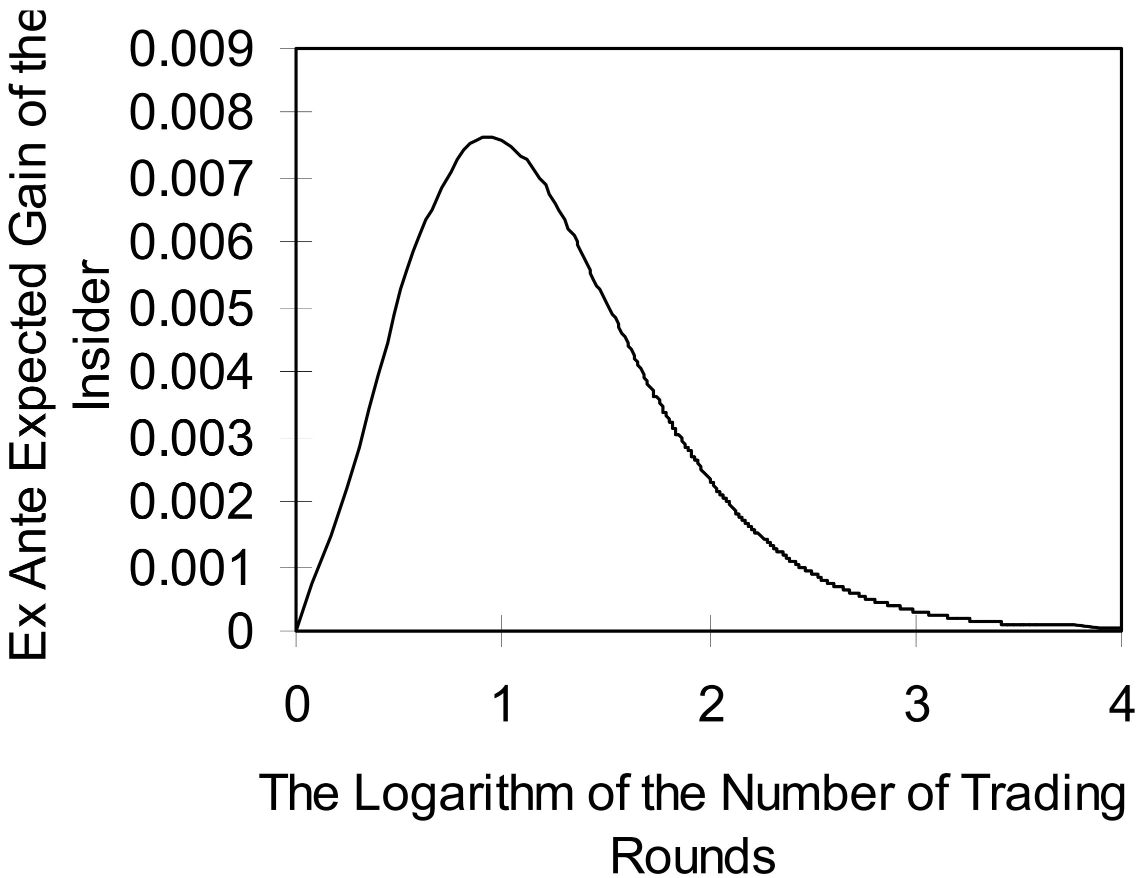
Figure 1: Percentage ex ante expected gain as a function of $\log N$ when $\rho = 1$. The largest $N = 10000$; $\sigma_u^2 = 1$; $\Sigma_0 = 1$.

Figure 2: Ex ante expected gain as a function of ρ. $\sigma_u^2 = 1$; $\Sigma_0 = 1$; $N = 1$, 5, 10, 20, 50.

Figure 3: Percentage ex post expected gain as a function of v. $\sigma_u^2 = 1$; $\Sigma_0 = 1$; $\rho = 0.5$; $N = 10$.

Figure 4: Inverse market depth as a function of calendar time. $\sigma_u^2 = 1$; $\Sigma_0 = 1$; $N = 10$; $\rho = 0$, 0.25, 0.75, 1.

Figure 5: Error variance of price as a function of calendar time: $\sigma_u^2 = 1$; $\Sigma_0 = 1$; $N = 10$; $\rho = 0$, 0.25, 0.75, 1.



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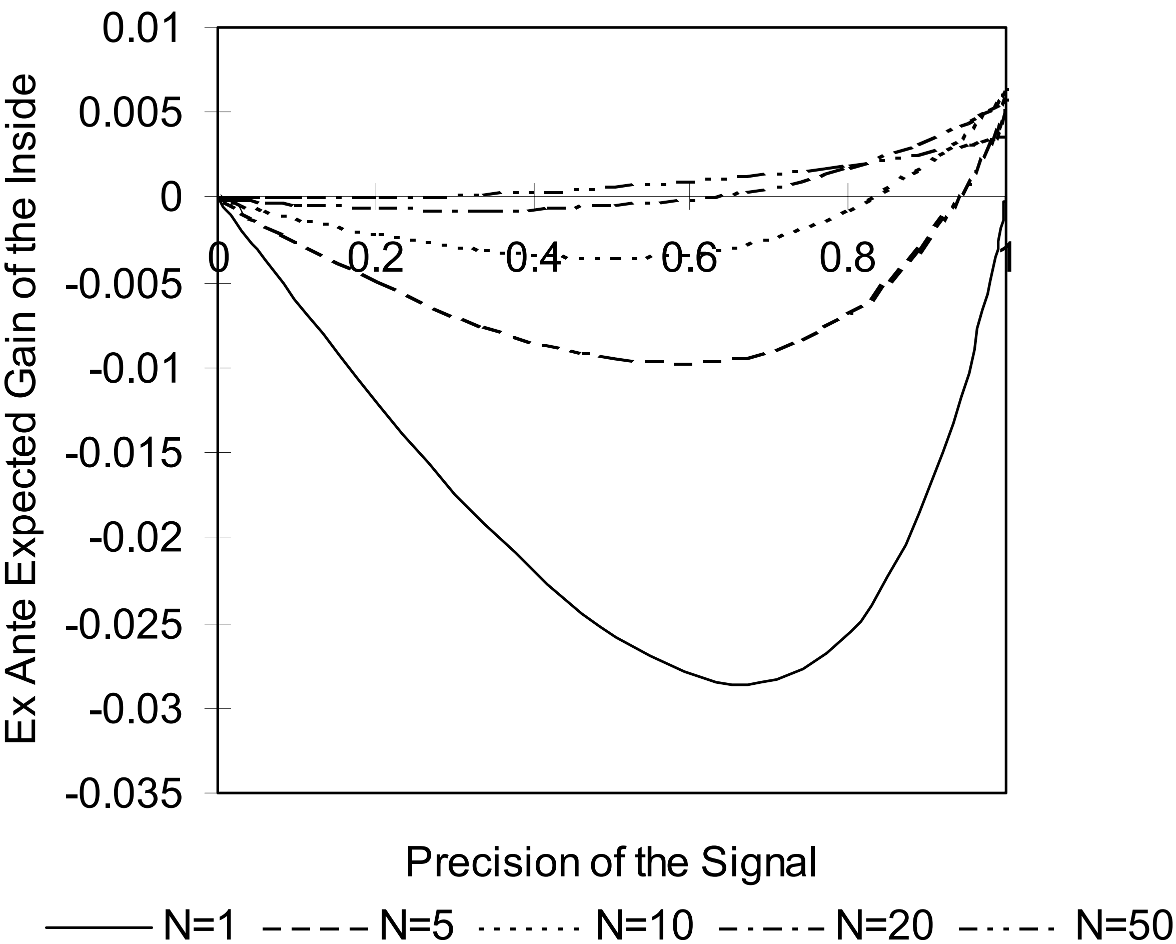

Figure 2: Ex ante expected gain as a function of ρ. $\sigma_u^2 = 1$; $\Sigma_0 = 1$; $N = 1, 5, 10, 20, 50$.

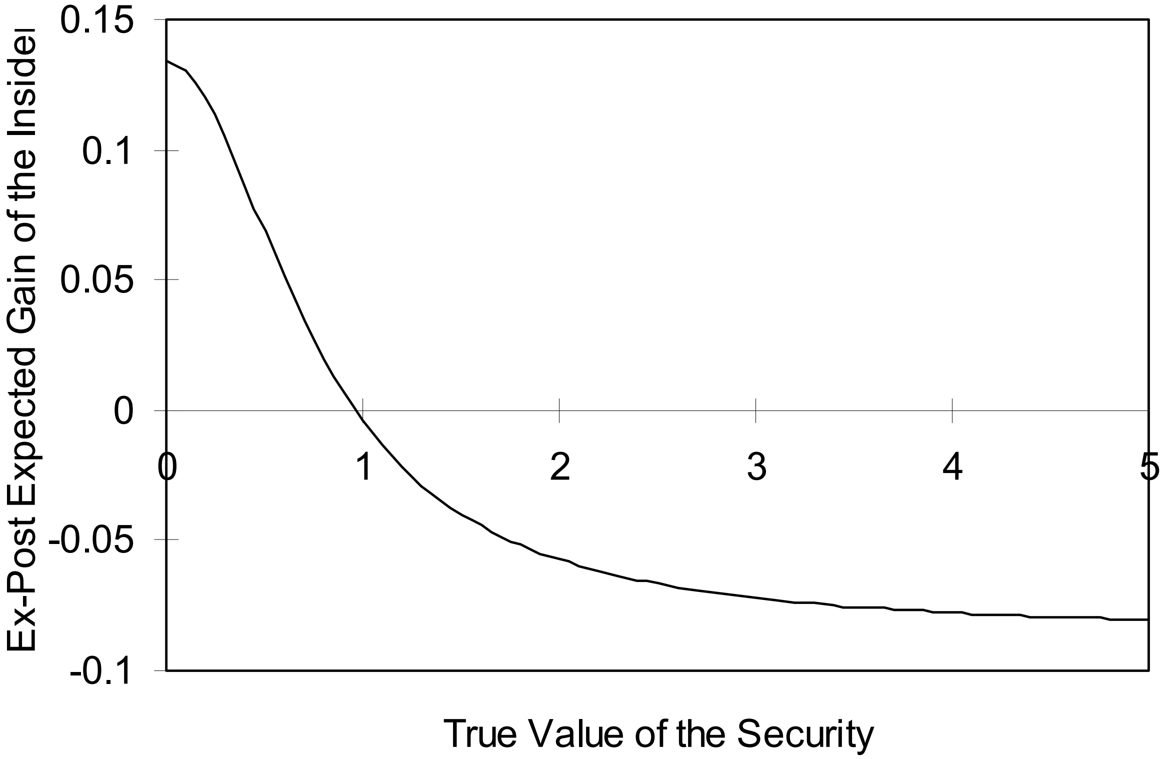

Figure 3: Percentage ex post expected gain as a function of v. $\sigma_u^2 = 1$; $\Sigma_0 = 1$; $\rho = 0.5$; $N = 10$.

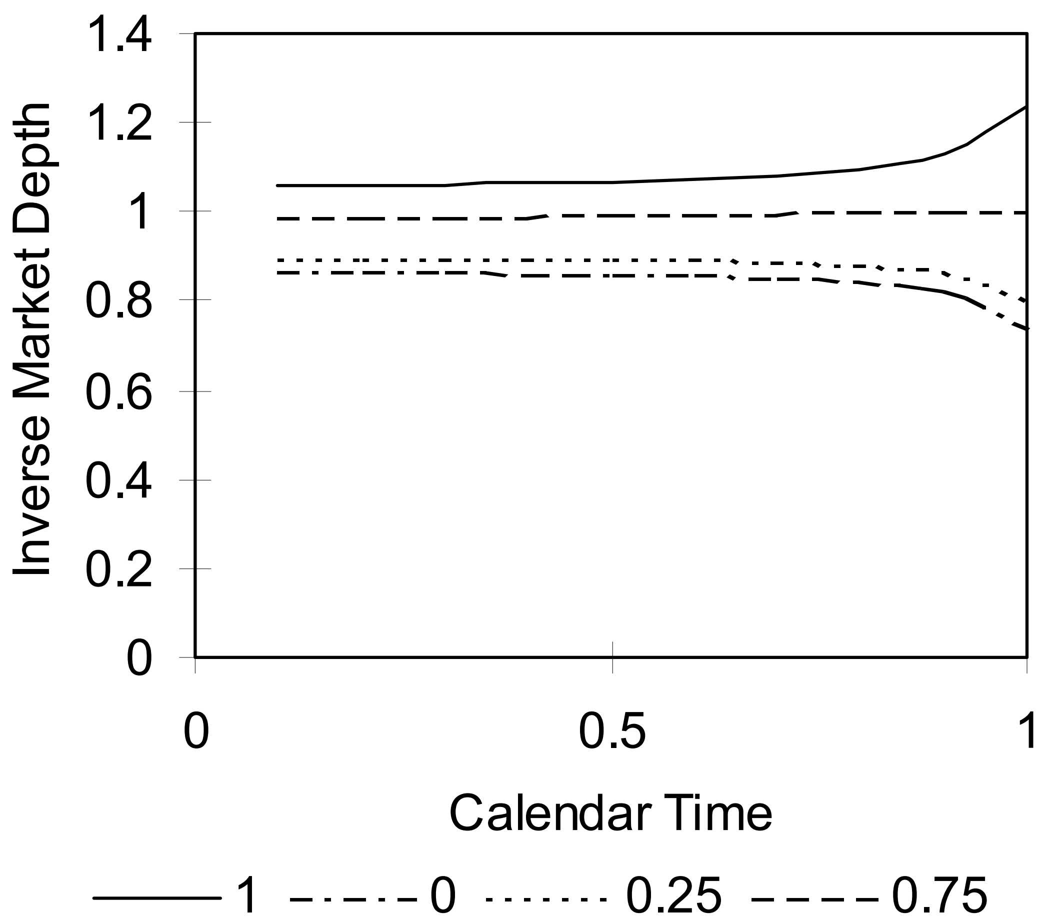

Figure 4: Inverse market depth as a function of calendar time. $\sigma_u^2 = 1$; $\Sigma_0 = 1$; $N = 10$; $\rho = 0$, 0.25, 0.75, 1.

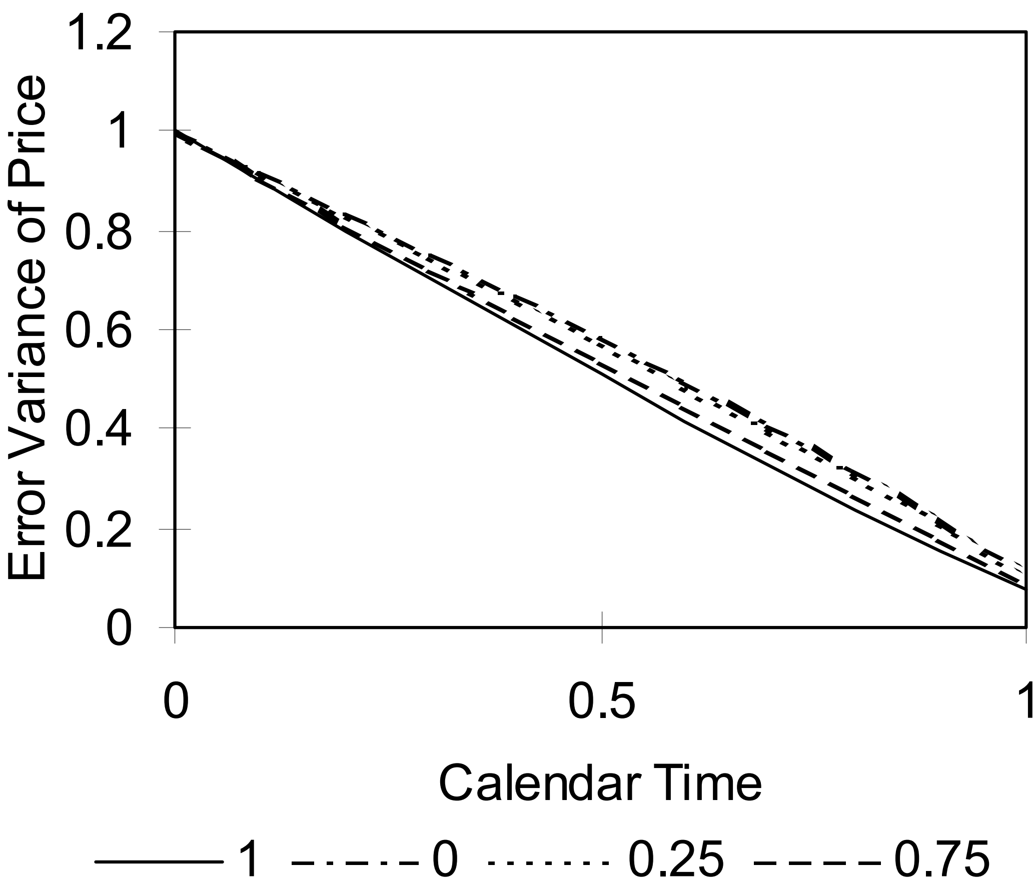

Figure 5: Error variance of price as a function of calendar time: $\sigma_u^2 = 1$; $\Sigma_0 = 1$; $N = 10$; $\rho = 0$, 0.25, 0.75, 1.