FE621HW2

October 10, 2020

[1]:
```
import os
import time
import numpy as np
from numpy import log as ln
import pandas as pd
from pandas import DataFrame
import pandas_datareader as dr
import datetime as dt
from scipy import misc
from scipy.stats import norm
import matplotlib.pyplot as plt
import pandas_market_calendars as calen
from yahoo_fin import stock_info as si
from yahoo_fin import options
```

1 Download Data and Calculate Implied Volatility

[2]:
```
#download option data
#get the maturities
amzn_maturi = options.get_expiration_dates("amzn")
print(amzn_maturi)
```

```
['October 16, 2020', 'October 23, 2020', 'October 30, 2020', 'November 6, 2020',
'November 13, 2020', 'November 20, 2020', 'December 18, 2020', 'January 15,
2021', 'February 19, 2021', 'March 19, 2021', 'April 16, 2021', 'June 18, 2021',
'July 16, 2021', 'August 20, 2021', 'September 17, 2021', 'January 21, 2022',
'June 17, 2022', 'January 20, 2023']
```

[3]:
```
# get three maturities
amzn_maturi=['November 13, 2020','December 18, 2020','January 15, 2021']

# get live price
amzn_live=si.get_live_price("amzn")
print('amzn live')
print(amzn_live)

# #download amzn option chian and save them to csv files separately
```

```
for date in amzn_maturi:
     call = options.get_calls("amzn",date)
     call.to_csv('amzn_call_'+date+'.csv')
     put = options.get_puts("amzn",date)
     put.to_csv('amzn_put_'+date+'.csv')
```

```
amzn live
3286.64990234375
```

[4]:
```
# Federal funds (effective)
# r=1.58/100
```

[5]:
```
#modify the forms
amzn_maturi=['November 13, 2020','December 18, 2020','January 15, 2021']
for type in ['call','put']:
    for date in amzn_maturi:
        file=pd.read_csv('amzn_'+type+'_'+date+'.csv')
        file['S0']=amzn_live
        file['moneyness']='nan'
        file['aver_price']=0.0
        file['cal_vol']=0.0
        file['BS']=0.0
        file['Bino_Euro']=0.0
        file['Bino_Ame']=0.0

        for i in file.index:

            file['aver_price'][i]=(file['Bid'][i]+file['Ask'][i])/2
            if file['S0'][i]/file['Strike'][i]>=0.95 and file['S0'][i]/
 ↪file['Strike'][i]<=1.05:
                file['moneyness'][i]='at the money'

        file=file[file['moneyness']=='at the money']
        file.to_csv('amzn_'+type+'_ATM_'+date+'.csv',index=False)
```

```
/Users/yuechenjiang/opt/anaconda3/lib/python3.7/site-
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[6]:
```
#Black-Scholes formulas

def bs_formula(S0,sig,T,K,r,option_type):

    d1=(ln(S0/K)+(r+(sig**2)/2)*T)/(sig*np.sqrt(T))
    d2=d1-(sig*np.sqrt(T))

    if option_type=='call':
        c=S0*norm.cdf(d1)-K*np.exp(-r*T)*norm.cdf(d2)
        return c
    elif option_type=='put':
        p=K*np.exp(-r*T)*norm.cdf(-d2)-S0*norm.cdf(-d1)
        return p
    else:
        print('Error')

#bisection method
def f(x):
    return bs_formula(S0,x,T,K,r,option_type)-price

def bisection_method(a, b, tol):
    if f(a)*f(b)> 0:
        #end function, no root.
        print('No root found.')
    else:
        while (b - a)/2.0 > tol:
            midpoint = (a + b)/2.0
            if f(midpoint)== 0:
                return(midpoint) #The midpoint is the x-intercept/root.
            elif f(a)*f(midpoint)< 0: # Increasing but below 0 case
                b = midpoint
            else:
                a = midpoint

        return(midpoint)
```

[7]:
```
#calculate implied volatility
nyse = calen.get_calendar('NYSE')

for type in ['call','put']:
    for date in amzn_maturi:
        file=pd.read_csv('amzn_'+type+'_ATM_'+date+'.csv')

        option_type=type
```

```
        schedule = nyse.schedule(start_date='2020-2-27', end_date=date)
        len(schedule)
        tol=10**(-6)
        T=len(schedule)/252
        r=1.58/100
        for i in file.index:
            S0=file['S0'][i]
            K=file['Strike'][i]
            price=file['aver_price'][i]
            file['cal_vol'][i]=bisection_method(0.01, 1,tol)

        file.to_csv('amzn_'+type+'_ATM_'+date+'.csv',index=False)
```

```
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```

1.1

Construct code to calculate option values using an additive binomial tree. For this part you need four versions: European and American as well as Call and Put. You may use the same tree construction (and function) for all options.

1.1.1 Binomial General Additive European

[8]:
```
def additiveBinomialEuro(K,T,S0,sig,r,N,option_type):
    # Equal jump size
    dt=T/N
    nu=r-0.5*sig**2
    dxu=np.sqrt(sig**2*dt+(nu*dt)**2)
    dxd=-dxu
    pu=0.5+0.5*(nu*dt/dxu)
    disc=np.exp(-r*dt)
    dpu=disc*pu
    dpd=disc*(1-pu)
    edxud=np.exp(dxu-dxd)
    edxd=np.exp(dxd)
    St=[0]*(N+1)
    C=[0]*(N+1)
    St[0]=S0*np.exp(N*dxd)

    for j in range(1,(N+1)):
        St[j]=St[j-1]*edxud

    if option_type=='call':
        sign=1
    elif option_type=='put':
        sign=-1
    else:
        print('Error')
        return

    for j in range(0,(N+1)):
            C[j] = max(0,sign*(St[j]-K))

    for i in range((N-1),-1,-1):
        for j in range(0,(i+1)):
            C[j]=dpu*C[j+1]+dpd*C[j]

    return (C[0])
```

1.1.2 Binomial General Additive American

[9]:
```
def additiveBinomialAmerican(K,T,S0,sig,r,N,option_type):
    # Equal jump size
    dt=T/N
    nu=r-0.5*sig**2
    dxu=np.sqrt(sig**2*dt+(nu*dt)**2)
    dxd=-dxu
    pu=0.5+0.5*(nu*dt/dxu)
    disc=np.exp(-r*dt)
    dpu=disc*pu
    dpd=disc*(1-pu)
    edxud=np.exp(dxu-dxd)
    edxd=np.exp(dxd)
    St=[0]*(N+1)
    C=[0]*(N+1)
    St[0]=S0*np.exp(N*dxd)

    for j in range(1,(N+1)):
        St[j]=St[j-1]*edxud

    if option_type=='call':
        sign=1
    elif option_type=='put':
        sign=-1
    else:
        print('wrong type')
        return

    for j in range(0,(N+1)):
            C[j] = max(0,sign*(St[j]-K))

    for i in range((N-1),-1,-1):
        for j in range(0,(i+1)):
            C[j]=dpu*C[j+1]+dpd*C[j]
            St[j]=St[j]/edxd
            C[j]=max(C[j],sign*(St[j]-K))

    return (C[0])
```

1.2

Download Option prices. Calculate the option price (European Calls and Puts) using the binomial tree, and compare the results with the Black–Scholes price. Use at least 200 steps in your tree construction. Treat the options as American as well and plot these values side by side with the European and Black Scholes values. When you create the plot do not forget to plot the bid-ask values as well.

From the downloaded data, select November 13th, 2020, December 18th, 2020, and January 15th, 2020. The time to maturity is one, two, and three months, and the strike price is close to the AMZN live price option. There are 20 options for call and put.

1.2.1 calciulate the option price by binomial tree

[10]:
```
nyse = calen.get_calendar('NYSE')
for type in ['call','put']:
    for date in amzn_maturi:
        file=pd.read_csv('amzn_'+type+'_ATM_'+date+'.csv')
        option_type=type
        N=200 #steps
        schedule = nyse.schedule(start_date='2020-2-27', end_date=date)
        len(schedule)
        T=len(schedule)/252
        r=1.58/100
        for i in file.index:
            S0=file['S0'][i]
            K=file['Strike'][i]
            sig=file['cal_vol'][i]
            file['BS'][i]=bs_formula(S0,sig,T,K,r,option_type)
            file['Bino_Euro'][i]=additiveBinomialEuro(K,T,S0,sig,r,N,option_type)
            file['Bino_Ame'][i]=additiveBinomialAmerican(K,T,S0,sig,r,N,option_type)

        file.to_csv('amzn_'+type+'_ATM_'+date+'.csv',index=False)
```

```
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```

```
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docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
```

[11]:
```
#report the result
amzn_maturi=['November 13, 2020','December 18, 2020','January 15, 2021']
for type in ['call','put']:
    for date in amzn_maturi:
        file=pd.read_csv('amzn_'+type+'_ATM_'+date+'.csv')
        file=file[0:20]
        file=file[['Contract␣
↪Name','Strike','Bid','Ask','cal_vol','BS','Bino_Euro','Bino_Ame']]
        #get table
        file.to_csv('amzn_'+type+'_report_'+date+'.csv',index=False)
```

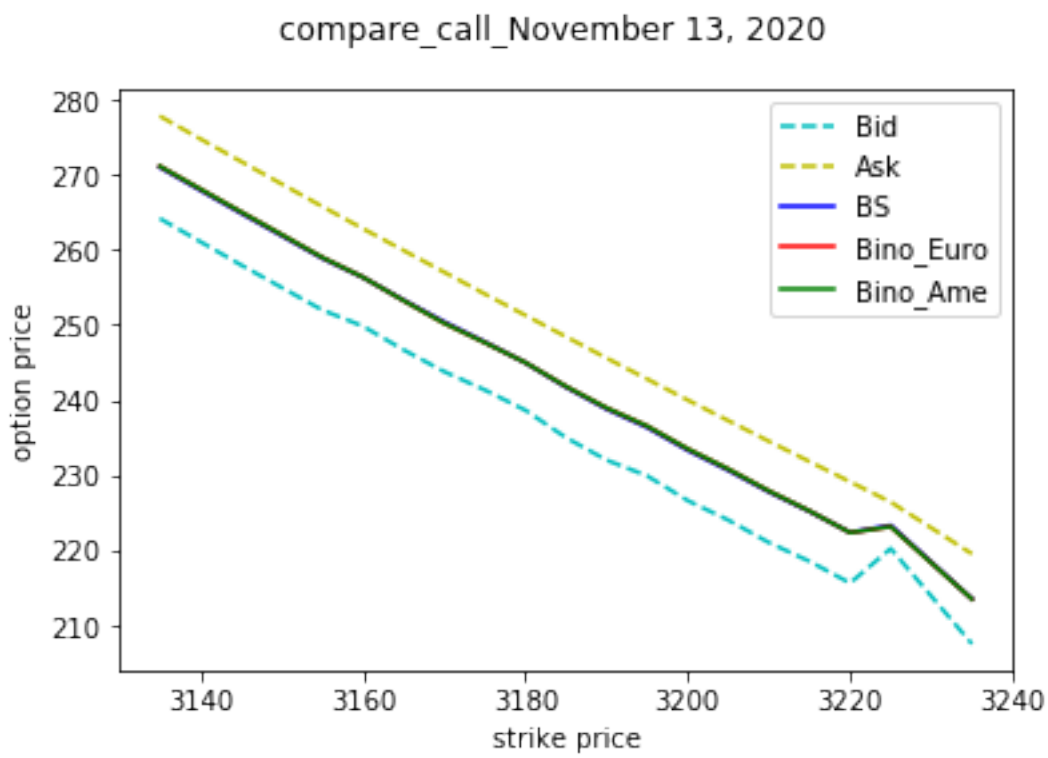
[12]:
```
#plot

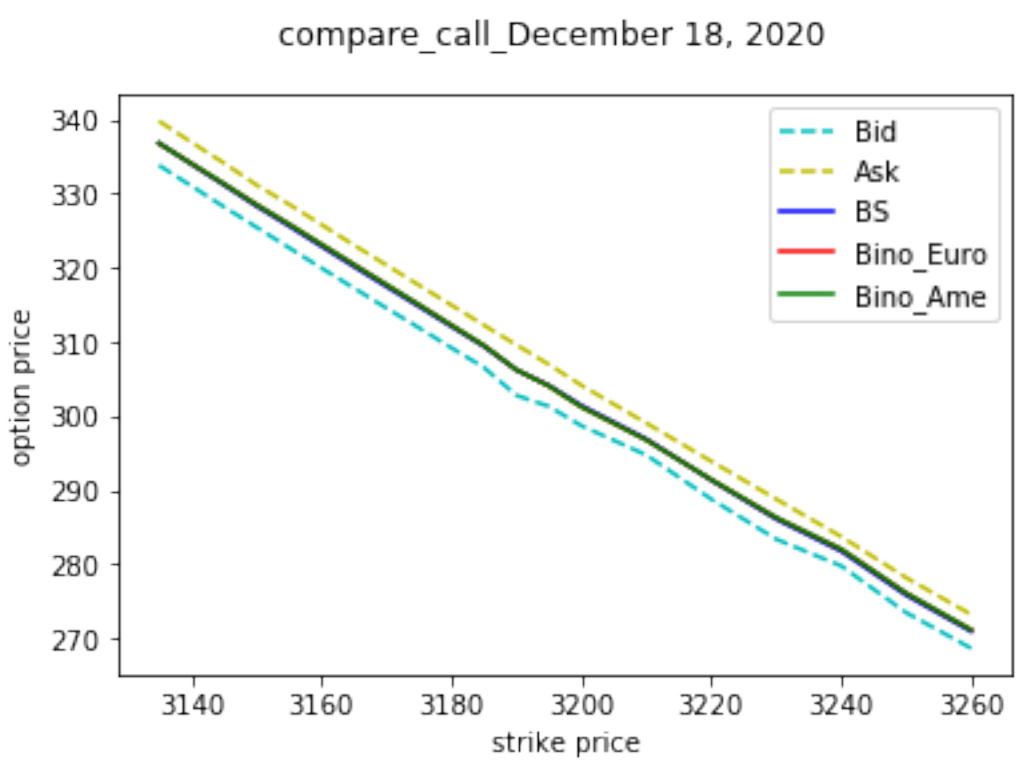
for type in ['call','put']:
    for date in amzn_maturi:
        file=pd.read_csv('amzn_'+type+'_report_'+date+'.csv')

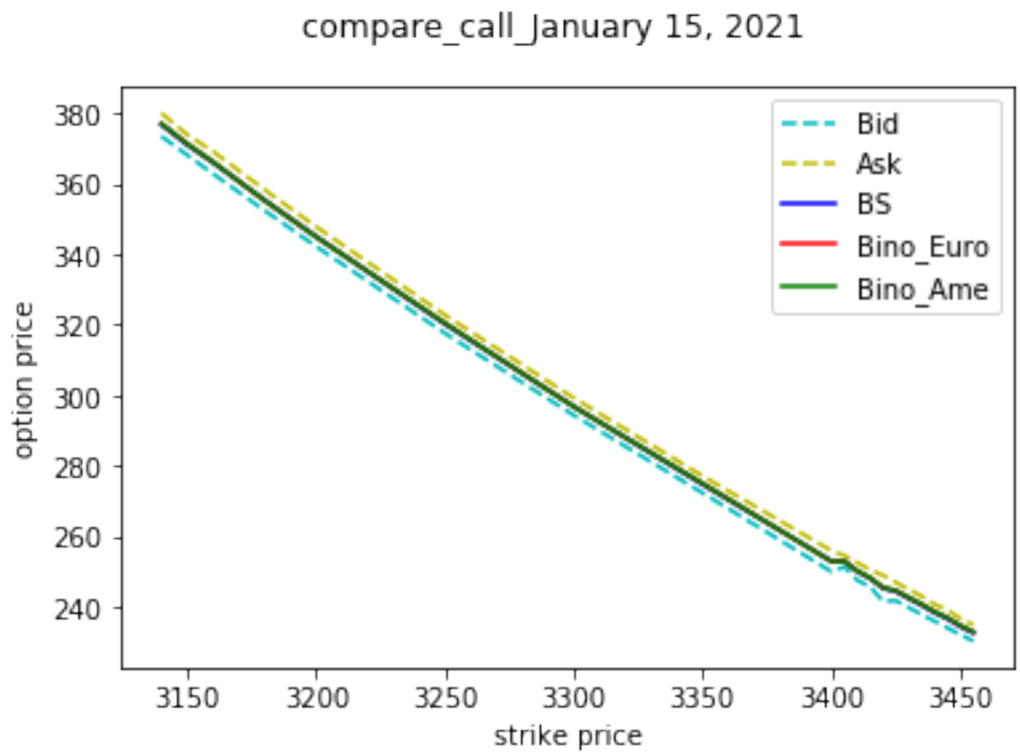
        plt.plot(file['Strike'],file['Bid'],'--',color='c')
        plt.plot(file['Strike'],file['Ask'],'--',color='y')
        plt.plot(file['Strike'],file['BS'],'b')
        plt.plot(file['Strike'],file['Bino_Euro'],'r')
        plt.plot(file['Strike'],file['Bino_Ame'],'g')

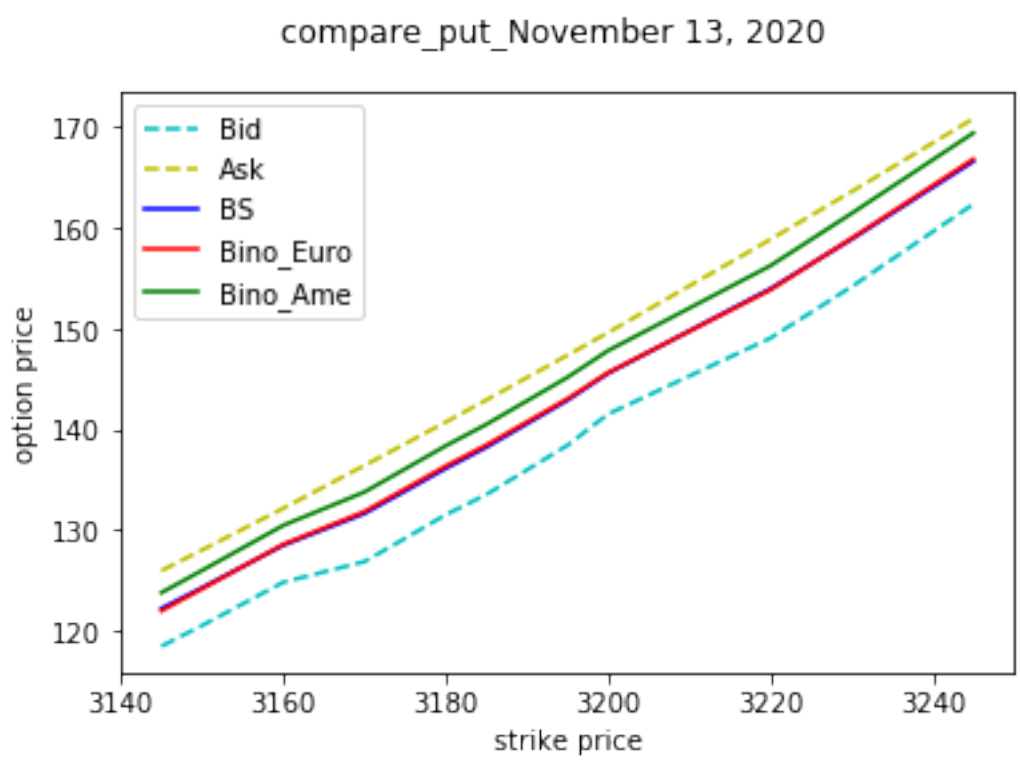
        plt.legend(labels=['Bid','Ask','BS','Bino_Euro','Bino_Ame'])
        plt.xlabel('strike price')
        plt.ylabel('option price')
        plt.suptitle('compare_'+type+'_'+date)

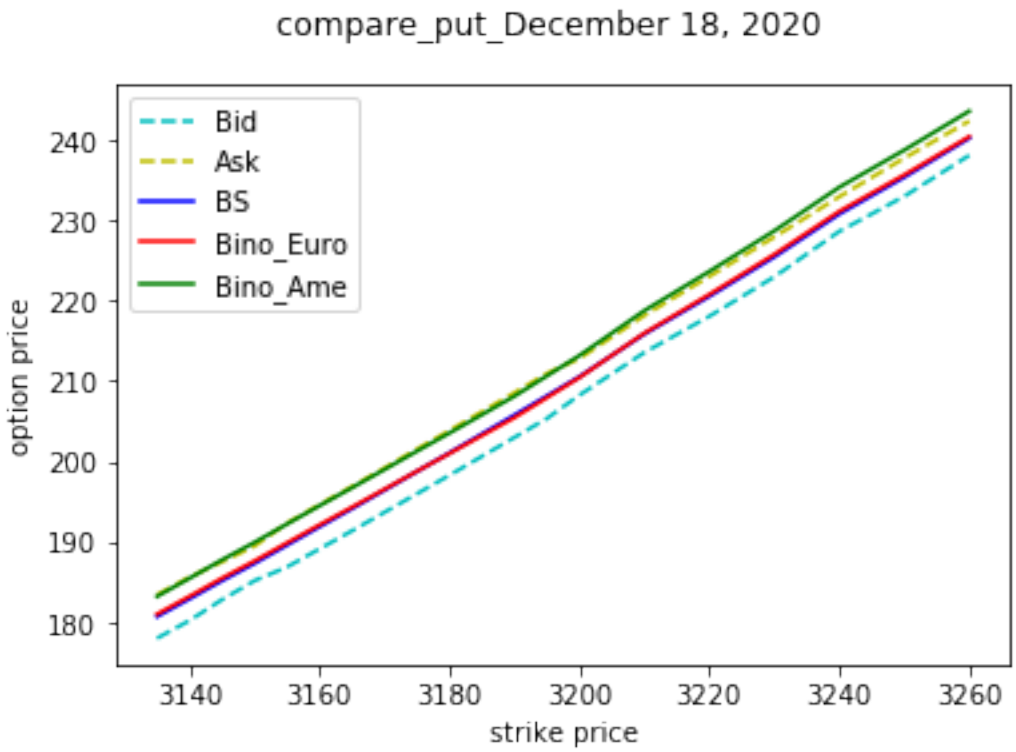
        plt.show()
```

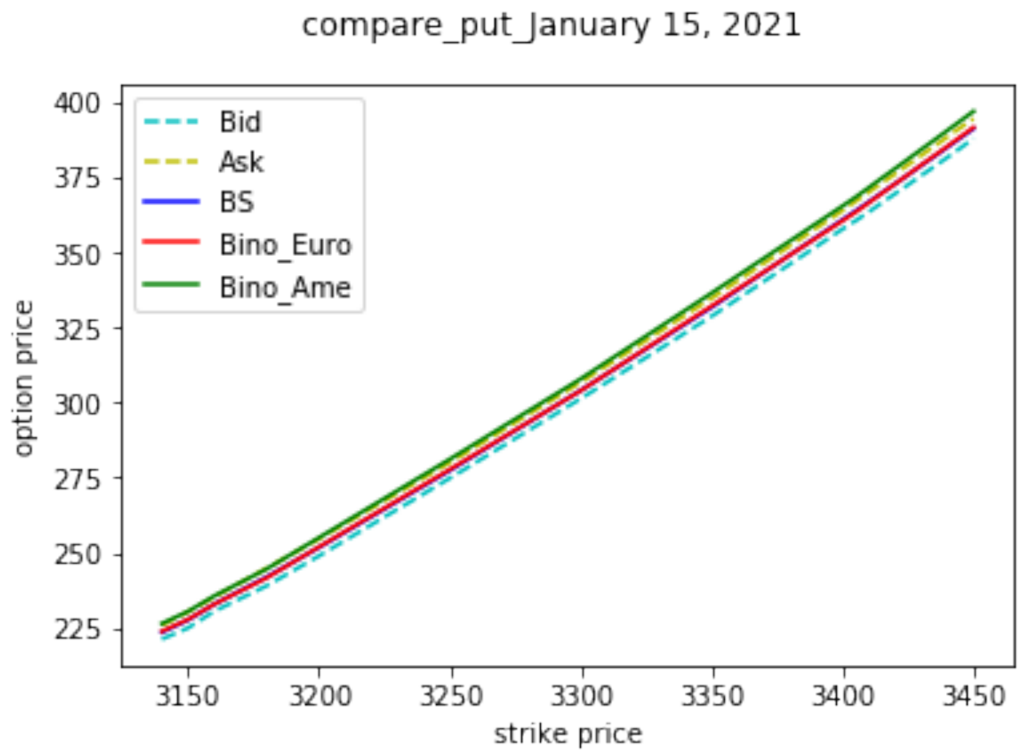

compare_call_November 13, 2020
Bid
Ask
BS
Bino_Euro
Bino_Ame
option price
strike price
280
270
260
250
240
230
220
210
3140
3160
3180
3200
3220
3240


compare_call_December 18, 2020
Bid
Ask
BS
Bino_Euro
Bino_Ame
option price
strike price
340
330
320
310
300
290
280
270
3140
3160
3180
3200
3220
3240
3260


compare_call_January 15, 2021
Bid
Ask
BS
Bino_Euro
Bino_Ame
option price
strike price
380
360
340
320
300
280
260
240
3150
3200
3250
3300
3350
3400
3450


compare_put_November 13, 2020
Bid
Ask
BS
Bino_Euro
Bino_Ame
option price
strike price
170
160
150
140
130
120
3140
3160
3180
3200
3220
3240


compare_put_December 18, 2020
Bid
Ask
BS
Bino_Euro
Bino_Ame
option price
strike price
240
230
220
210
200
190
180
3140
3160
3180
3200
3220
3240
3260



The data running result is in the zip file

1.3 Comment of the table in the previous part

As is shown in the tables and graphs, the option prices computed by Black–Scholes, European binomial tree and American binomial tree are very close and they are all located between Bid-Ask prices.

For call options, the European prices equal to American prices, because we don't consider the dividends and it is never optimal to exercise an American call option on a non-dividend-paying stock prior to the option's expiration. So, in this case, American call options are equivalent to European call options.

For put options, American option prices are higher than European option prices, because under some circumstances the early exercise of an American put option on a non-dividend-paying stock is optimal.

1.4

Consider N {10, 20, 30, 40, 50, 100, 150, 200, 250, 300, 350, 400}. Compute and plot the absolute error for the European Put N for as a function of N N^* the number of steps in the tree:

$$\epsilon_N = |P^{BSM}(S_0, K, T, r; \sigma) - P_N^{BinomTree}(S_0, K, T, r; \sigma)$$

where$P^{BSM}(S_0, K, T, r; \sigma)andP_N^{BinomTree}(S_0, K, T, r; \sigma)$are the Black-Scholes–Merton price and the price calculated using a binomial tree with N steps, respectively. What do you observe?

1.4.1 Compute and plot the absolute error for the European Put

[13]:
```
def error_Euro_P(N):
    return␣
 ↪abs(bs_formula(S0,sig,T,K,r,'put')-additiveBinomialEuro(K,T,S0,sig,r,N,'put'))

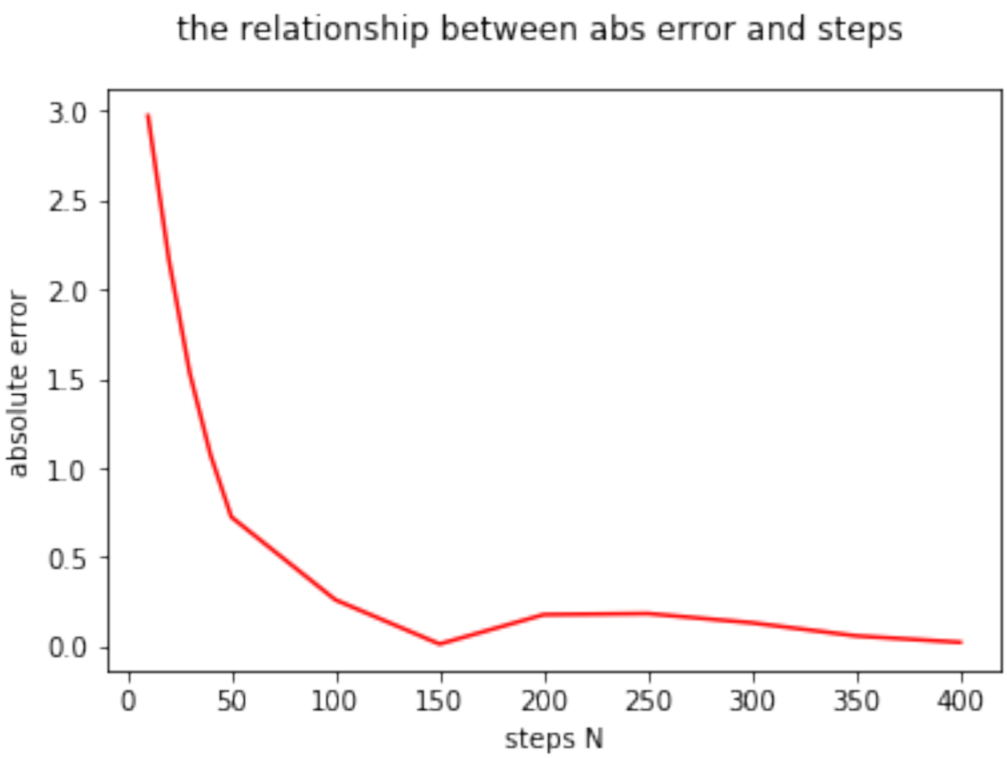
#select a set of value to show
nyse = calen.get_calendar('NYSE')
schedule = nyse.schedule(start_date='2020-2-27', end_date='2020-10-09')
len(schedule)
T=len(schedule)/252
S0=1930.11
sig=0.32
K=1840
r=1.58/100

N_list=[10, 20, 30, 40, 50, 100, 150, 200, 250, 300, 350, 400]
error_list=[]

for N in N_list:
    error=error_Euro_P(N)
    error_list.append(error)

plt.plot(N_list,error_list,'r')

plt.xlabel('steps N')
plt.ylabel('absolute error')
plt.suptitle('the relationship between abs error and steps')
plt.show()
```



2 Problem 2: Implied Volatlity

Using the binomial tree for American Calls and Puts, calculate the implied volatility corresponding to the data you have downloaded in part (b). You will need to use the bisection or Newton/secant method of finding roots with the respective binomial trees. Compare these values of the implied volatility with the volatilities calculated using the usual Black Scholes formula (as in Homework 1). Write detailed observations.

2.1 Bisection Method

[14]:
```
def f(x):
    return additiveBinomialAmerican(K,T,S0,x,r,N,option_type)-price

def bisection_method(a, b, tol):
    if f(a)*f(b)> 0:
        #end function, no root.
        print('No root found')
    else:
        while (b - a)/2.0 > tol:
            midpoint = (a + b)/2.0
            if f(midpoint)== 0:
                return(midpoint) #The midpoint is the x-intercept/root.
```

```
            elif f(a)*f(midpoint)< 0: # Increasing but below 0 case
                b = midpoint
            else:
                a = midpoint

        return(midpoint)
```

[15]:
```
nyse = calen.get_calendar('NYSE')
for type in ['call','put']:
    for date in amzn_maturi:
        file=pd.read_csv('amzn_'+type+'_ATM_'+date+'.csv')
        file=file[0:20]
        file['Bino_Ame_vol']=0.0
        option_type=type
        schedule = nyse.schedule(start_date='2020-2-27', end_date=date)
        len(schedule)
        tol=10**(-6)
        T=len(schedule)/252
        r=1.58/100
        N=200
        for i in file.index:
            S0=file['S0'][i]
            K=file['Strike'][i]
            price=file['aver_price'][i]
            file['Bino_Ame_vol'][i]=bisection_method(0.01, 1,tol)

        file=file[['Contract Name','Strike','cal_vol','Bino_Ame_vol']]
        file.to_csv('amzn_'+type+'_comparevol_'+date+'.csv',index=False)
```

```
/Users/yuechenjiang/opt/anaconda3/lib/python3.7/site-
packages/ipykernel_launcher.py:18: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
/Users/yuechenjiang/opt/anaconda3/lib/python3.7/site-
packages/ipykernel_launcher.py:18: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
/Users/yuechenjiang/opt/anaconda3/lib/python3.7/site-
packages/ipykernel_launcher.py:18: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
```

```
/Users/yuechenjiang/opt/anaconda3/lib/python3.7/site-
packages/ipykernel_launcher.py:18: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
/Users/yuechenjiang/opt/anaconda3/lib/python3.7/site-
packages/ipykernel_launcher.py:18: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
/Users/yuechenjiang/opt/anaconda3/lib/python3.7/site-
packages/ipykernel_launcher.py:18: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

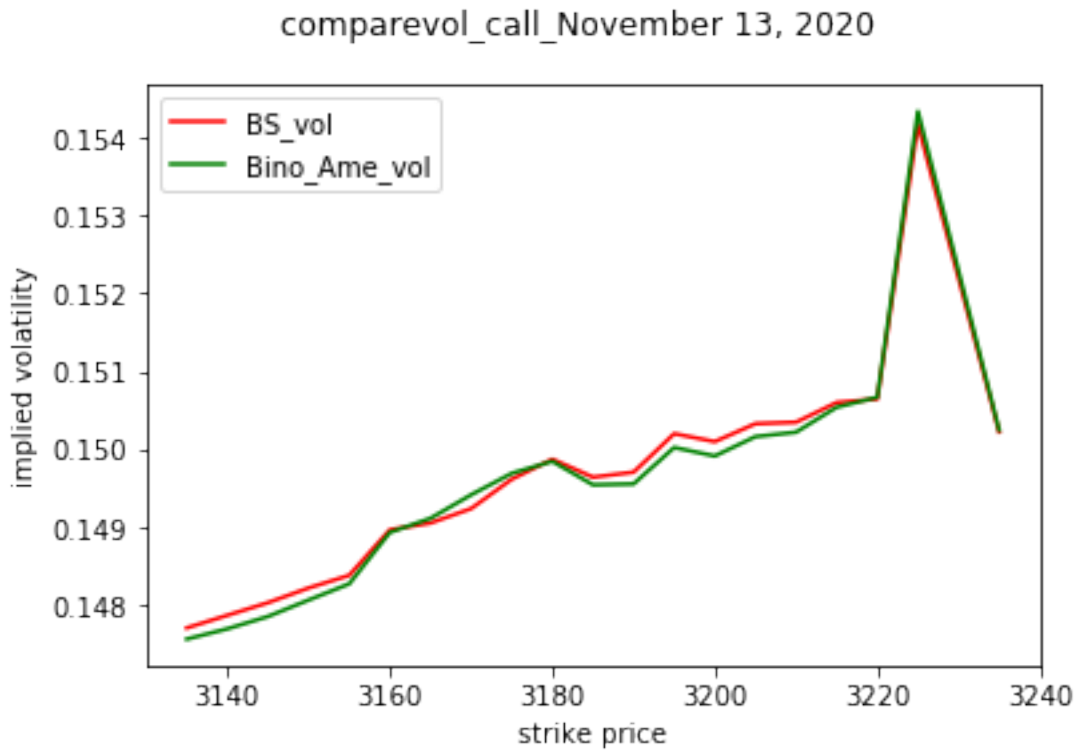
See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
```

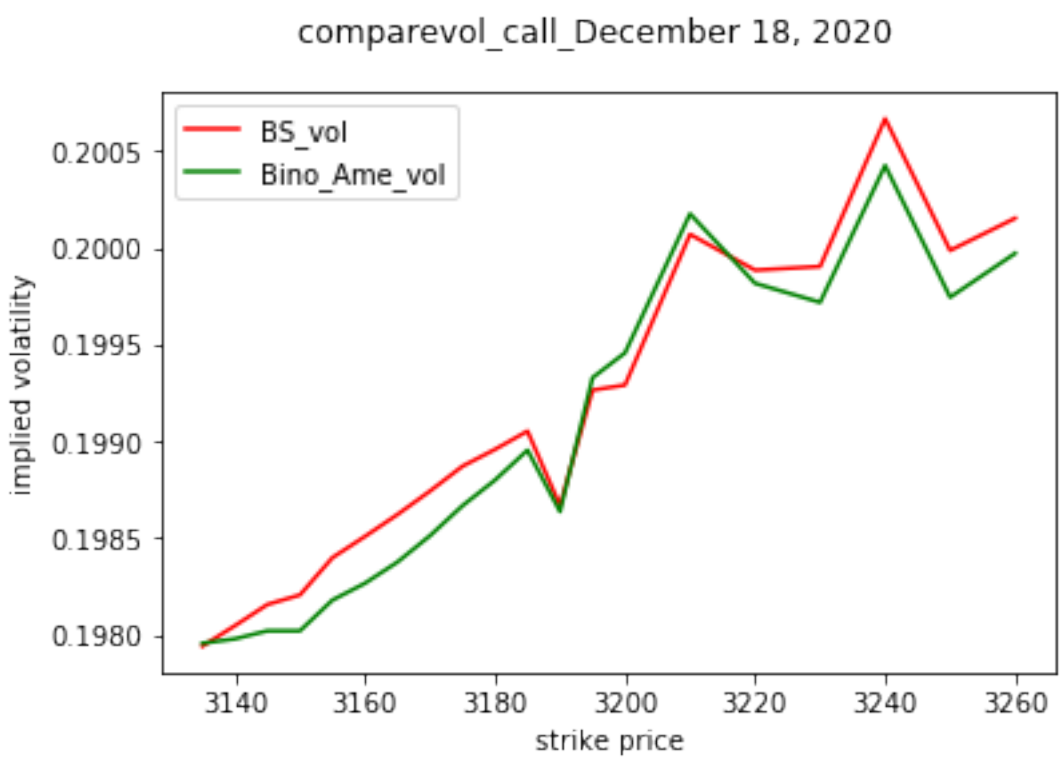
2.2 plot and compare

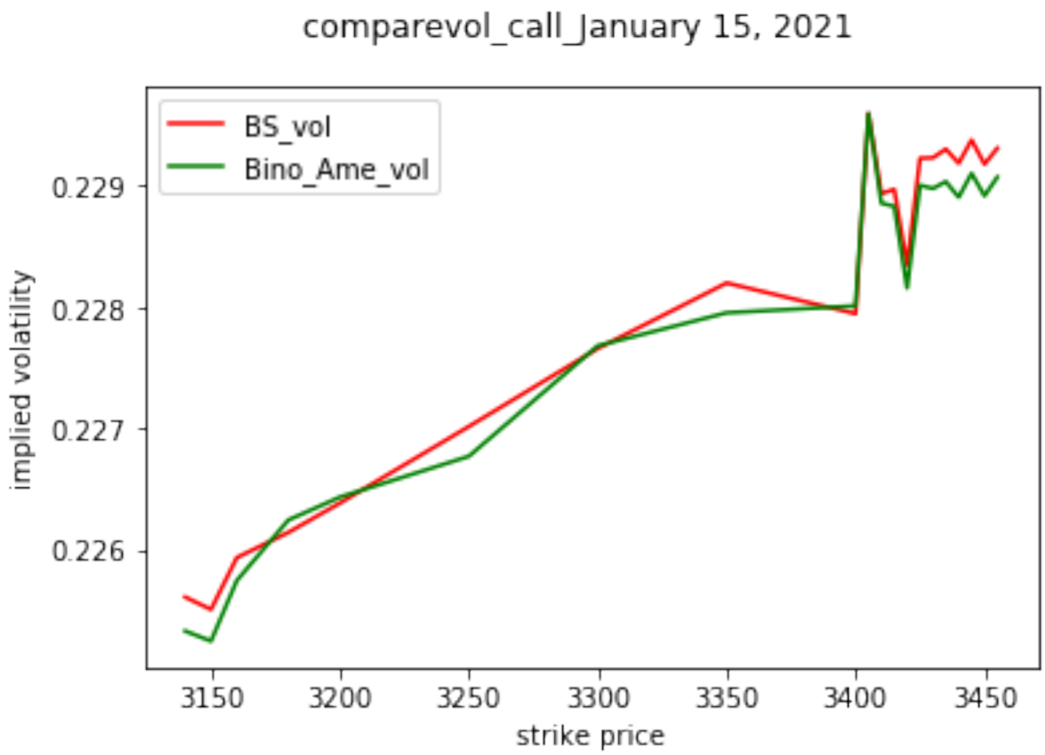
[16]:
```
for type in ['call','put']:
    for date in amzn_maturi:
        file=pd.read_csv('amzn_'+type+'_comparevol_'+date+'.csv')
        plt.plot(file['Strike'],file['cal_vol'],'r')
        plt.plot(file['Strike'],file['Bino_Ame_vol'],'g')

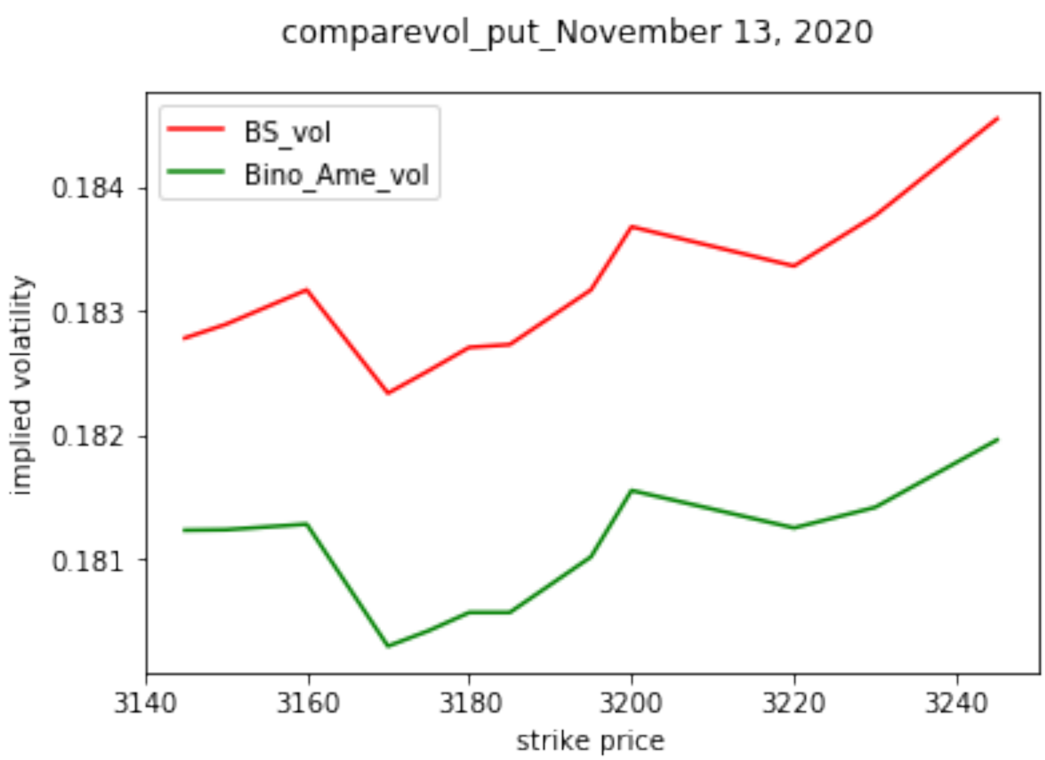
        plt.legend(labels=['BS_vol','Bino_Ame_vol'])
        plt.xlabel('strike price')
        plt.ylabel('implied volatility')
        plt.suptitle('comparevol_'+type+'_'+date)

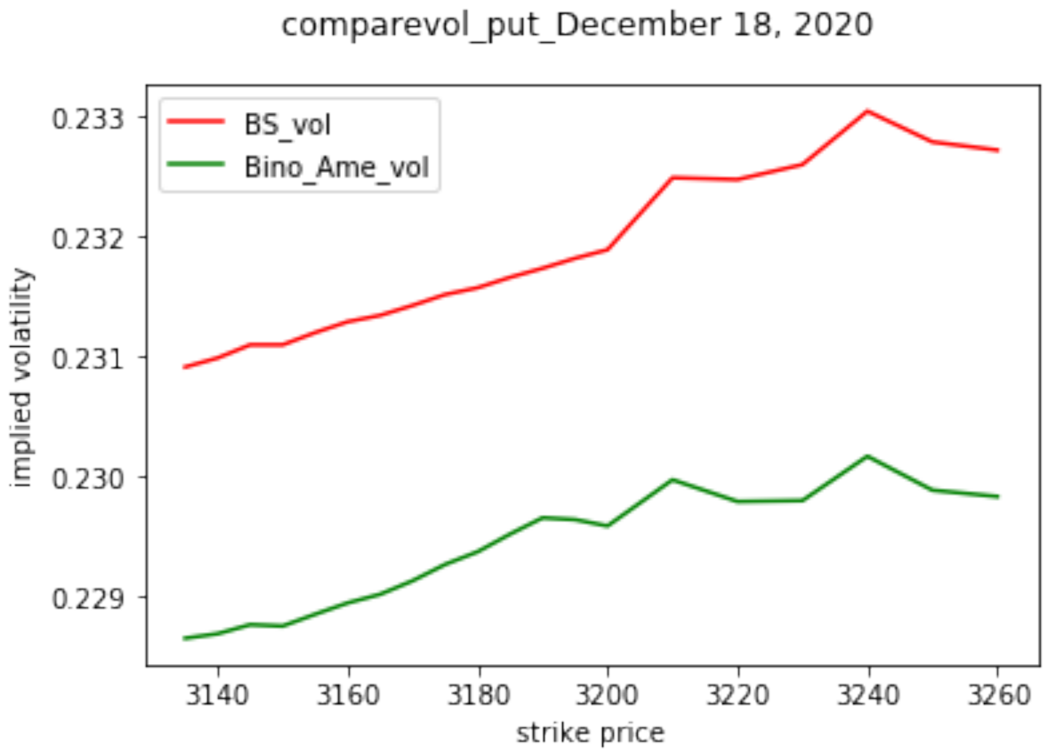
        plt.show()
```

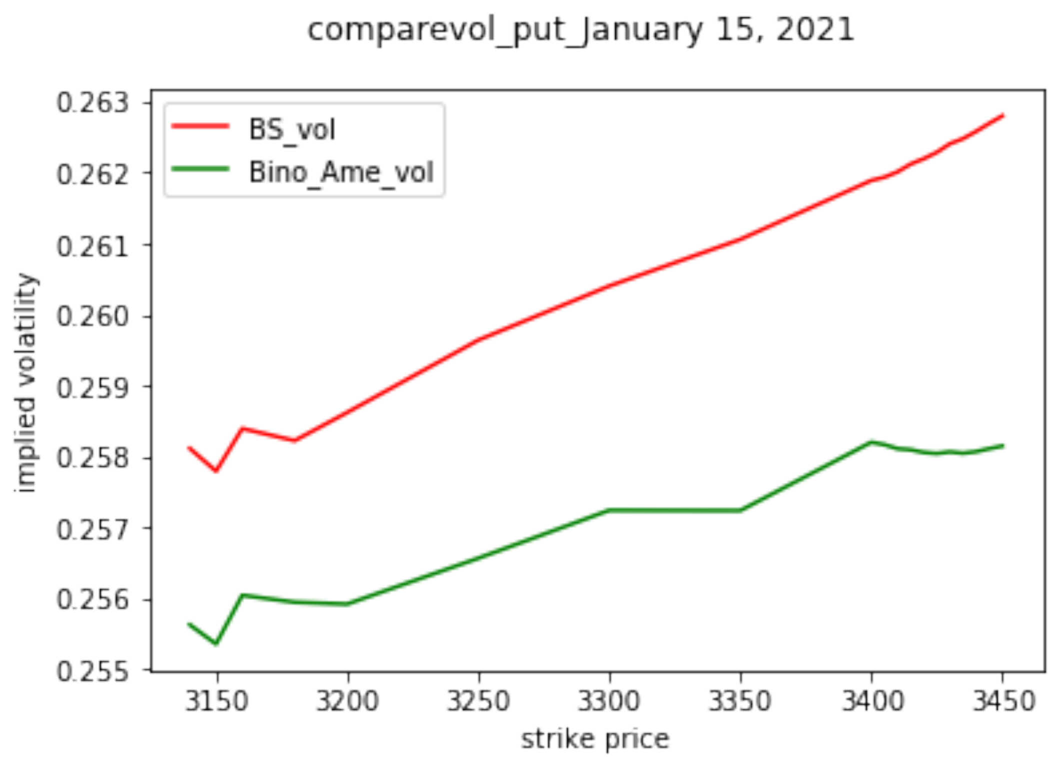

comparevol_call_November 13, 2020
BS_vol
Bino_Ame_vol
implied volatility
0.154
0.153
0.152
0.151
0.150
0.149
0.148
3140
3160
3180
3200
3220
3240
strike price


comparevol_call_December 18, 2020
BS_vol
Bino_Ame_vol
implied volatility
0.2005
0.2000
0.1995
0.1990
0.1985
0.1980
3140
3160
3180
3200
3220
3240
3260
strike price


comparevol_call_January 15, 2021
BS_vol
Bino_Ame_vol
0.229
0.228
0.227
0.226
implied volatility
3150
3200
3250
3300
3350
3400
3450
strike price


comparevol_put_November 13, 2020
BS_vol
Bino_Ame_vol
0.184
0.183
0.182
0.181
implied volatility
3140
3160
3180
3200
3220
3240
strike price


comparevol_put_December 18, 2020
BS_vol
Bino_Ame_vol
implied volatility
0.233
0.232
0.231
0.230
0.229
3140
3160
3180
3200
3220
3240
3260
strike price



3 Problem 3

3.1

Implement a trinomial tree to price European, American Call and Put options.

3.1.1 Trinomial European Option

[17]:
```
def trinomial_European(K,T,S0,sig,r,div,N,option_type):
    dt=T/N
    dx=sig*np.sqrt(3*dt)
    nu=r-div-0.5*sig**2
    edx= np.exp(dx)
    pu=0.5*((sig**2*dt+nu**2*dt**2)/(dx**2)+nu*dt/dx)
    pd=0.5*((sig**2*dt+nu**2*dt**2)/(dx**2)-nu*dt/dx)
    pm =1-pu-pd
    disc= np.exp(-r*dt)
    St=np.zeros(shape=(N+1,2*N+1))
    C=np.zeros(shape=(N+1,2*N+1))

    for i in range(0,N+1):
        for j in range(0,(2*i+1)):
```

```
            St[i,j]=S0*edx**(i-j)

    if option_type==0:
        sign=1
    elif option_type==1:
        sign=-1
    else:
        print('Error')
        return

    for i in range(0,(N+1)):
        for j in range(0,(2*i+1)):
             C[i,j]=max(sign*(St[i,j]-K),0)

    for i in range(N-1,-1,-1):
        for j in range(0,2*N-1):
            C[i,j]=disc*(pu*C[i+1,j]+pm*C[i+1,j+1]+pd*C[i+1,j+2])

    return C[0,0]
```

3.1.2 Trinomial American Option

[18]:
```
def trinomial_American(K,T,S0,sig,r,div,N,option_type):
    dt=T/N
    dx=sig*np.sqrt(3*dt)
    nu=r-div-0.5*sig**2
    edx= np.exp(dx)
    pu=0.5*((sig**2*dt+nu**2*dt**2)/(dx**2)+nu*dt/dx)
    pd=0.5*((sig**2*dt+nu**2*dt**2)/(dx**2)-nu*dt/dx)
    pm =1-pu-pd
    disc= np.exp(-r*dt)
    St=np.zeros(shape=(N+1,2*N+1))
    C=np.zeros(shape=(N+1,2*N+1))

    for i in range(0,N+1):
        for j in range(0,(2*i+1)):
            St[i,j]=S0*edx**(i-j)

    if option_type==0:
        sign=1
    elif option_type==1:
        sign=-1
    else:
        print('Error')
        return

    for i in range(0,(N+1)):
```

```
        for j in range(0,(2*i+1)):
            C[i,j]=max(sign*(St[i,j]-K),0)

    for i in range(N-1,-1,-1):
        for j in range(0,2*N-1):
            
↪C[i,j]=max(disc*(pu*C[i+1,j]+pm*C[i+1,j+1]+pd*C[i+1,j+2]),sign*(St[i,j]-K))

    return C[0,0]
```

3.2

Consider $S_0 = 100, K = 100, T = 1year, \sigma = 20\%, r = 5\%, \delta = 0.02$.Repeat the methods in problem 1 b) to d) with these parameters. Use at least N = 200 time steps and you do not need to download data. Create a table containing all results and comment.

3.2.1 Black-Scholes formulas when considering dividends

[19]:
```
def bs_formula(S0,sig,T,K,r,div,option_type):

    d1=(ln(S0/K)+(r-div+(sig**2)/2)*T)/(sig*np.sqrt(T))
    d2=d1-(sig*np.sqrt(T))

    if option_type==0:
        c=S0*np.exp(-div*T)*norm.cdf(d1)-K*np.exp(-r*T)*norm.cdf(d2)
        return c
    elif option_type==1:
        p=K*np.exp(-r*T)*norm.cdf(-d2)-S0*np.exp(-div*T)*norm.cdf(-d1)
        return p
    else:
        print("Error")
```

[20]:
```
if __name__ == "__main__":
    K = 100
    S0 = 100
    T = 1
    sig = 0.2
    r = 0.05
    div = 0.02
    N = 500
    BS = []
    European = []
    American = []
    bs_call = bs_formula(S0,sig,T,K,r,div,0)
    bs_put = bs_formula(S0,sig,T,K,r,div,1)
    euro_call = trinomial_European(K,T,S0,sig,r,div,N,0)
    euro_put = trinomial_European(K,T,S0,sig,r,div,N,1)
```

```
    american_call = trinomial_American(K,T,S0,sig,r,div,N,0)
    american_put = trinomial_American(K,T,S0,sig,r,div,N,1)
    BS.append(bs_call)
    European.append(euro_call)
    American.append(american_call)
    BS.append(bs_put)
    European.append(euro_put)
    American.append(american_put)
    option_types = ['call','put']
    d = {'option_type':option_types,'BS':BS,'Euro':European,'AMER':American}
    d = pd.DataFrame(data=d)
    print(d)
```

```
  option_type        BS      Euro      AMER
0        call  9.227006  9.223214  9.223214
1         put  6.330081  6.326289  6.657063
```

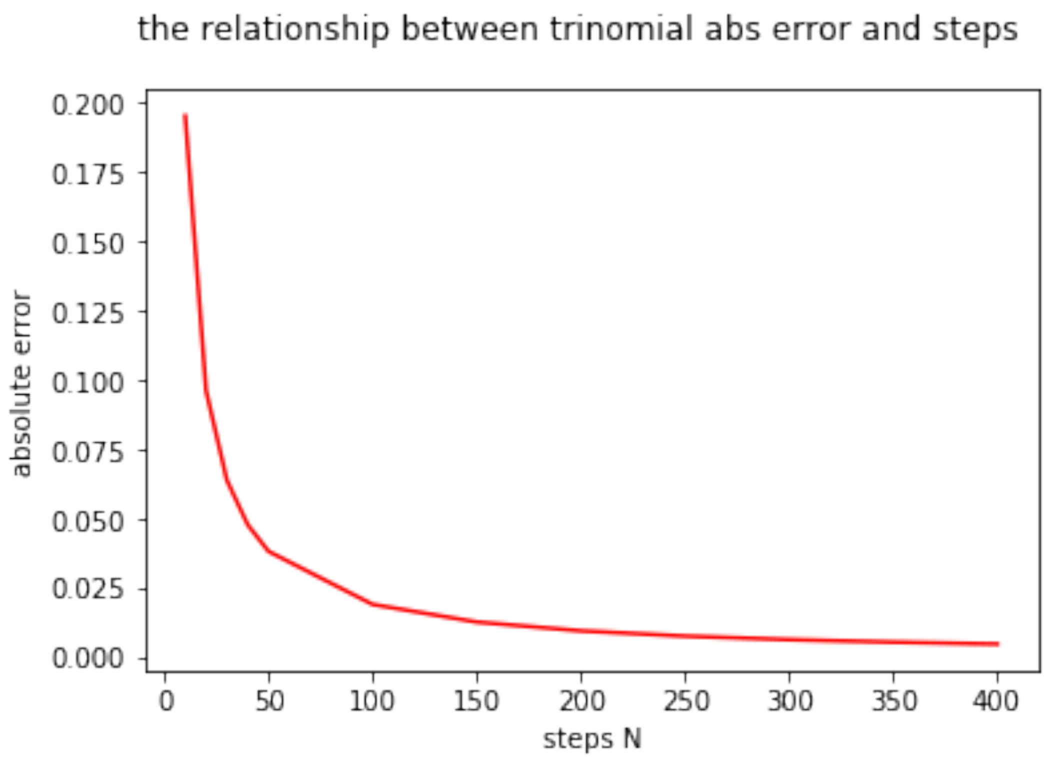
As is shown in the table, the option prices calculated by European trinomial tree are very close to that of BS formula. Besides, with dividends, both American call and put options prices are higher than that of European options, which is because that it could be better to exercise American options before expiration on a dividend-paying stock.

[21]:
```
def error_triEuro_P(N):
    return␣
 ↪abs(bs_formula(S0,sig,T,K,r,div,1)-trinomial_European(K,T,S0,sig,r,div,N,1))
```

[22]:
```
if __name__ == "__main__":
    K=100
    S0=100
    T=1
    sig=0.2
    r=0.05
    div=0.02
    N_list=[10, 20, 30, 40, 50, 100, 150, 200, 250, 300, 350, 400]
    error_list=[]

    for N in N_list:
        error=error_triEuro_P(N)
        error_list.append(error)
```

[23]:
```
plt.plot(N_list,error_list,'r')
plt.xlabel('steps N')
plt.ylabel('absolute error')
plt.suptitle('the relationship between trinomial abs error and steps')
plt.show()
```



4 Problem 4: Pricing Exotic Options

4.1

Construct a binomial tree to calculate the price of an European Up-and-Out call option. Use $S_0 = 10$, strike $K = 10$, maturity $T = 0.3$, volatility $\sigma = 0.2$, short rate $r = 0.01$, dividends $\delta = 0$, and barrier $H = 11$. Use as many steps in your tree as you think are necessary.

[24]:
```
def Bino_Euro_Up_Out_Call(K,T,S0,sig,r,H,N):
    # Equal jump size
    dt=T/N
    nu=r-0.5*sig**2
    dxu=np.sqrt(sig**2*dt+(nu*dt)**2)
    dxd=-dxu
    pu=0.5+0.5*(nu*dt/dxu)
    disc=np.exp(-r*dt)
    dpu=disc*pu
    dpd=disc*(1-pu)
    edxud=np.exp(dxu-dxd)
    edxd=np.exp(dxd)
    St=[0]*(N+1)
    C=[0]*(N+1)
```

```
    St[0]=S0*np.exp(N*dxd)

    for j in range(1,(N+1)):
        St[j]=St[j-1]*edxud

    for j in range(0,(N+1)):
        if St[j]<H:
            C[j] = max(0,St[j]-K)
        else:
            C[j]=0

    for i in range((N-1),-1,-1):
        for j in range(0,(i+1)):
            if St[j]<H:
                C[j]=dpu*C[j+1]+dpd*C[j]
            else:
                C[j]=0

    return (C[0])
```

[25]:
```
if __name__ == "__main__":
    S0=10
    K=10
    T=0.3
    sig=0.2
    r=0.01
    H=11
    N=400
    print(Bino_Euro_Up_Out_Call(K,T,S0,sig,r,H,N))
```

```
0.1336371505621503
```

4.2

For the European Up-and-Out Call option explicit formulas exist. For example, implement the formula (5.2) from [2] and compare your results with part (a). Use the same parameters as before. Are your results matching?

[26]:
```
def bs_formula(S0,sig,T,K,r,option_type):

    d1=(ln(S0/K)+(r+(sig**2)/2)*T)/(sig*np.sqrt(T))
    d2=d1-(sig*np.sqrt(T))

    if option_type==0:
        c=S0*norm.cdf(d1)-K*np.exp(-r*T)*norm.cdf(d2)
        return c
    elif option_type==1:
```

```
        p=K*np.exp(-r*T)*norm.cdf(-d2)-S0*norm.cdf(-d1)
        return p
    else:
        print("Error")
```

[27]:
```
def C_bs(S0,K):
    return bs_formula(S0,sig,T,K,r,0)

def P_bs(S0,K):
    return bs_formula(S0,sig,T,K,r,1)


def d_bs(S0,K):
    return (ln(S0/K)+nu*T)/(sig*np.sqrt(T))
```

[28]:
```
# formula 5.2
def Up_Out_call_bs(K,T,S0,sig,r,H):
    return C_bs(S0,K)-C_bs(S0,H)-(H-K)*np.exp(-r*T)*norm.cdf(d_bs(S0,H))-(H/
 ↪S0)**(2*nu/(sig**2))*(C_bs(H**2/S0,K)-C_bs(H**2/S0,H)-(H-K)*np.
 ↪exp(-r*T)*norm.cdf(d_bs(H,S0)))
```

[29]:
```
if __name__ == "__main__":
    S0=10
    K=10
    T=0.3
    sig=0.2
    r=0.01
    H=11
    N=400
    nu=r-0.5*sig**2

    print(Up_Out_call_bs(K,T,S0,sig,r,H))
```

```
0.05309279660325303
```

Analytical result: 0.05309279660325303
Binomial result: 0.1336371505621503
When comparing the results of binomial tree and analytical method, I find the results don't match. I don't think the binomial tree ideal for pricing barrier option. There might be some problems with its accuracy and simplicity of the tree structure.

4.3

Price an European Up-and-In call option, using the same parameters as before.

[30]:
```
# formula 5.1
def Up_In_call_bs(K,T,S0,sig,r,H):
```

```
    return (H/S0)**(2*nu/(sig**2))*(P_bs(H**2/S0,K)-P_bs(H**2/S0,H)+(H-K)*np.
↪exp(-r*T)*norm.cdf(-d_bs(H,S0)))+C_bs(S0,H)+(H-K)*np.exp(-r*T)*norm.
↪cdf(d_bs(S0,H))
```

[31]:
```
if __name__ == "__main__":
    ␣
↪I_by_parity=bs_formula(S0,sig,T,K,r,0)-Bino_Euro_Up_Out_Call(K,T,S0,sig,r,H,N)
    print(Up_In_call_bs(K,T,S0,sig,r,H))
    print(I_by_parity)
```

```
0.3981948482776454
0.31765049431874837
```

Show the result:
Analytical method: 0.3981948482776454
In-Out parity: 0.31765049431874837

4.4 Calculate the price of an AMERICAN Up and In Put option

[32]:
```
def American_U_I_P(K,T,S0,sig,r,H,N):
    # Equal jump size
    dt=T/N
    nu=r-0.5*sig**2
    dxu=np.sqrt(sig**2*dt+(nu*dt)**2)
    dxd=-dxu
    pu=0.5+0.5*(nu*dt/dxu)
    disc=np.exp(-r*dt)
    dpu=disc*pu
    dpd=disc*(1-pu)
    edxud=np.exp(dxu-dxd)
    edxd=np.exp(dxd)
    St=[0]*(N+1)
    C=[0]*(N+1)

    St[0]=S0*np.exp(N*dxd)


    for j in range(1,(N+1)):
        St[j]=St[j-1]*edxud


    for j in range(0,(N+1)):
        if St[j]>H:
            C[j] = max(0,K-St[j])
        else:
            C[j]=0

    for i in range((N-1),-1,-1):
```

```
        for j in range(0,(i+1)):
            St[j]=St[j]/edxd
            if St[j]>H:
                C[j]=dpu*C[j+1]+dpd*C[j]
                C[j]=max(C[j],K-St[j])
            else:
                C[j]=0

    return (C[0])
```

[33]:
```
if __name__ == "__main__":
    S0=10
    K=10
    T=0.3
    sig=0.2
    r=0.01
    H=11
    N=800
    print(American_U_I_P(K,T,S0,sig,r,H,N))
```

```
0
```

Bonus Problem 6(100 points). A multinomial recombining tree for general Stochastic Volatility models. We consider here an interesting method of option pricing under general assumptions, involving a multinomial recombining tree and particle filtering techniques. Please read the paper [4], and pay special attention to sections 3 and 4.1, 4.2.

(a) Using synthetic parameters, i.e., chosen by you, estimate the probability distribution for the volatility process Y_t at discrete time points $t_1, t_2, \ldots, t_n$. To this end, implement the particle filter described in Section 3 of [4]. You should store from this step the particles $\{\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_n\}$ together with their corresponding probabilities $\{p_1, p_2, \ldots, p_n\}$.

We could apply the Heston Model and using the formula sets below:

$$\begin{cases} dX_t = \left(r - \frac{Y_t}{2}\right) dt + \sqrt{Y_t} dZ_t^1 \\ dY_t = \alpha(\nu - Y_t) dt + \psi(Y_t) dZ_t^2 \end{cases} \tag{1}$$

In that <which includes supposing numbers for each unknown factors>:

X_t	Lognormal of stock price $X_t = log(S_t)$
Y_t	Volatility
r	Interest Rate: <set $r = 0.0125$>
α	Mean reversion speed factor: < $\alpha = M/h$>
ν	Mean reversion level factor: < $\nu = 1$>
$\psi(Y_t)$	The variance of volatility
Z_t^1, Z_t^2	They are i.i.d and $dZ_t = \sqrt{T} dW_t$

Then, we suppose $h = t_{i+1} - t_i$ and subdivide time in to M segments. Each part will be:

$$dt = \frac{t_{i+1} - t_i}{M} = \frac{h}{M} \tag{2}$$

We use counting number j, where $j = 0,1,2 \ldots \ldots, (M-1)$.
After that, we consider to change function (1) to discrete mode and plug in Equation (2):

$$\begin{cases} X_{t_i,j+1} - X_{t_i,j} = \left(r - \frac{Y_{t_i,j}}{2}\right)\frac{h}{M} + \sqrt{{Y_{t_i,j}}_t}\sqrt{\frac{h}{M}} dW_t^1 \\ Y_{t_i,j+1} - Y_{t_i,j} = \alpha\left(\nu - Y_{t_i,j}\right)\frac{h}{M} + \psi\left(Y_{t_i,j}\right)\sqrt{\frac{h}{M}} dW_t^2 \end{cases} \tag{3}$$

We have to indicate something in advance, stock price could be observed in advance, which means that X_{t_i} is a series of known numbers. We set $X_{t_i} = x_i$, which means the stock price at time t_i. We also make empirical distribution to the possibility of observed values as p_i. We will apply n times to repeat the following parts and apply $\left\{Y_i^j\right\}_{j=1}^n$, which means we will calculate n $Y_i s$ at time t_i.

About Dirac Point mass, according to the files, we define: <suppose $\sum x = A$>

$$\delta_x = \begin{cases} 0 & x \notin \Omega \\ \dfrac{1}{A} & x \in \Omega \end{cases}$$

And we could get:

$$\int_{-\infty}^{\infty} \delta_x dx = 1$$

According to the article, we define a function:

$$\phi(x) = \begin{cases} 1 - |x| & -1 < x < 1 \\ 0 & -1 < x < 1 \end{cases}$$

And:

$$\phi_n(x) = \sqrt[3]{n}\phi(\sqrt[3]{n}x) = \begin{cases} \sqrt[3]{n}(1 - |\sqrt[3]{n}x|) & -\dfrac{1}{\sqrt[3]{n}} < x < \dfrac{1}{\sqrt[3]{n}} \\ 0 & -1 < x < 1 \end{cases} \tag{3}$$

Because $X_{t_i,j}$ and $Y_{t_i,j}$ are i.i.ds. Thus, we could create a function which relates to the distribution of $X_{t_i,j}$ and a "selection function" of $Y_{t_i,j}$.
We make the distribution of X_{t_i} as the following function, Set,

$$C = \sum_{j=1}^{n} \phi_n \left\{ X_{t_i}^{j} - x_i \right\}$$

And let the summation of this function into 1, which makes it could be understand as probability. Still we use the "selection" of Dirac Point mass, which could be write as:

$$\int_{-\infty}^{\infty} f(x)\delta(x - t_0)dx = f(t_0)$$

Thus, we get the function, which is used to estimate the probability of $Y_{t_i}^{j}$ from the distribution of $X_{t_i}^{j}$ <set $\delta\left(x - Y_{t_i}^{j}\right) = \delta_{\left\{Y_{t_i}^{j}\right\}}$>

$$\phi_i^n = \begin{cases} \dfrac{1}{C}\displaystyle\sum_{j=1}^{n} \phi_n \left\{ X_{t_i}^{j} - x_i \right\} \delta_{\left\{Y_{t_i}^{j}\right\}} & for\ C > 0 \\ \delta_{\{0\}} & otherwise \end{cases} \tag{4}$$

Here, we make it the simple form of expectation:

$$\overline{p_J} = \frac{\phi_n \left\{ X_{t_i}^{j} - x_i \right\}}{C} = \frac{\phi_n \left\{ X_{t_i}^{j} - x_i \right\}}{\sum_{j=1}^{n} \phi_n \left\{ X_{t_i}^{j} - x_i \right\}}$$

$$\overline{Y_J} = Y_{t_i}^{j}$$