Portfolio Theory

FE-690

Report

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Contents

1. Download Data from yahoo finance

We choose the data of 200 days(long-term) starting from:

①   First date of starting 200-day: 2007-01-03 (before crisis)

②   Second date of starting 200-day 2008-01-02 (during crisis)

③   Third date of starting 200-day 2009-01-02 (after crisis)

④   Back-test Date: 2020-09-25

The short-term and mid-term data are a part of the long-term period.

2. Get the time series data from French FAMA 3-factor models

2.1 The return of ETFs

Names	FXE	EWJ	GLD	QQQ	SPY	SHV
1	-1.499409	2.6408551	-2.52889	4.7409786	0.53053	0.13800
2	-1.603904	-6.446007	-6.001635	-1.191573	-1.99408	-0.04598
3	0.3459296	1.4306309	1.288023	0.1710524	1.15627	0.13796
4	-0.403022	1.6002702	1.5294268	1.2534036	-0.21247	-0.0230
5	-0.941854	-3.710248	-1.068194	2.9478485	0.83289	-0.0460
6	-0.906848	-1.255375	0.165047514	2.5773359	1.09516	0.09194
…	…	…	…	…	…	…

Table 2-1: The part of annualized return (in 2007)[1]

2.2 Risk-free Rate

We use the effective zero-coupon bond as the risk-free rate. Because of the violently fluctuation around the 2008 crisis, we decide to use the zero-coupon bond as the risk-free rate for each month. Because this could effectively reflect the Interest Rate changes during that terrible period.

2.3 Market Return (S&P 500)

We use the S&P 500 index to reflect the most general market in the US and hope this would also show the changes among global countries. Due to the market in US is the most active in the world. The S&P 500 index is the diversified enough so that it could represent the US stock market. We select S&P 500 index to calculate the market return.

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[1] See attachment file: Ret1.csv(for 2007); Ret2.csv(for 2008); Ret3.csv(for 2009)

2.4 Calculate r_{SMB}

For small minus big in ***French FAMA 3-factor model***, we use the total average return of 20% top capitalization ETF minus that of 20% bottom capitalization ETF. we choose best 3 capitalization ETFs and worst 3 ETFs since there are 12 ETFs. As for how to rank the ETF in terms of capitalization, we use the total transaction value (Close price multiply volume) for sorting our ETF. [2][3]

3. Cross Sectional Model

Name	FXE	EWJ	GLD	QQQ	SPY	SHV
FXE	0.820242	0.314153	0.492075	0.425616	0.419475	0.001395
EWJ	0.314153	4.675093	1.713107	1.628637	1.635762	0.003569
GLD	0.492075	1.713107	6.112127	2.487064	2.495758	0.0050004
QQQ	0.425616	1.628637	2.487064	6.138711	2.781972	0.0093036
SPY	0.419475	1.635762	2.495758	2.781972	4.975794	0.0073976
SHV	0.001395	0.003569	0.0050004	0.009304	0.007398	0.0385088
…	…	…	…	…	…	…

Table 3-1. A part of covariance matrix [4]

We apply the linear regression to estimate the parameter in matrix B and compute the diagonal matrix consist of the variance of residual. Then we compute the covariance of returns.

4. Weight of each ETF with Minimize Vol

Name	Beta=0	Beta=0.5	Beta=1	MinVol
FXE	-0.007674	1.116433	1.807788	0.329186
EWJ	-0.080685	1.116433	1.807788	-0.036357
GLD	0.217183	1.116433	1.807788	0.1210132
QQQ	0.0283304	-1.116433	-1.807788	0.093441
SPY	-0.071499	-1.116433	-1.807787	-0.069317
SHV	-0.113251	1.116433	-0.489828	0.332219
DBA	-0.044189	-1.116433	-1.807788	0.1036126
USO	0.361117	1.116433	1.807788	0.0262679
ILF	0.2627769	-1.116433	-1.807788	0.0054548
EPP	0.275891	-1.099003	-1.795327	0.02383009
FEZ	-0.103507	-1.116433	-1.807788	-0.0317113
XBI	0.275508	1.116433	1.807788	0.1023613

Table 4-1. The weights of different strategies of Long-term in 2007

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[2] The Best 3 ETFs are: GLD, QQQ, SPY

[3] The Worst 3 ETFs are: XBI, DBA, SHV

[4] See attachment file: .Cross_Sectional.csv

Name	Beta=0	Beta=0.5	Beta=1	MinVol
FXE	0.2252296	-1.92322	-3.893891	0.115872
EWJ	-0.142118	1.92322	3.8938902	0.039063
GLD	0.5522004	1.92322	3.8938902	-0.00163
QQQ	-0.130219	-1.92322	-3.893891	-0.030181
SPY	-0.213073	-0.92412	-2.893145	-0.035639
SHV	0.226342	-1.92322	-3.893891	0.9999999
DBA	-0.031304	-1.92322	-3.89389	-0.044931
USO	0.426914	-1.92322	-3.89389	-0.001785
ILF	-0.001048	1.923218	3.89389	0.068795
EPP	-0.019059	1.92322	3.89388	-0.105794
FEZ	-0.405765	1.92322	3.89389	-0.185804
XBI	0.511899	1.92322	3.89389	0.182034

Table 4-2. The weights of different strategies of Long-term in 2008

Name	Beta=0	Beta=0.5	Beta=1	MinVol
FXE	0.181603	-0.999618	-0.999999	0.010733
EWJ	-0.306543	0.999999	0.9999998	-0.096497
GLD	0.772258	0.999999	0.376534	0.088521
QQQ	-0.017187	-0.999999	-0.999999	0.076336
SPY	-0.295956	-0.311181	-0.999995	-0.066215
SHV	0.142937	-0.967081	-0.999999	0.881463
DBA	0.004199	-0.722122	-0.999999	0.0303399
USO	0.197235	-0.999999	0.6234633	-0.011993
ILF	0.23521	0.999999	1.000000	0.105615
EPP	0.221316	0.999999	0.999999	0.076447
FEZ	-0.470291	0.999999	0.999999	-0.050881
XBI	0.335218	0.999999	0.999998	-0.043869

Table 4-3. The weights of different strategies of Long-term in 2009

5. Performance and Risk reporting of Comparing Plans

	FXE	EWJ	GLD	QQQ	SPY	SHV	DBA
Geo Daily Mean Return	0.10535	-0.00264	0.248990	0.272818	0.104468	0.013155	0.202581
Daily Min Return	-2.11833	-6.44601	-9.875164	-10.26866	-9.764411	-1.071692	-10.14761
Max Drawdown	1.99554	2.079895	2.264601	2.432708	2.326706	4.359003	1.5094098
Volatility	0.9056	2.162349	2.472070	2.4775126	2.230589	0.196205	3.403777
Sharpe Ratio	0.09848	-0.00515	0.098454	0.107879	0.043314	-0.023256	0.061029
Skewness	-0.04934	-0.03354	-0.492383	-0.55737	-0.75957	-3.565555	0.513382
Kurtosis	-0.2971	0.401473	1.695972	1.328072	2.647229	14.213192	5.521043
VaR	-1.45933	-4.00698	-3.984067	-4.620341	-3.821527	-0.341254	-5.067558
CVaR	-1.78778	-4.78967	-5.988426	-5.701501	-5.960438	-0.80409	-7.676834

Table 5-1 The summarizing table for comparison with the underlying in 2007(1)

	USO	ILF	EPP	FEZ	XBI	Opt_Port	SPY
Geo Daily Mean Return	0.402892	0.523849	0.400853	0.182749	0.338805	0.166627	0.104468
Daily Min Return	-10.2537	-19.86847	-14.62607	-11.84961	-9.980058	-2.465430	-9.764411
Max Drawdown	1.771730	2.219231	2.425971	2.074983	2.334511	1.981708	2.326706
Volatility	4.131199	5.052567	3.853878	2.917695	2.737981	0.766164	2.230589
Sharpe Ratio	0.101432	0.110262	0.107103	0.062354	0.122692	0.195780	0.043314
Skewness	-0.06551	-0.478230	-0.612765	-0.324569	-0.177258	-0.351054	-0.75957
Kurtosis	0.052309	1.561388	1.627300	2.052645	1.008964	0.356642	2.647229
VaR	-6.81239	-8.587442	-6.789470	-4.653478	-4.439265	-1.234751	-3.821527
CVaR	-8.40089	-12.32531	-9.507402	-7.051163	-6.041961	-1.575789	-5.960438

Table 5-1 The summarizing table for comparison with the underlying in 2007(2)

Comment on Table 5-1: Consider the final output in 2007, we could find that the geometric mean return, daily minimize return and maximum drawdown are in the range of the maximum and minimize one of the 13 ETFs. The small volatility ratio and large sharp ratio which comparing with most of 13 single ETF shows us the small uncertainty and possibly have the largest returning among ETFs. A small negative Kurtosis of portfolio means both higher and lower returns have a higher likelihood to happen. All the numbers reflects that, our portfolio could have the much stable earning with less risk. Value of Var and CVaR have verify this conclusion.

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	Beta=0	Beta=0.5	Beta=1	MinVol	S&P 500
Geo Daily Mean Return	0.525965	-1.091319	-2.190820	0.166627	0.073729
Daily Min Return	-13.59301	-41.77871	-67.96706	-2.46543	-7.412407
Max Drawdown	2.413005	1.932511	1.936625	1.981708	2.01512
Volatility	3.336601	13.645401	22.176521	0.766164	2.647928
Sharpe Ratio	0.158960	-0.054064	-0.055046	0.195780	0.026404
Skewness	-0.527875	0.15833	0.156355	-0.351054	-0.331472
Kurtosis	1.250934	1.02361	1.008917	0.356642	0.909711
VaR	-5.377754	-22.62284	-37.46982	-1.234751	-5.092071
CVaR	-7.532463	-31.20354	-50.71362	-1.575789	-6.471271

Table 5-2. Comparison with between the 2 built strategies and S&P 500 in 2007

Comment on Table 5-2: This sheet mainly shows us the return of each situation. We would like to maximum our return under certain risk (Beta) and minimize volatility with fixed return (Rho). Above, we show the results by changing the Value of Beta. If we change our Beta from 0 to 0.5 to 1.0 or even more, which shows us the medium value of the daily minimize returns. Although the Geometric return has a little bit growth, but not much. Comparing with the values of S&P500, all 3 Bata value has greater sharp Value than S&P500, which stands that it could have much better return than market which S&P500 represent. There is a little bit higher skewness shows that it may have lower probability for lower return. To sum up, all the portfolios show better performance than S&P500.
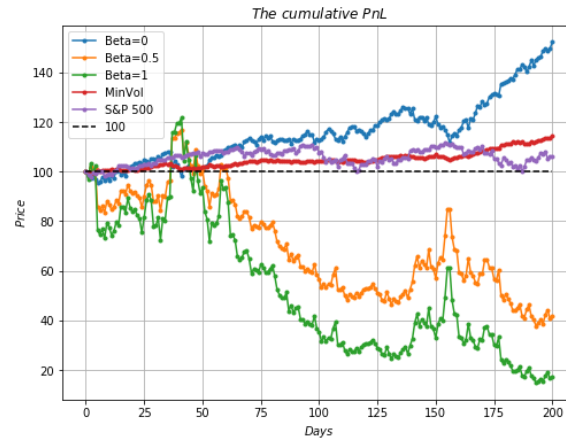
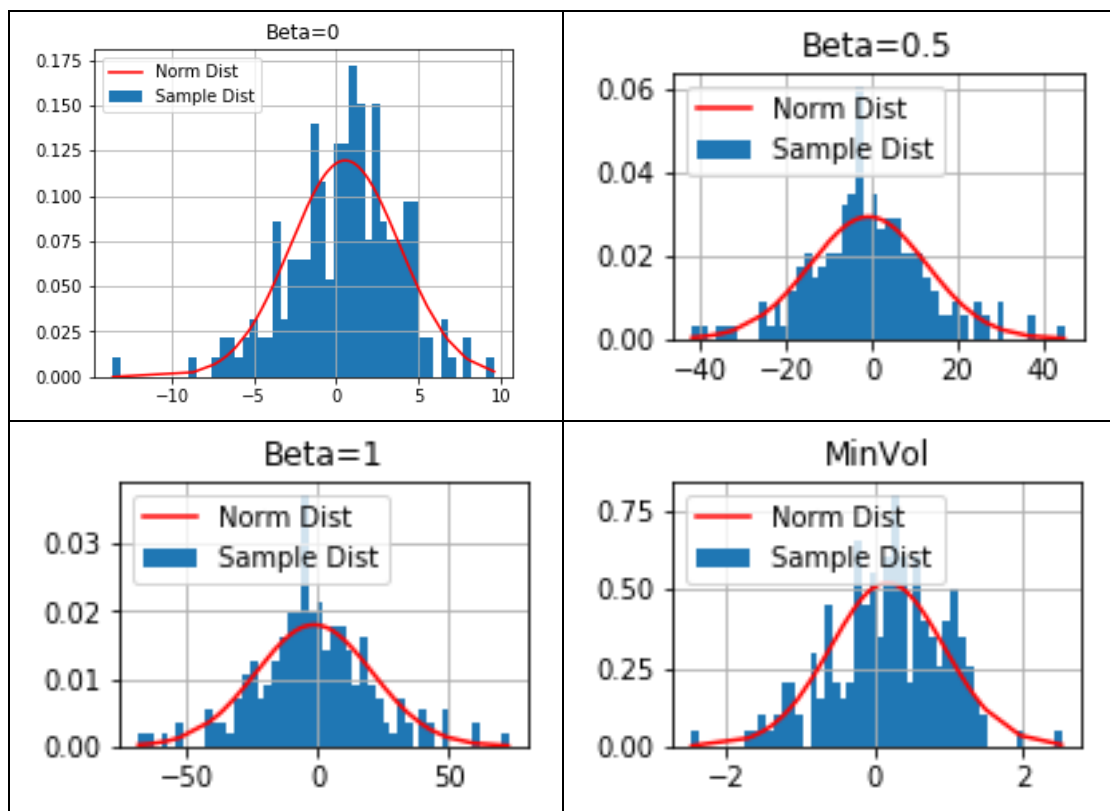**Figure 5-1. The cumulative PnL of long-term model in 2007**

Figure 5-2. The Distribution of daily Returns in 2007

Comment on Figure 5-1 & Figure 5-2: Beta shows the relationship between our portfolio with the risk from global market. Lower Beta means the lower relationship with global risks, or on the other side, we may have greater return with greater returns and stronger relation with market risks. The plot above shows the shape of high kurtosis and fat tail, which proved that it is relatively healthy market.

	FXE	EWJ	GLD	QQQ	SPY	SHV	DBA
Geo Daily Mean Return	-0.112603	-0.500673	-0.084849	-0.55733	-0.543665	0.005452	-0.350472
Daily Min Return	-6.429315	-26.01922	-18.57620	-22.38917	-24.61192	-0.904981	-21.51494
Max Drawdown	2.037832	1.657800	1.658116	1.736198	1.678025	2.138729	2.512770
Volatility	1.846713	5.789925	4.873951	5.463495	5.307145	0.170231	5.843203
Sharpe Ratio	-0.066933	-0.078144	-0.011408	-0.094398	-0.095296	-0.072170	-0.051305
Skewness	-0.042952	1.035079	0.505229	0.296680	0.806175	-1.144777	-0.374796
Kurtosis	0.883315	12.911341	5.751784	6.299916	13.736068	8.402613	1.072220
VaR	-3.173846	-8.920168	-7.679605	-9.121059	-7.526585	-0.385903	-11.66329
CVaR	-4.179752	-13.63812	-10.94755	-14.00587	-13.97456	-0.540975	-14.88232

Table 5-3 The summarizing table for comparison with the underlying in 2008(1)

	USO	ILF	EPP	FEZ	XBI	Opt_Port	SPY
Geo Daily Mean Return	-0.349047	-0.807190	-0.75486	-0.769306	-0.173432	0.154374	-0.543665
Daily Min Return	-26.44706	-48.66667	-28.05453	-28.57730	-17.28350	-3.150900	-24.61192
Max Drawdown	2.337796	1.741705	1.676419	1.651910	1.521214	1.893072	1.678025
Volatility	6.393997	9.758582	6.871601	6.308598	5.332841	0.963818	5.307146
Sharpe Ratio	-0.044577	-0.065228	-0.098759	-0.112323	-0.02536	0.143622	-0.095296
Skewness	-0.196530	0.745335	0.334774	0.992047	0.940474	0.249618	0.806175
Kurtosis	1.193474	13.80333	9.033500	14.77476	7.571912	1.534995	13.736068
VaR	-10.41546	-15.06590	-12.30613	-10.67675	-8.494604	-1.311541	-7.526585
CVaR	-14.46385	-25.74107	-18.59885	-16.58828	-12.44403	-1.903866	-13.97456

Table 5-3 The summarizing table for comparison with the underlying in 2008(2)

Comments on Table 5-3: It could be easily found in the sheet, except the portfolio, almost every ETF shows the negative returns in Geometric mean return. The minimum Daily return is much lower than 2007, due to the influence of financial crisis in 2008. Comparing with other ETFs, portfolio has the lowest volatility and mall positive skewness, which means the low risks of uncertainty and higher potential of getting much higher returns. A high sharp ratio means the high return among the other ETFs. To sum up, the portfolio have a stable perform in 2008, small negative VaR and CVaR also shows the small possibility of losing money during this period.

	Beta=0	Beta=0.5	Beta=1	MinVol	S&P 500
Geo Daily Mean Return	0.257084	-6.464159		0.154374	-0.559698
Daily Min Return	-17.65388	-188.6013	-356.7001	-3.150900	-22.58745
Max Drawdown	1.782871	1.549096	1.54183	1.893072	1.780220
Volatility	5.420333	45.476021	87.48792	0.963818	6.995403
Sharpe Ratio	0.054886	-0.050225	-0.046949	0.143622	-0.068625
Skewness	0.292770	1.734880	1.739422	0.249618	0.190941
Kurtosis	2.209376	17.30591	16.92798	1.534995	3.357388
VaR	-8.102512	-73.78977	-141.7072	-1.311541	-12.97348
CVaR	-11.73767	-100.5883	-191.0106	-1.903866	-17.86786

Table 5-4. Comparison with between the 2 built strategies and S&P 500 in 2008

Comments on Table 5-4: If we change our Beta from 0 to 0.5 to 1.0 or even more, which shows us the small Geometric return and minimize daily return. This is the same trend as market performs. Our portfolios have the opposite performance that is quite stable. Our minimum volatility portfolio has a positive daily return with low volatility, VaR, CVaR. In Additional, positive skewness means higher potential of getting much higher returns. S&P index shows the 2008 have a terrible financial crisis, which do great damage to global market. It seems a good choice to escape from this collapsing market.
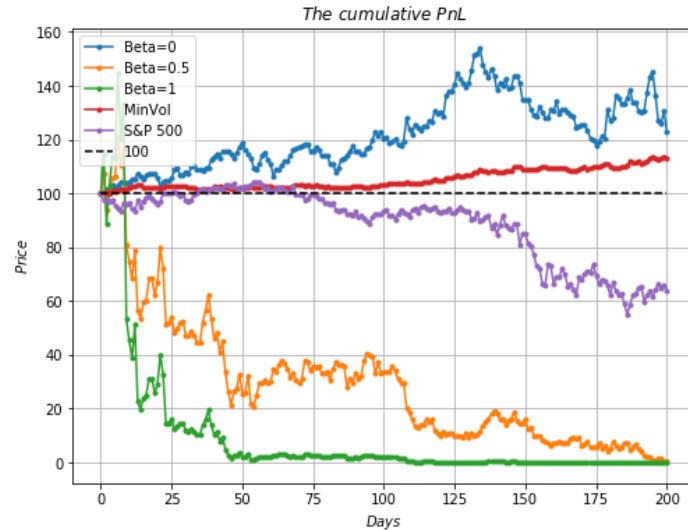
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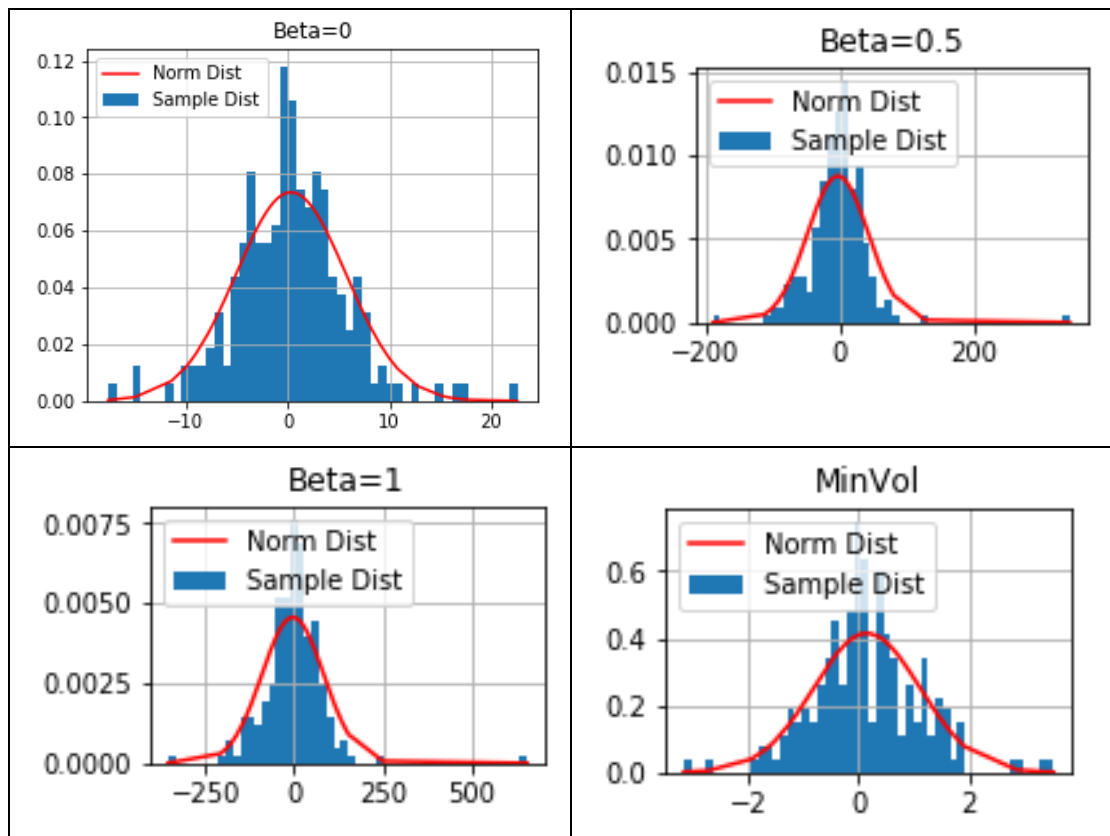**Figure 5-3. The cumulative PnL of long-term model in 2008**

Figure 5-4. The Distribution of daily Returns in 2008

Comment on Figure 5-3 & Figure 5-4: Everything changes at around 75 days, the higher beta and higher loses in such market. In our minimized volatility model, investors could have little returns, which is much better than maximum return model. It is more obvious in Figure 6-4 of high kurtosis and fat tail. About the other 3 beta values, during the 2008, it should keep the Beta in a low value even negative value in order to avoid risks

	FXE	EWJ	GLD	QQQ	SPY	SHV	DBA
Geo Daily Mean Return	0.093058	0.042121	0.237090	0.414240	0.208087	-0.001926	-0.011099
Daily Min Return	-7.687385	-10.78320	-9.413716	-12.40654	-13.19656	-0.272115	-16.82420
Max Drawdown	1.837507	1.551381	1.806291	1.803382	1.734888	2.331725	2.556632
Volatility	2.118370	4.354482	3.277630	4.272672	4.524641	0.072716	4.265590
Sharpe Ratio	0.039744	0.014214	0.073417	0.101270	0.051051	-0.271067	0.001746
Skewness	-0.049684	0.412089	0.117541	0.083556	0.062683	-0.622952	-0.297116
Kurtosis	1.769237	1.984675	0.561331	1.149262	1.647481	2.079917	0.784925
VaR	-3.496894	-6.952856	-5.059145	-7.396450	-7.863615	-0.158658	-8.109727
CVaR	-4.728233	-9.172984	-6.879332	-8.777740	-10.33353	-0.196509	-9.893531

Table 5-5 The summarizing table for comparison with the underlying in 2009(1)

	USO	ILF	EPP	FEZ	XBI	Opt_Port	SPY
Geo Daily Mean Return	0.168198	0.711472	0.606641	0.270962	-0.063309	0.149653	0.208087
Daily Min Return	-26.71141	-18.70458	-18.61923	-21.41666	-13.24946	-2.202038	-13.19656
Max Drawdown	2.221986	2.004759	1.864739	2.059582	1.657517	2.076521	1.734888
Volatility	8.051465	7.036924	6.175081	6.383292	4.613803	0.625053	4.524641
Sharpe Ratio	0.034781	0.112602	0.107636	0.052431	-0.008427	0.212198	0.051051
Skewness	-0.205362	-0.193510	-0.054143	-0.330875	0.337159	-0.011739	0.062683
Kurtosis	0.865252	0.603663	0.733221	1.114589	2.321142	1.074867	1.647481
VaR	-12.50675	-11.45613	-10.47068	-12.11484	-7.164035	-0.726945	-7.863615
CVaR	-18.94799	-16.00818	-13.22931	-15.33364	-10.24368	-1.259395	-10.33353

Table 5-5 The summarizing table for comparison with the underlying in 2009(2)

Comments on Table 5-5: Consider the final output in 2009, we could find that the geometric mean return, daily minimize return and maximum drawdown are in the range of the maximum and minimize one of the 13 ETFs. The small volatility ratio and large sharp ratio which comparing with most of 13 single ETF shows us the small uncertainty and larger returning among ETFs. A small negative Kurtosis of portfolio means both higher and lower returns have a higher likelihood to happen. All the numbers reflect that, our portfolio could have the much stable earning with less risk. Value of Var and CVaR have verify this conclusion. All the comparing of 3 years data, which strongly proved that the minimize volatility model shows the power of lowering the financial risks. The market is recovering from the crisis in 2008.

	Beta=0	Beta=0.5	Beta=1	MinVol	S&P 500
Geo Daily Mean Return	0.312104	0.695798	0.527964	0.149653	0.580122
Daily Min Return	-13.64307	-40.37530	-70.99890	-2.202038	-10.69744
Max Drawdown	1.979463	1.685289	2.110468	2.076521	1.604738
Volatility	3.713653	16.225873	20.240889	0.625053	3.908179
Sharpe Ratio	0.086664	0.073973	0.066416	0.212198	0.151613
Skewness	-0.414506	0.102440	-0.322451	-0.011739	0.435928
Kurtosis	1.792463	0.548297	0.855883	1.074867	2.658861
VaR	-5.522091	-26.07807	-37.59047	-0.726945	-5.963635
CVaR	-8.859254	-32.95418	-46.64795	-1.259395	-7.912937

Table 5-6. Comparison with between the 2 built strategies and S&P 500 in 2009

Comment on Table 5-6: Above, we show the results by changing the Value of Beta. If we change our Beta from 0 to 0.5 to 1.0 or even more, which shows us the medium value of the daily minimize returns and greater relationship with the market. This means the market is getting better at that time. The minimum volatility portfolio has a relatively lower daily return with low volatility and extremely low VaR, CVaR. There is a little bit higher skewness shows that it may have lower probability for lower return. To sum up, high volatility in S&P 500 means a possibility of unstable financial market and which finally proved that it is a good method to create a minimize volatility portfolio to deal with the unstable market. Or we could change our beta Value according with market situation.
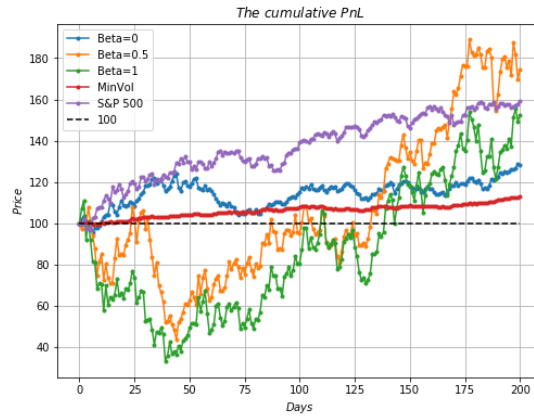
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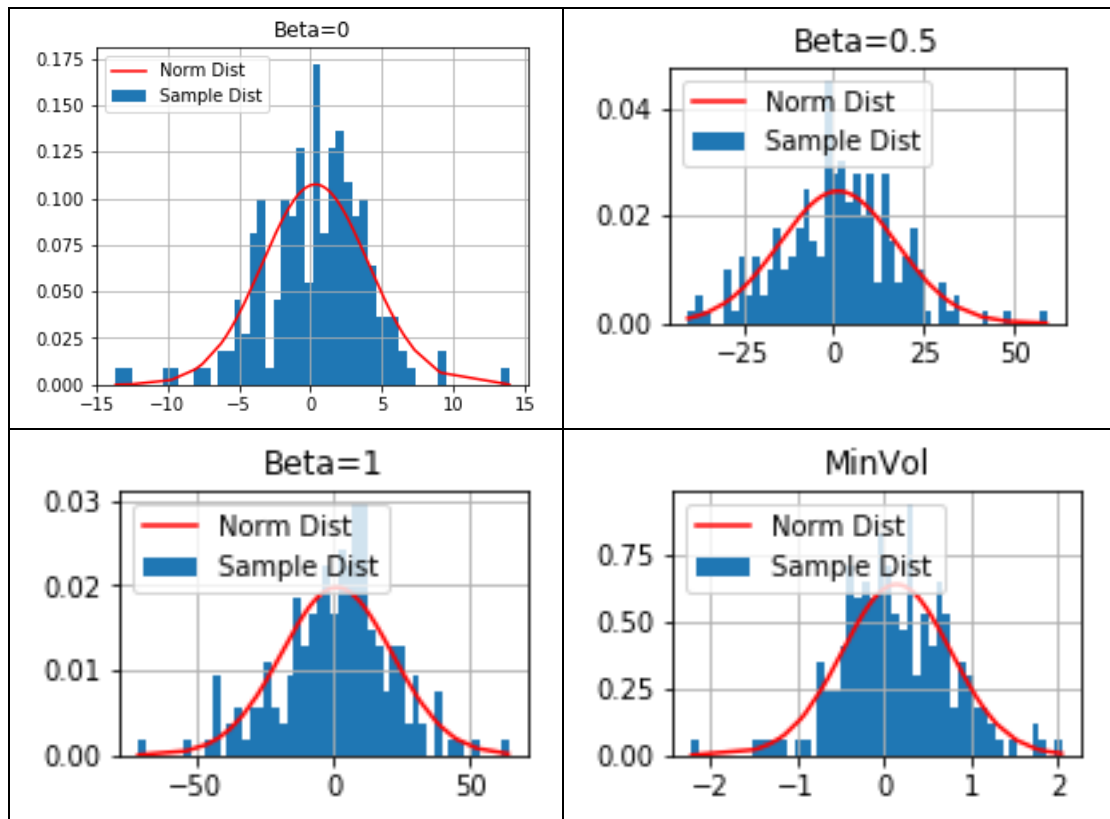**Figure 5-5. The cumulative PnL of long-term model in 2009**

Figure 5-6. The Distribution of daily Returns in 2009

Comment on Figure 5-5 & Figure 5-6: This plot is much similar with the plot in 2007. The higher Beta value means to be willing to take much more risks to get more returns. Higher beta means higher relations with the market, which is the opposite in 2008. More risks mean more loss has been disappeared. In 2009, the economic has shown a trend of recovering.

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6. Comparing Long-term, Mid-term, short-term models
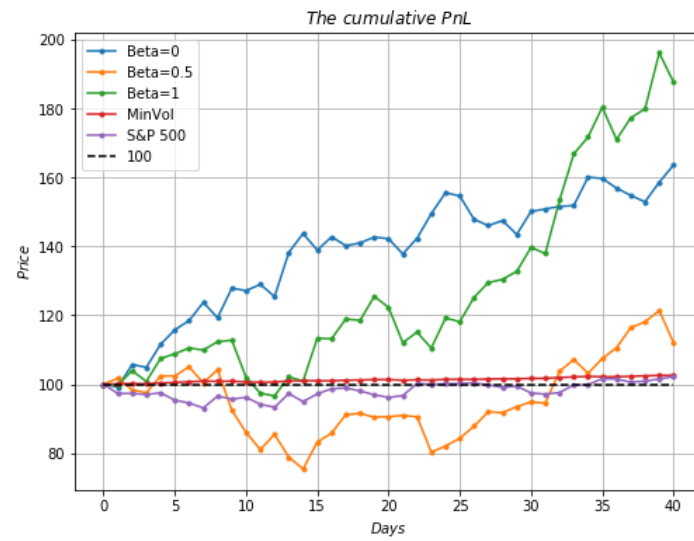
Figure 6-1. Short-term PnL in 2008
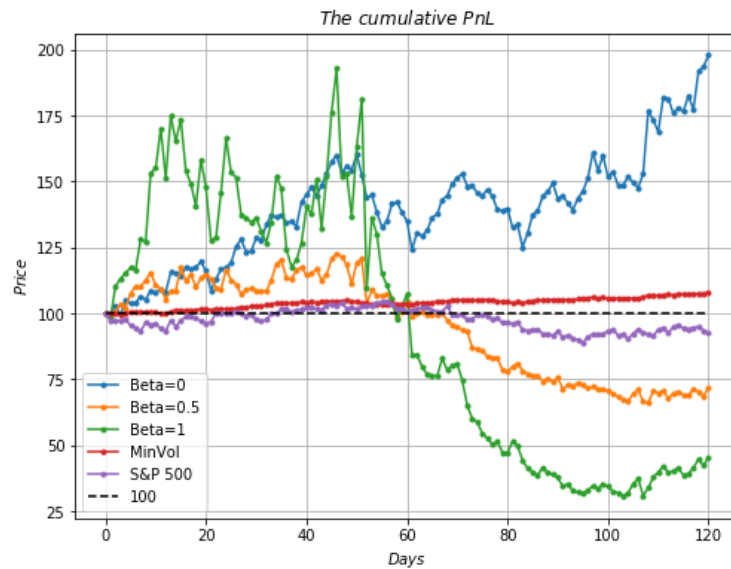
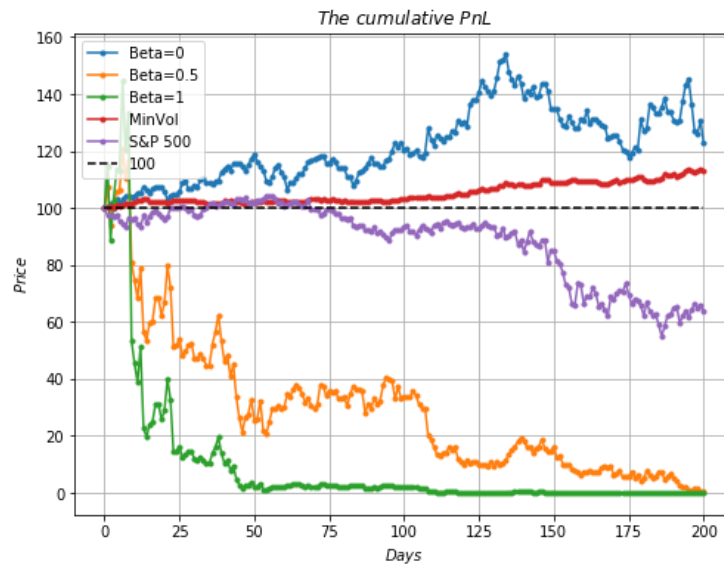**Figure 6-2. Mid-term PnL in 2008**

Figure 6-3. Long-term PnL in 2008

Comments on Figure 6-(1 to 3): Comparing the short-term (40 Days), mid-term (120 Days) and long-term (200 Days) model using the data in 2008. Market price will not change a lot in short-term and middle-term model, however, when it comes to long-term model, it is in a bad situation due to 2008 financial crisis. ST will have a quick response to market changes while MT and LT will not be sensitive to market changes. Thus, when we foresee the potential risks in the future, we should increase the frequency of adjusting the portfolio to erase the damage that would happen.
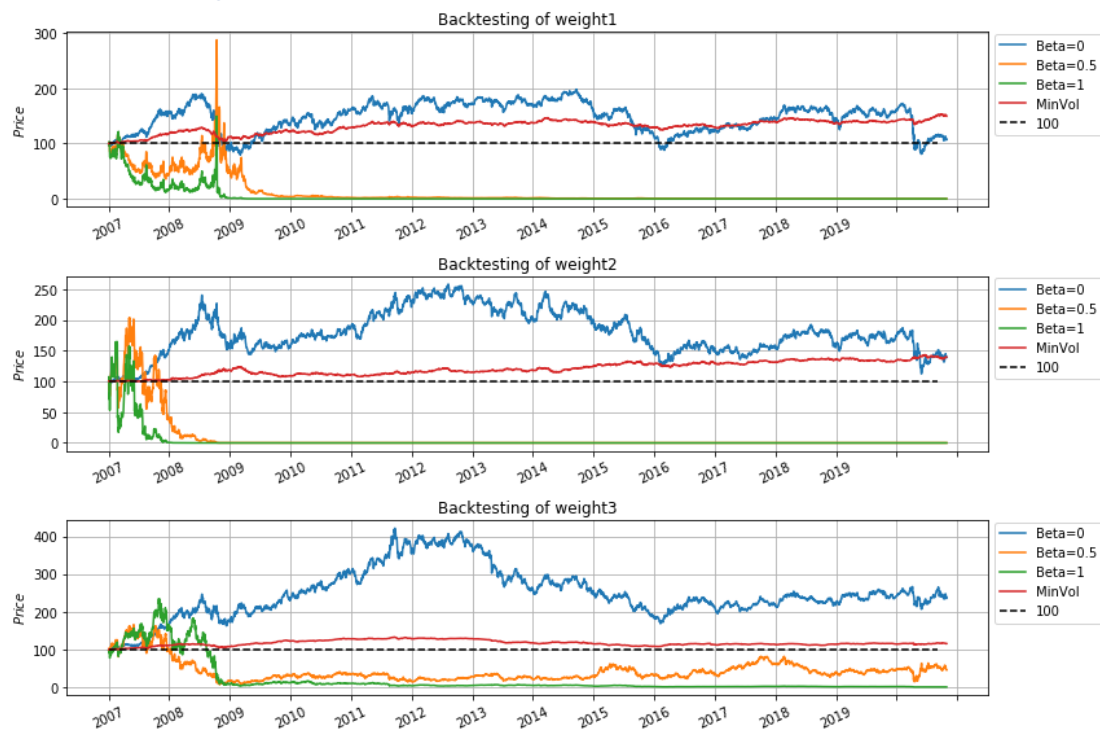
7. Back-testing from 2020-09-25

Figure 7-1. The back-testing with different weights

Comments on Figure-7:

We used the LT method of 2007, 2008 and 2009 to back-test the portfolio price from 2020-09-25, and get some conclusions:

① the minimize volatility method performs good in the whole period, which influenced little by the real market. Comparing with other portfolio in the market, it seems more stable but with low expected returns.

② when it related to the beta value, the absolute value of beta means the relativity with the real market. No matter the positive or the negative, it should depend on the real market, because at that time, it shows strong relationship with market. Under that situation, we should keep beta in a level around 0 when we have no idea about which direction market will continue.