Physics 406 Homework

$\mathbb{E}$van $\mathbb{C}$arpenter

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1 Homework 1

Problem 1.

problem 1

Problem 2.

problem 2

Problem 3.

problem 3

Problem 4.

Suppose that a particle moving in one dimension is confined to $x > 0$, and it's energy is $E = \frac{p^2}{2m} + mgx$ Make a sketch to indicate what region of classical phase space is accessible to this particle if its energy lies between E_0 and $E_0 + \delta E_0$.

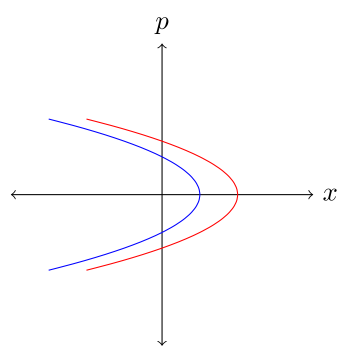

Figure 1: Particle constrained

2 Homework 2

Problem 1.

(a) Show that the number of states $\phi(E)$ with energy less than E, for a particle of mass m in a cubical box of side L is:

$$\phi(E) = \frac{\pi}{6}\left(\frac{L}{\pi\hbar}\right)^3 (2mE)^{3/2}$$

Hint: Use the energy levels 2.1.3 in Reif and treat the n as continuous variables.

(b) Calculate $\Omega(E)$

(c) A nitrogen molecule at room temperature has a typical energy of 6×10^{-14}ergs. Calculate $\phi(E)$ for a particle in a box of side length 10cm. Also calculate $\Omega(E)$ assuming $\delta E = 10^{-24}$ergs

Problem 2. *Reif 2.4*

Consider an isolated system consisting of a large number N of weakly interacting localized particles of spin $\frac{1}{2}$. Each particle has a magnetic moment μ which can point either parallel or antiparallel to an applied field H. The energy od the system is then $E = -(n_1 - n_2)\mu H$, where n_1 is the number of spins aligned parallel to H and n_2 is the number of spins aligned antiparallel to H.

(a) Consider the energy range between $E + \delta E$ where δE is much smaller than E, but E is still microscopically large, so $\mu H \ll \delta E \ll E$. What is $\Omega(E)$ (the total number of states in the energy range)?

(b) Write down an expression for $\ln(\Omega(E))$ as a function of E. Simplify this expression by using Stirling's formula in it's simplest form:

$$\ln(n!) \approx n \ln(n) - n$$

(c) Assume that the energy E is in a region where $\Omega(E)$ is appreciable $\rightarrow$ that it is not close to the extreme possible values $\pm N\mu H$ which it can assume. In this case apply a Gaussian approximation to part (a) to obtain a simple expression for $\Omega(E)$ as a function of E.

Problem 3. *Reif 2.5*

Consider the infinitesimal quantity

$$A(x, y)dx + B(x, y)dy \equiv đF$$

(a) Suppose $đF$ is an exact differential so that $F = F(x, y)$. Show that A, B must satisfy the condition:

$$\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}$$

(b) If $đF$ is an exact differential, show that the integral $\int đF$ evaluated along any clsoed path on the xy plane must vanish.

Problem 4. *Reif 2.7*

(a) Consider a particle confined to a cubical box. The possible energy levels are given by

$$E = \frac{(\hbar\pi)^2}{2m}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right]$$

Show that the force exerted by the particle in this state on a wall perpendicular to the x axis is given by

$$F_x = -\frac{\partial E}{\partial L_x}$$

while the length L_x is changed quasi-statically by an amount dL_x.

(b) Calculate explicitly the pressure on this wall. By averaging over all posible states, find an expression for the mean pressure on this wall (Hint: Exploit the property that $\overline{n_x^2} = \overline{n_y^2} = \overline{n_z^2}$ must be true by symmetry.) Show that the mean pressure can be simply expressed in terms of mean energy $\overline{E}$ of the particle and the volume $V = L_x L_y L_z$ of the box.