Statistical Mechanics

Physics 406 at **University of Michigan**

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Winter Semester 2022

Lecture 1. (Jan 05) *States, Probability and Binomial Distribution*

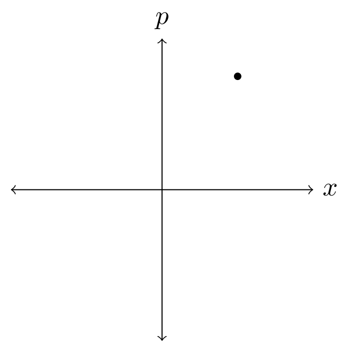

Figure 1: Phase space of 1-D particle

Lecture 2. (Jan 10) *Ensembles*

Lagrange multipliers

$$S = -k \sum_r p_r \ln(p_r) \tag{0.1}$$

Microcanonical ensemble: All accessible microstate are equally probable

Lecture 3. (Jan 12) *Finding total microstate*

N particles in volume V with energy between $E, E + \delta E$. Counting number of microstate by using phase space
simplifying example : a 1-D particle has only x and p. Plot in phase space Example in harmonic oscillator with ellipse and shading in phi(E) and Omega(E) Include text in caption explaining equations below it.
Moving to 3-D talk about degrees of freedom and volume of h_0.
Integrating to get Phi(E) with multiintegrals and then taylor approx to get Omega
Quantum Description–¿ specify microstate with quantum numbers
example with simp harmon oscill

Lecture 4. (Jan 19) *More on Microcanonical Ensemble*

$\Omega(E) = \#$ of states with energy between $E + \delta E$
Describing energy levels of each particle, think N-cube
Now particle can interact!! Mechanical interactions and thermal interactions(both macro descriptions).
1 isolated system at equilibrium→ same system but with partition, now 2 systems. A^0 is comprised of A, A'.
Macro parameters of A^0 are for both states (N,V,E,T,. . .).

Thermal Interaction External parameters of A, A' are fixed but mean energy transferred from one system to the other as a result of purely thermal interactions called heat. Probabilities of energy states can change when systems interact $P(r)$

Mechanical Interaction External Parameters of A, A' change, one does *work* on the other! This causes the mean energies of A, A' to change.

$$\overline{E} = \sum_r p_r E_r$$

Pure thermal and purely mech example in inf sqwell

Lecture 5. (Jan 24)

Pure thermal interaction changes p_r
Pure mechanical interaction changes E_r

1 Derivative Crusher Algorithm

Derivative crusher algorithm taught by Ratindranath Akhoury

Variable	Easy to keep fixed?	Easy to measure change?
S	**Yes**, Adiabatic processes	**No**, change in S depends on change in Q, which is hard to measure
T	**Yes**, with a heat bath or feedback system	**Yes**, by using a thermometer
V	**No**, not easily, thermal expansion of the container is a byproduct of the container being thermally conductive, the volume changes with heat	**Yes**
P	**Yes**, keeping equilibrium with atmosphere	**Yes**, barometer

Table 1: T, P are the easiest quantities to measure and control, making $G(T, P)$ the easiest fundamental relationship to work with.

Looking at $G(T, P) = E - TS - PV$ more closely:

$$\left(\frac{\partial G}{\partial T}\right)_P = -S \qquad \left(\frac{\partial G}{\partial P}\right)_T = V$$

$$\left(\frac{\partial^2 G}{\partial T^2}\right)_P = -\left(\frac{\partial S}{\partial T}\right)_P \equiv \frac{C_p}{T} \qquad \left(\frac{\partial^2 G}{\partial P^2}\right)_T = \left(\frac{\partial V}{\partial P}\right)_T \equiv \kappa V$$

$$\left[\frac{\partial}{\partial T}\left(\frac{\partial G}{\partial P}\right)_T\right]_P = \left[\frac{\partial}{\partial P}\left(\frac{\partial G}{\partial T}\right)_P\right]_T = \alpha V$$

C_p is specific heat at constant pressure.
κ is isothermal compressibility.
α is volume coefficient of expansion.

1.1 Math Background

A partial derivative $\left(\frac{\partial x}{\partial y}\right)_z$ where $\{x, y, z\}$ can be any of $\{S, T, P, V, E, F, G, H\}$ can be expressed in terms of in terms of $1^{st}2^{nd}$ derivatives