# Derivative Crusher Algorithm
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**Idea:** A partial derivative with thermodynamically significant variables $\left(\frac{\partial x}{\partial y}\right)_z$ where $\{x, y, z\}$ can be any of $\{S, T, P, V, E, F, G, H\}$ can be expressed in terms of in terms of first and second derivatives of fundamental relations. This re-arranging of expressions is very useful in thermodynamics and statistical mechanics, as it allows us to find simple relations between physical quantities in a thermodynamical system. The Derivative Crusher Algorithm is an algorithm that "crushes" a thermodynamically significant partial derivative into useful and easily measureable quantities. Developed and taught by Professor Ratindranath Akhoury.

## 1 Background

| Variable | Easy to keep fixed? | Easy to measure change? |
|---|---|---|
| $S$ | **Yes**, Adiabatic processes | **No**, change in S depends on change in Q, which is hard to measure |
| $T$ | **Yes**, with a heat bath or feedback system | **Yes**, by using a thermometer |
| $V$ | **No**, not easily, thermal expansion of the container is a byproduct of the container being thermally conductive, the volume changes with heat | **Yes** |
| $P$ | **Yes**, keeping equilibrium with atmosphere | **Yes**, barometer |

Table 1: $T, P$ are the easiest quantities to measure and control, making $G(T, P)$ the easiest fundamental relationship to work with.

Looking at $G(T, P) = E - TS - PV$ more closely:

$$\left(\frac{\partial G}{\partial T}\right)_P = -S \qquad \left(\frac{\partial G}{\partial P}\right)_T = V$$

$$\left(\frac{\partial^2 G}{\partial T^2}\right)_P = -\left(\frac{\partial S}{\partial T}\right)_P \equiv \frac{C_p}{T} \qquad \left(\frac{\partial^2 G}{\partial P^2}\right)_T = \left(\frac{\partial V}{\partial P}\right)_T \equiv \kappa V$$

$$\left[\frac{\partial}{\partial T}\left(\frac{\partial G}{\partial P}\right)_T\right] = \left[\frac{\partial}{\partial P}\left(\frac{\partial G}{\partial T}\right)_P\right]_T = \alpha V$$

$C_p$ is specific heat at constant pressure.
$\kappa$ is isothermal compressibility.
$\alpha$ is volume coefficient of expansion.

### 1.1 Background

The derivative crusher algorithm is made of three partial derivative operations:

1. Bringing $y$ to the numerator:
$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

2. Bringing $z$ to the numerator:
$$\left(\frac{\partial x}{\partial y}\right)_z = -\frac{\left(\frac{\partial z}{\partial y}\right)_x}{\left(\frac{\partial z}{\partial x}\right)_y}$$

3. Introduce new variable $w$:
$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{\left(\frac{\partial x}{\partial w}\right)_z}{\left(\frac{\partial y}{\partial w}\right)_z}$$

## 2 Steps

1. Bring potentials $E, F, G, H$ to the numerator and eliminate them using:
$$dE = TdS - PdV \qquad dF = -SdT - PdV$$
$$dG = -SdT + VdP \qquad dH = TdS + VdP$$

2. Bring $S$ to numerator using Maxwell Relations depending on what $z$ is in $\left(\frac{\partial x}{\partial y}\right)_z$
$$z = T \qquad \boxed{\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \quad , \quad \left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P}$$
$$z = P, V \quad \boxed{\left(\frac{\partial S}{\partial T}\right)_P = \frac{C_P}{T} \quad , \quad \left(\frac{\partial S}{\partial T}\right)_V = \frac{C_v}{T}}$$

3. Bring $V$ to numerator in order to get $\alpha$ or $\kappa$.

4. Eliminate $C_v$ in favor of $C_p, \alpha, \kappa$