

# Physics 457 Hw 1

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## Problem 1.

Make the following unit conversions, and show all work. Note that this tests how you use natural units where  $c = \hbar = k_B = 1$ . Thus, there is one fundamental dimension, which we can take to be energy and express it in GeV to various powers.

- a) 1 GeV, in joules (energy)
- b) 1 GeV, in Kelvin (temperature)
- c) 1 GeV, in kilograms (mass)
- d) 1 GeV<sup>-1</sup>, in meters (length)
- e) 1 GeV<sup>-1</sup>, in seconds (time)
- f) 1 GeV<sup>4</sup>, in kg/m<sup>3</sup> (mass density)

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a)  $1 \text{ GeV} = 1.602 \times 10^{-10} \text{ J}$

b)  $k_B = 8.617333262 \times 10^{-5} \text{ eV/K} = 1 \rightarrow 8.617333262 \times 10^{-14} \text{ GeV} = 1 \text{ K} \rightarrow 1 \text{ GeV} = 1.16045 \times 10^{13} \text{ K}$

c)  $E = mc^2$  so  $E/c^2 = m \rightarrow 1 \text{ GeV} = 1.783 \times 10^{-27} \text{ kg}$

d)  $E = \frac{\hbar c}{\lambda}$  so  $1 \text{ GeV}^{-1} = \frac{\lambda}{\hbar c} \rightarrow 1 \text{ GeV}^{-1} = 1.973 \times 10^{-16} \text{ m}$

e)  $\hbar = 6.58212 \times 10^{-16} \text{ eVs} \rightarrow \frac{\hbar}{\text{eV}} = 6.58212 \times 10^{-16} \text{ s}$   
 $\rightarrow \frac{1}{\text{eV}} = 6.58212 \times 10^{-16} \text{ s} \rightarrow 1 \text{ GeV}^{-1} = 6.58212 \times 10^{-25} \text{ s}$

f) Using results from c),d), the answer is:

$$\frac{1 \text{ GeV}}{1 \text{ GeV}^{-3}} = \frac{1.783 \times 10^{-27} \text{ kg}}{(1.973 \times 10^{-16} \text{ m})^3}$$
$$1 \text{ GeV}^4 = 2.322 \times 10^{20} \text{ kg/m}^3$$

**Problem 2.**

Unstable particles appear to live longer if moving, therefore can travel a longer distance after creation. Consider the following problems and calculate the flight distances.

- Calculate the flight distance of a muon with 100 GeV energy. Note that the muon lifetime (in its rest frame) is  $2.2\mu s$ .
- Calculate the flight distance of a  $B^+$ -meson with 20 GeV energy if its lifetime is  $1.6 \times 10^{-12}$  s and its mass is 5.38 GeV.
- Pions are produced in the upper atmosphere when a proton from outer space hits a proton in the atmosphere. The pions then decay into muons:

$$\begin{aligned}\pi^- &\rightarrow \mu^- + \bar{\nu}_\mu \\ \pi^+ &\rightarrow \mu^+ + \nu_\mu\end{aligned}$$

But the lifetime of the pion ( $2.6 \times 10^{-8}$  s) is much shorter than that of the muon. If the pion is produced at 800 meters above the ground, can it reach the ground if its speed is  $0.998c$ ?

- Given the energy and  $\Delta t_{lab}$  and knowing

$$\begin{aligned}d_{flight} &= v_{lab} \Delta t_{lab} = \boxed{v \gamma \Delta t_\mu} \\ \gamma &= E/m_\mu \cong 943.4 \implies v \cong 1(c) \\ \Delta t_\mu &= 2.2\mu s \\ m_\mu &= 0.1056 \text{ GeV}/c^2\end{aligned}$$

from lecture, I can calculate:

$$d_{flight} = 943.4 * 3 \times 10^8 \frac{m}{s} * 2.2 \times 10^{-6} s = 622,644 m \rightarrow \boxed{d_{flight} = 622 \text{ km}}$$

- $\gamma = \frac{E}{m_{B^+}} = 3.71747 \rightarrow v \cong 0.96314c$

$$\boxed{d_{flight} = 0.001717 \text{ m}}$$

- $\gamma = \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2} \cong 15.819$

$$d_{flight} = 0.998c \times 15.819 \times 2.6 \times 10^{-8} s = \boxed{123.056 \text{ m}}$$

**Problem 3.**

Antiprotons were first created at Lawrence Berkeley National Lab (LBL) in 1955 by a proton beam hitting a proton target with the following reaction:

$$p + p \rightarrow 3p + \bar{p}$$

What is the minimum total energy  $E$  of the proton beam to allow this reaction? Please give your answer in unit of proton mass  $m_p$ . (Hint: using center-of-mass energy  $E_{CM}$  conservation, and assume the final particles are produced at rest.)

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