Physics 457 Hw 1

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Due: 01-19-2022

Problem 1.

Make the following unit conversions, and show all work. Note that this tests how you use natural units where $c = \hbar = k_B = 1$. Thus, there is one fundamental dimension, which we can take to be energy and express it in GeV to various powers.

a) 1 GeV, in joules (energy)

b) 1 GeV, in Kelvin (temperature)

c) 1 GeV, in kilograms (mass)

d) 1 GeV^{-1}, in meters (length)

e) 1 GeV^{-1}, in seconds (time)

f) 1 GeV^4, in kg/m^3 (mass density)

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a) $1\ GeV = 1.602 \times 10^{-10}\ J$

b) $k_B = 8.617333262 \times 10^{-5} eV/K = 1 \rightarrow 8.617333262 \times 10^{-14}\,\text{GeV} = 1K \rightarrow \boxed{1\,\text{GeV} = 1.16045 \times 10^{13} K}$

c) $E = mc^2$ so $E/c^2 = m \rightarrow \boxed{1\,\text{GeV} = 1.783 \times 10^{-27} kg}$

d) $E = \frac{hc}{\lambda}$ so $1\,\text{GeV}^{-1} = \frac{\lambda}{hc} \rightarrow \boxed{1\,\text{GeV}^{-1} = 1.973 \times 10^{-16} m}$

e) $\hbar = 6.58212 \times 10^{-16} eV\,s \rightarrow \frac{\hbar}{eV} = 6.58212 \times 10^{-16} s$
$\rightarrow \frac{1}{eV} = 6.58212 \times 10^{-16} s \rightarrow \boxed{\text{GeV}^{-1} = 6.58212 \times 10^{-25} s}$

f) Using results from c),d), the answer is:

$$\frac{1\,\text{GeV}}{1\,\text{GeV}^{-3}} = \frac{1.783 \times 10^{-27} kg}{(1.973 \times 10^{-16} m)^3}$$

$$\boxed{1\,\text{GeV}^4 = 2.322 \times 10^{20} kg/m^3}$$

Problem 2.

Unstable particles appear to live longer if moving, therefore can travel a longer distance after creation. Consider the following problems and calculate the flight distances.

a. Calculate the flight distance of a muon with 100 GeV energy. Note that the muon lifetime (in its rest frame) is $2.2\mu s$.

b. Calculate the flight distance of a B$^+$-meson with 20 GeV energy if its lifetime is 1.6×10^{-12} s and its mass is 5.38 GeV.

c. Pions are produced in the upper atmosphere when a proton from outer space hits a proton in the atmosphere. The pions then decay into muons:

$$\pi^- \to \mu^- + \overline{\nu}_\mu$$
$$\pi^+ \to \mu^+ + \nu_\mu$$

But the lifetime of the pion (2.6 $\times 10^{-8}$ s) is much shorter than that of the muon. If the pion is produced at 800 meters above the ground, can it reach the ground if its speed is 0.998c?

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a. Given the energy and Δt_{lab} and knowing

$$d_{flight} = v_{lab}\Delta t_{lab} = \boxed{v\gamma\Delta t_\mu}$$
$$\gamma = E/m_\mu \approxeq 943.4 \implies v \approxeq 1(c)$$
$$\Delta t_\mu = 2.2\mu s$$
$$m_\mu = 0.1056\,\text{GeV}/c^2$$

from lecture, I can calculate:

$$d_{flight} = 943.4 * 3 \times 10^8 \frac{m}{s} * 2.2 \times 10^{-6} s = 622,644m \to d_{flight} = 622 \text{ km}$$

b. $\gamma = \frac{E}{m_{B^+}c^2} = 3.71747 \to v \approxeq 0.96314c$

$$d_{flight} = 0.001717 \text{ m}$$

c. $\gamma = \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2} \approxeq 15.819$

$$d_{flight} = 0.998c \times 15.819 \times 2.6 \times 10^{-8} s = 123.056 \text{ m}$$

No, the pion will not reach the ground!

Problem 3.

Antiprotons were first created at Lawrence Berkeley National Lab (LBL) in 1955 by a proton beam hitting a proton target with the following reaction:

$$p + p \to 3p + \overline{p}$$

What is the minimum total energy E of the proton beam to allow this reaction? Please give your answer in unit of proton mass m_p. (Hint: using center-of-mass energy E_{CM} conservation, and assume the final particles are produced at rest.)

$$\begin{aligned}
E_{CM} &= \sqrt{2E_{beam}m_pc^2} = \sqrt{2E_Bm_pc^2} \\
E_{CM} + m_pc^2 + 0 + m_pc^2 &= 3m_pc^2 + m_{\overline{p}}c^2 = 4m_pc^2 \\
E_{CM} + 2m_pc^2 &= 4m_pc^2 \\
\sqrt{2E_Bm_pc^2} + 2m_pc^2 &= 4m_pc^2 \\
\sqrt{2E_Bm_pc^2} &= 2m_pc^2 \\
2E_Bm_pc^2 &= 4m_p^2c^4 \\
E_B &= 2m_pc^2 \\
&\boxed{E_{Beam} = 2m_p}
\end{aligned}$$