Computational Statistics HW3

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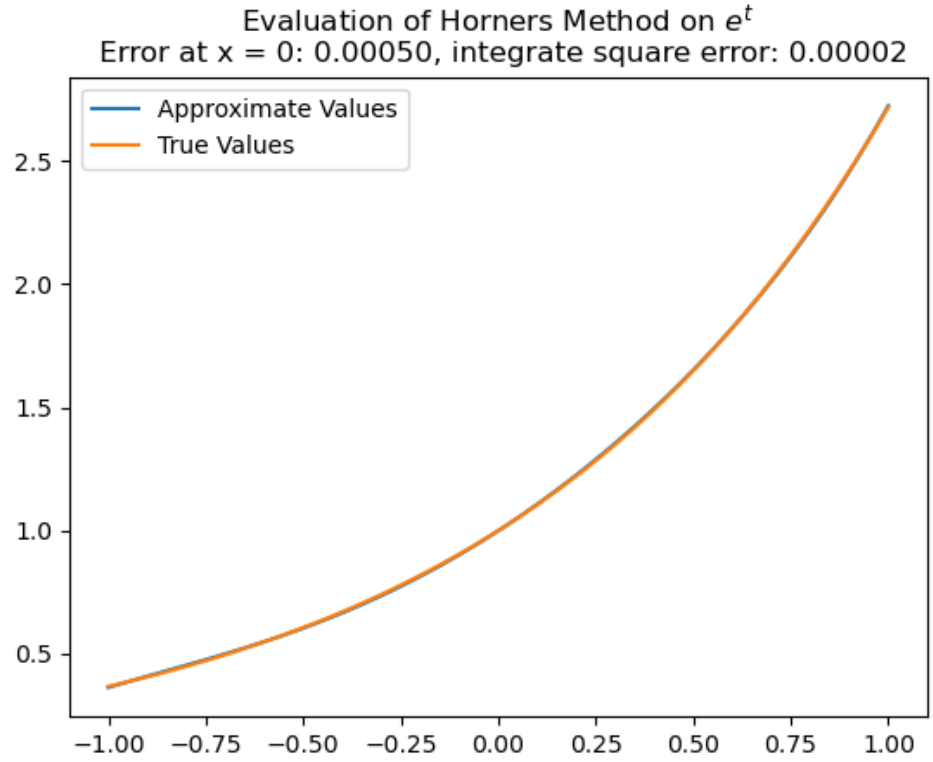
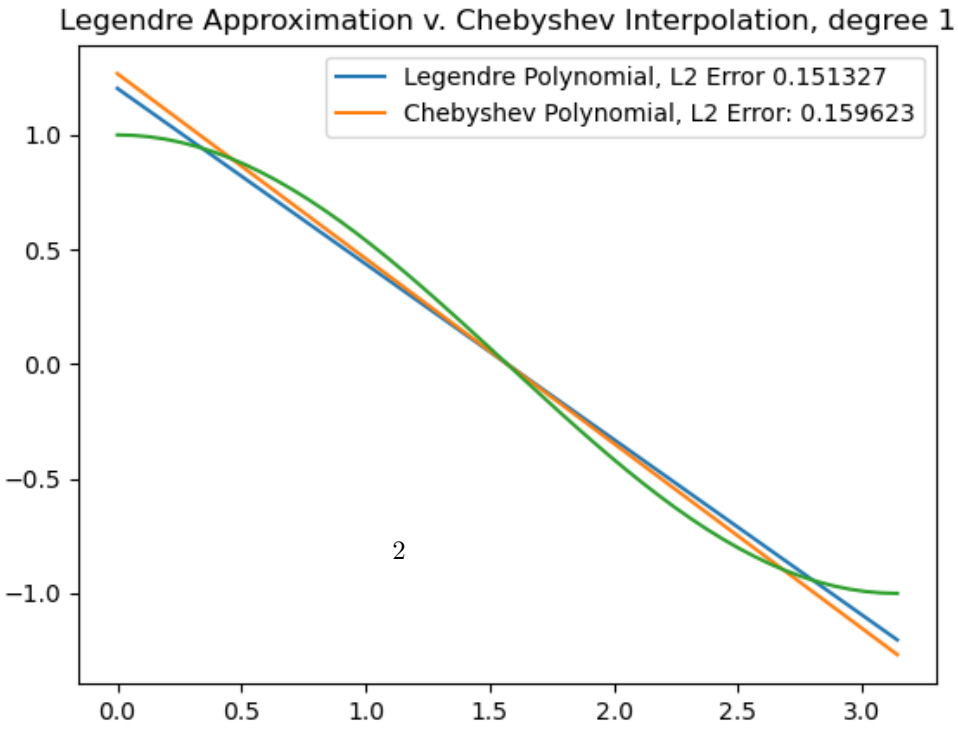
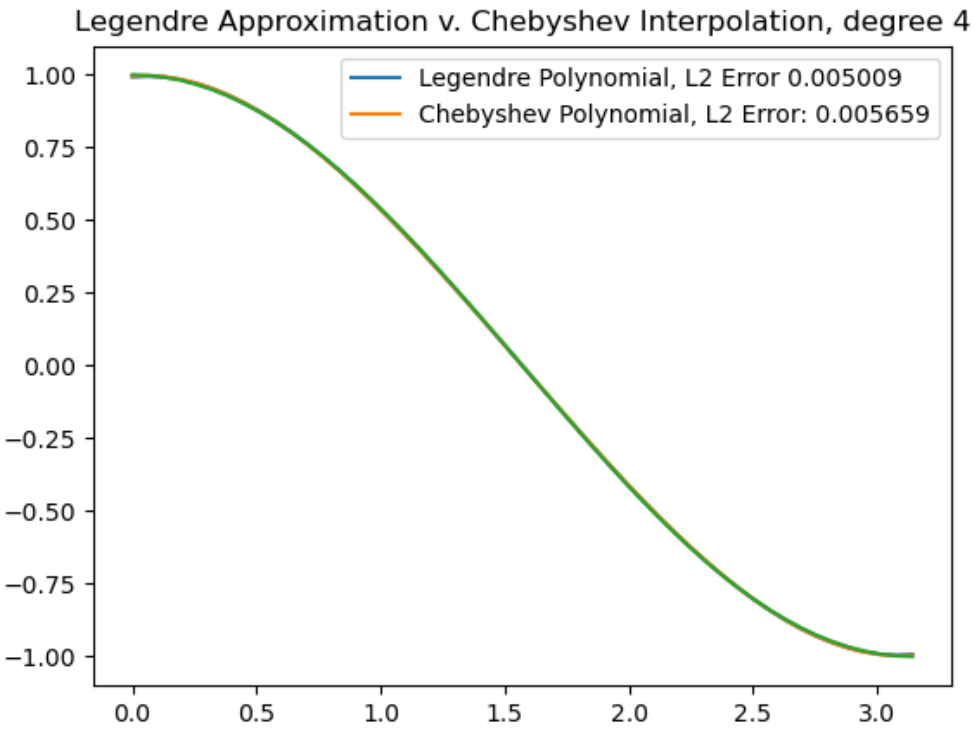
Problem 1

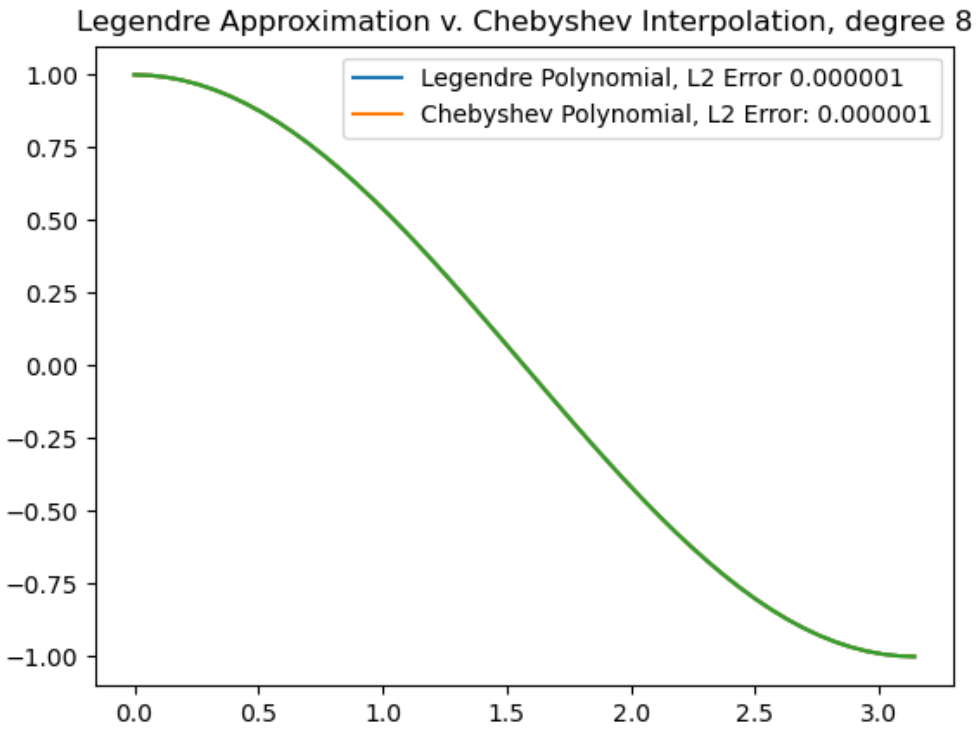
```python
def horners(coef, x, r, t, s):
#Polynomial evaluation through horner method
#Input: Polynomial coefficients, evaluation point
#Output: Approximation of function at evaluation point
    deg = coef.shape[0]
    f_k2 = coef[-1]
    f_k1 = coef[-2] + f_k2*(r*x-s)
    for k in range(deg-2, 0, -1):
        f_k = coef[k] + f_k1*(r*x-s) - f_k2*t
        f_k2 = f_k1
        f_k1 = f_k
    p_0 = coef[0] + f_k1*x - f_k2
    return p_0
```

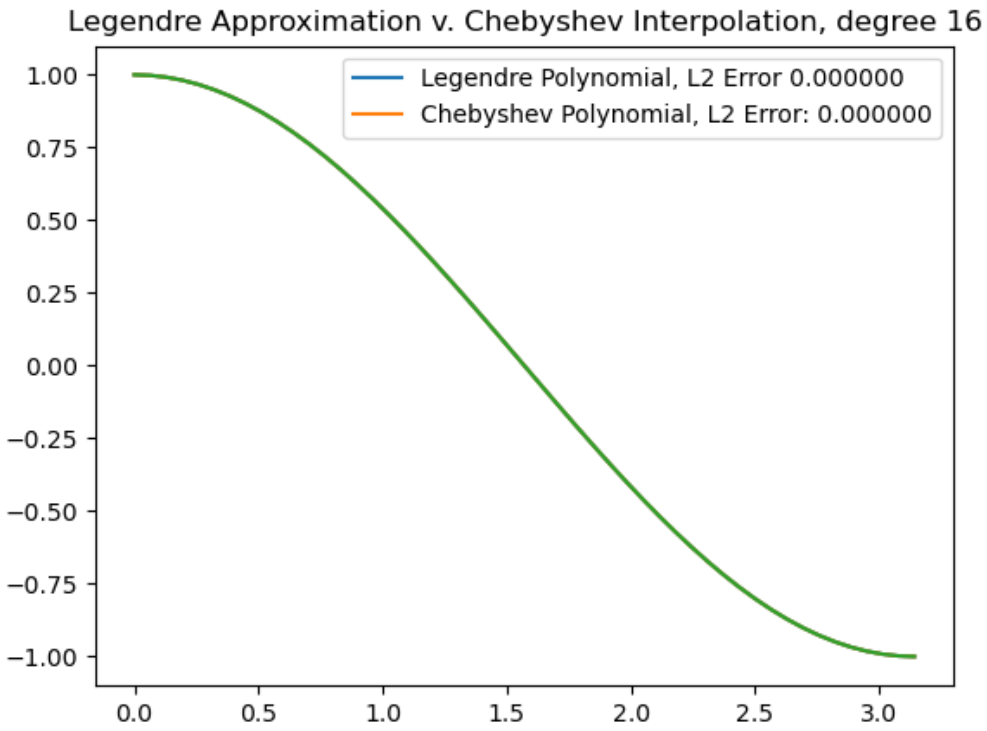

Evaluation of Horners Method on e^t
Error at x = 0: 0.00050, integrate square error: 0.00002
Approximate Values
True Values
2.5
2.0
1.5
1.0
0.5
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

Problem 2


Legendre Approximation v. Chebyshev Interpolation, degree 1
Legendre Polynomial, L2 Error 0.151327
Chebyshev Polynomial, L2 Error: 0.159623
1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0


Legendre Approximation v. Chebyshev Interpolation, degree 4
Legendre Polynomial, L2 Error 0.005009
Chebyshev Polynomial, L2 Error: 0.005659
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0


Legendre Approximation v. Chebyshev Interpolation, degree 8
Legendre Polynomial, L2 Error 0.000001
Chebyshev Polynomial, L2 Error: 0.000001
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0



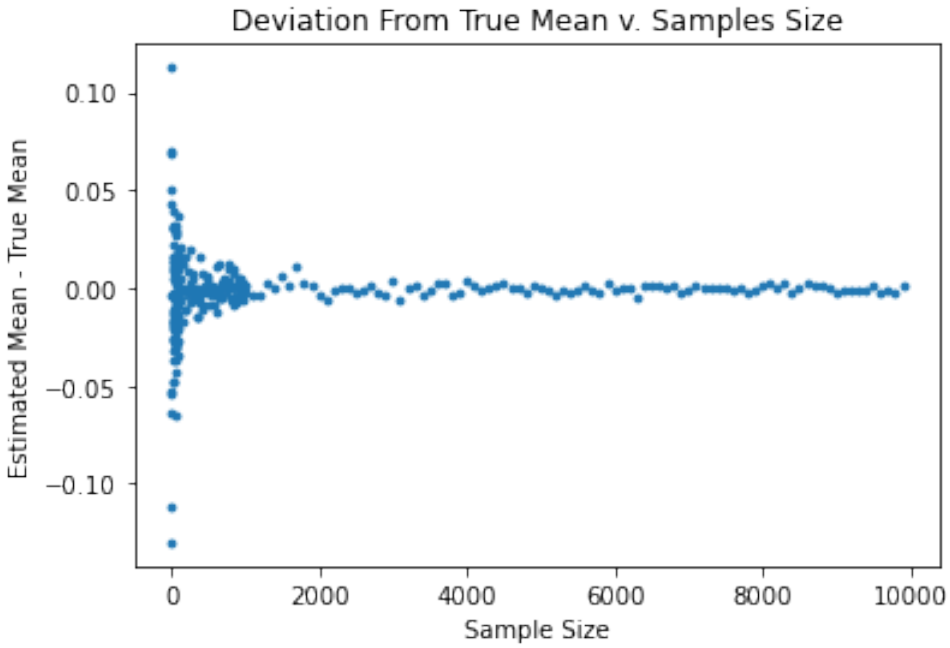
Degree	L2 Error Cheb	L2 Error Leg
1	.151327	.159623
2	.151327	.174876
4	.005009	.005659
8	.000001	.000001
16	.000000	.000000

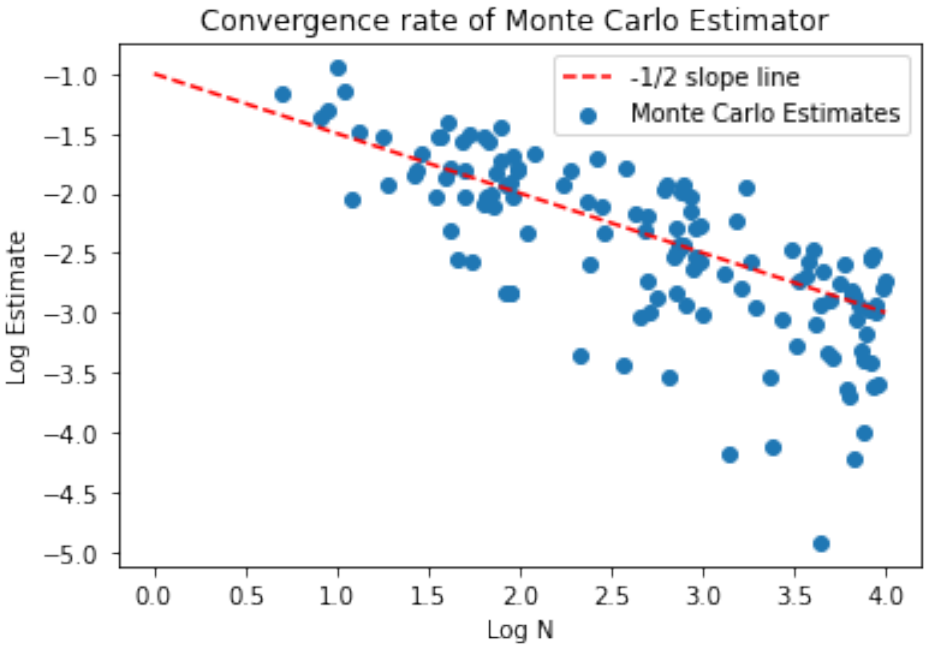
Problem 3

x	value	quadrature nodes
1	0.765198	13.0
2	0.223891	17.0
4	-0.397150	21.0
8	0.171651	29.0
16	-0.174899	41.0
32	0.138079	63.0
64	0.092590	103.0
128	0.001472	175.0
256	-0.036653	315.0

Problem 4

```
The value of F(.25, 3, 3) is 0.10351562500000008
The value of F(.5, 4, 5) is 0.6367187500000001
```




Convergence rate of Monte Carlo Estimator
-1/2 slope line
Monte Carlo Estimates
Log Estimate
Log N
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0