Simple Macro

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1 Simple world

There are N people in the world. Each day a person has to decide whether to be farmer or worker. As worker, a person gets daily wage w and produces a fixed number γ of manufactured goods. As farmer, the person produces η units of agricultural goods every day. η is heterogeneous: the first farmer has $\eta = 1$, the second $\eta = 2$, and the last $\eta = N$. Personal agricultural production η is not a function of how many other people farm, it is a personal parameter of each agent.

For L_m workers, total daily production is:

$$q_m = \gamma L_m$$

A farmer will become a worker only if wages are above or equal its personal η. Because I expect the "worst" farmers (those with the lowest η) to become workers first, daily wages are:

$$w_m = \frac{L_m(L_m + 1)}{2}$$

(which is just the sum of the first L_m natural number). Since $L_a + L_m = N$ and we assume that only the best farmers remain so, total agricultural production is:

$$q_a = \frac{N(N+1)}{2} - \frac{L_m(L_m+1)}{2}$$

which simplifies in:

$$q_a = \frac{L_a(N+1) - L_a L_m}{2}$$

Every person has the same utility function:

$$U = q_a^{.5} q_m^{.5}$$

Which has marginal rate of substitution of one to one. So setting MRS equal to price ratio, and remembering that a is the monetary commodity so its price is one:

$$q_a = p_m q_m$$

For workers, the budget constraint is:

$$q_a + p_m q_m = \frac{L_m(L_m+1)}{2}$$
$$2p_m q_m = \frac{L_m(L_m+1)}{2}$$
$$q_m = \frac{L_m(L_m+1)}{4p_m}$$

A farmer budget constraint is contingent on its own personal production:

$$2p_m q_m = \eta_i$$
$$q_m = \frac{\eta_i}{2p_m}$$

So if we try to aggregate the demands we have L_m worker demands and $N - L_m$ farmer demands:

$$q_m = L_m * \frac{L_m(L_m+1)}{4p_m} + \sum_{i=L_m+1}^{N} \frac{\eta_i}{2p_m}$$

The sum of all ηs is just q_a, total agricultural production:

$$q_m = L_m \times \frac{L_m(L_m+1)}{4p_m} + \frac{q_a}{2p_m}$$

Flip it so that we have price as a function of quantity:

$$p_m = \frac{2q_a + L_m^3 + L_m^2}{4q_m}$$

You can also exploit the fact that $L_m = \frac{q_m}{\gamma}$ And the result is the messy:

$$p = \frac{2q_a\gamma^3 + q_m^2(\gamma + q_m)}{4q_m\gamma^3}$$

This is problematic because the demand function is not decreasing in q_m. This must be the income effect biting my ass.

Starting from

$$q_m = L_m \times \frac{L_m(L_m+1)}{4p_m} + \frac{q_a}{2p_m}$$

We know that

$$q_a = \frac{N(N+1)}{2} - \frac{L_m(L_m+1)}{2}$$

So

$$q_m = L_m \times \frac{L_m(L_m+1)}{4p_m} + \frac{N(N+1)}{4p_m} - \frac{L_m(L_m+1)}{4p_m}$$
$$4p_m q_m = (L_m - 1)L_m(L_m+1) + N(N+1)$$
$$4p_m q_m = (L_m^3 - L) + N(N+1)$$
$$p_m = \frac{L_m^3 - L_m + N^2 + N}{4q_m}$$