A Cybernetic Model of Macroeconomic Disequilibrium

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Abstract In "On Keynesian Economics and the Economics of Keynes" (1968) Leijonhufvud described his cybernetic vision for macro-economic microfoundations based on the dynamics that brought the economy from one equilibrium to another. Here I implement that vision in agent-based model.
I focus on the difference in dynamics between price-led adjustments ("Marshallian" in the original text) and quantity-led adjustments ("Keynesian"). In a partial equilibrium model the two dynamics lead to the same equilibrium at the same speed through the same path.
In a general equilibrium model the Keynesian dynamics overshoots and undershoots with consequent over-employment and unemployment. This result occurs in a classical Walrasian world but where equilibria are attractor points rather than the only state the economy is allowed to be in. The agents within the model act by trial and error using very limited knowledge which I simulate here through the use of PI controllers. This allows me to study the effect of changing the adjustment speed with which agents grope during the disequilibrium and so study the effect that policies like increasing labor flexibility have on prices and outputs.

1 Introduction

I extend here the Zero-Knowledge traders methodology (Carrella, 2014) to macroeconomics. By modeling the economy as a process that agents try to control, I can study the effect that higher flexibility and adjustment speed have on the economy as a whole. More flexibility is not always beneficial as it can aggravate the disequilibrium and the social costs associated with a recession.

I use my model to study reforming the labor market during a recession. In Europe labor market reforms were touted both before (Siebert, 1997) and after (Bertola, 2014) the economic crisis as a way to boost economic growth. The point of reforming the labor market is to increase labor mobility and therefore productivity. This chapter challenges the notion that increasing either mobility or productivity is beneficial during a recession. I show how increasing mobility actually deepens the recession and output undershooting if it coincides with an exogenous drop in demand.

I base my model on Leijonhufvud's "Keynes and the Keynesians" where the difference between Keynesian and Marshallian economics is how agents adapt to disequilibrium (Leijonhufvud, 1972). To Leijonhufvud, Marshallian agents react to mismatches in demand by first adjusting prices and only later changing production. Keynesian agents instead react first by adjusting quantities and only later prices. I implement this idea and show how these differences have no effect in microeconomics but do so in macroeconomics.

This is not the first attempt to model the microfoundations of "Keynes and the Keynesians". Leijonhufvud himself cited the search models in "Information Costs, Pricing and Unemployment" (Alchian, 1969) and Clower's false trades (Clower, 1965) as a way to achieve his vision. This chapter follows in the false trades tradition of allowing exchanges at wrong prices but I provide a simple trial and error agent that corrects itself over time. It is the dynamics generated by this trial and error pricing that differentiate Keynesian disequilibrium from the Marshallian one.

2 Literature Review

I classify agents in macroeconomics on a spectrum that goes from complete feedback to complete feed-forward. Feedback agents are reactive, they manipulate control variables by inferring over time what their effect is on the other model variables. Feed-forward agents know perfectly the model and set all the control variables at the beginning of the simulation after having solved for the optimal path.

Modern economics focuses mostly on feed-forward agents. The Ramsey-Cass-Koopmans model (Ramsey, 1928)(Cass, 1965)(Koopmans, 1963) is an example of a pure feed-forward agent. In this model the agent is omniscient and chooses the saving rate for any instant of its infinite life by optimizing utility given the lifetime budget constraint. This omniscience is a fundamental driver of the model as it explains, for example, why permanent taxes do not crowd out investments while temporary taxes do (Romer, 2011).

In Prescott's Real Business Cycle growth model (Prescott, 1986) the agent is an imperfect feed-forward control. In this model there are auto-regressive random technological shocks that cannot be predicted ahead of time. The agent then has a large feed-forward element that finds the optimal distribution of control strategies to implement and a small feedback process to choose the control strategies from this distribution as the random shocks occur.

Feed-forwarding optimization with feedback adaptation to uncertainty remains the standard macroeconomic approach to this day. The more uncertainty a model has, the larger the feedback element of the agent is but the focus is always on feed-forwarding. Learning models as in Evans and Honkapohja, 2009 are emblematic of this: agents don't have model knowledge but rather than managing this uncertainty they employ feedback econometrics to learn the model just so that they can then use the usual feed-forward control strategies on what they learned.

The only agent in economics that is still pure feedback is the central bank: the Taylor rule (Taylor, 1993) is a simple feedback and adaptive rule to set interest rates. It is in fact a simplified PI controller (Hawkins, Speakes, & Hamilton, 2014). Real economists allow simulated economists some slack in assuming not only that they face uncertainty but that they are never able to reduce it by learning the full model.

The modern focus on feed-forwarding is surely a reaction to the feedback oriented methodology that preceded it. The Keynesian IS-LM (Modigliani, 1944) were almost pure feedback models. Consumption would be a fixed proportion of income, workers would be reacting the same fixed way to changes in prices and therefore could be fooled over and over again into generating a Philips curve (Heijdra & Van der Ploeg, 2002). Explicitly cybernetic models shared this top-down fixed feedback approach (Tustin, 1957) (Phillips, 2000) (Cochrane & Graham, 1976) (Lange, 1970). Leijonhufvud called it the "Keynesian Revolution that didn't come off" (Aoki & Leijonhufvud, 1976) but the approach survives in the field of system dynamics (Sterman, 2000).

My agents are feedback only. The difference with the past is that my feedback mechanisms are there to allow a flexible agent to adapt to shocks rather than linking in a fixed way economic components as the IS-LM models wanted to do.

3 Microeconomics

3.1 Marshallian Agents

This is a brief summary of the Zero-Knowledge trader methodology. The unit of time is a "market day" as in Hicks (Leijonhufvud, 1984) . Take a simple market for one type of good. Each market day, a firm produces y^s_t units of good, consumers buy y^d_t units at price p_t. The Marshallian firm is a price-maker that takes production as given and changes p_t every day in order to make production equal demand that is:

$$y^s = y^d \tag{1}$$

The agent has no knowledge of market demand and how his own price p_t affects it. It knows only that higher prices imply lower demand. It proceeds then by trial and error: it sets a price p_t and computes the error $e_t = y^s - y^d$ and uses it to set p_{t+1}. I simulate the trial and error process by a PI controller:

$$p_{t+1} = \alpha e_t + \beta \sum_{i=0}^{t} e_i \tag{2}$$

By manipulating p_t the Marshallian agent changes demand y^d until it equals supply y^s. Within each market day the Marshallian agent treats its own good supply as given but over time it can use the price it discovers to guide production. At the end of each day there is a small fixed probability (in this simulation $\frac{1}{20}$) to change supply y^s by adjusting labor hired. The decision is simple marginal optimization: increase production while Marginal Benefit > Marginal Costs and viceversa. The firm again adjusts by trial and error and with a separate PI controller whose error is $e_t = \frac{\text{Marginal Benefit}}{\text{Marginal Cost}} - 1$.

In the original Zero-Knowledge paper I go through more complicated scenarios with multiple firms and markets, monopoly power and learning. But this minimal setup is enough for this application. Here a firm has only two degrees of freedom, price set p and labor hired L. Each set by an independent PI controller.

3.2 Keynesian Agents

Keynesian firms function exactly as Marshallian ones except that they reverse the speed and the error of the two PI controllers. A Keynesian firm changes L every day trying to make $y^s = y^d$ and changes p with the a small fixed probability trying to make Marginal Benefit = Marginal Cost.

Functionally the Keynesian firm tries to match supply and demand within a market day by changing y^s directly rather than changing p and therefore y^d as the Marshallian one.

3.3 In a partial-equilibrium scenario Keynesian and Marshallian agents perform equally

There is one firm in the economy. It faces the exogenous daily linear demand

$$y_t^d = 100 - p_t \tag{3}$$

One person hired produces one unit of good a day:

$$y_t^s = L_t \tag{4}$$

There is infinite labor supply at $w = \$50$. The perfect competitive solution is $L = 50, y = 50$

I run 1000 simulations for a Marshallian and a Keynesian firm each setting their PI parameters $\alpha, \beta \in [0.05, 0.2]$ and random initial price and labor $p_0, L_0 \in [1, 100]$. Firms in all simulations always find the equilibrium as shown in figure 1.

Define equilibrium day as the simulation day when the firm produces within 0.5 units of the equilibrium production. Figure 2 compares the equilibrium day distribution of Keynesian and Marshallian firms. A two-sided Kolmogorov-Smirnoff test fails to reject that the two samples come from the same distribution (p-value is 0.263) In a partial equilibrium microeconomic scenario Keynesian and Marshallian firms performs equally well and at equal speed.

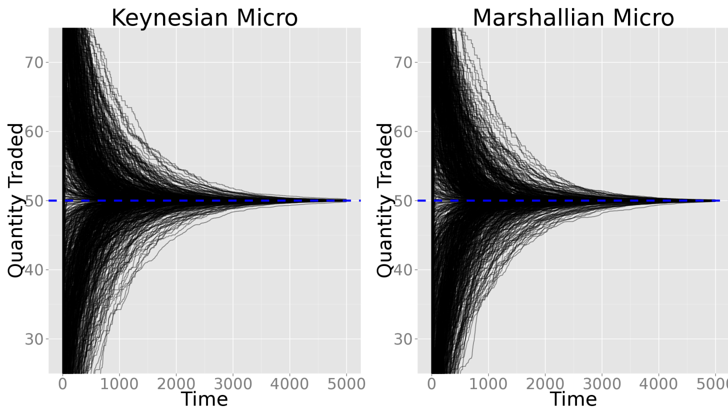

Figure 1: The path of y traded for a 1000 Keynesian and Marshallian simulations. Regardless of initial conditions and PI parameters, the runs all reach equilibrium.

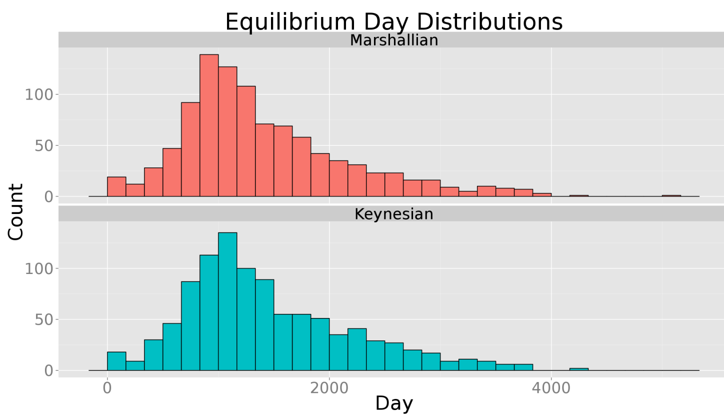

Figure 2: The empirical distribution of equilibrium time for the Keynesian and Marshallian microeconomic simulations. There is no difference between them.

4 Macroeconomics

4.1 Both Marshallian and Keynesian firms are able to reach equilibrium in a simple macro model

Here I present a minimal macroeconomic model and show how Marshallian and Keynesian dynamics diverge. The main reason they do is that Keynesian adjustment has side effects. Keynesian firms manipulate labor directly; in microeconomics hiring and firing workers affected how many goods were produced but in macroeconomics the good demand is equal to labor income so that hiring and firing workers affect how many goods are demanded as well. Marshallian firms instead manipulate prices which moves the demand without affecting supply.

There is a single firm in the world. It is programmed to act as in perfect competition and targets Marginal Benefits=Marginal Costs. It produces a single good with daily production function:

$$Y^S = a\sqrt{L} - b \tag{5}$$

It has access to an infinite supply of labor L at $w = 1$.

The demand for the output is equal to the real wages paid:

$$Y^D = \frac{L}{p} \tag{6}$$

Unsold output spoils, unused labor income is never saved. This market has the following unique equilibrium:

$$L = \frac{4b^2}{a^2} \tag{7}$$

$$p = \frac{2\sqrt{L}}{a} \tag{8}$$

$$y = -b \tag{9}$$

When $a = 0.5$ and $b = 1$ the solution is:

$$L = 16 \tag{10}$$

$$p = 16 \tag{11}$$

$$y = 1 \tag{12}$$

The computer simulation proceeds just like the previous microeconomic section except that demand here is endogenous and equal to wages paid Two sample runs are shown in the figure 3.

I run 100 simulations each for Keynesian and Marshallian firms, where the p and i parameters of the controllers are random $\sim U[0.05, 0.2]$. Both Keynesian and Marshallian firms are always able to achieve equilibrium.

4.2 Keynesian and Marshallian firms generate very different dynamics when reacting to a demand shock

As initial conditions matter, rather than studying the dynamics toward equilibrium ab ovo, I first let the model reach equilibrium then subject it to a demand shock and see how the firms differ in adapting to it. I run the same simulation as before, but after 10,000 days the output demand is shocked by s\$:

$$Y = \frac{L}{p} - s \tag{13}$$

When $s = 0.2$ the new equilibrium becomes:

$$L = 10.24 \tag{14}$$

$$p = 12.8 \tag{15}$$

$$Y = 0.6 \tag{16}$$

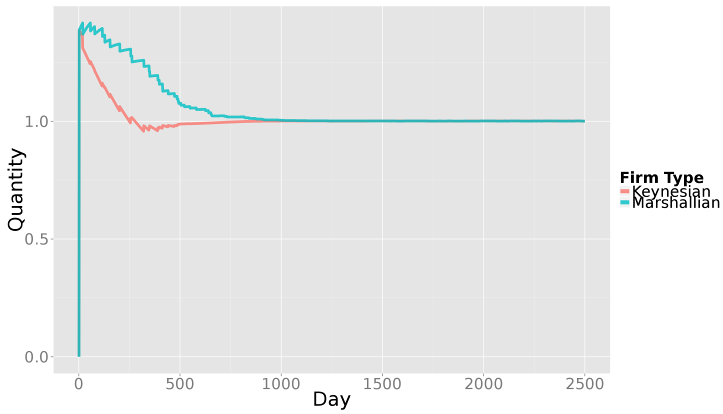

Figure 3: Two sample runs of the economy Y with a Keynesian and a Marshallian firm.

Figure 4 shows the difference in adjustment dynamics between Keynesian and Marshallian firms. Marshallian firms react to the sudden drop in demand by lowering price so that quantity traded briefly recovers after the shock. Eventually though the lower prices feed into the profit maximization PI which cuts production towards the new equilibrium. Keynesian firms instead react to the drop in demand by immediately firing workers. While firing workers lowers supply it also decreases demand because unemployed workers don't consume. The Keynesian firm can't change supply without changing demand as well.

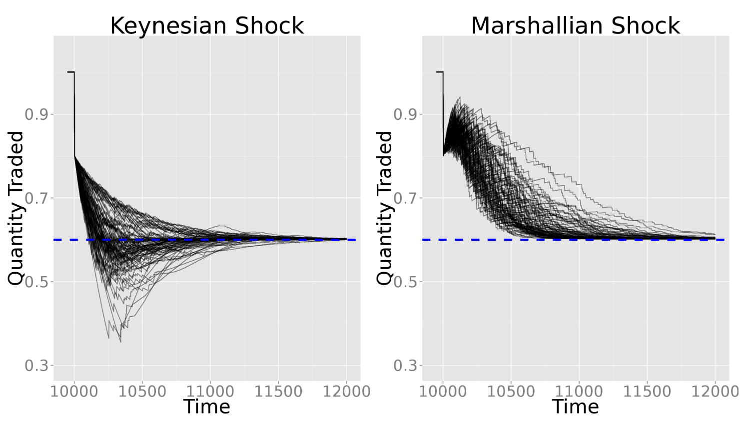

Figure 4: A comparison between the adjustment dynamics after a demand shock of Keynesian and Marshallian firms. The Keynesian runs often undershoot and have larger output contractions than the same Marshallian firms in spite of the pre-shock and after-shock equilibria being the same

Keynesian firms reach the new equilibrium faster. Define equilibrium time as after how many days the output settles within 0.05 of equilibrium. Average equilibrium time is 570.2 days for a Keynesian firm and 808.37 days for a Marshallian one (which is a statistically significant difference). Moreover Keynesian firms tend to stay closer to equilibrium overall. To see this define deviation of output y from equilibrium y^* as:

$$\log(t) * (y_t - y^*)^2 \tag{17}$$

Then the average deviation for Keynesian economy is 4.076 while it is 20.971 in the Marshallian economy. Figure 5 shows the difference. On the other hand output drops 10% or more below the new equilibrium in 29 Keynesian runs out of 100 . Marshallian firms never undershoot.

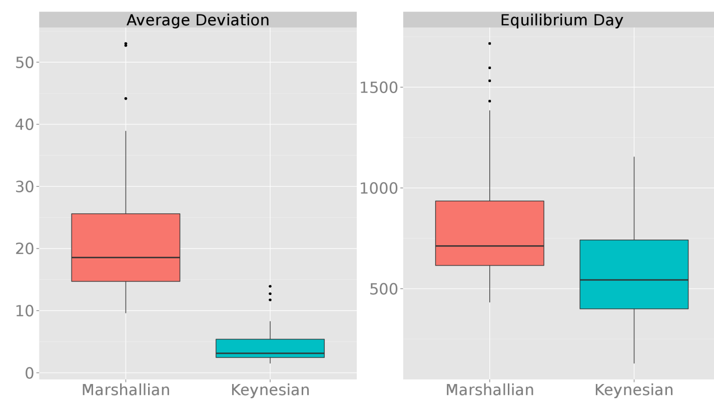

Figure 5: Box-plot comparison of deviation and equilibrium day between Keynesian and Marshallian macro

Keynesian adjustment is less efficient and creates larger social losses in spite of reaching equilibrium faster. In figure 6 I compare firm profits and labor income during disequilibrium versus what they would be if the adjustment was immediate. Labor income is higher in the Marshallian world (on average 2010.957$ per run compared to 1024.894$ in the Keynesian world). This is because the disequilibrium involves firing unnecessary workers and the longer it takes the more the workers benefit from the disequilibrium. What is less obvious is that the Marshallian firm is also better off than the Keynesian one as figure 6 shows.

The reason Marshallian firms can over-produce for longer and still make consistently less losses than Keynesian firms is that Marshallian disequilibrium dynamics are less wasteful. To see this focus on market day equilibria, each day the difference between what is produced and what is sold is wasted. Figure 7 shows

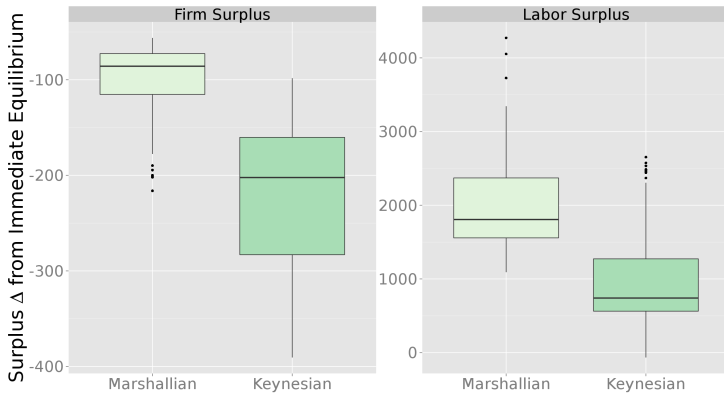

Figure 6: The difference in surpluses between Marshallian and Keynesian firms. The surplus is measured as a difference in \$ (or wage units) compared to what it would be if it moved immediately to the new equilibrium

the daily waste and in particular how it is larger with Keynesian firms. Keynesian firms over-produce and waste because of their inability to match demand to supply quickly as any cut in production cuts demand as well. Marshallian firm takes longer to get to the new equilibrium but proceeds over a more efficient path where demand and supply match most of the time. Keynesian firms get to equilibrium faster but demand and supply never match until the equilibrium is reached.

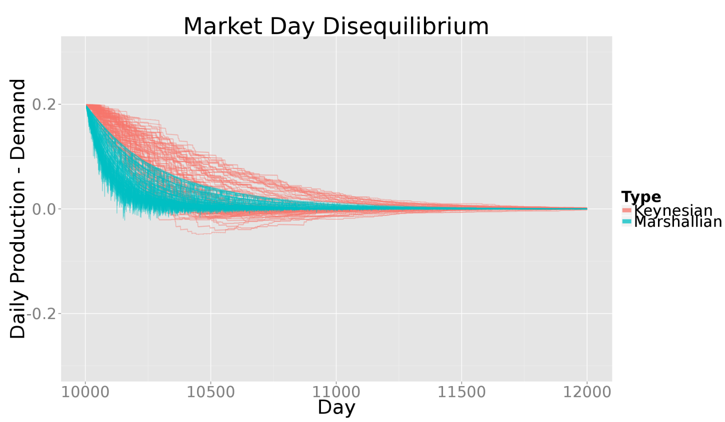

Figure 7: The difference between what is produced and what is sold each day, regardless of what the profit-maximizing equilibrium is. The larger the deviation from 0 the more the waste.

Overproduction is the signal that pushes Keynesian firms quickly to the new equilibrium, but it is a wasteful and expensive signal that costs more to society than the slower Marshallian alternative.

5 Labor Reforms and the Zero-Knowledge Agents

5.1 Increasing labor flexibility during a recession makes it worse

In this section I model the world as Keynesian. I do so because price rigidities are a well established empirical fact (Klenow & Malin, 2010). It is also advantageous to model labor market reforms and speed in the Keynesian world since the PID controlling production targets (and therefore labor) is not sticky.

I model labor flexibility in two ways. First, increasing flexibility may mean faster hiring and firing. I can replicate this in the model by increasing the parameters of the PI controlling the workforce so that it adjusts more aggressively. Alternatively increasing flexibility may mean increasing the productivity of labor. I can replicate this in the model by increasing the a parameter of the production function.

Assume the world is Keynesian. Assume the same shock to demand as the previous section. Here I simulate what happens if concurrent to the demand shock there is a flexibility shock to the firm where its PI parameters double. I compare the same simulation with the same random seed with and without the flexibility shock. Notice that the economic equilibria has not changed, the difference can only be in dynamics.

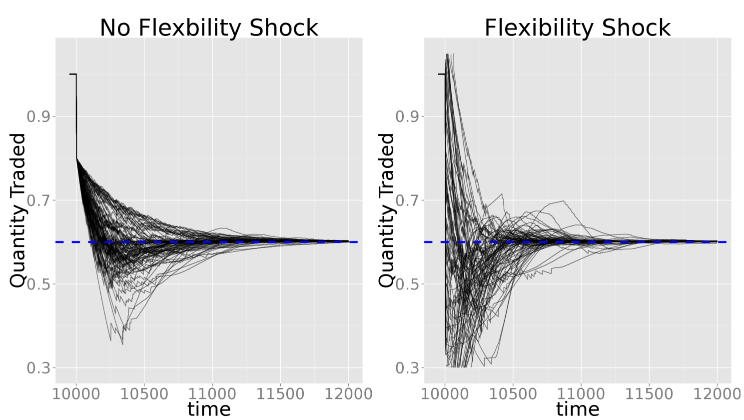

Figure 8: 100 Keynesian runs as in figure 4 and the same runs where concurrent to the demand shock we double the PI labor parameters. Overshooting becomes more likely and deeper. 10 runs fail to reach equilibrium when their flexibility is increased

Figure 8 shows the effect of increasing flexibility together with the demand shock. Higher flexibility results in higher chance of overshooting, 88 runs out of 100 have output dropping more than 10% below equilibrium (compared to 29 without flexibility shock). Moreover in 10 runs the overshooting is so severe that the run ends on $Y = 0$ (which is a steady state) and never reaches the equilibrium. Figure 9 shows that the deviation from equilibrium is higher with higher flexibility (because of the severity of the overshooting) while there is no statistical significant difference in equilibrium time.

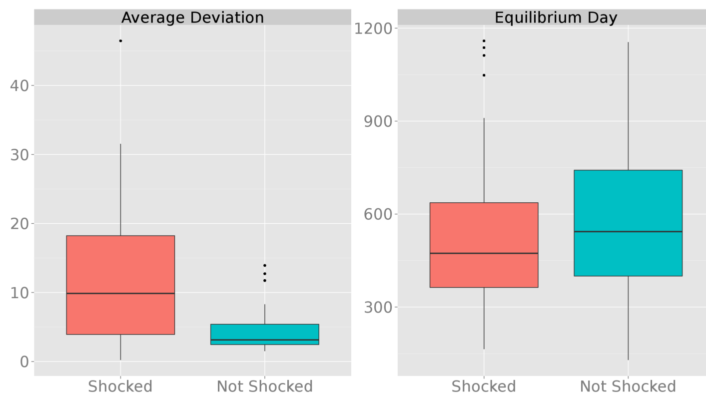

Figure 9: Comparison between the Keynesian equilibrium metrics with and without flexibility shock.

Figure 10 shows how labor surplus is lower when there is a flexibility shock. Overshooting is so severe that on average the labor surplus is negative. Note first that labor surplus was positive in the previous section: because the new equilibrium requires fewer workers and the agent takes time to get to the new equilibrium point some workers that should have been fired instantly profited from the disequilibrium. Higher flexibility fire workers faster, which reduces benefits from disequilibrium and when overshooting it fires too many so that labor overall is hurt by the disequilibrium rather than profiting from it. Firm surplus is higher with more flexibility; the difference in means is statistically significant.

More generally, what the right labor flexibility is in terms of speed is a tuning problem. What we want are the controller parameters that move the economy to the new equilibrium as fast as possible while minimizing overshooting. This is an empirical question and the answer depends on the kind of original equilibrium, production function, shock and every other parameter. It is not the case that more flexibility and speed always make for a better economy.

Turn to flexibility as an alias for productivity, assume again a Keynesian world. Concurrent with a demand shock the productivity a increases from 0.5

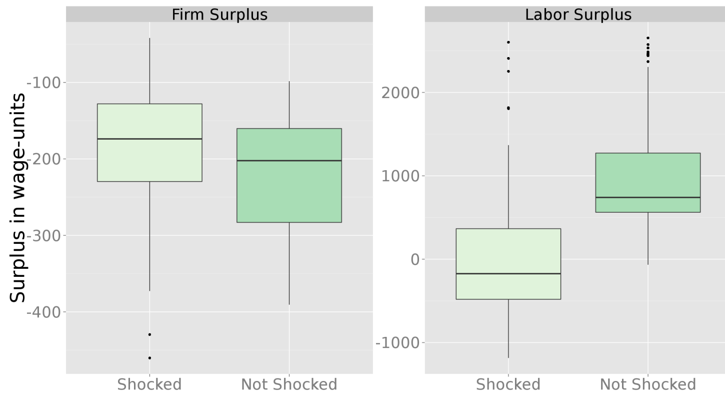

Figure 10: Box-plot of surplus differences between runs with and without flexibility shock. The values are $ (or equivalently wage-units) differences between surpluses and what would the surplus be if the system immediately moved to the new equilibrium

to 0.6. This changes the equilibrium L and p but not optimal output Y:

$$L = 7.11 \tag{18}$$

$$p = 8.88 \tag{19}$$

$$Y = 0.6 \tag{20}$$

Again I run 100 simulations with and without productivity shock, keeping fixed random seeds for comparison. In this case the only change is in the new equilibrium conditions, PI controllers are invariate. Figure 11 compares the two dynamics.

Increasing productivity makes the approach to equilibrium worse as shown in figure 12. More runs undershoot, 68 out of 100, and output deviation from equilibrium is higher with no improvement in equilibrium time. As shown in figure 13 there are no meaningful improvements in disequilibrium surplus for either firms or labor although it is hard to judge the overall effect because the equilibrium values the two sets of runs are compared to are different.

While improving productivity is always a good long term policy there is no validation from this model that raising it makes disequilibrium any better.

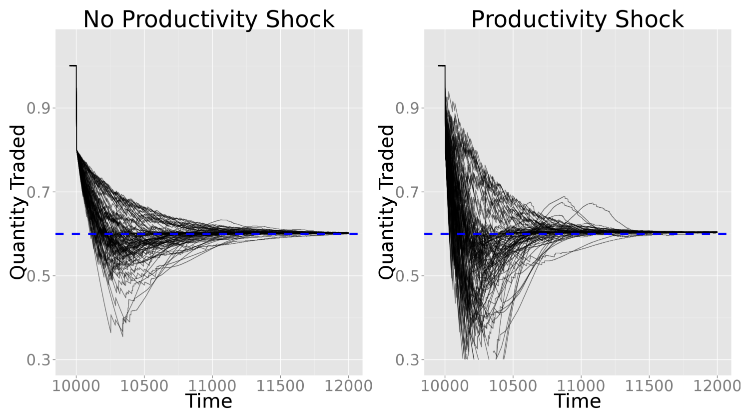

Figure 11: The dynamics of 100 Keynesian simulations with paired random seeds and how they deal with demand shock with and without productivity shock.

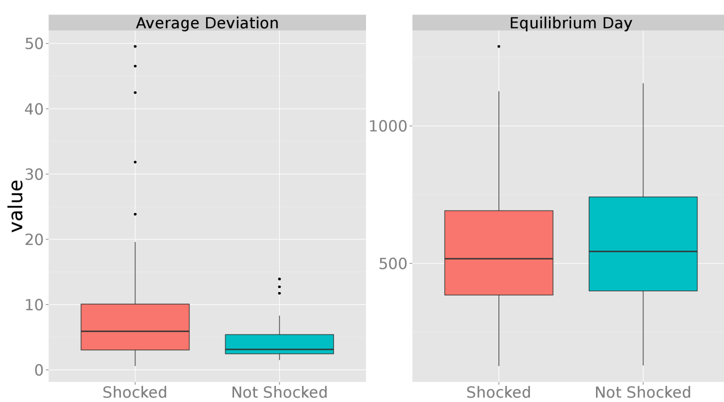

Figure 12: Comparison between the Keynesian equilibrium metrics with and without productivity shock.

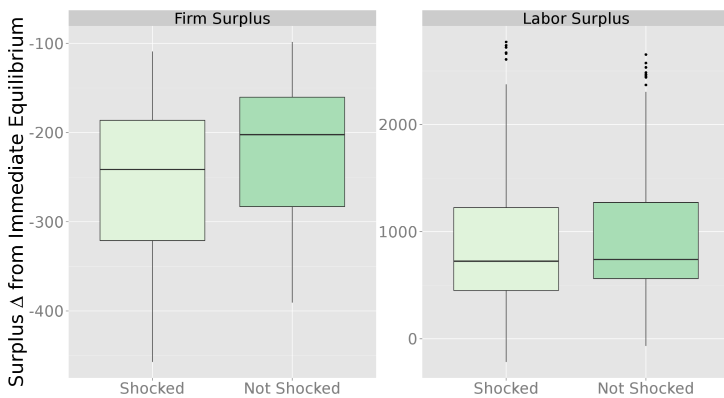

Figure 13: Box-plot of surplus differences between runs with and without productivity shock. Notice that the productivity shock changes the equilibrium p and l so that the two classes of surpluses are compared to two different optimal points

6 Conclusion

To highlight disequilibrium dynamics this model was made very simple. Some of the assumptions present should be removed in future work. The first large assumption I made is infinite fixed wage labor supply. Previous chapters did not assume this. I did so here in order to simplify the decision process of the firm; in this model the firm only sets one price (output) and one production target. Had I added wages it would have made it impossible to compare Marshallian and Keynesian dynamics since there would be two prices to set concurrently. In that circumstance Marshallian firms would be quicker since they would set p and w quickly and target L slowly while Keynesian would have to set L quickly while p and w slowly.

One could argue that we can still use fixed wages around the equilibrium and salvage the shock comparisons in section 4.2 by either assuming efficiency wages or some form of downward rigid wages as Modigliani's IS-LM (Modigliani, 1944). But these would have to be micro-founded rather than just assumed.

The second large assumption is the lack of utility micro-foundations. If the consumer has a lexicographic utility where it prefers a world with no waste (that is demand equals supply) and splits ties according to the world that produces the most, then the simulation would be utility maximizing. But this is non-standard utility formulation and general results must not depend on these. I also gave no explanation for the demand shock.

The third limitation of the chapter is the lack of agents. Previous papers on the methodology had multiple firms competing with one another but here there is a single firm taking all the decisions. This was primarily to avoid any noise in the simulation except those caused by demand shocks. The same weakness is present regarding consumers and workers. A single force supplies labor and consumes wages; there are no distribution effects and no asymmetric cost to unemployment. All these assumptions are, I believe, minor. They are employed to remove noise from the model and further highlight the difference between Keynesian and Marshallian firms.

I believe this paper's result are timely. I show how increases in productivity and labor flexibility during a recession while improving the final economic equilibrium worsen the path the economy takes towards it. This kind of results can only be achieved through agent-based economics and simulation. This kind of results can only be achieved by focusing on disequilibrium. And these results are needed to chart a complete policy response to economic crises.

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