ITQ Proposal

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I am putting out this feeler to get some feedback on how to model ITQ reservation prices. Feel free to rip it to shreds

Single Species

A fisher is wondering how much he is willing to pay for an additional "unit" of quota. After accounting for all other costs (transportation, crew and so on) the fisher knows he makes Π from catching and selling an additional unit of fish. The quota costs λ.
It would seem then that the monetary revenue from buying a new unit of quota is:

$$\text{Revenue from Buying} = \begin{cases} \Pi - \lambda, & \text{if quota is used} \\ -\lambda, & \text{otherwise} \end{cases}$$

There is a risk in buying a quota and not using it before the season ends in which case we just wasted λ.

The expected value of buying a quota (for a risk neutral fisher) is then:

$$E[\text{Value of Quota}] = \Pr(\text{Needed})(\Pi - \lambda) + \Pr(\overline{\text{Needed}})(-\lambda)$$

And we can solve for the λ^* which is the price that makes fishers indifferent between owning or not a quota

$$\lambda^* = \Pr(\text{Needed})\Pi$$

Focusing on what price would the fisher be willing to sell a owned quota gives this revenue function:

$$\text{Revenue from Selling} = \begin{cases} \lambda - \Pi, & \text{if quota is needed} \\ \lambda, & \text{otherwise} \end{cases}$$

Which, for a risk neutral agent gives back exactly the same λ^*.

Agents in the model know or can guess the following:

$$\begin{aligned} q &= \text{Quota owned} \\ c &= \text{Daily catch} \\ t &= \text{Day of the season} \\ T &= \text{Season length} \end{aligned}$$

Then the probability that you will need that unit of quota is just:

$$1 - \Pr(c \leq \frac{q}{T-t})$$

That is that catches per day times days left in the season is higher than the number of quotas you have.

Two Species

This scenario is a lot trickier. The fisher can't perfectly target only one species and will at any tow catch a mix of the two. The problem is that not having quota for one species will ban the fisher from selling any catch from any species.

Imagine for a second that each time the fisher catches 1 unit of species 1 it also catches on average x_2 units of species 2. The benefit of owning an additional unit of quota of species 1 then is not just that you can sell that unit of that fish but also that you can sell x_2 units of species 2, provided you own x_2 units of quotas for species 2.

In short, we are in this situation:

$$\text{Revenue from Buying} = \begin{cases} \Pi_1 - \lambda_1 + x_2(\Pi_2 - \lambda_2), & \text{if quota is used} \\ -\lambda_1, & \text{otherwise} \end{cases}$$

For a risk neutral fisher the quota price that makes him indifferent between buying or not is:

$$\lambda_1^* = \Pr(\text{Needed}) \left(\Pi_1 + x_2(\Pi_2 - \lambda_2)\right)$$

Notice how, depending on the profits from the other species Π_2 and its quota prices λ_2, the reservation price could be higher or lower than it was for the one species example.
Similarly for species 2:

$$\lambda_2^* = \Pr(\text{Needed}) \left(\Pi_2 + x_1(\Pi_1 - \lambda_1)\right)$$

Now ($\Pr(\text{Needed})$) can be estimated much like the one species example. We can estimate x_2 too easily as the ratio of some averaged daily catches for each fish:

$$x_2 = \frac{c_2}{c_1}$$
$$x_1 = \frac{c_1}{c_2}$$

Multiple Species

Same process as above will get:

$$\lambda_i^* = \Pr(\text{Needed}) \left(\Pi_i + \sum_{j \neq i} \frac{c_j}{c_i}(\Pi_j - \lambda_j)\right)$$

Discrete Margins

While I wrote the previous formulas in terms of "fishing an additional unit of specie i" the truth is that margins don't work like this in the fishing model. Agents can't just fish marginally more fish for marginally more cost: they fish in discrete steps (a trip). This makes the definition of unit profit Π_i important to avoid double counting costs.

Imagine that the average trip nets you a vector of biomass caught for each species $[x_1, x_2, \ldots, x_S]$. The profit you make for this trip is:

$$\sum x_i p_i - k - \sum x_i \lambda_i$$

Where k is the cost of running a trip. We can do a bit of algebra to make it easier:

$$\Pi_{\text{TOT}} = \sum x_i p_i - \frac{\sum x_i k}{\sum x_i} - \sum x_i \lambda_i$$

$$\Pi_{\text{TOT}} = \sum x_i p_i - \sum_i \frac{x_i k}{\sum_j x_j} - \sum x_i \lambda_i$$

$$\Pi_{\text{TOT}} = \sum x_i (p_i - \frac{k}{\sum_j x_j} - \lambda_i)$$

So as long as we define:

$$\Pi_i = p_i - \frac{k}{\sum_j x_j}$$

That is unit profits for specie i is just the price of selling one unit of specie i minus the average cost of the trip catch, then we can just find that the average profits in terms of x_1 (for example) is:

$$\frac{\Pi_{\text{TOT}}}{x_1} = \Pi_1 - \lambda_1 + \sum_{i \neq 1} \frac{x_i}{x_1} (\Pi_i - \lambda_i)$$

Which is the same formula we plugged in the previous sections.

Last detail to remember if you are looking at the coe is that Π_i is actually computed circuitously as $(p_i x_i - \frac{x_i k}{\sum_j x_j}) \frac{1}{x_i}$ (same definition, one more step).