Sticky Prices Microfoundations in a Supply Chain Agent Based Model

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Abstract

I build a simple supply chain model with minimal rationality agents and show how sticky prices are necessary to achieve correct prices and quantity. Stickiness is necessary because the effect of price changes takes time to propagate within a supply chain and changing prices quickly leads to noise and wrong equilibria. I then extend the model to deal with inventories, monopolies and learning.

1 Introduction

Why would firms decide to change prices once a year when they can do it every day? We usually assume sticky prices are a compromise between the advantages of price flexibility and the reality of price adjustment costs. If it weren't for these costs firms would be more flexible. I build a simple model where price stickiness is not only superior to flexible pricing but is necessary to achieve equilibrium.

I build an agent-based model where agents price their output by trial and error in a simple feedback loop, trying to match today's sales with today's production. In a single sector economy the faster the price changes, the quicker the agents reach equilibrium. This is because price changes have immediate effects. With multiple sectors there is a delay between a price change and its effects; this delay ruins agents' feedback causing their prices to overshoot and undershoot out of control. Agents can easily manage by slowing down the price changes so that their feedback is not fooled by supply chain delays.

The main advantage of this model is the ability to explain price stickiness without the need of adjustments and menu costs or kinks in demand. Price-stickiness here is not a poor substitute of total flexibility, it is necessary for the agents to deal with a slowly adapting world.

I introduce zero-knowledge traders in section 3 and use it to provide examples of how delays break trial-and-error pricing and how stickiness can solve it. The delay in this section is exogenous and arbitrary but after introducing production in section 4.1 I build a two sector supply-chain model in section 4.2

where delays are endogenous. I described a more complex version of the zero-knowledge framework in Carrella (Forthcoming) but I expand it here to deal with inventories in section 5 and learning in section 7.

The source code is available[1] on an open-source AFL 3.0 license.

2 Literature Review

The firm surveys by Blinder (1998) and Silvia Fabiani et al. (2006) are both extensive literature reviews on price stickiness' microfoundations and empirical tests of the most important theories. Both papers highlight the importance of returning customers' goodwill (Okun, 1981), coordination between firms (Clower, 1965) and long term contracts.

My paper is most similar to Blanchard (1982). In both papers price inertia is due to the desynchronization between firms in a supply-chain. While the results of the papers are similar the causality is reversed. In that paper production adapts instantaneously but prices are set slowly and asynchronously which causes price inertia within the supply chain. In my paper production adapts slowly so that while prices can be set very quickly it is counterproductive to do so, since suppliers and customers need time to adapt.

This paper is also related to the modern macroeconomic literature on multi-sector "price-complementarity" (Carvalho & Lee, 2011) as the cause of price stickiness. In my model it would be more accurate to talk about "production-complementarity"; prices are signals firms can send to adjust production mismatches but there is a long lag between the price being set and it fully affecting production. This delay and the lack of knowledge on the effects of the price change are the causes of sticky prices.

In sticky-information models (Mankiw & Reis, 2002) the lack of knowledge alone can cause price-stickiness, as information about shocks is expensive. In my model information gathering is not a separate costly activity, it is a by-product of taking action. Firms never know what the profit-maximizing prices and quantities are, they can only experiment and see what the results are. Sticky prices become necessary when there is a large delay between running the experiment (setting a price) and see its result (changes in behavior along the supply-line).

3 Zero Knowledge Traders and Delays

3.1 Simple agents working through feedback

Zero knowledge traders price their goods in a feedback loop. A trader receive y^* goods to sell each day. She sets sale price p and attracts y paying customers. If there are fewer customers than goods to sell today, the trader lowers the price

[1] https://github.com/CarrKnight/MacroIIDiscrete

tomorrow. And viceversa. Defining the daily error as

$$e_t = y^* - y_t \tag{1a}$$
$$e_t = \text{Inflow} - \text{Outflow} \tag{1b}$$
$$e_t = \text{Netflow} \tag{1c}$$

The trader adjusts tomorrow's prices through a PID controller rule:

$$p_{t+1} = ae_t + b\sum_{i=1}^{t} e_i + c(e_t - e_{t-1}) \tag{2}$$

This simple feedback is enough for agents to find the correct prices as shown in Figure 1.

Figure 1: A sample of a trader iteratively finding the correct prices when having 50 units of goods to sell and facing the daily linear demand $q = 101 - p$. This trader is using a PID controller with parameters $a = b = .1$ and $c = 0$.

This feedback loop simulates naïve trial and error pricing. As with all experimentation, PID control works better when trial results are informative and unambiguous. A simple way to mislead the agents is to add a time delay δ between a price being set and the quantity demanded q adjusting to it. Mathematically the trader faces the demand:

$$q_t = f(p_{t-\delta})$$

Delays mean that even when the trader guesses the right price it takes δ days to yield the right quantity. This fools the trader in thinking the price is wrong and convinces her to tweak further her price. Depending on δ, the delay can slow down the approach to real prices (as in figure 2) or prevent it entirely (as in figure 3).

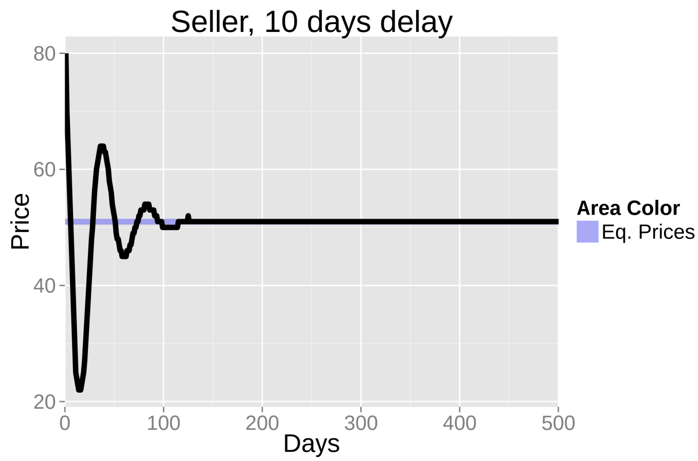

Figure 2: The same trader of Figure 1 now faces demand $q_t = 101 - p_{t-10}$. The trader takes longer to find the right price.

Figure 3: The same trader of figure 1 now faces demand $q_t = 101 - p_{t-20}$. The trader never finds the right price.

3.2 Dealing with delays

The simplest way to deal with delays is to slow down the price adjustment accordingly. Knowing it takes δ days for prices to take effect, the trader can update prices with equation 2 only every δ days. Effectively, sticky prices. As shown in figure 4 this adjustment is enough to fix the feedback loop of the trader. A similar way to deal with delays is to be more timid in adjusting prices day by day. The trader activates equation 2 every day,but scales a and b down; effectively the seller adjusts her prices less aggressively. As an example see figure 5

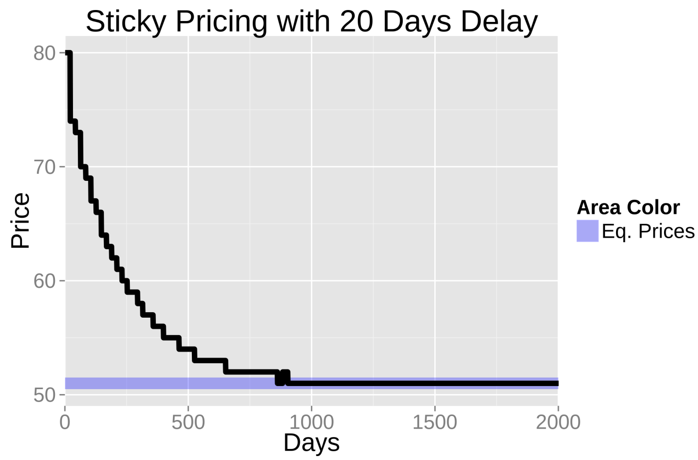

Figure 4: The same setting of figure 3 but this time the trader adjusts her prices only every 20 days. The result is the same approach as figure 1 but with a longer time frame

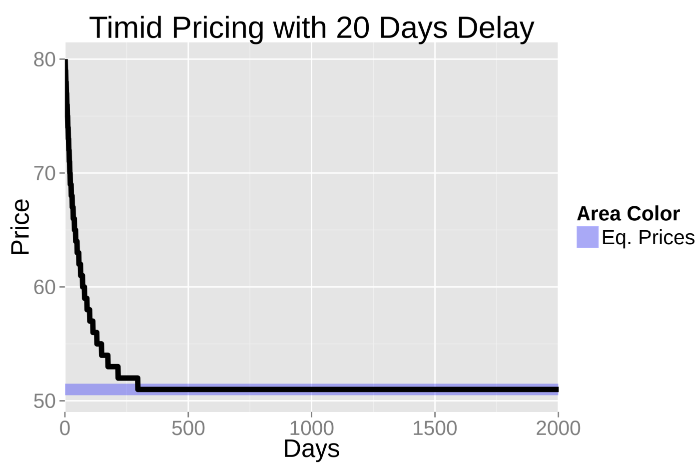

Figure 5: The same setting of Figure 3 but the PID controller has $a = b = .01$, 10 times smaller than the original.

Define stickiness the number of days the PID controller waits before adjusting prices. Define timidity as the number dividing the baseline PI parameters (so a timidity of 10 with a baseline of .1 means that the PI parameters $a = b = \frac{.1}{10} = .01$). Fix the demand delay δ to 50. Figure 6 shows which combinations of timidy and stickiness achieve correct prices over 5 experimental runs. Define the daily distance from the correct price as:

$$\sum_{t=1}^{T} (p_t - 51)^2$$

Figure 7 shows for which combination the distance is minimized. That is which combination of timidy and stickiness achieves the fastest convergence to correct prices.

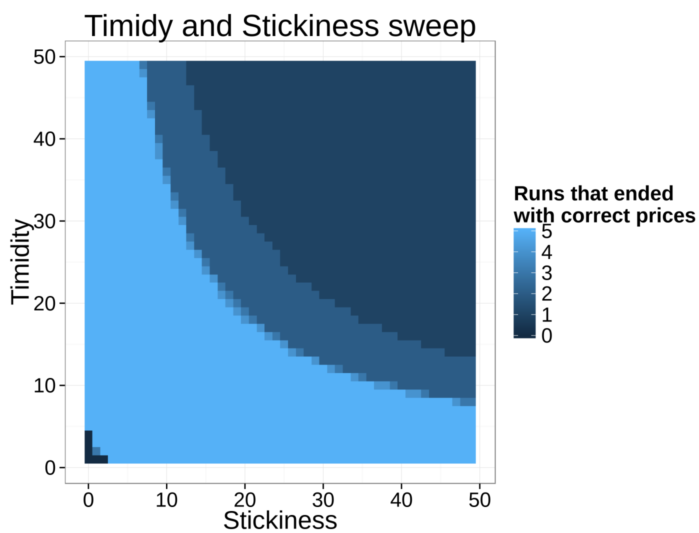

Figure 6: Run the model 5 times for 15000 market days with fixed PID parameters and speed but different initial prices. Controllers that are too fast or too slow fail in at least some cases. Demand delay is 50 days

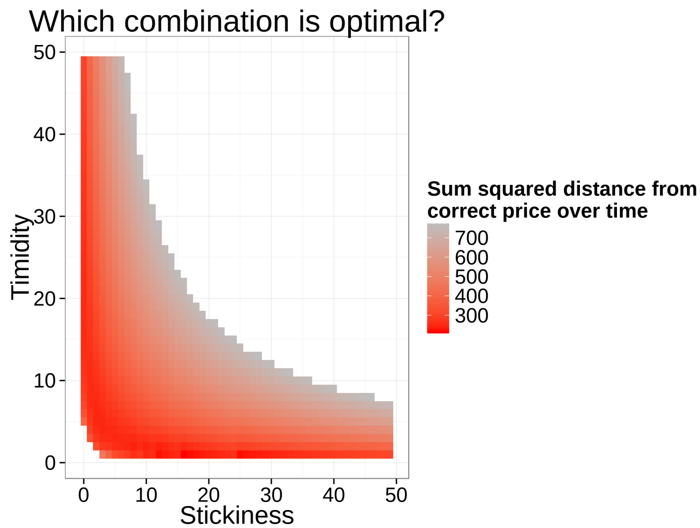

Figure 7: Average sum squared distance over 5 simulations run. The minimum distance is achieved by stickiness of 16 and timidity of 1. Only the succesful combinations from figure 6 are considered. Demand delay is 50 days

Agents working by trial and error benefit from acting slowly and timidly whenever price changes take time to have an effect. Because there is no menu costs, the agents are indifferent between adjusting the price often but slowly or seldom but aggressively. The weakness of this section is that the delay is exogenous and unrealistic. I take care of this in the next section.

4 Firms, Production and Supply Chains

4.1 Zero Knowledge Firms in Isolation

Zero Knowledge firms hire workers, buy and consume inputs, produce and sell output. The firms use the PID Control from section 3.1 to set their sale price and wage offers. When buying inputs the firm simply buys the cheapest available that day. Firms' production is linear with respect to workers hired L_t:

$$F(L_t) = L_t$$

The firm has to decide how much to produce each day; equivalently, the firm has to decide how many workers to hire. The simplest way to do so is to raise production as long as:

$$\text{MB} > \text{MC}$$

A firm producing one type of good priced p_t and consuming labor as only input with unit wage w_t should increase production as long as:

$$p_t + \Delta^p > w_t + \Delta^w \tag{3}$$

Where Δ^p is the price impact: the presumably negative change to sale prices from increasing production; Δ^w is the wage impact: the presumably positive change in wages from hiring more workers.

Notice that p_t and w_t are prices and wages set by independent PID controls. As shown in section 3.1 it takes some time for PID controls to find the correct prices. As a consequence it takes time for the prices found to correctly reflect the true marginal benefits and marginal costs. It follows that the decision to raise or lower production should be taken infrequently in order to be based on correct prices. Define T as the decision period: how many days pass before the firm checks marginal benefits and costs and decide a new production quota.

The firm must also know what the price impacts are. I will deal with their endogenous discovery in section 7. Until then I'll simply assume they are known.

Take a firm facing the daily demand function: $q = 101 - p$, with daily production function $q = f(L) = L$ and wage curve $w = 14 + L$. A firm acting as a monopolist would maximize profits by producing 22 units a day and selling them at \$79. The decision period T is set to 20 for this run. Figure 8 shows a sample run of a zero-knowledge monopolist firm.

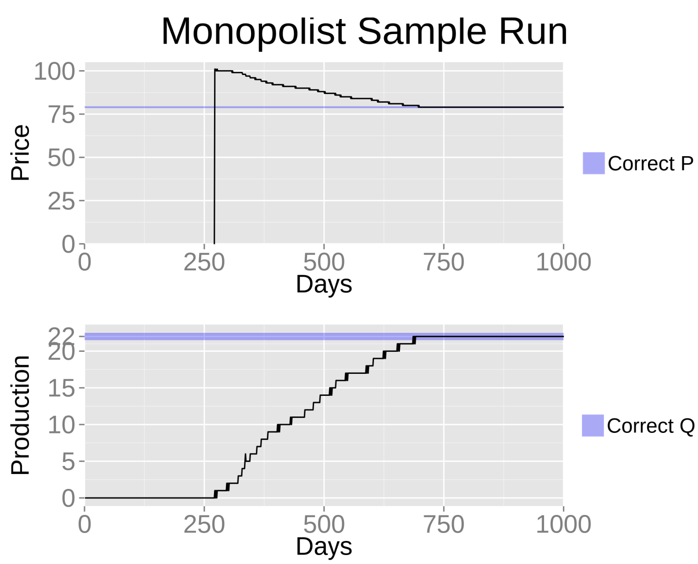

Figure 8: A sample run with a monopolist firm. It reaches the correct price and quantity

If there are multiple firms locked in perfect competition, the zero profit solution is a daily total production of 44 units sold at \$58. Figure 9 shows a sample run with 5 competitors. When competing zero-knowledge firms create noise since they have no knowledge of the opponents decisions. Notice that the number of firms competing is not important for the correct result. If I run the model with a single firm but set its price impacts in equation 3 to 0 I would get a noiseless perfect competitive solution. Perfect competition is a state of mind. At least until section 7.

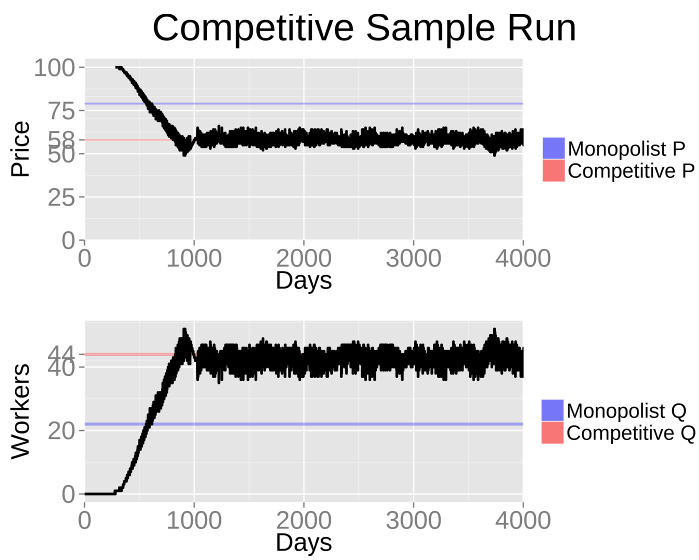

Figure 9: A sample run with a 5 competitive firm. There is noise but centered around the correct price and quantity.

4.2 Zero Knowledge Firms in a Simple Supply Chain

Take a supply chain made of two sectors: wood and furniture. There is a final daily demand for furniture which is exogenous and fixed at:

$$q_F = 102 - p_F$$

Daily production of one unit of furniture requires one worker and consumes one unit of wood:

$$q_F = \min(L_F, q_W)$$

Daily production of one unit of wood requires one worker.

$$q_W = L_W$$

Each sector has its own independent linear labor supply:

$$w_W = L_W$$
$$w_F = L_F$$

Helpfully, there are infinite trees waiting to be cut.

In this section further assume that the wood sector is a monopolist while the furniture market is competitive. I go through all the market-structure permutations in section 6. Solving for the market equilibrium yields the following:

$$q_F = q_W = 17 \quad (4a)$$
$$w_W = w_F = 17 \quad (4b)$$
$$p_W = 68 \quad (4c)$$
$$p_F = 85 \quad (4d)$$

The theoretical demand for wood from the furniture sector is

$$p_W = 102 - 2q_F \quad (5)$$

Before running the full model it is important to know what would be the best PID parameters for a monopolist facing the demand curve in equation 5. Figure 10 shows the search for the optimal PID controller facing an undelayed demand function as in equation 5; the best parameters are $a = 0.08$ and $b = .17$. Now the only difference in running the full model is that the demand faced by the monopolist wood supplier will be made up of other zero-knowledge firms.

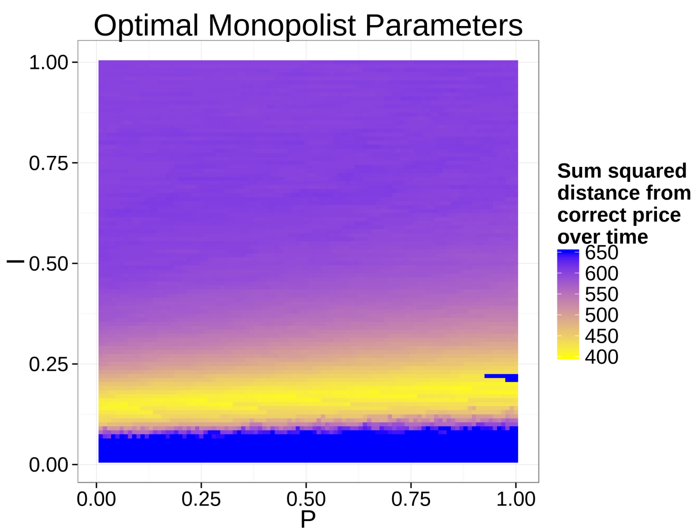

Figure 10: The average squared distance from correct prices when a monopolist faces demand $p = 102 - 2q$ and labor supply $w = L$. Each cell represent a pair of parameters used by the monopolist's sales PID control. The optimal parameter pair is $P = 0.08$ and $I = 0.17$.

We run the supply chain model where the monopolist uses the optimal PID parameters in figure 11. It is clear that the supply-chain neither achieves equilibrium nor orbits around it. While the PID parameters chosen were optimal when facing the instantenously reacting demand curve in equation 5 they work extremely poorly when they face the same demand implemented by other zero-knowledge firms. The issue is delay. It takes time for wood-consumers to deal with new prices and change their wood consumption. By the time they adjust, the supplier has changed sale price again.

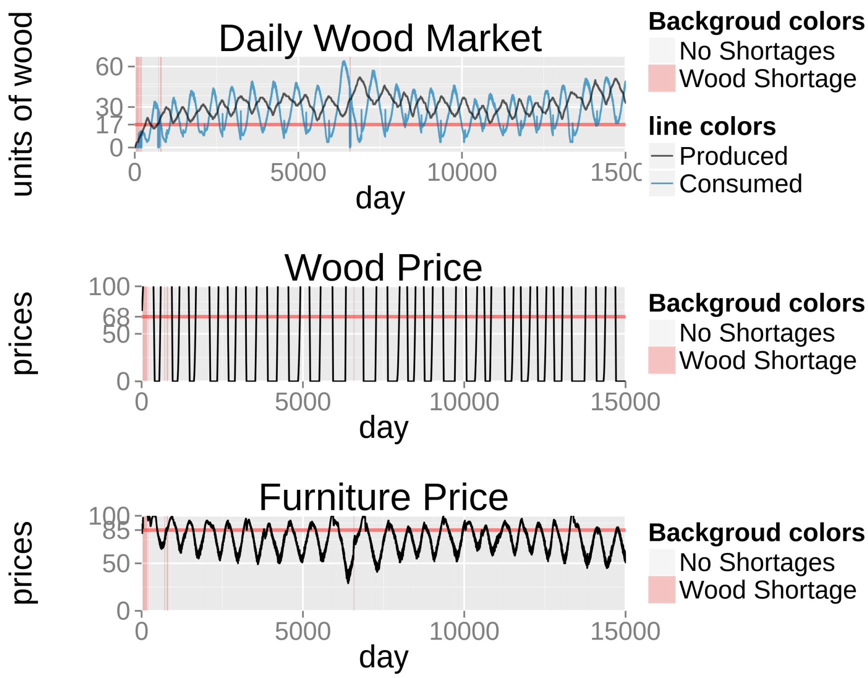

Figure 11: A sample run of the supply chain model using the PID parameters that work best in a single-sector scenario

Now add price stickiness. Force the wood monopolist to change its prices only every 100 days. The results are shown in figure 12. Stickiness solves all supply chain problems: production and prices in both sectors are the correct ones and remain in equilibrium. This is because sticky prices allow the customers to adapt to each supplier decision.

Figure 12: A sample run of the supply chain model where the wood producer uses sticky prices.

There are two endogenous sources of delays in the supply chain. The first delay is the time it takes between a firm making the decision to change its production quota and the controllers adapting to it by finding new wages and prices. The second delay source is the decision period T of the furniture producers which delays their response to change in prices of the wood supplier. The larger T the more the wood supplier prices need to be sticky to balance. Figure 13 shows the relationship between the stickiness and T.

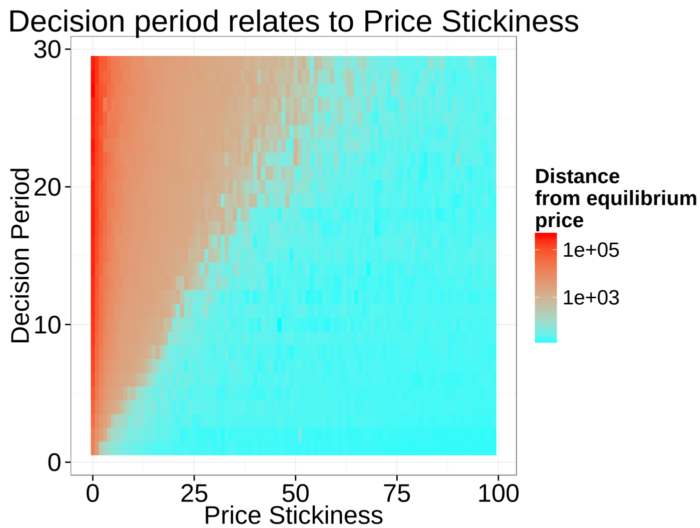

Figure 13: The decision period T of firms is an important source of delays in the system; the higher T the higher the price stickiness needs to be in order to balance it. Each tile represents a 5-runs average squared distance from the correct price over the whole run.

In section 6 I vary the market structure of the supply chain. Before then I introduce a slightly more complicated PID control that allows for inventory targeting.

5 Inventories

5.1 Modifying the PID Control to Deal with Inventories

Assume sellers also have an inventory target i^*: the number of goods they would like to have in stock at the end of each day. There is a tension between targeting stocks and flows at the same time. PID controls discover prices by zeroing netflows but inventory accumulation requires prolonged positive netflow. Inventory shrinkage requires prolonged negative netflow.

Define the inventory error at day t as:

$$e^i_t = i^* - i_t$$
$$e_t = \text{Target Inventory} - \text{Inventory Today}$$

Then the daily netflow necessary to achieve inventory target in arbitrary T days is:

$$\frac{1}{T} e^i_t$$

Now modify the PID error in equations 1 as follows:

$$e_t = y^* - y_t - \frac{1}{T}e_t^i \tag{6a}$$

$$e_t = \text{Inflow} - \text{Outflow} - \text{Target Netflow} \tag{6b}$$

$$e_t = \text{Netflow} - \text{Target Netflow} \tag{6c}$$

Fix a T (in this paper 30 days) and zero-knowledge traders can keep using PID adjustment (equation 2) using the new error definition of equation 6a.

There are two reasons to expand Zero-Knowledge firms to deal with inventories. Firstly, firms with inventories can avoid having to count the number of customers they attract. This is useful in models and markets where that is not feasible. Firms with inventories can find the right prices just by monitoring their own change in inventory; that is I can further modify equation 6a as:

$$e_t = i_t - i_{t-1} - \frac{1}{T}e_t^i$$

Secondly, inventories accumulate PID errors and pose problems for stickiness . Sticky prices, by construction, allow for long periods where the daily production is not equal to the number of customers. This implies an unwanted accumulation (or shrinkage) of inventory. When monitoring only flows, that's not an issue. When stocks are also important sticky prices can cause adjustment issues.

Figure 14 shows a sample run of a seller discovering the correct prices and at the same time maintaining a fixed inventory level of 100. In figure 15 I add a small delay, $\delta = 10$, which is enough to create persistent noise. The issue can be fixed as in section 3.2: making prices comparatively sticky as in figure 16. Notice that while price and inventory targets are reached, it takes about 10 times more days than the no-inventory examples.

Figure 14: A sample run of a zero knowledge seller with inventory target of 100 facing, like in figure 1 the daily linear demand $q = 101 - p$. This trader is using a PID controller with parameters $a = b = .1$ and $c = 0$

Figure 15: A sample run of a zero knowledge seller with inventory target of 100 facing, like in figure 2 the daily linear demand $q_t = 101 - p_{t-10}$. This trader is using a PID controller with parameters $a = b = .1$ and $c = 0$. This seller takes longer to rech the same price and cannot cancel the noise completely.

Figure 16: The same setup as in figure 15 but this time the zero-knowledge seller uses sticky prices: changes them every 10 days. It achieves both targets

Production decisions as expressed in section 4.1 are unaffected by the inventory targeting. Figure 17 shows a sample run with a monopolist facing demand curve: $q = 101 - p$ and labor supply curve $w = 14 + L$. The zero knowledge monopolist finds profit maximizing prices and quantity while at the same time holding 100 units of goods in inventory. Figure 18 shows a sample run of 5 zero-knowledge firms which result in zero-profits prices and quantities.

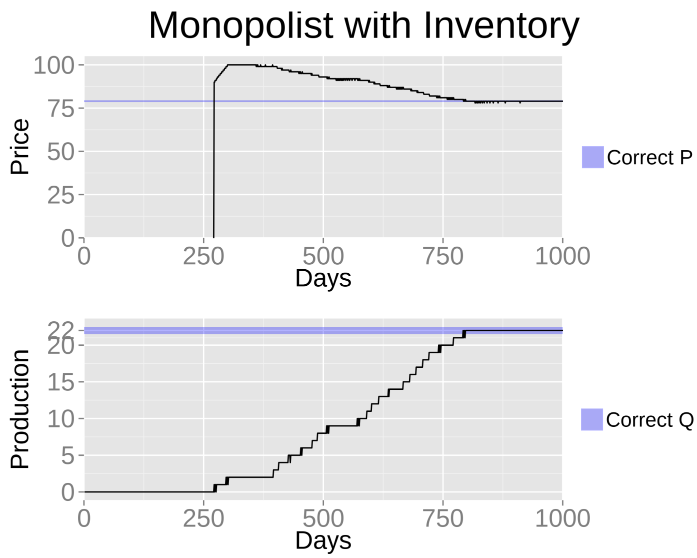

Figure 17: A sample run with a monopolist targeting 100 units of inventory. It reaches the profit-maximizing price and quantity as the run in figure 8

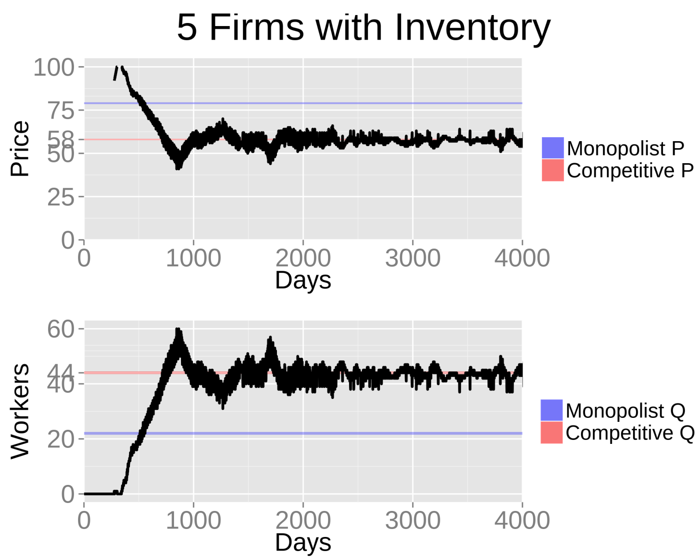

Figure 18: A sample run with a 5 zero knowledge competitors each targeting 100 units of inventory. It reaches the zero profit prices and quantities as the run in figure 9

6 Market Structure

In equations 4 I expressed the solution where the wood sector is a monopolist while the furniture sector is competitive. If the wood sector is competitive while the furniture is monopolistic the equilibrium is:

$$q_F = q_W = 17 \tag{7a}$$
$$w_W = w_F = 17 \tag{7b}$$
$$p_W = 17 \tag{7c}$$
$$p_F = 85 \tag{7d}$$

If both sectors are competitive the no-profit equilibrium should be:

$$q_F = q_W = 34 \tag{8a}$$
$$w_W = w_F = 34 \tag{8b}$$
$$p_W = 34 \tag{8c}$$
$$p_F = 68 \tag{8d}$$

I run 100 simulations for each market structure (competitive means 5 firms in the same sector). Each simulation runs for 15000 market days. All wood producers have inventory target 1000, all food producers have target 100, regardless of market structure. All input producers use sticky prices (100 days each price change), regardless of market structure.

In general the model behaves as predicted by theory. Figure 19 shows the distribution of input prices at the end of the simulation; figure 20 shows the output prices; figure 21 shows the quantity produced.

Figure 19: The price of wood (first sector) for 300 simulated runs, 100 for each market structure. The vertical dashed lines represent the theoretical equilibrium. Each datum in the histogram is the average price of the last 500 days of simulation.

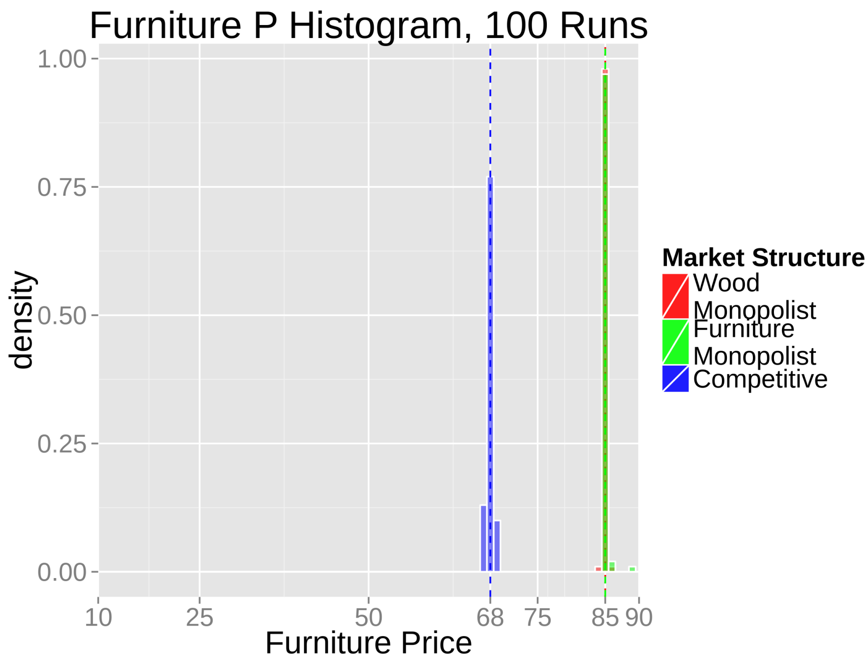

Figure 20: The price of furniture (second sector) for 300 simulated runs, 100 for each market structure. The vertical dashed lines represent the theoretical equilibrium. Each datum in the histogram is the average price of the last 500 days of simulation.

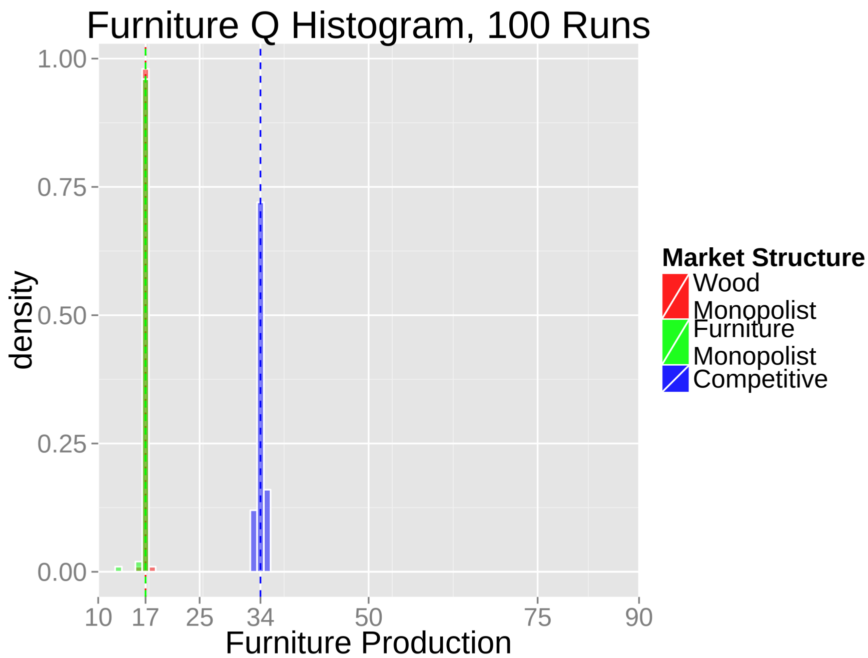

Figure 21: The units of furniture produced daily at the end of 300 simulated runs, 100 for each market structure. The vertical dashed lines represent the theoretical equilibrium. Each datum in the histogram is the average daily production over the last 500 days of simulation.

7 Learning

7.1 Learning in a one-sector economy

A zero-knowledge firm should be able to learn on its own whether it is a monopolist or not and, if it is, it should be able to find out the slope of the supply or demand curve it faces. This helps the firm determine the price impact functions and therefore marginal costs and benefits.

The firm can take advantage of the data generated by its PID controls. In each market the firm each day sets a price and observes a quantity sold or bought; learning then is the act of gathering this data and regressing it.

Because the PID controls keep generating one new observation each day in each market it makes sense to implement the regressions by a Recursive Least Squares filter(Welch & Bishop, 1995). For example the zero-knowledge firm has to discover the change in prices given an increase in production Δ^p. Estimate the linear demand function $p_t = \alpha + \Delta^p q$. There are two parameters: $\vec{\beta} = (\alpha, \Delta^p)$. Each day the firm observes the price it has offered and the quantity traded $y_t = p_t, \vec{x}_t = (1, q_t)$. The current estimation of $\vec{\beta}$ is $\hat{\vec{\beta}}_{t-1} = \left(\hat{\alpha}_{t-1}, \hat{\Delta}^p_{t-1}\right)$. Update it with the new observation in four steps:

$$\vec{k} = \mathbf{P}_{t-1}\vec{x}^T \left(\vec{x}P_{t-1}\vec{x}^T + 1\right)^{-1} \quad \text{Constructing the Kalman gain} \tag{9a}$$

$$\epsilon_t = y_t - x\hat{\vec{\beta}}_{t-1} \quad \text{Finding the prediction error} \tag{9b}$$

$$\hat{\vec{\beta}}_t = \hat{\vec{\beta}}_{t-1} + \vec{k}\epsilon_t \quad \text{Updating predictor given error} \tag{9c}$$

$$\mathbf{P}_t = (I - \vec{k}\vec{x}_t)\mathbf{P}_{t-1} \quad \text{Updating covariance matrix} \tag{9d}$$

Where $\mathbf{P}_t$ is the 2×2 covariance matrix. Functionally $\mathbf{P}_0$ is a Bayesian prior which is set at $10^4 I$ for all simulations.

Combining learning with inventory targeting I have truly zero-knowledge agents. Learning is fast and efficient in one-sector markets. Figure 22 and figure 23 show the results of running 100 simulations with a learning monopolist and 100 simulations with 5 learning competitors. Notice that there is initially no difference between a "monopolist" and a "competitor", it is the endogenous discovery of price impacts that drives firms' behavior.

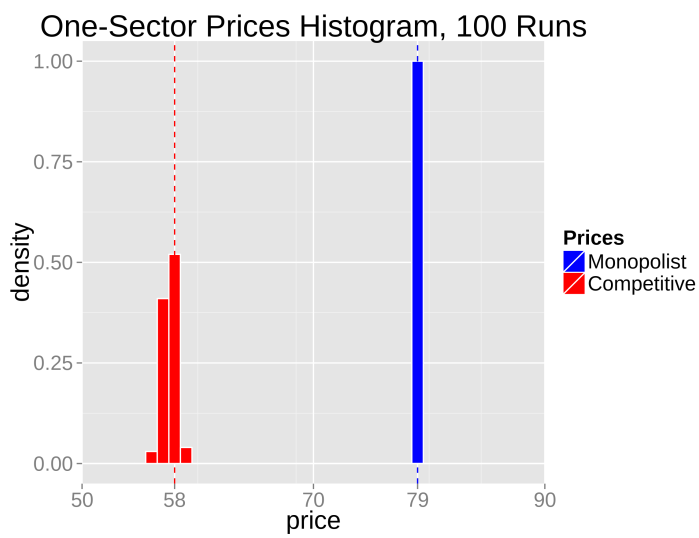

Figure 22: The histogram of prices from running 100 monopolist and 100 competitive (5 firms) scenarios. All firms need to learn the price and wages impact. All firms target inventory (100 units of output). Each observation in the histogram is the average of the last 500 days' prices of that particular simulation. Prices are very accurate

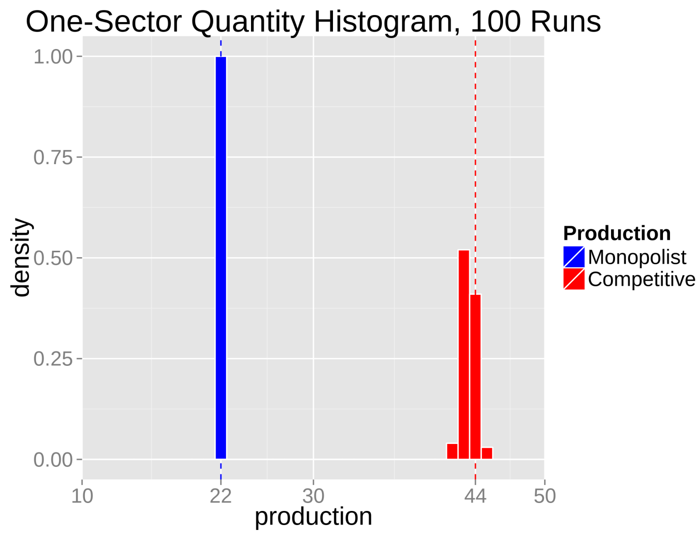

Figure 23: The histogram production from running 100 monopolist and 100 competitive (5 firms) scenarios. All firms need to learn the price and wages impact. All firms target inventory (100 units of output). Each observation in the histogram is the average of the last 500 days' production of that particular simulation. Quantities are very accurate

7.2 Learning in a supply-chain

Learning is far more problematic in a supply-chain. Firstly, if there is a delay δ between setting a price p_t and it affecting quantity traded, the zero-knowledge firm should regress $p_{t-\delta}$ over q_t. But this is impossible as the delay is unknown. Secondly, because of stickiness, the firm often sets prices p_t that are not the market clearing ones. Because learning works by regressing observed pairs p_t, q_t, the results are often useless. Finally, with inventory targeting and stickiness, the prices set by the zero-knowledge firm stay above or below market-clearing in order to accumulate or dispose inventories. This creates very strong error autocorrelations (if inventories were not enough yesterday, they are probably not enough today as well).

Because of this I found useful letting zero-knowledge firms using sticky prices to regress not p_t, q_t but their moving averages $\text{MA}(p_t), \text{MA}(q_t)$ of size 500. Figures 24 25 26 show the results of 100 simulations for each market structure. All are using inventory targeting and sticky prices. The results are far more dispersed, usually because one of the recursive least squares filters failed to learn the correct the right slope.

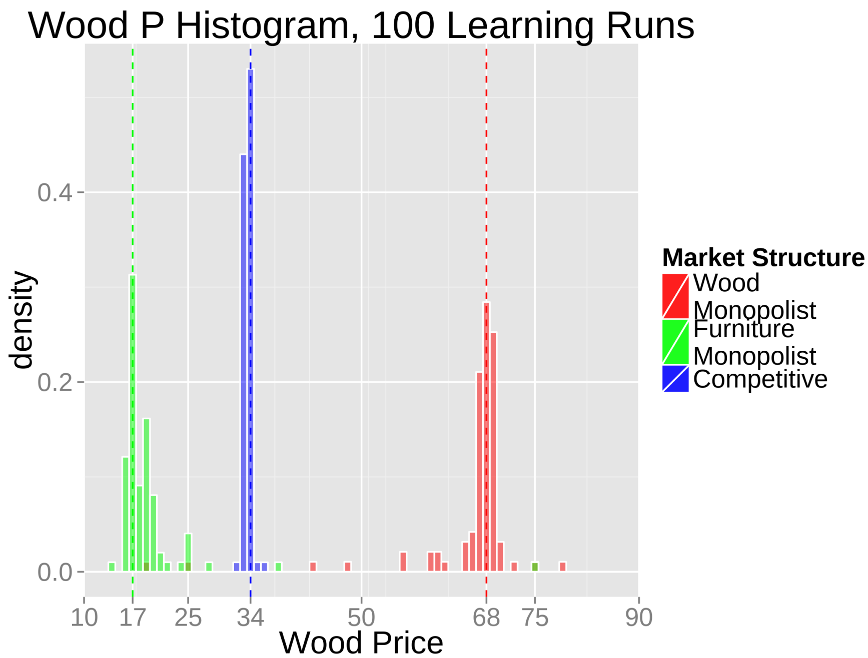

Figure 24: The price of wood (first sector) for 300 simulated runs, 100 for each market structure. The dashed vertical lines represent the theoretical equilibrium. Each datum in the histogram is the average price of the last 500 days of simulation.

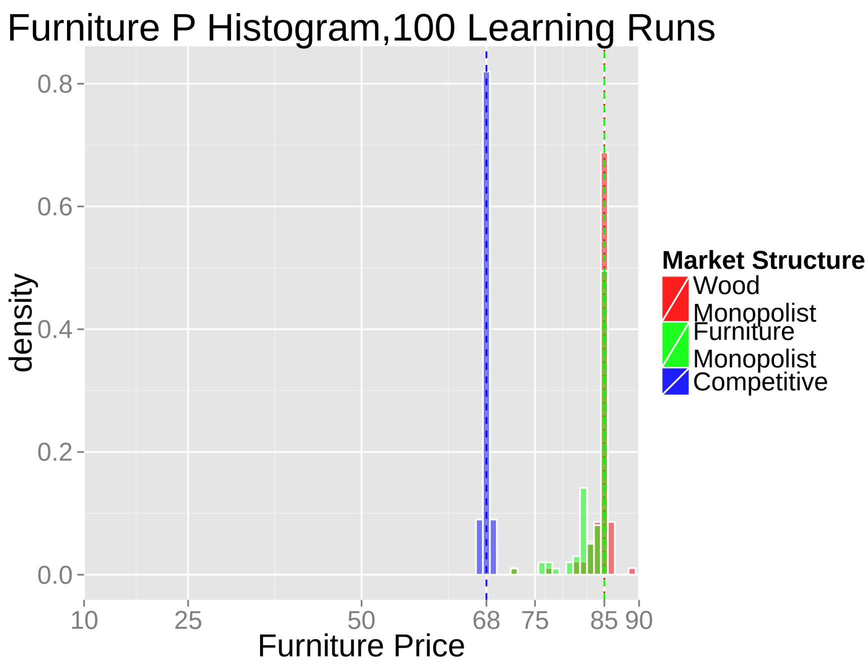

Figure 25: The price of furniture (second sector) for 300 simulated runs, 100 for each market structure. The dashed vertical lines represent the theoretical equilibrium. Each datum in the histogram is the average price of the last 500 days of simulation.

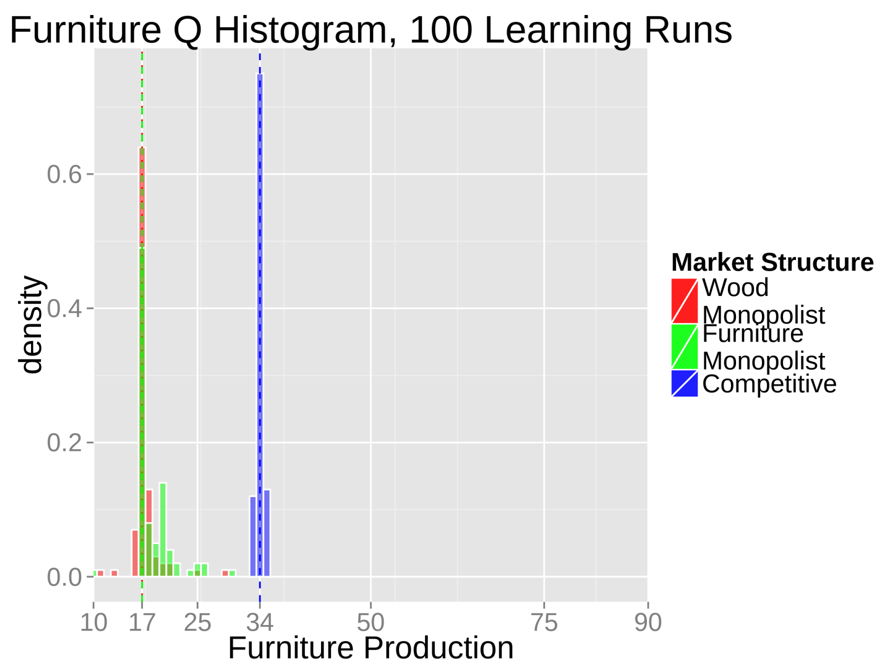

Figure 26: The units of furniture produced daily at the end of 300 simulated runs, 100 for each market structure. The dashed vertical lines represent the theoretical equilibrium. Each datum in the histogram is the average daily production over the last 500 days of simulation.

8 Weaknesses of the Model and Conclusions

The model has many weaknesses that will need to be addressed. Learning works poorly with delays and inventories. In section 7.2 I managed the seasonality by filtering the data with an arbitrary 500 days moving average. While this low-pass filtering attenuates some of the seasonality, it is very rudimentary and can be improved.

Another issue is the lack of endogenous tuning of PID parameters. I set the PID parameters and stickiness at the beginning of each simulation. I tried to justify every decision through parameter sweeps but it's clear that such information wouldn't be available to the agents themselves. The problem with just taking an auto-tuning method from the control theory literature and applying it here is that as soon multiple PID compete the model stops being linear time invariant, which is a pre-requisite for most tuning techniques.

The total flexibility that buyers have by always choosing the cheapest seller creates some unnatural dynamics in competitive markets. In a simple competitive setting like in figure 9 each firm produces only a part of the total output.

But while in aggregate quantity and prices are correct, in reality each market day only one firm, the price-leader, trades. Any given market day has a firm with the lowest price, that firm will attract all the buyers. Because it sells more than it produces the firm will raise the price the next day while all its competitors will lower it since they sold nothing. The daily competitive market is actually made by an ever changing price-leader firm that holds the monopoly for that day but only by charging the most competitiv price.

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