MAT3007 Assignment 2

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Student Number: 120090272 **Assignment:** Assignment 2

Problem 1

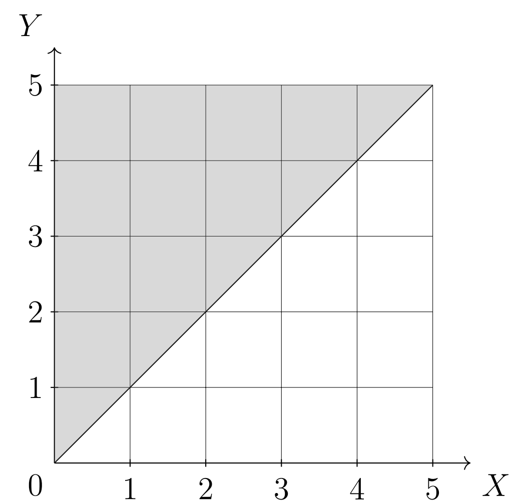
Solution:

1. False

Reason: In some cases, LP can be feasible but optimal value is unbounded.

Example: minimize: $c^T x = [1, -1][x_1, x_2]^T$
subject to: $Ax = b \Rightarrow [-1, 1, -1][x_1, x_2, x_3]^T = 0$
$x_1, x_2, x_3 \geq 0$

In this case, the graph of LP is as below.
and in this case, the optimal value is feasible but unbounded, so the answer is False.



2. False

Reason: More than m variables can be positive.

Example: minimize: $c^T x = [-1, -1][x_1, x_2]^T$
subject to: $[1, 1, 1, 1][x_1, x_2, x_3, x_4]^T = 0$
$x_1, x_2, x_3, x_4 \geq 0$

In the solutions of this linear program, $x_1 + x_2 = 1$ and both larger than 0, in this case 2 variables are positive, which is larger than the scale of A for m=1, so the answer is False.

3. True

Reason: If there is more than one optimal solution, then there are uncountably many optimal solutions.

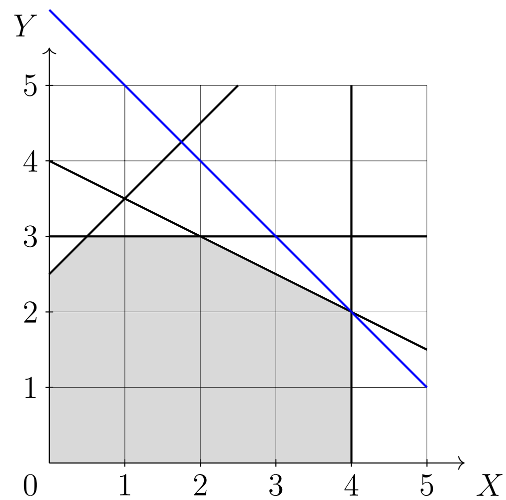
Proof: minimize: $c^T x$
subject to: $Ax = b$
$x \geq 0$

For feasible set is a convex set, objective function is linear.
Assume that x_1, x_2 are two optimal solutions of a multi-solution linear program with $x_1 \neq x_2$.
So we can have: $Ax_1 = Ax_2 = b$, and exist an x_3 in convex set such that $x_3 = \lambda x_1 + (1-\lambda)x_2$
multiply the equation with matrix A: $Ax_3 = \lambda b + (1-\lambda)b = b \Rightarrow Ax_3 = b$
also $c^T x_3 = \lambda c^T x_1 + (1-\lambda)c^T x_2 = c^T x_1 = c^T x_2$
so we can infer from the equation that x_3 is also an optimal solution, and so as others.
So, if there is more than one optimal solution, then there are uncountably many optimal solutions, the answer is True.

Problem 2

Solution:

maximize: $x_1 + x_2$
subject to: $-x_1 + x_2 \leq 2.5, \quad x_1 + 2x_2 \leq 8$
$0 \leq x_1 \leq 4, \quad 0 \leq x_2 \leq 3$
The graph of the LP is as below,
Active constraints: $x_1 \leq 4$ and $x_1 + 2x_2 \leq 8$
Vertices of feasible region: (0,2.5),(0.5,3),(2,3),(4,2),(4,0),(0,0)



Problem 3

Solution:

1. the standard from:
minimize: $-x_1 - 4x_2 - x_3$
subject to:
$$\begin{aligned} 2x_1 + 2x_2 + x_3 + s_1 \quad &= 4 \\ x_1 \quad - x_3 \quad - s_2 &= 1 \\ x_1, x_2, x_3, s_1, s_2 &\geq 0 \end{aligned}$$

2. Write the constraints as: $Ax = b \Rightarrow \begin{bmatrix} 2 & 2 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & -1 \end{bmatrix} [x_1 x_2 x_3 s_1 s_2]^T = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

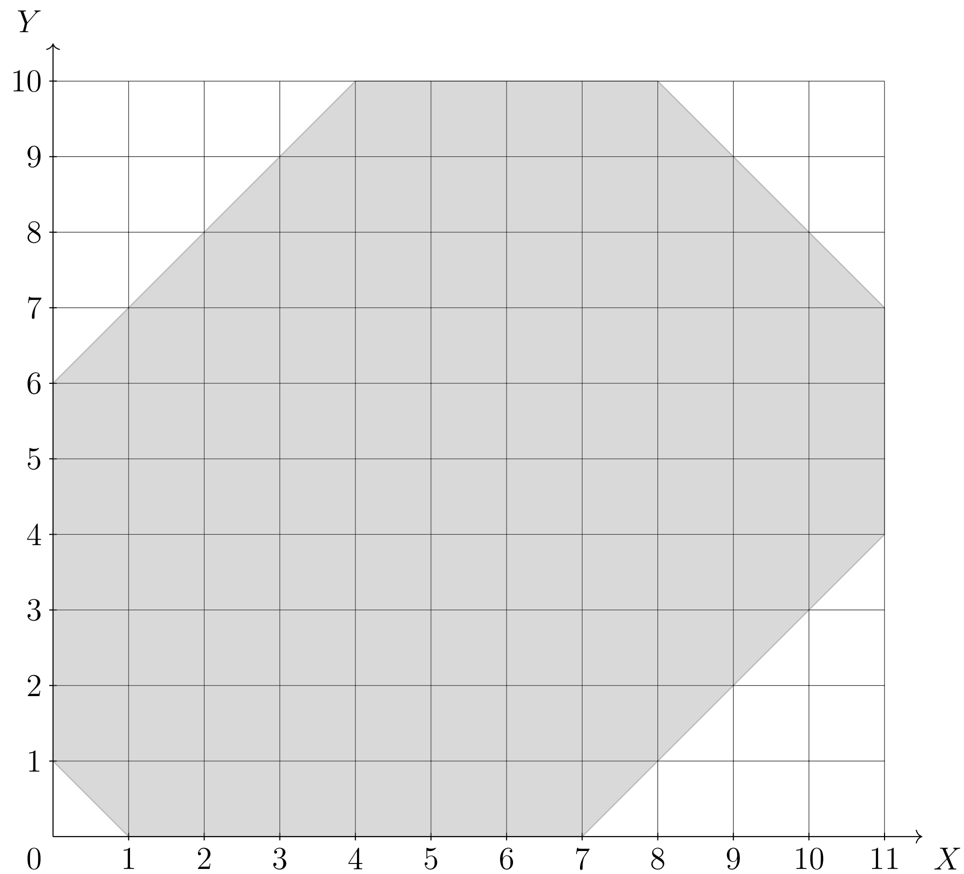
Solution = x = $A_{B(m)}^{-1} b$
The indices: $[A_1, A_2],\ [A_1, A_3],\ [A_1, A_4],\ [A_1, A_5],\ [A_2, A_3],\ [A_2, A_5],\ [A_3, A_4],\ [A_3, A_5],\ [A_4, A_5]$
Basic Solutions: $[1, 1, 0, 0, 0],\ [\frac{5}{3}, 0, \frac{2}{3}, 0, 0],\ [1, 0, 0, 2, 0],\ [2, 0, 0, 0, 1],\ [0, \frac{5}{2}, -1, 0, 0],\ [0, 2, 0, 0, -1],$
$[0, 0, -1, 5, 0],\ [0, 0, 4, 0, -5],\ [0, 0, 0, 4, -1]$
Basic Feasible Solutions: $[1, 1, 0, 0, 0],\ [\frac{5}{3}, 0, \frac{2}{3}, 0, 0],\ [1, 0, 0, 2, 0],\ [2, 0, 0, 0, 1]$
Values: 5, $\frac{7}{3}$, 1, 2
So the Basic Solutions and Basic Feasible Solutions are above.

3. From problem 2 we infer that the optimal solution is $x_1 = x_2 = 1, x_3 = 0$, and the optmal value is 5.

Problem 5

Solution:

The plot of the points are as below:



The functions of the constraints are:

$-x_1 + x_2 \leq 6$

$x_2 \leq 10$

$x_1 + x_2 \leq 18$

$x_1 \leq 11$

$x_1 - x_2 \leq 7$

$x_2 \geq 0$

$-x_1 - x_2 \leq -1$

So we can generate LP:

minimize: -r
subject to:
A * y <= b;
A * (repmat(y,1,8) + Direction' * r) <= repmat(b,1,8);
r >= 0;

In the above equations, Direction is an matrix consists the information of direction.
And the constraints are:
b=[0,6,10,18,11,7,0,-1]

$$A = \begin{bmatrix} -1 & -1 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & -1 & -1 & -1 \end{bmatrix}^T$$

solved by the software, the optimal center is at about(5.4, 4.8), and max radius r = 4.5962.

Problem 4

Solution:

Solved by the sofware we find the optimal solution is: [0, 0, 5, 5, 5] It means that if we purchase security 3, 4, 5, we will at least have 1 dollar profit

[The code of Q4 and Q5]:

```
price = [0.75; 0.35; 0.40; 0.75; 0.65];
share_limit = [10; 5; 10; 10; 5];
payoff = [1, 1, 1, 0, 0;
          0, 0, 0, 1, 1;
          1, 0, 1, 0, 1;
          1, 1, 1, 1, 0;
          0, 1, 0, 1, 1];
results = eye(5);

cvx_begin quiet
variables x(5) outcome;
    minimize -outcome;
    subject to:
        0<= x <= share_limit;
        w = x' * (payoff * results - repmat(price, 1, 5));
        repmat(outcome, 1, 5) <= w;
cvx_end
```

```
A = [-1, 0;
      -1,  1;
       0,  1;
       1,  1;
       1,  0;
       1, -1;
       0, -1;
      -1, -1];
b = [ 0;
      6;
     10;
     18;
     11;
      7;
      0;
     -1];
Direction = [1,       0;
             0,       1;
            -1,       0;
             0,      -1;
     sqrt(1/2), sqrt(1/2);
     sqrt(1/2), -sqrt(1/2);
    -sqrt(1/2), sqrt(1/2);
    -sqrt(1/2), -sqrt(1/2)];
```

```
cvx_begin quiet
variables r y(2); % generate variables r and y
    minimize -r;
    subject to:
        A * y <= b;
        A * (repmat(y,1,8) + Direction' * r) <= repmat(b,1,8);
        r >= 0;
cvx_end
```