Networked Life

Q1 Tutorial

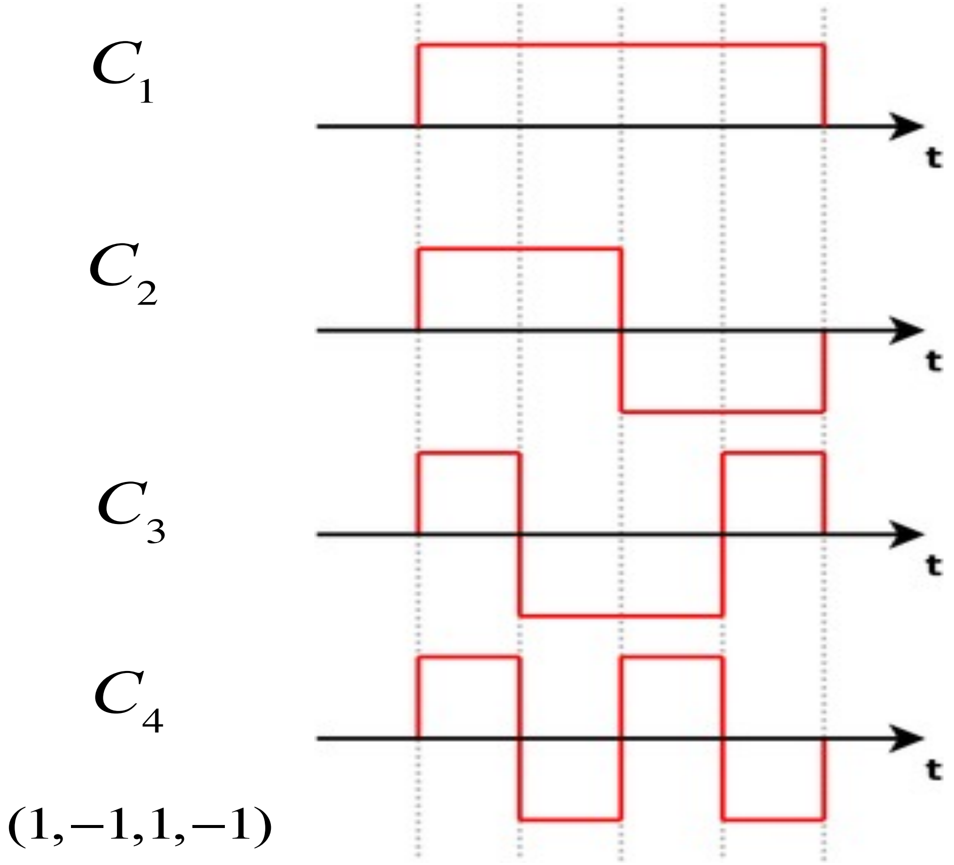
- CDMA
- Games
- Feasible power example

CDMA concepts and examples

A CDMA primer

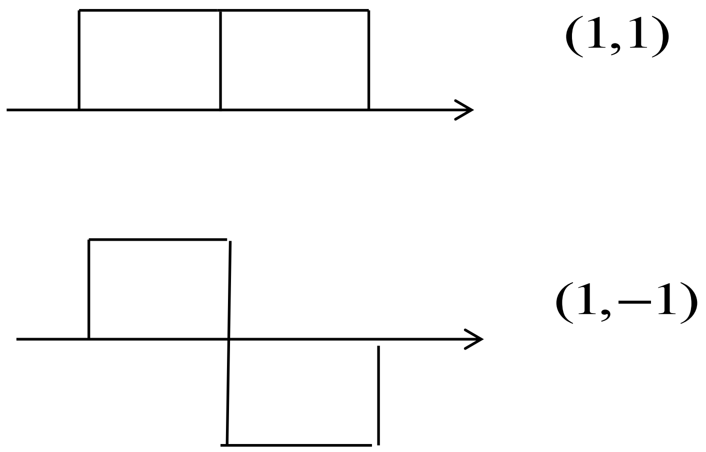
- How synchronous CDMA works
- https://en.wikipedia.org/wiki/Code-division_multiple_access

Example of orthogonal codes



Orthogonal: $C_i \otimes C_j = 0$

$\otimes$: inner product of the bit vectors

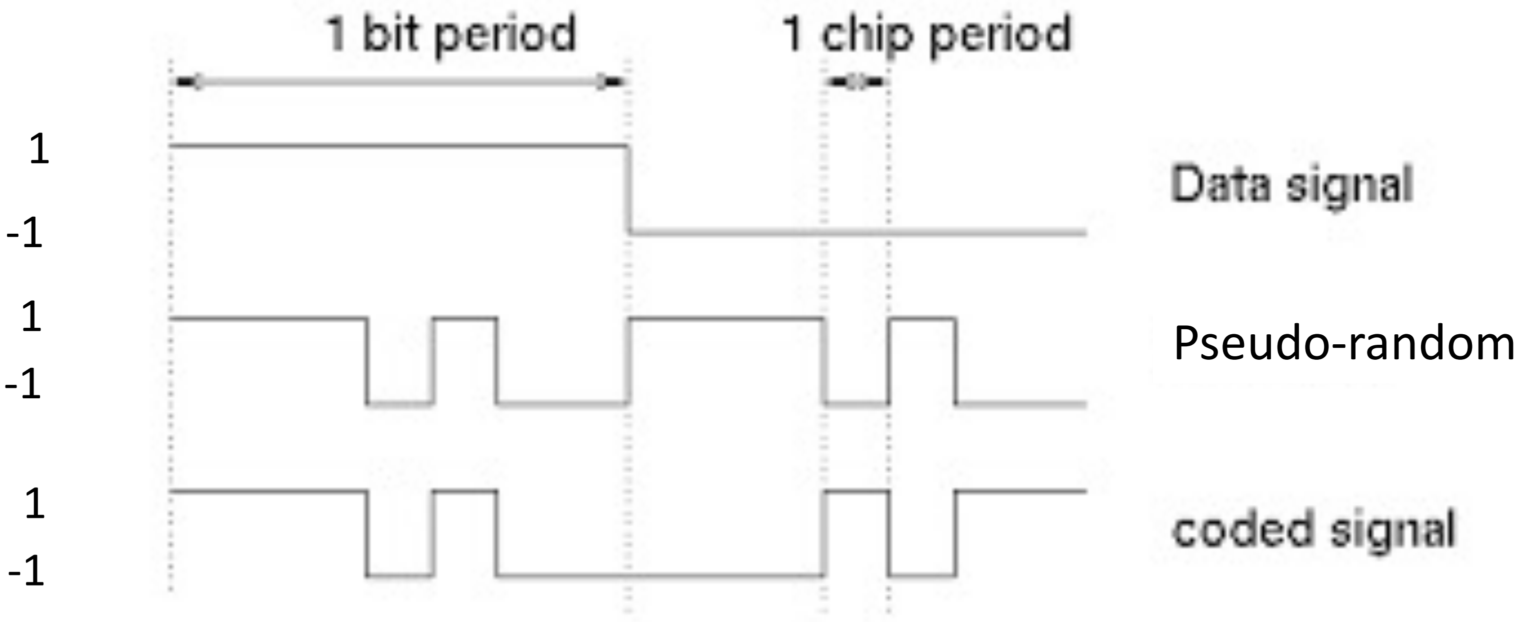


codes of length 2

codes of length 4

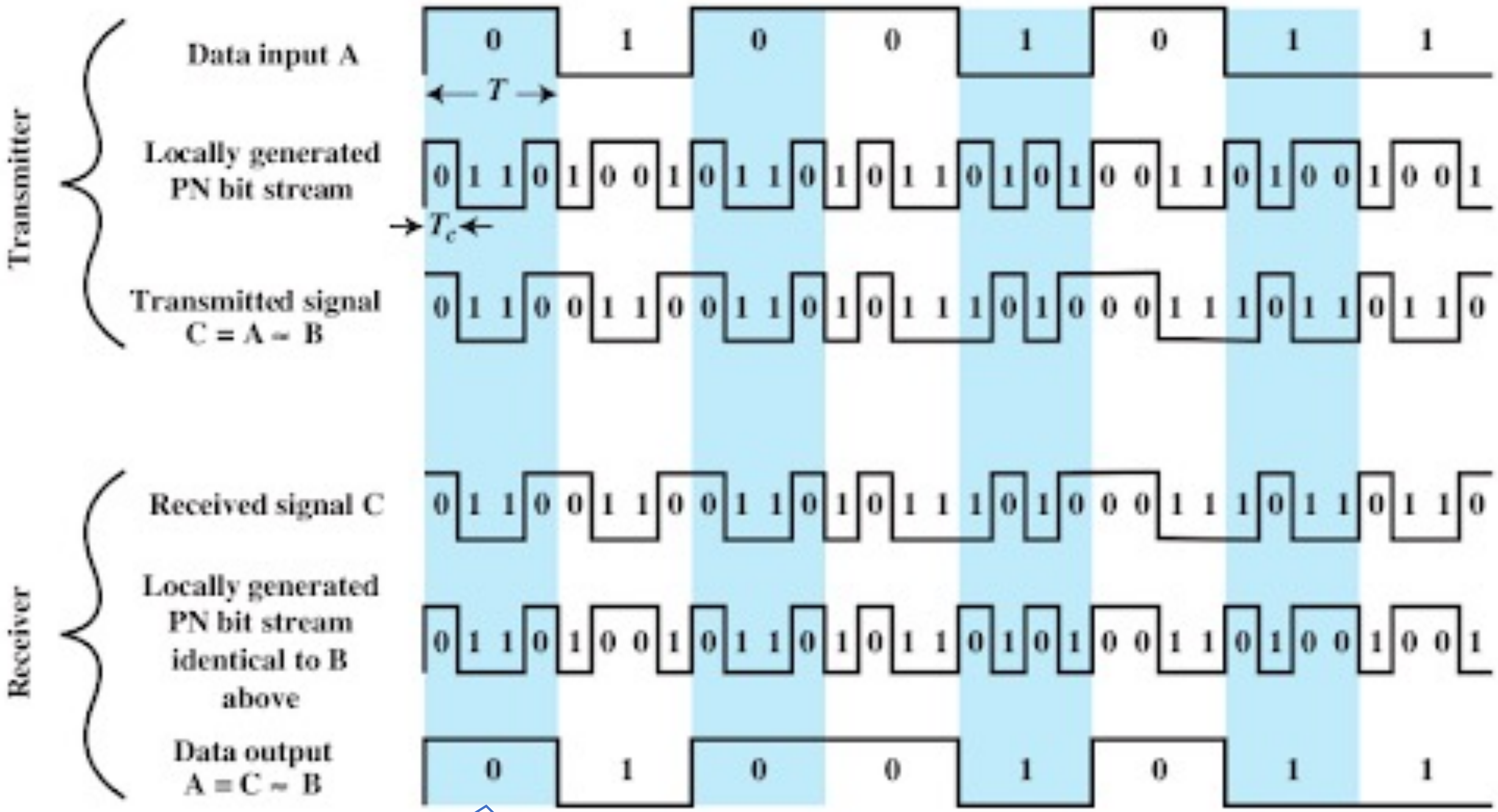
$C_i \otimes C_j = 0, C_i \otimes C_i = 4$

Generation of CDMA signal



Example of encoding and decoding

we encode bits as voltage values:
bit 0: +1v
bit 1: -1v



pointwise product of code×data

inner product ⊗ of code ⊗ data

Case1: Both senders send

Step	Encode sender0	Encode sender1
0	code0 = (1, –1), data0 = (1, 0, 1, 1)	code1 = (1, 1), data1 = (0, 0, 1, 1)
1	encode0 = 2(1, 0, 1, 1) – (1, 1, 1, 1) = (1, –1, 1, 1)	encode1 = 2(0, 0, 1, 1) – (1, 1, 1, 1) = (–1, –1, 1, 1)
2	signal0 = encode0 ⊗ code0 = (1, –1, 1, 1) ⊗ (1, –1) = (1, –1, –1, 1, 1, –1, 1, –1)	signal1 = encode1 ⊗ code1 = (–1, –1, 1, 1) ⊗ (1, 1) = (–1, –1, –1, –1, 1, 1, 1, 1)

Signal = signal0 + signal1 = (0, -2, -2, 0, 2, 0, 2, 0)

Step	Decode sender0	Decode sender1
0	code0 = (1, –1), signal = (0, –2, –2, 0, 2, 0, 2, 0)	code1 = (1, 1), signal = (0, –2, –2, 0, 2, 0, 2, 0)
1	decode0 = pattern.vector0	decode1 = pattern.vector1
2	decode0 = ((0, –2), (–2, 0), (2, 0), (2, 0)).(1, –1)	decode1 = ((0, –2), (–2, 0), (2, 0), (2, 0)).(1, 1)
3	decode0 = ((0 + 2), (–2 + 0), (2 + 0), (2 + 0))	decode1 = ((0 – 2), (–2 + 0), (2 + 0), (2 + 0))
4	data0=(2, –2, 2, 2), meaning (1, 0, 1, 1)	data1=(–2, –2, 2, 2), meaning (0, 0, 1, 1)

Case2: only sender0 sends

Step	Encode sender0
0	code0 = (1, –1), data0 = (1, 0, 1, 1)
1	encode0 = 2(1, 0, 1, 1) – (1, 1, 1, 1) = (1, –1, 1, 1)
2	signal0 = encode0 ⊗ code0 = (1, –1, 1, 1) ⊗ (1, –1) = (1, –1, –1, 1, 1, –1, 1, –1)

Signal = signal0

Step	Decode sender0	Decode sender1
0	code0 = (1, –1), signal = (1, –1, –1, 1, 1, –1, 1, –1)	code1 = (1, 1), signal = (1, –1, –1, 1, 1, –1, 1, –1)
1	decode0 = pattern.vector0	decode1 = pattern.vector1
2	decode0 = ((1, –1), (–1, 1), (1, –1), (1, –1)).(1, –1)	decode1 = ((1, –1), (–1, 1), (1, –1), (1, –1)).(1, 1)
3	decode0 = ((1 + 1), (–1 – 1),(1 + 1), (1 + 1))	decode1 = ((1 – 1), (–1 + 1),(1 – 1), (1 – 1))
4	data0 = (2, –2, 2, 2), meaning (1, 0, 1, 1)	data1 = (0, 0, 0, 0), meaning no data

Non-cooperative behaviour: games and equilibria

What is a game

- A game is huge abstraction of reality. It forgets many behavioral aspects and approximate all this in the strategy set and the payoff functions
- 3-tuple definition:
  - Players
  - Strategy space per player
  - Payoff function per player
- The most commonly used solution concepts are equilibrium concepts, most famously the Nash equilibrium

Example: Prisoner's dilemma

- Game: Player A, Player B,   Strategies: {*deny, confess*}
- Payoff matrix: $U_A(a,b), U_B(a,b)$
- Game description: strategic form, extensive form (dynamics)

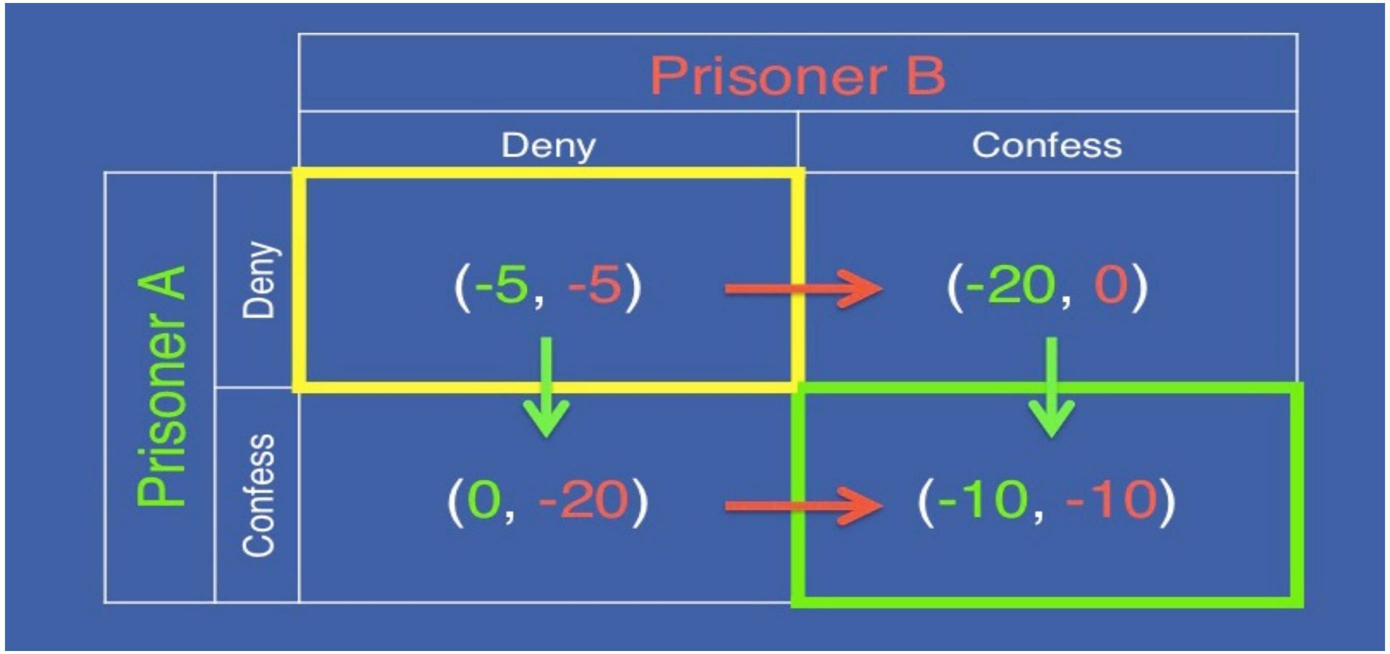
Simultaneous games
Sequential games

		Prisoner B	
		Deny	Confess
Prisoner A	Deny	(-5, -5)	
	Confess		

$$U_A(deny, deny) = -5$$
$$U_B(deny, deny) = -5$$

Example: Prisoner's dilemma

- Game: Player A, Player B,   Strategies: {*deny, confess*}
- Best response strategies: $s \in BR_A(.,b)$ iff $U_A(s,b) \geq U_A(s',b)$ for all $s' \in S_A$



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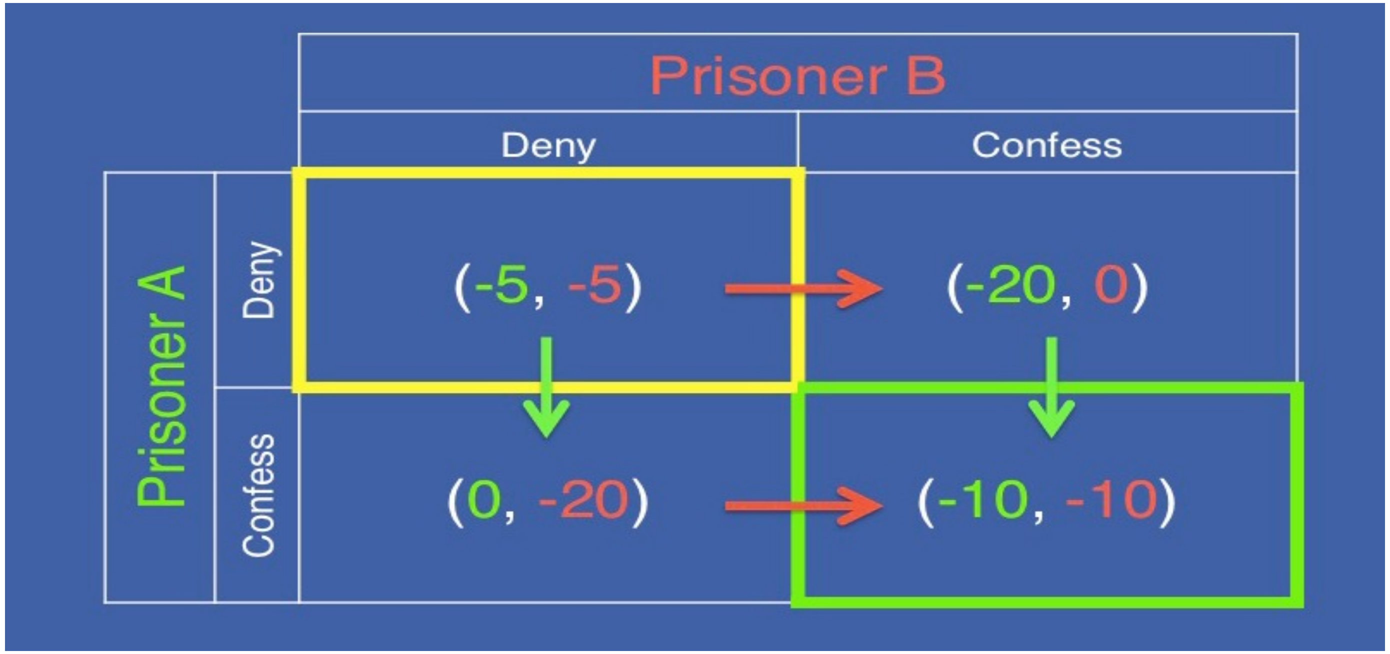


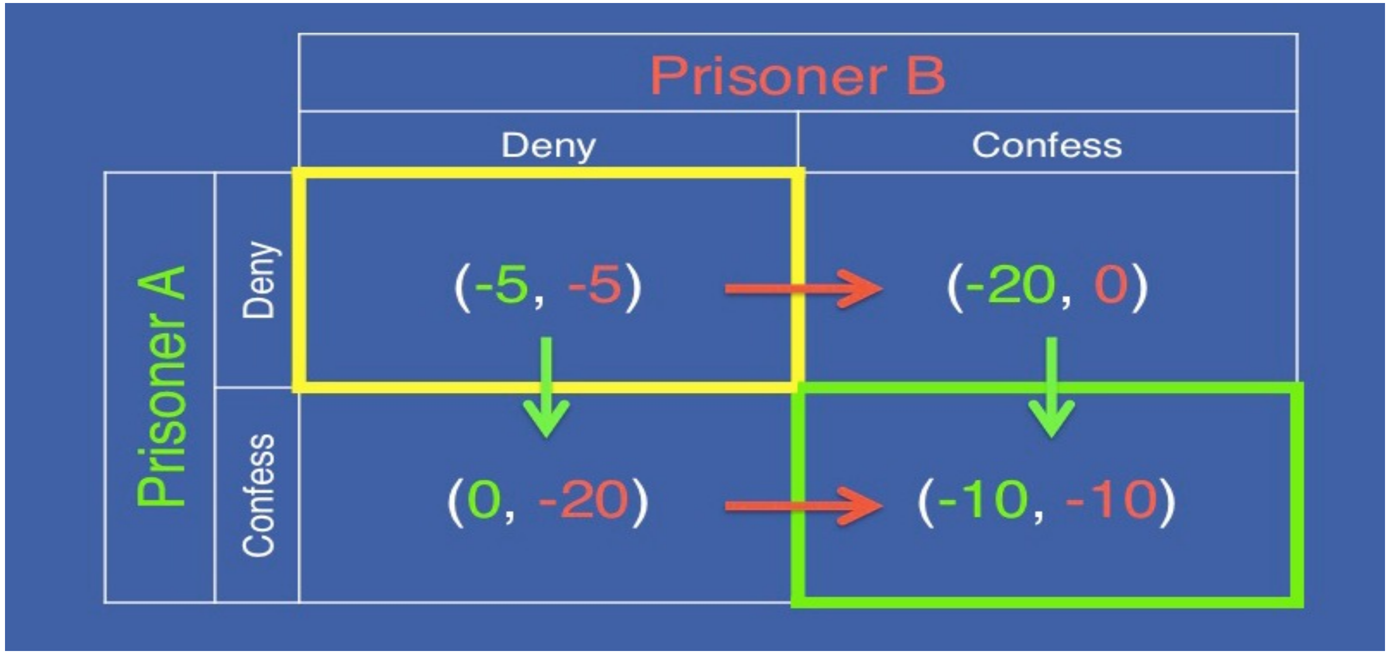
Nash Equilibrium NE:
(a,b) is a NE iff
$a \in BR_A(.,b)$,
$b \in BR_B(a,.)$

The most commonly used solution concept for a game

may apply a **refinement** to narrow down the solutions

Example: Prisoner's dilemma

- Game: Player A, Player B, Strategies: {*deny, confess*}
- Dominant strategy: $s \in D_A$ iff $U_A(s,b) \geq U_A(s',b)$ for all $s' \in S_A$, $b \in S_B$

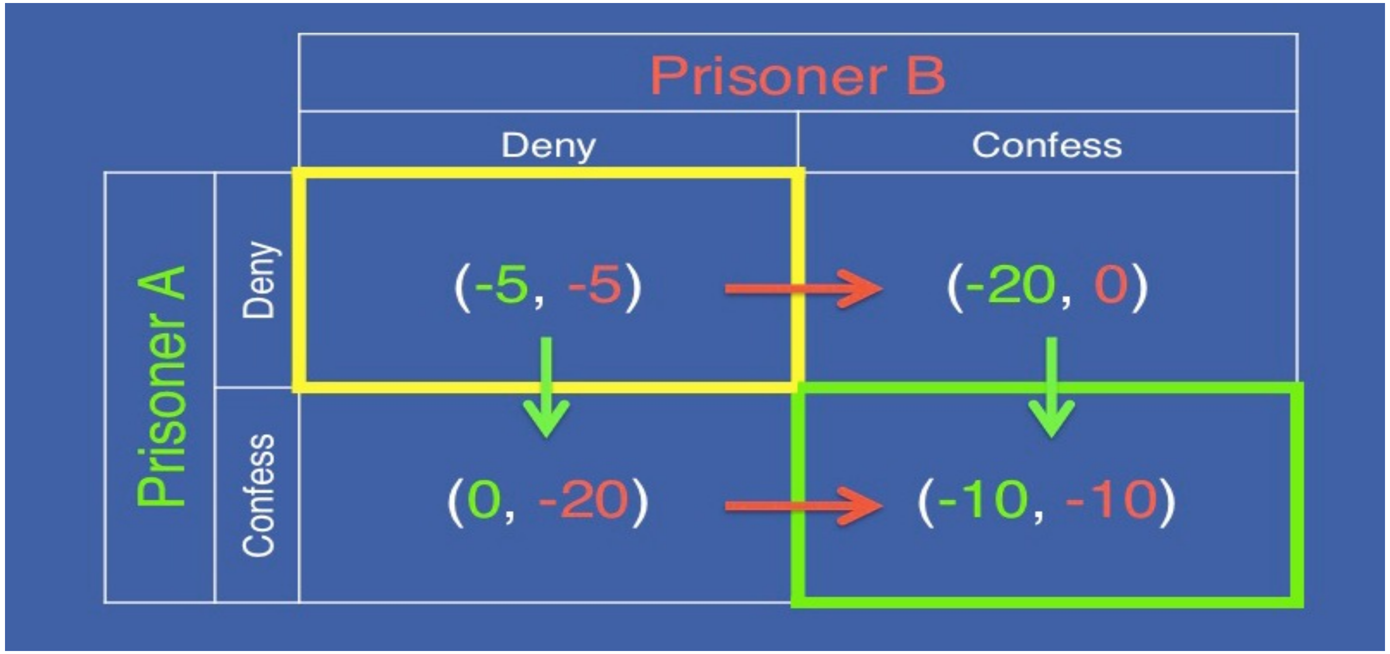


$confess \in D_A$,
$confess \in D_B$

Solution: (confess, confess)
We eliminate all strictly dominated strategies

Example: Prisoner's dilemma

- Game: Player A, Player B,   Strategies: {*deny, confess*}
- Socially optimal strategies: maximize $U_A + U_B$
- Pareto optimal strategies



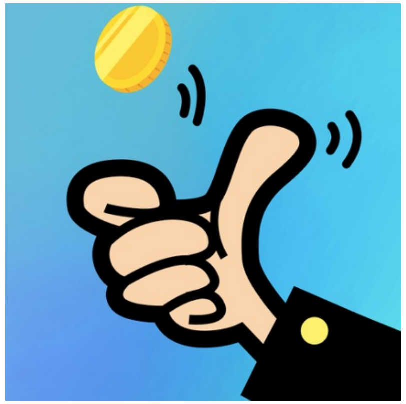
Example: the hawk – dove game

	Player 2		
Player 1		Compromise	Don't compromise
	Compromise	(0,0)	(-1,1)
	Don't compromise	(1,-1)	(-10,-10)

Find the Nash Equilibria in pure strategies

Mixed strategies

Generalized coins: many faces



- Flip a (generalized) coin to pick my action
- My strategy = **type of coin** (probabilities)
- Example: Player A has two pure strategies $\{a, b\}$
- Her mixed strategy is the probability p to choose strategy a
- In general: if k pure strategies, a mixed strategy is any vector

$$(p_1, p_2, \dots, p_k), \text{ s.t. } p_i \geq 0 \text{ and } p_1 + \cdots + p_k = 1$$

Example of randomization: matching pennies

		Column		
		p Heads	$1-p$ Tails	
Row	q Heads	(1, –1)	(–1, 1)	$1p + -1(1-p) = 2p - 1$
	$1-q$ Tails	(–1, 1)	(1, –1)	$-1p + 1(1-p) = 1 - 2p$

How to find the NE: if I randomize, I must be indifferent between the expected payoff resulting form any of the outcome

Assume an equilibrium with (q, p)

For row player to be indifferent between choosing H and T: $1p + (-1)(1-p) = (-1)p + 1(1-p)$ hence column players must use $p = 0.5$. Similarly, $q = 0.5$

Example of randomization: coordination game

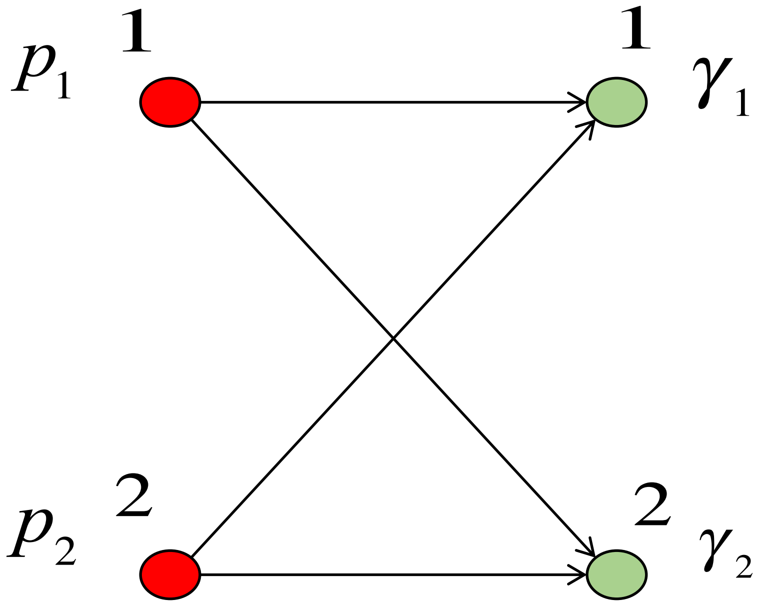
coin of row player:	coin of column player:	1/3	2/3
		Action Movie	Romance Movie
2/3	Action Movie	(2,1)	(0,0)
1/3	Romance Movie	(0,0)	(1,2)

3 Nash equlibria: (A, A), (R, R), $\left((\underbrace{\frac{2}{3}, \frac{1}{3}}_{\text{row player}}), (\overbrace{\frac{1}{3}, \frac{2}{3}}^{\text{column player}}) \right)$

Feasible power region example

Feasible power set example

Given the gains of the system, noises, and minimum values γ_i for SIR_i, are there *feasible* transmit powers to satisfy $SIR_i \geq \gamma_i$?

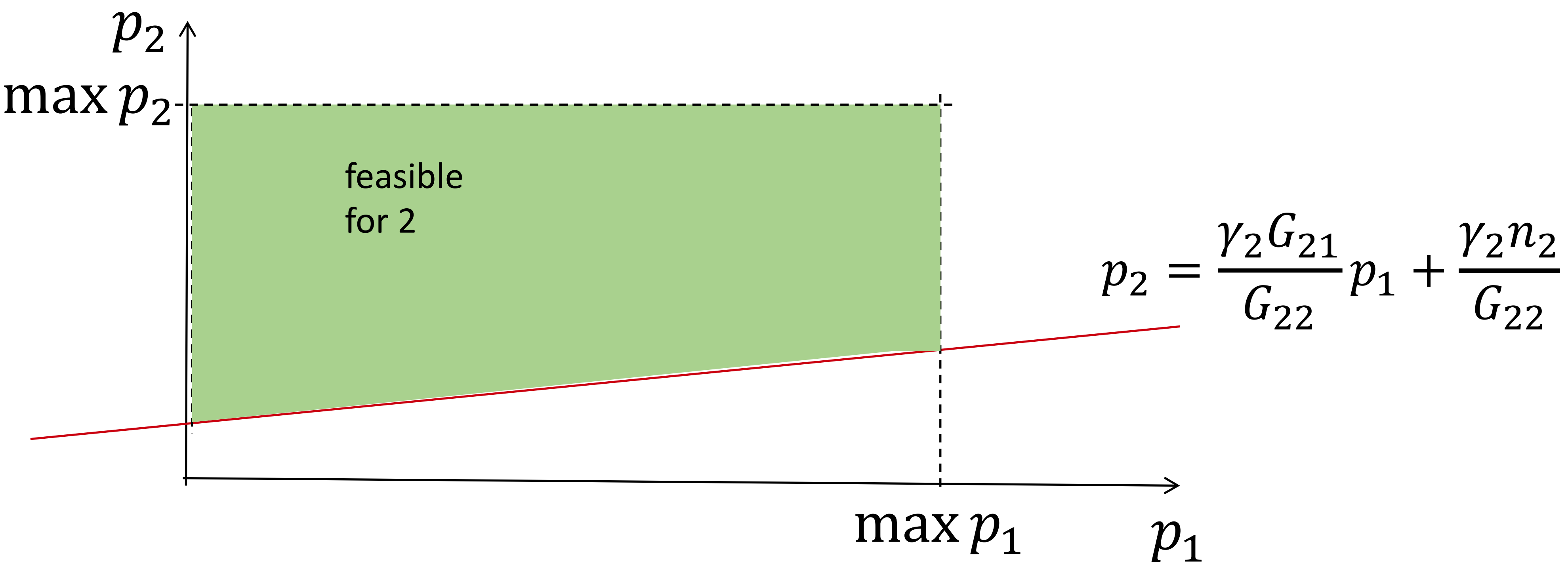


$$SIR_1 = \frac{G_{11}p_1}{G_{12}p_2+n_1}, \quad SIR_2 = \frac{G_{22}p_2}{G_{21}p_1+n_2}$$

We want $SIR_1 \geq \gamma_1, SIR_2 \geq \gamma_2$

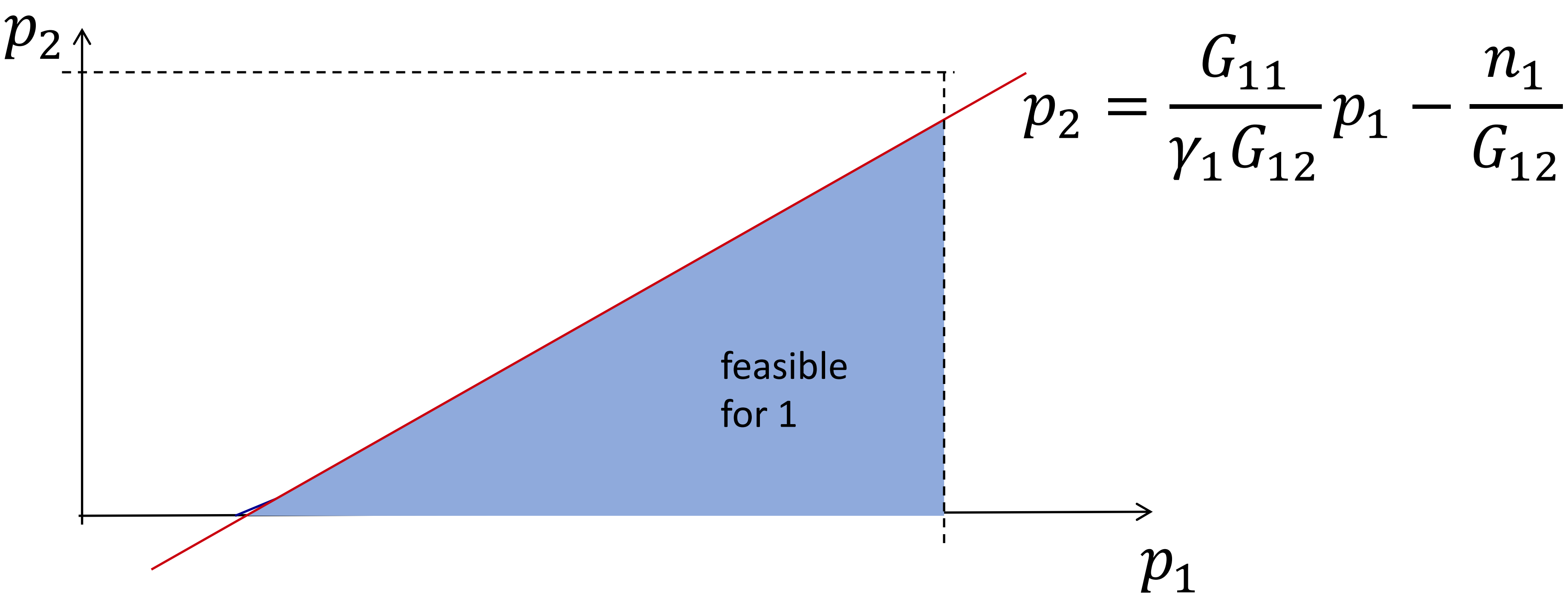
Conditions for a solution?

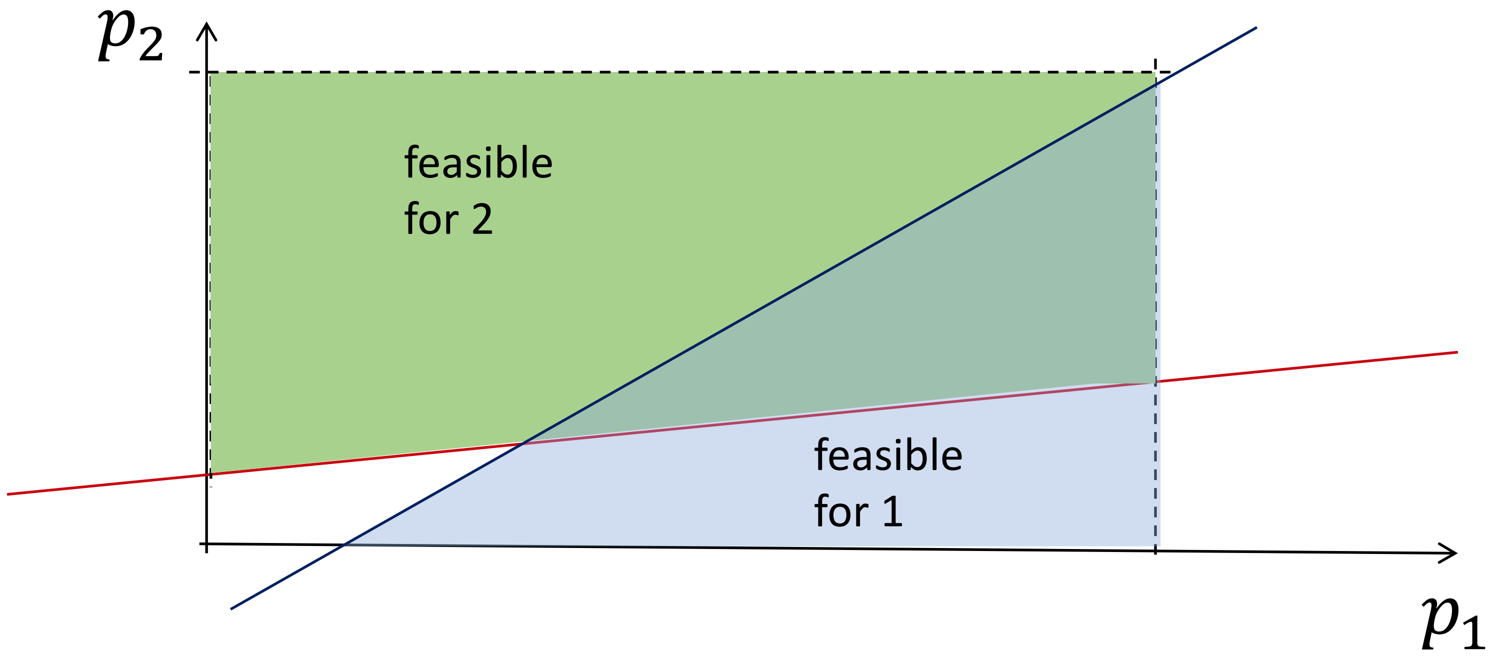
$$\frac{G_{22}p_2}{G_{21}p_1+n_2} \geq \gamma_2 \Rightarrow G_{22}p_2 \geq \gamma_2(G_{21}p_1 + n_2) \Rightarrow p_2 \geq \frac{\gamma_2 G_{21}}{G_{22}}p_1 + \frac{\gamma_2 n_2}{G_{22}}$$

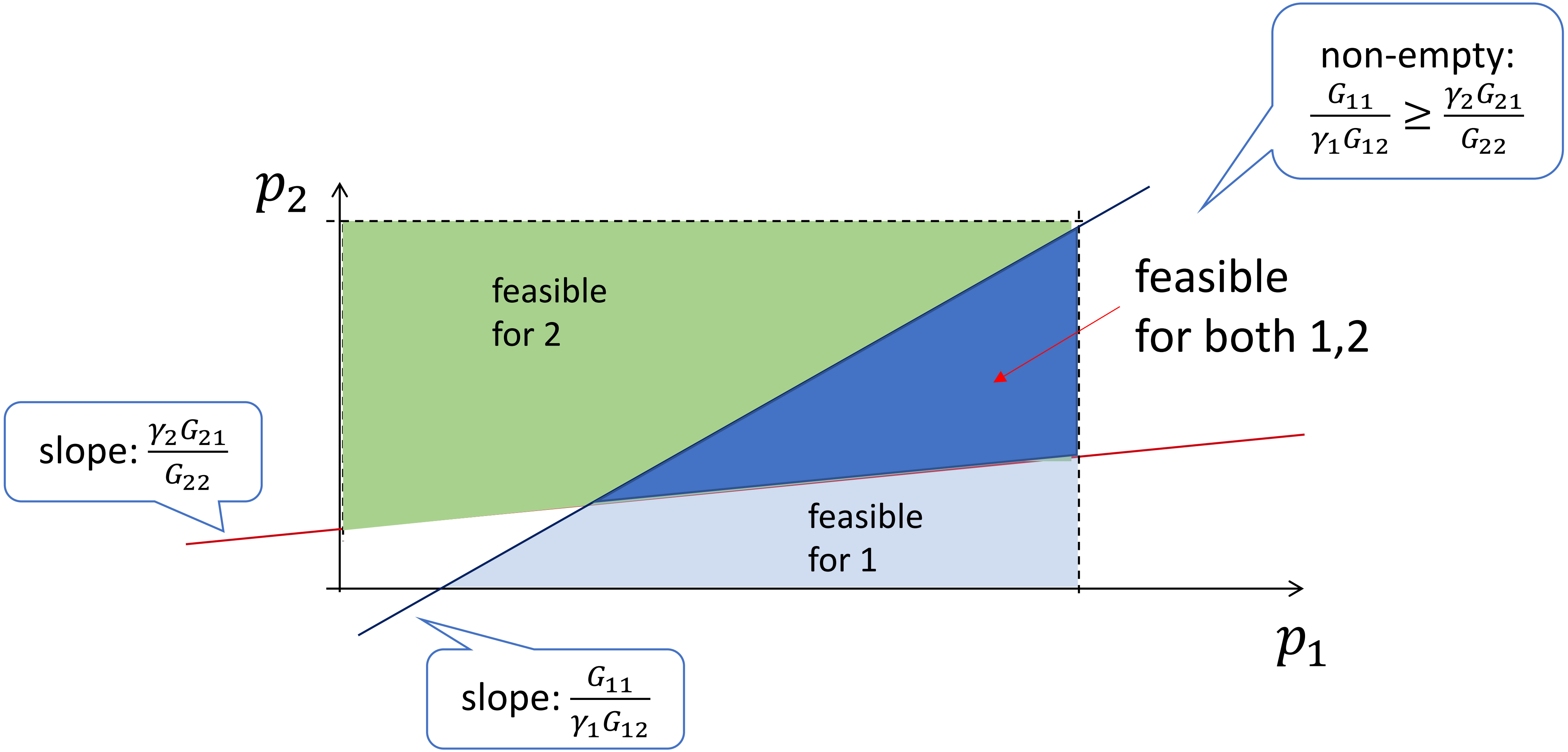


assume there are max power constraints

$$\frac{G_{11}p_1}{G_{12}p_2+n_1} \geq \gamma_1 \Rightarrow G_{12}p_2+n_1 \leq \frac{G_{11}p_1}{\gamma_1} \Rightarrow p_2 \leq \frac{G_{11}}{\gamma_1 G_{12}}p_1 - \frac{n_1}{G_{12}}$$




p_2
feasible
for 2
feasible
for 1
p_1



non-empty: $\frac{G_{11}}{\gamma_1 G_{12}} \geq \frac{\gamma_2 G_{21}}{G_{22}}$
p_2
feasible
for 2
feasible
for both 1,2
slope: $\frac{\gamma_2 G_{21}}{G_{22}}$
feasible
for 1
p_1
slope: $\frac{G_{11}}{\gamma_1 G_{12}}$

Numerical example

$$G_{11} = 1,\ G_{22} = 1,$$
$$G_{12} = G_{21} = 0.2$$
$$\gamma_1 = 5, \gamma_2 = 1, n = 0.3$$

$$SIR_i = \frac{G_{ii} p_i}{G_{ij} p_j + n}$$