DDA4260 Assignment 2

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Student Number: 120090272 **Assignment:** Assignment 2

Exercise 1: Baseline predictor

Solution:

mean rating: $\bar{r} = \frac{\sum_{u,i} r_{ui}}{C^{train}} = 3.125$ for predicted rating: $\hat{r_{ui}} = \bar{r} + b_u + b_i$
we want to find the optimal value for b_u and b_i such that:

$$min_{b_u,b_i} \sum_{(u,i)\in\omega} (b_u + b_i - (r_{ui} - \bar{r}))^2$$

$C = R - \bar{R}$. So we find matrix A with the cell with rating:

$$A = \begin{bmatrix} 1&0&0&0&0&1&0&0&0 \\ 1&0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&0&0&0&1 \\ 0&1&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0&1 \\ 0&0&1&0&0&1&0&0&0 \\ 0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&1&0 \\ 0&0&1&0&0&0&0&0&1 \\ 0&0&0&1&0&1&0&0&0 \\ 0&0&0&1&0&0&1&0&0 \\ 0&0&0&1&0&0&0&0&1 \\ 0&0&0&0&1&1&0&0&0 \\ 0&0&0&0&1&0&1&0&0 \\ 0&0&0&0&1&0&0&1&0 \end{bmatrix}, \quad C = \begin{bmatrix} 1.875 \\ 1.875 \\ 0.875 \\ -2.125 \\ -2.125 \\ 0.875 \\ 0.875 \\ -2.125 \\ -1.125 \\ 0.875 \\ -0.125 \\ 0.875 \\ -0.125 \\ -2.125 \\ 1.875 \\ -0.125 \end{bmatrix}, \quad b^* = (A^T A)^{-1} A^T c$$

solving with sofware, we find that:
$b^* = [1.5202, -1.2071, -0.3889, 0.0657, 0.0657, -0.1907, -0.0088, -0.3725, 0.6275]$

and $\hat{R} = \begin{bmatrix} 4.45 & 4.64 & 4.27 & 5.27 \\ 1.72 & 1.91 & 1.55 & 2.55 \\ 2.55 & 2.73 & 2.36 & 3.36 \\ 3.00 & 3.18 & 2.82 & 3.82 \\ 3.00 & 3.18 & 2.82 & 3.82 \end{bmatrix}$

Exercise 2: Neiborhood predictor

Solution:

calculating the difference of movies: we find:

$$dif = \begin{bmatrix} 0 & -0.9245 & -0.2350 & 0.0908 \\ -0.9425 & 0 & 0.8015 & -0.8177 \\ -0.2350 & 0.8015 & 0 & -0.9790 \\ 0.0908 & -8177 & -0.9790 & 0 \end{bmatrix}$$

So the neibor of movie A is B and C, the neibor of movie B is A and D, the neibor of movie C is B and D, the neibor of movie D is B and C.

solved that: $\hat{R^N} = \begin{bmatrix} 5 & 4.9355 & 5 & 4 \\ 2.5638 & 1 & 1 & 4 \\ 4 & 1 & 2 & 4 \\ 3 & 4 & 3.6364 & 3 \\ 1 & 5 & 3 & 2.8916 \end{bmatrix}$

Exercise 3: Least Squares

Solution:(1)

to solve minimization problem, $min_b||Ab-c||^2 = min_b(Ab-c)^T(Ab-c)$
$\frac{d}{db}(Ab-c)^T(Ab-c) = 2(Ab-c)^T A$
$\Rightarrow A^T(Ab-c) = 0$
$\Rightarrow A^T Ab = A^T c$
$\Rightarrow b = (A^T A)^{-1} A^T c$
Given that $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$, and $c = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 3 \end{bmatrix}$, using software to solve this problem:
we have: $b = [1.0357, 0.2143, 0.5357]$

Solution:(2)

to minimize $||Ab-c||_2^2 + \lambda||b||_2^2$:
take the derivative of $||Ab-c||_2^2 + \lambda||b||_2^2$ with respect to b,
we find that $n = (A^T A + \lambda I)^{-1} A^T c$
solving with the help of python, we find the solution of plot as below:

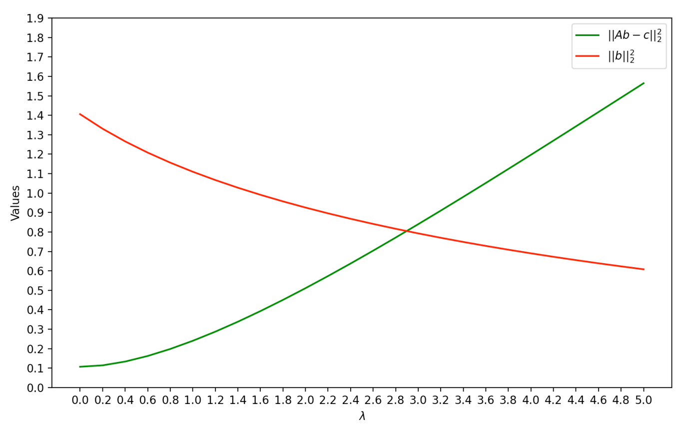

Figure 1: figure of lambda

the solution of b is:

```python
import numpy as np
import matplotlib.pyplot as plt
A = np.array([[1, 0, 2],[1, 1, 0],[0, 2, 1],[2, 1, 1]])
c = np.array([2, 1, 1, 3])
I = np.identity(3)
b_list=[]
o_list=[]
x=[]
for i in range(0, 52, 2):
    x.append(0.1*i)
    B = A.T @ A + 0.1*i*I
    X = np.linalg.pinv(B) @ A.T
    b_hat = X @ c
    M = A@b_hat-c
    b_list.append(b_hat.T @ b_hat)
    o_list.append(M.T @ M)
plt.figure(figsize=(10,6),dpi = 100)
plt.plot(x,o_list,color = 'green',linestyle = '-',label = r'$||Ab - c||_2^2$')
plt.plot(x,b_list,color = 'r',linestyle = '-',label = r'$||b||_2^2$')
plt.ylabel('Values')
plt.xlabel(r'$\lambda$')
plt.xticks(np.arange(0,5.2,0.2))
plt.yticks(np.arange(0,5,0.1))
plt.legend()
plt.show()
```

```
[array([1.03571429, 0.21428571, 0.53571429]),
 array([0.99429306, 0.22546419, 0.53974761]),
 array([0.95792147, 0.23476112, 0.54125481]),
 array([0.92556404, 0.24249423, 0.54094866]),
 array([0.89646465, 0.24891775, 0.53932179]),
 array([0.8700565 , 0.25423729, 0.53672316]),
 array([0.84590517, 0.25862069, 0.53340517]),
 array([0.82367124, 0.26220615, 0.52955359]),
 array([0.80308501, 0.26510832, 0.52530723]),
 array([0.78392903, 0.26742301, 0.52077113]),
 array([0.76602564, 0.26923077, 0.51602564]),
 array([0.74922802, 0.27059971, 0.51113278]),
 array([0.73341352, 0.27158774, 0.5061408 ]),
 array([0.71847874, 0.27224436, 0.50108744]),
 array([0.70433573, 0.272612  , 0.4960024 ]),
 array([0.69090909, 0.27272727, 0.49090909]),
 array([0.67813374, 0.27262181, 0.48582605]),
 array([0.6659531 , 0.27232308, 0.48076791]),
 array([0.6543177 , 0.27185501, 0.47574627]),
 array([0.64318404, 0.27123849, 0.47077024]),
 array([0.63251366, 0.2704918 , 0.46584699]),
 array([0.62227241, 0.26963103, 0.46098209]),
 array([0.6124298 , 0.26867031, 0.4561798 ]),
 array([0.60295851, 0.26762211, 0.45144336]),
 array([0.59383398, 0.26649746, 0.44677516]),
 array([0.58503401, 0.26530612, 0.44217687])]
```

Figure 2: value of b