Stochastic Differential Equations

Introduction

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Outline

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1. Background on Shrimpy
   - The Problem
   - Statistics
2. Modeling
3. Stochastic Calculus
   - Stochastic Integration
   - Stochastic Differential Equations
   - Random Thoughts

S · U · N · Y POTSDAM
St. Lawrence Academy · 1816

The Problem

We gonna talk about stuff.

The Issues

We gonna talk about the issues about talking about stuff.

Random Stuff

Random stuff is like totally out there.

Random Stuff

Random stuff is like totally out there.
It could just be totally surprising.

Random Stuff

Random stuff is like totally out there.
It could just be totally surprising.
Unexpected even, you know what I mean?

Statistics Do Not Lie

You can totally trust the statistics.

Statistics Do Not Lie

You can totally trust the statistics.
Well... usually

- We could make a type I error.
- Or it could be a type II error.

Statistics Do Not Lie

You can totally trust the statistics.
Then again maybe the hypothesis test does not even make sense.

Statistics Do Not Lie

You can totally trust the statistics.
Then again maybe the hypothesis test does not even make sense.
Then you are really hosed.

Outline

Nondimensionalization

The initial model is:

$$\frac{dx}{ds} = rx\left(1 - \frac{x}{k}\right) - \alpha xy,$$
$$\frac{dy}{ds} = \rho y\left(1 - \frac{y}{l}\right) - \beta xy.$$

The nondimensionalized system is:

$$\frac{dx}{ds} = rx(1 - x) - \alpha xy,$$
$$\frac{dy}{ds} = y(1 - y) - \beta xy.$$

Itô's Formula

Itô's Formula is used in Itô Calculus to find the differential of a time-dependent function of a stochastic process.

Itô's Formula

$$\partial x_t = \frac{\partial x}{\partial t} \cdot dt + \frac{\partial x}{\partial B} \cdot dB + \frac{1}{2}\left(\frac{\partial^2 x}{\partial B^2} \cdot dt\right)$$

Heun's Method

- Heun's method is a numerical procedure for approximating ordinary differential equations with a given initial value.
- First you calculate the intermediate value $\tilde{y}_{i+1}$ and then the final approximation y_{i+1} at the next generation point.

$$\tilde{y}_{i+1} = y_i + \Delta t \; f(t_i, y_i)$$
$$y_{i+1} = y_i + \frac{\Delta t}{2}\left[f(y_i, t_i) + f(\tilde{y}_{i+1}, t_{i+1})\right]$$

Outline

Overview

The governing Equations:

$$\begin{aligned} \dot{a} &= L_1 a + N_1(a, g), \\ \dot{g} &= L_2 g + N_2(a, g). \end{aligned}$$

The "Usual Scaling"

$$\begin{aligned} x &\rightarrow \bar{X}\xi, \\ t &\rightarrow \bar{T}s. \end{aligned}$$

The Finite Difference Approximation

- Usual centered difference scheme.

The Finite Difference Approximation

- Usual wave speed problem.
- Usual centered difference scheme.

The Finite Difference Approximation

- Usual ease of use.
- Usual wave speed problem.

Don't that beat all?

The Finite Difference Approximation

- Usual stability issues.
- Usual ease of use.
- Usual wave speed problem.

Don't that beat all?

Itô's Formula

Itô is like totally cool!

Comparison

This is what the left looks like

This is what the right looks like

Random Thoughts

But what about Barney and PBS?

Barney?

Is it okay to trust your kids with Barney?

Random Thoughts

But what about Barney and PBS?

Barney?

Is it okay to trust your kids with Barney?

No, not Barney!

Probably not.