Optimal normalization policy under behavioral expectations*

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Abstract

We characterize optimal normalization policy in a framework in which agents' expectations can deviate from the rational expectations benchmark and the central bank faces cost-push shocks. When interest rate fluctuations are costless, our findings indicate that the interest rate is the primary tool for managing inflationary pressures, consistently outperforming balance sheet adjustments, regardless of the expectations formation process. However, under de-anchored expectations, an increasing role for balance sheet management arises when interest rate fluctuations become costly. Finally, our analysis reveals that expectations significantly influence the optimal interest rate trajectory, whereas their impact on the optimal balance sheet path is comparatively minimal.

Keywords: Optimal monetary policy, de-anchored expectations, normalization strategy
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1 Introduction

In response to historically low interest rates following the global financial crisis, central banks in advanced economies expanded their balance sheets to historically high levels. The exit from the effective lower bound (ELB) and the return to conventional policy tools that followed the recent inflation surge have raised concerns about whether large balance sheets should be maintained. As shown in models with rational expectations (RE), the balance sheet entails higher costs and is less effective as a policy instrument in normal times (see Sims and Wu, 2021). At the same time, the effectiveness of standard policy tools, such as the policy rate, can deteriorate when expectations deviate from RE. This paper therefore examines whether an increased role for balance sheet adjustments to achieve stabilization is justified when agents' expectations are de-anchored, using a small-scale model.

We use the four-equation New Keynesian model of Sims et al. (2023) and characterize the optimal normalization policy, the latter involving the setting of both the short-term policy rate and the central bank balance sheet. We extend the model to incorporate a resource cost for central bank bond holdings as in Kabaca et al. (2023). In our model, agents may hold de-anchored expectations, defined as the misperception of the long-run mean of macroeconomic variables, which are influenced by short-term forecast errors, following the framework of Eusepi et al. (2020).

We emphasize the following main takeaways. First, we underscore the primary role of the short-term policy rate in addressing inflationary pressures after a cost-push shock, which we find to be more effective than the balance sheet in curbing inflation, independent of the private sector's expectations. Second, in scenarios where both policy instruments are set optimally following a cost-push shock, expectations influence the optimal interest rate path considerably, necessitating greater monetary policy tightening when agents hold de-anchored expectations. In contrast, expectations play a limited role in shaping the optimal balance sheet trajectory. Third, when variations in the short-term rate become costly, central bank balance sheet management takes on a greater role under de-anchored expectations, helping to dampen the necessary short-term rate hikes after a cost-push shock. Finally, when credit shocks arise, it is the central bank balance sheet, and not the short-term policy rate, the better-suited tool for stabilization, regardless of the expectations formation mechanism. Thus, the primacy of the short-term policy rate depends on the nature of the shock.

We derive optimal normalization policy under rational and de-anchored expectations relying on a model-consistent welfare criterion, in the spirit of Rotemberg and Woodford (1997). Similar to Molnár and Santoro (2014), we show analytically that the policy rate should react more aggressively to inflation, after a cost-push shock, when expectations are de-anchored than when they are rational.[1] Importantly, we demonstrate that the optimal interest rate rule under RE is a limiting case of that under de-anchored expectations. Similarly, we derive the optimal path for the central bank balance sheet. We highlight that the balance sheet is a less effective tool for responding to cost-push shocks due to the conflicting supply and demand channels of balance sheet policy.[2] Moreover, its optimal path is only weakly affected by the expectations formation mechanism. Finally, we show that the inflation-output gap trade-off becomes more severe under de-anchored expectations.

The post-pandemic surge in inflation, driven to a significant extent by cost-push (or supply-side) shocks, led central banks of advanced economies to initiate a rate hiking cycle while still operating with a significantly expanded balance sheet. We simulate the model to generate an environment that reflects these conditions. This allows us to explore the dynamics and to quantify the welfare costs of each policy instrument using a welfare-relevant measure. Additionally, we allow for the cost-push shock to be persistent as opposed to our analytical results where $i.i.d.$ shocks are considered for tractability. We show that the welfare costs from setting only the short-term rate optimally after an adverse cost-push shock are limited in comparison to the welfare costs of balance sheet adjustments, regardless of the expectations formation mechanism. We further extend our quantitative analysis beyond our analytical results along two dimensions, namely by penalizing short-term interest rate variations, first, and second, by considering a scenario in which the economy is hit by credit and cost-push shocks simultaneously.

Under de-anchored expectations, the balance sheet plays a more prominent role when variations in the short-term policy rate are costly. Optimal policy in this case prescribes a milder interest rate hike than when variations are not costly, following a cost-push shock. To offset this milder response, the central bank must shrink its balance sheet more. Under RE instead, costly variations in the short-term rate lead to negligible changes in the optimal

[1] Gáti (2023) and Gaspar et al. (2006) also show numerically that the policy rate should react more aggressively to inflation when expectations become de-anchored or when agents have adaptive expectations.

[2] The role of the supply versus the demand channel of balance sheet policies is discussed in Sims et al. (2023). See also Boehl et al. (2022) for a quantitative analysis for the US.

policy paths and the resulting allocations.

Finally, our results highlight that in the presence of large credit shocks, central banks may need to moderate the pace of quantitative tightening (QT) to mitigate the negative effects on financial stability. When responding optimally to a combination of cost-push and credit shocks, the central bank tends to rely on the policy rate for inflation control and the balance sheet for maintaining financial stability. This finding holds regardless of the expectations formation mechanism, indicating that the preference for the short-term rate as a stabilization tool depends on the nature of the shock, being more effective for cost-push shocks, while the balance sheet is better suited to address credit disturbances.

1.1 Related literature

The recent policy normalization of the major central banks has led to a growing literature on optimal monetary policy normalization. Benigno and Benigno (2022) show that QT should start before the liftoff of the policy rate. Focusing on the portfolio balance channel, Cantore and Meichtry (2023) show instead that rate hikes should start prior to QT. Using a model with a banking sector, Karadi and Nakov (2021) argue that the optimal balance sheet normalization should be gradual due to the slowing down of the banking sector recapitalization. Our paper is distinct from the above contributions in that it considers optimal normalization policy under de-anchored expectations.

The literature on optimal monetary policy under bounded rationality is vast (see Gaspar et al., 2006; Dennis and Ravenna, 2008; Gaspar et al., 2010; Molnár and Santoro, 2014; Eusepi and Preston, 2018; Mele et al., 2020; Hommes et al., 2023; Gáti, 2023). Molnár and Santoro (2014) and Gaspar et al. (2010) show that an interemporal trade-off arises under optimal policy when agents learn adaptively. This coincides with a more aggressive optimal interest rate trajectory, as in Gáti (2023) and Molnár and Santoro (2014), compared to the RE benchmark.[3] Accounting for long-run interest rate expectations, Eusepi et al. (2020) show instead that aggressive policy responses can be sub-optimal due to the induced volatility in long-term interest rates.

Sims and Wu (2020) explore the substitutability between conventional monetary policy

[3]Using an estimated New Keynesian model with endogenous forecast switching, Fischer (2022) finds that the optimal response to changes in inflation is significantly higher when expectations are endogenously anchored compared to RE.

and QE at the effective lower bound (ELB) and find that, when the policy rate is fixed, QE can indeed serve as an effective substitute to achieve price stability, albeit with different implications for the output gap compared with conventional policy.

The paper is organized as follows. Section 2 describes the extension of the model by Sims et al. (2023) to account for costs associated with central bank asset holdings. Section 3 presents analytical results from the design of optimal monetary policy under RE and de-anchored expectations. Section 4 presents our simulations from optimal policy. Section 5 concludes.

2 Model

The model is based on Sims et al. (2023), which adds a role for asset purchases by the central bank in a standard three-equation New Keynesian model. The non-reduced framework consists of two types of households, the patient (referred to as the *parent*) and the impatient (the *child*), financial intermediaries modeled as in Gertler and Karadi (2011), and a standard production side of the economy split into three sectors (final output, retail output, and wholesale output). To this baseline model, we add a cost of QE/QT, as a proxy for the unmodeled distortions and political costs of maintaining a positive central bank balance sheet as in Karadi and Nakov (2021) and Kabaca et al. (2023). In its log-linearized form, the model boils down to four equations.[4] The IS equation and the New Keynesian Phillips curve are specified as follows:

$$\begin{aligned} x_t = \tilde{\mathbb{E}}_t x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - \tilde{\mathbb{E}}_t \pi_{t+1} - r_t^f\right) - \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\left(\tilde{\mathbb{E}}_t qe_{t+1} - qe_t\right) \\ - z\bar{b}^{FI}\left(\tilde{\mathbb{E}}_t \theta_{t+1} - \theta_t\right) \end{aligned} \tag{1}$$

$$\pi_t = \beta\tilde{\mathbb{E}}_t \pi_{t+1} + \gamma\zeta x_t - \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) qe_t - \frac{\gamma\sigma z}{1-z}\bar{b}^{FI}\theta_t + cp_t \tag{2}$$

The variable π_t denotes inflation, while $x_t = y_t - y_t^*$ is the output gap, with y_t^* being potential output.[5] The short-term nominal interest rate is denoted r_t^s. The real value of

[4]Further details on the model and the introduction of the QE/QT cost in the baseline model can be found in the online Appendix A.

[5]Note that, unlike in the standard New Keynesian model, in Sims et al. (2023), the equilibrium level of potential output is consistent with price flexibility and no credit shocks, as both frictions distort the competitive

the central bank bond portfolio is captured by qe_t.[6] The term $\tilde{\mathbb{E}}_t$ captures the fact that private agents might have non-rational expectations, which we discuss in detail below. The parameter σ is the inverse of the intertemporal elasticity of substitution, β is the subjective discount factor, γ is the elasticity of inflation with respect to real marginal costs and ζ is the elasticity of real marginal costs with respect to the output gap. The parameter $z \in [0, 1)$ represents the share of impatient households in the total population, while $\bar{b}^{cb}$ and $\bar{b}^{FI}$ are the steady state long-term bond holdings of the central bank and financial intermediaries, respectively, as a share of the total outstanding long-term bonds. The cost of QE/QT is captured by τ, and $\frac{QE}{Y}$ is the steady-state ratio of central bank bond holdings to GDP.

In the model, the *child* finances her consumption by issuing long-term bonds that the central bank and financial intermediaries can buy. Subsequently, the *child* receives full bailout from the *parent* to service her debt. This creates two effects: on the demand side, a rise in the central bank's bond portfolio, qe_t, boosts the consumption of the *child*, and thereby aggregate consumption. This increases the output gap, x_t, and inflation. On the supply side, balance sheet policy exerts instead a negative impact on inflation. This arises from the full bailout that the *parent* provides to the *child*. The more the *child* borrows, the stronger the negative wealth effect incurred by the *parent* who in response supplies more labor. As a result, real wages and, hence, real marginal costs decrease, creating a downward pressure on inflation as opposed to the demand channel.

The economy is subject to shocks to the natural rate, r_t^f, to price cost-push shocks, cp_t, and to credit shocks, θ_t. These shock processes are exogenous AR(1) processes, with persistence ρ_j and standard deviation s_j for $j = \{f, cp, \theta\}$:

$$r_t^f = \rho_f r_{t-1}^f + s_f \varepsilon_{f,t}, \quad \varepsilon_{f,t} \sim N(0,1) \tag{3}$$

$$cp_t = \rho_{cp} cp_{t-1} + s_{cp} \varepsilon_{cp,t}, \quad \varepsilon_{cp,t} \sim N(0,1) \tag{4}$$

equilibrium.

[6]Although we are interested in normalization policy and hence in QT, we use the notation of Sims et al. (2023) whose focus instead is on QE. Given our focus on QT, we assume that a central bank has positive asset holdings at the steady state. In her speech on March 27, 2023, at an event organized by Columbia University and SGH Macro Advisors, Isabel Schnabel stated that "*the size of our balance sheet will not return to the levels seen before the global financial crisis*" and concluded by stating "*Ultimately, our obligation to act in line with the principle of an open market economy implies that, in the steady state, the size of our balance sheet should only be as large as necessary to ensure sufficient liquidity provision and effectively steer short-term interest rates towards levels that are consistent with price stability over the medium term*". Given the modeling of the central bank balance sheet in Sims et al. (2023), we treat as plausible the assumption of positive central bank asset holdings at the steady state.

$$\theta_t = \rho_\theta \theta_{t-1} + s_\theta \varepsilon_{\theta,t}, \quad \varepsilon_{\theta,t} \sim N(0,1) \tag{5}$$

Our analysis focuses primarily on a fully optimal normalization policy in which both monetary policy instruments, namely the short-term policy rate and central bank asset holdings, are set optimally. We also compare the main scenario in which both instruments are used optimally with suboptimal scenarios in which only one of the two instruments is set optimally.[7] This allows us to explore the efficacy of each instrument in isolation to achieve the central bank's objectives. We distinguish two suboptimal scenarios. In the first, we look at the optimal interest rate policy only, where we assume that the central bank bond portfolio follows a stationary AR(1) process, as specified in equation (7) below. In the second, we examine the optimal path of QT under a predetermined interest rate policy given by a Taylor rule, as specified in equation (6) below, also taking into account the zero lower bound (ZLB):

$$r_t^s = \max\left[0\,;\phi_\pi \pi_t + \phi_x x_t + s_r \varepsilon_{r,t}\right], \quad \varepsilon_{r,t} \sim N(0,1) \tag{6}$$

$$qe_t = \rho_q qe_{t-1} + s_q \varepsilon_{q,t}, \quad \varepsilon_{q,t} \sim N(0,1) \tag{7}$$

with ρ_q the persistence of the QE/QT process, and s_r and s_q the standard deviations of the monetary policy shocks. The parameters ϕ_π and ϕ_x govern the reaction of the short-term nominal rate to changes in inflation and the output gap.

Expectations In this model, some or all private agents can have non-rational expectations. The economy is populated by two types of agents, namely those with RE (or anchored agents) and those with de-anchored expectations. Agents with RE know the actual law of motion of the economy.[8] Anchored agents have expectations that are well-aligned with the central bank's stated targets. Importantly, they understand the communication (e.g., forward guidance) of the central bank. This allows the policymaker to commit to a specific policy strategy, which these agents will understand and factor into their expectations. Anchored agents forecast the evolution of the economy using the minimum state variable

[7]We do so by looking at the welfare costs of using one monetary policy tool optimally relative to using both in Section 4. We also report simulations when the interest rate only is set optimally in Section 4, while the case of the balance sheet only is reported in the online Appendix C.3.

[8]In a scenario where a fraction of agents have RE, while others do not, agents with RE thus understand that there is a share of agents with de-anchored expectations. Moreover, they know the share of agents with de-anchored expectations with certainty.

solution of the model. Their expectations thus have the following law of motion:

$$\tilde{\mathbb{E}}_t^a\left(z_{t+1}\right)=\left(\mathbf{a}+\mathbf{b}y_t+\mathbf{c}w_t\right) \tag{8}$$

where $z_t=[\pi_t,x_t,qe_t,\theta_t]'$ gathers the expectational variables, the vector $\mathbf{a}$ represents the constant terms including the central bank's inflation target, the matrix $\mathbf{b}$ maps the impact of contemporaneous variables, $y_t=[qe_t,r_t^f,cp_t,\theta_t]'$, and the matrix $\mathbf{c}$ accounts for the impact of each shock, $w_t=[\varepsilon_{f,t},\varepsilon_{cp,t},\varepsilon_{\theta,t},\varepsilon_{r,t},\varepsilon_{q,t}]'$, on expectations.[9]

The remaining share of agents has de-anchored expectations and forecast the long-run mean of endogenous variables through simple constant gain learning, as in Eusepi et al. (2020) and Molnár and Santoro (2014). The beliefs of de-anchored agents follow:

$$\mathbb{E}_t^d z_{t+1}\equiv\omega_t^z=\omega_{t-1}^z+\bar{g}\left(z_{t-1}-\omega_{t-1}^z\right) \tag{9}$$

in which $\bar{g}$ is the constant-gain learning parameter, which governs the extent to which expectations are affected by short-term forecast errors. These agents misperceive the unobserved long-run mean (or drift) ω_t^z of variables $z_t=[\pi_t,x_t,qe_t,\theta_t]'$, and revise their expectations based on previous short-term forecast errors $\left(z_{t-1}-\omega_{t-1}^z\right)$. They are therefore backward-looking, and do not necessarily believe that the central bank can credibly commit to reaching its announced target. As Eusepi and Preston (2018) point out, under constant gain learning, the learning process does not converge point-wise to rational expectations as the estimated drift is constantly revised with new information.[10] For values of parameter $\bar{g}$ sufficiently close to zero, stability conditions yield eigenvalues lying inside the unit circle for the law of motion of beliefs.[11]

Aggregate expectations are therefore the sum of anchored and de-anchored agents' ex-

[9]Note that central bank asset holdings, qe_t, appear in vector y_t as long as qe_t is assumed to follow an AR(1) process. When we consider the fully optimal normalization policy where both r_t^s and qe_t are set optimally, the vector y_t boils down to $y_t=[r_t^f,cp_t,\theta_t]'$. In the case where both agents with rational and agents with de-anchored expectations are included, the vector of contemporaneous variable becomes $y_t=[\pi_t,x_t,qe_t,r_t^f,cp_t,\theta_t,\boldsymbol{\omega}_t^z]'$ and thus contains $\boldsymbol{\omega}_t^z$, which relates to the beliefs of de-anchored agents about variables $z_t=[\pi_t,x_t,qe_t,\theta_t]'$. The existence of de-anchored expectations also leads to the inclusion of additional endogenous variables $\{\pi_t,x_t\}$.

[10]In the literature on optimal monetary policy with boundedly rational agents, Molnár and Santoro (2014) and Gaspar et al. (2010) derive optimal policy also assuming constant gain learning while Gáti (2023), Eusepi and Preston (2018) assume that agents' learning process converges point-wise to rational expectations.

[11]As shown Evans and Honkapohja (2001) the condition of stability of inflation expectations under constant

pectations, weighted by their relative shares $n^{a,z}$ and $n^{d,z}$, respectively:

$$\begin{aligned}\tilde{\mathbb{E}} z_{t+1} &= n^{a,z}\, \tilde{\mathbb{E}}^{a}\left(z_{t+1}\right) + n^{d,z}\, \tilde{\mathbb{E}}^{d}\left(z_{t+1}\right) \\ &= n^{a,z}\Big[\mathbf{a} + \mathbf{b} y_t + \mathbf{c} w_t\Big] \; + n^{d,z}\, \Big[\omega_{t-1}^{z} + \bar{g}\left(z_{t-1} - \omega_{t-1}^{z}\right)\Big]\end{aligned} \tag{11}$$

with $n^{a,z} + n^{d,z} = 1$.[12] In our main results in Section 3 and Section 4, we focus on the two extreme cases in which either all agents have rational expectations (i.e. $n^{a,z} = 1$ and $n^{d,z} = 0$), or they all have de-anchored expectations (i.e. $n^{a,z} = 0$ and $n^{d,z} = 1$). In online Appendix C.4, we allow the fractions to vary, such that the economy is populated by both rational and de-anchored agents.[13]

Our modeling of the expectations formation is in line with the Euler-equation approach, as in Bullard and Mitra (2002), Ferrero (2007) and Orphanides and Williams (2007, 2008). This approach asserts decision rules in which only one-period-ahead expectations matter for agents' decisions. We acknowledge that this poses some limitations, the main related to the standard characterization of the transmission mechanism of monetary policy embedded in the New Keynesian framework. The model stipulates that not only the current interest rate but also the entire future sequence of expected one-period rates affects agents' decisions today. The anticipated utility approach (Kreps, 1998; Sargent, 1999; Preston, 2005; Woodford, 2013) instead accounts for this feature as it is consistent with solving infinite-horizon intertemporal decision problems that depend on long-term beliefs about policy. Hence, in this case, commitment to a given policy rate path, for instance, affects agents' expectations today even though they might be boundedly rational. However, as regards the central bank's balance sheet decisions, as we show in Section 3 and Section 4, expectations play a limited role. As regards, the short-term rate though we show that expectations do matter and affect the aggressiveness with which the central bank reacts to inflation.[14]

gain learning in a simple three equation New-Keynesian model is determined by the eigenvalue:

$$1 - \bar{g}\frac{1 - \phi_{\pi}^{-1}}{1 - \beta} \tag{10}$$

We consider parameterizations of the constant gain learning parameter that guarantee stability of agents' beliefs.

[12]In this paper, we calibrate these fractions at a fixed level, uniformly applied across all variables.

[13]In this case, the shares of each type of expectations formation are uniform across the two types of agents, the *parent* and the *child*.

[14]Eusepi et al. (2020) show that under the anticipated utility approach, the interest rate should respond less aggressively than under optimal discretion with RE to inflation fluctuations when long-term interest

Baseline calibration For our optimal policy exercises, we mainly parameterize the model following Sims et al. (2023). The subjective discount factor, β, and intertemporal elasticity of substitution, $1/\sigma$, follow standard values from the existing literature. The consumption share of the *child* in total consumption z is set to one-third.[15] The steady state bond holdings for the central bank and financial intermediaries, $\bar{b}^{cb} = 0.3$ and $\bar{b}^{FI} = 0.7$, are derived from the calibrations of other steady state parameters. The parameters γ and ζ are based on structural parameters of the non-linear model, including the Calvo pricing parameter, as described in Sims et al. (2023).[16] The resource cost of QE/QT follows Kabaca et al. (2023). We set the gain, $\bar{g}$, equal to the value estimated in Eusepi et al. (2020), in the period after 1999. When the short-term interest rate is not set optimally, we assume that it follows a Taylor rule as in (6) with $\phi_\pi = 1.5$ and $\phi_x = 0$. We set all standard deviation of shocks equal to 0.01, while first-order autocorrelation of all shocks are set to 0.8. The calibration of the parameters is summarized in Table 1.

rate expectations matter. Instead, they show that when the central bank can control the output gap a more aggressive response to inflation is optimal. In a recent contribution Gáti (2023) using the anticipated utility approach and assuming time-varying de-anchoring of inflation expectations finds that optimal policy moves the interest rate aggressively when expectations de-anchor, a result in line with the Euler-equation approach (see Ferrero, 2007; Orphanides and Williams, 2005). In fact, Gáti (2023) shows that her conclusions remain unchanged under Euler-equation learning.

[15] Sims et al. (2023) described this parameter as calibrated in such a way that it approximately reflects the proportion of durable consumption and private investment in the overall private non-government domestic spending.

[16] In particular, $\gamma = \frac{(1-\phi)(1-\phi\beta)}{\phi}$ in which $\phi \in (0,1]$ is the probability of non-price adjustment. Additionally, $\zeta = \frac{\chi(1-z)+\sigma}{1-z}$ in which χ is the inverse Frisch labor supply elasticity for the *parent*.

Table 1: Calibrated Parameters

Parameter	Value or Target	Description
β	0.995	Discount Factor
z	0.33	Consumption share of the child
σ	1	Inverse elasticity of substitution
$\bar{b}^{cb}$	0.3	Steady state share of central bank's LT bond holdings
$\bar{b}^{FI}$	0.7	Steady state share of fin. intermediary's LT bond holdings
γ	0.086	Elasticity of inflation with regard to marginal cost
ζ	2.49	Elasticity of marginal cost with regard to the output gap
τ	0.01	Resource cost of QE
$\frac{QE}{4Y}$	0.1	Steady state ratio of QE to annualized output
ϕ_π	1.5	Inflation reaction coefficient in the Taylor rule
ϕ_x	0	Output gap reaction coefficient in the Taylor rule
ρ_q	0.8	Persistence of the QE/QT process
$\bar{g}$	0.02	Learning gain of de-anchored agents
ρ_f	0.8	Persistence of natural rate shocks
ρ_{cp}	0.8	Persistence of cost push shocks
ρ_θ	0.8	Persistence of credit shocks

3 Optimal normalization policy

In this section, we characterize the optimal normalization policy, namely the optimal setting of the short-term policy rate and the central bank asset holdings, first under rational expectations, in Subsection 3.1, and then under de-anchored expectations, in Subsection 3.2. In both cases, we assume that the central bank has full information about the way the private sector forms expectations, be it rational or de-anchored, and about the actual law of motion of the economy. The central bank maximizes the economy's welfare subject to the structural equations characterizing its dynamics. Our focus is on the design of optimal policy away from the ZLB. We derive the welfare loss function based on a second-order approximation of the utility functions of the two types of households, the *parent* and the *child*. Aggregate welfare is summarized as:

$$W_t = V_t + V_{b,t} \tag{12}$$

where now V_t and $V_{b,t}$ represent second-order approximations of the utility of the *parent* and the *child*, respectively. Our derivation of the welfare loss function is summarized in the following proposition.

Proposition 1. *The sum of the second-order approximations of the parent and child utilities is given by:*

$$W_t = W - U_C Y \Bigg\{ (1-z)\left(\frac{C}{2}\right) c_t^2 + z\left(\frac{C_b}{2}\right) c_{b,t}^2 + \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)}\pi_t^2 + \frac{1}{2}(\tilde{v}_t^p)^2 + \tau\frac{Qb^{cb}}{Y} qe_t + \tau Qb^{cb}\frac{Qb^{cb}}{2Y} qe_t^2 \Bigg\} + t.i.p. + O\left(||\xi^3||\right)$$

Proof. In online Appendix B.1. ■

A couple of important observations stand out from Proposition 1. First, fluctuations in the consumption of each type are costly. Second, the central bank's balance sheet, as measured by qe_t, enters the loss function. Fluctuations in it therefore matter as they are costly.[17] There is also an indirect channel which is related to the way changes in central bank's bond holdings affect the consumption of each group. If, for instance, the central bank decides to unwind its balance sheet, the consumption of the *child*, $c_{b,t}$, will shrink because the central bank will buy less of its bonds and hence the *child* will borrow less to finance its consumption ceteris paribus.[18] On the other hand, given that the *parent* provides full bailout to the *child*, lower borrowing implies lower bailout which in turn boosts *parent*'s consumption. The decision of the central bank about its balance sheet, qe_t, determines this trade-off. Observing the loss function, it is clear that the consumption shares captured by z and $1-z$ play a key role in determining whose consumption fluctuations weigh more in that decision. Moreover, the steady state asset holdings (i.e., the size of the balance sheet) of the central bank play a key role in the design of optimal monetary policy.

3.1 The case of rational expectations

To get analytical results that are more tractable, we derive here the optimal normalization policy under discretion. Later in our numerical exercises, we also display the case of commitment. Minimizing the welfare loss subject to the model structural equations leads to the

[17] In our setup, central bank's asset holdings are non-zero at the steady state. Given that they are costly, the steady state is distorted. Hence, optimal monetary policy is designed under a distorted steady state, in the spirit of Benigno and Woodford (2005).

[18] Recall that in the original model, the total stock of long-term bonds issued by the *child* are held by the financial intermediaries and the central bank.

following targeting criteria:[19]

$$c_t = -\gamma\zeta\,(1-z)\,\frac{\lambda_\pi}{\lambda_C}\,\pi_t \tag{13}$$

$$qe_t = -\frac{\tau QE}{2\lambda_{qe}Y\Psi} - \frac{\lambda_\pi}{\lambda_{qe}\Psi}\left[\left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\right]\pi_t - \frac{\lambda_{C_b}\bar{b}^{cb}\bar{b}^{FI}}{\lambda_{qe}\Psi}\theta_t \tag{14}$$

where $\Psi = 1 + (\bar{b}^{cb})^2\frac{\lambda_{C_b}}{\lambda_{qe}}$ and $\zeta = \frac{\chi(1-z)+\sigma}{1-z}$. The term $\lambda_\pi = \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)}$ is the weight on inflation stabilization in the welfare loss function defined in Proposition 1 above and λ_C,λ_{C_b}, $\lambda_{qe} > 0$ are the corresponding weights on the *parent*'s and the *child*'s consumption and on the size of the central bank balance sheet, respectively. The first targeting criterion in equation (13) states that in the face of inflationary pressures from a cost-push shock the central bank must respond by lowering the consumption of the *parent*. Given their stronger exposure to inflation relative to the *child* and their higher consumption share, $1-z$, it is natural for the central bank to trade their consumption off when trying to lower inflation. Given the full bailout they offer to the *child* assumed in Sims et al. (2023), the consumption of the latter depends primarily on the central bank balance sheet. In fact, as shown in Sims et al. (2023) and in our online Appendix A.7 the consumption of the *child* boils down to:

$$c_{b,t} = \bar{b}^{FI}\theta_t + \bar{b}^{cb}qe_t \tag{15}$$

Following inflationary pressures due to a cost-push shock, the central bank must shrink its balance sheet, qe_t, if it wants to curb inflation further, as captured by the optimality criterion in equation (14).[20] However, trading off the consumption of the *child* is heavier for the central bank, than that of the *parent*, due to the demand and the supply channels of central bank balance sheet policy working in opposite directions. The demand channel is captured by $\gamma\zeta$ which is the coefficient on the slackness term in the Phillips curve in (2). The term $\frac{\gamma\sigma}{1-z}$ represents the supply channel as it captures the direct negative impact of balance sheet policy on inflation, captured again in the Phillips curve equation (2).[21]

[19] Complete derivations are provided in the online Appendix B.2.

[20] The constant term in (14) arises because central bank balance sheet adjustments have been assumed to be costly distorting thereby the steady state. This also rationalizes the negative sign next to the constant term which has the interpretation that, for any given level of inflation, it is optimal for the central bank to hold less long-term bonds.

[21] In the model, the demand channel of central bank balance sheet policy operates through the effect of output on inflation. Unwinding the central bank balance sheet leads to a decline in economic activity and,

While the two channels operate in opposite directions, as indicated by the term $\gamma\zeta - \frac{\gamma\sigma}{1-z}$, the demand channel dominates because $\gamma\zeta > \frac{\gamma\sigma}{1-z}$. However, the supply channel of balance sheet policy tends to weaken the demand channel. As a result, controlling inflation via balance sheet policy requires larger policy interventions. Trading off the consumption of the *child* to control inflation becomes thus more costly, given also that fluctuations in central bank's balance sheet entail welfare costs. Trading off the consumption of the *parent*, as in (13), instead operates against the supply channel of balance sheet policy.[22]

Plugging the above optimality conditions in the Phillips curve (2), and assuming zero shock persistence for tractability ($\rho_\theta, \rho_{cp} = 0$), we obtain the equilibrium inflation under optimal discretionary policy:

$$\pi_t = -\frac{\tau QE}{2\lambda_{qe} Y \Psi \varsigma (1-\beta)} \varpi + \left[\left(\gamma\zeta - \frac{\gamma\sigma}{1-z} \right) z - \bar{b}^{cb} \frac{\lambda_{C_b} \varpi}{\lambda_{qe} \Psi} \right] \frac{\bar{b}^{FI}}{\varsigma} \theta_t + \frac{1}{\varsigma} cp_t \tag{16}$$

where Ψ, ς, and ϖ are all non-linear functions of structural parameters, specified in online Appendix B.2. Using (16) in (14) we obtain the equilibrium balance sheet under optimal discretionary policy, which we also derive in detail in the online Appendix B.2:

$$qe_t = \omega^{qe} - \omega^{qe}_\theta \theta_t - \omega^{qe}_{cp} cp_t \tag{17}$$

where:

$$\omega^{qe} = \frac{\tau QE}{2\lambda_{qe} Y \Psi} \left[\frac{\lambda_\pi}{\lambda_{qe} \Psi} \frac{\varpi^2}{\varsigma (1-\beta)} - 1 \right] \tag{18}$$

$$\omega^{qe}_\theta = \frac{\lambda_\pi}{\lambda_{qe} \Psi} \varpi \left[\left(\gamma\zeta - \frac{\gamma\sigma}{1-z} \right) z - \bar{b}^{cb} \frac{\lambda_{C_b} \varpi}{\lambda_{qe} \Psi} \right] \frac{\bar{b}^{FI}}{\varsigma} + \bar{b}^{cb} \frac{\lambda_{C_b}}{\lambda_{qe} \Psi} \bar{b}^{FI} \tag{19}$$

$$\omega^{qe}_{cp} = \frac{\lambda_\pi}{\lambda_{qe} \Psi} \frac{\varpi}{\varsigma} \tag{20}$$

through the Phillips curve, to disinflation. The supply channel of balance sheet policy instead results in a negative relationship between central bank asset holdings and inflation. This operates as follows. A decline in central bank long-term bond holdings implies that the *child* will borrow less. Given the assumption of a full bailout by the *parent*, the latter experiences a positive income effect stemming from a lower bailout due to lower borrowing by the *child*. The positive income effect leads to a decline in labor supply by the *parent* and thereby to a rise in real wages and marginal costs of firms. Hence, the supply channel suggests that shrinking the balance sheet is inflationary.

[22]The sensitivity of the inflation's response to central bank balance sheet shocks arising from a conflict between demand and supply channels, is also emphasized in larger structural models as Boehl et al. (2022) and Sims and Wu (2021). This is also examined in Sims et al. (2023) through the lens of a small-scale framework.

Combining the resource constraint of the economy with the optimal criterion (13) and (14) and the equilibrium inflation (16), we obtain an analogous expression for the output gap:[23]

$$x_t = \omega^x - \omega^x_\theta \theta_t - \omega^x_{cp} cp_t \tag{21}$$

where:

$$\begin{aligned}
\omega^x &= -(1-z)^2 \gamma\zeta \frac{\lambda_\pi}{\lambda_C}\omega^\pi + \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\omega^{qe} \\
\omega^x_\theta &= \left[(1-z)^2 \gamma\zeta \frac{\lambda_\pi}{\lambda_C}\omega^\pi_\theta + \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\omega^{qe}_\theta\right] z\bar{b}^{FI} \\
\omega^x_{cp} &= (1-z)^2 \gamma\zeta \frac{\lambda_\pi}{\lambda_C}\omega^\pi_{cp} + \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\omega^{qe}_{cp}
\end{aligned}$$

where ω^π, ω^π_θ and ω^π_{cp} are the constant term and the coefficients on the credit, θ_t, and the cost-push shock, cp_t, respectively, in the expression for the equilibrium inflation in (16). Combining the resource constraint and the optimality condition (13) with the IS equation (1) and solving for the policy rate, we can derive an interest rate rule that delivers the optimal paths for inflation and the output gap consistent with that described by the optimality condition (23) discussed below. Moreover, this interest rate rule is consistent with the optimality condition (14) describing the optimal path of the central bank balance sheet. The optimal interest rate rule considering the case of zero shock persistence ($\rho_\theta, \rho_{cp} = 0$) reads then as follows:

$$r^s_t = r^f_t + \gamma\sigma\zeta \frac{\lambda_\pi}{\lambda_C}(1-z)\pi_t \tag{22}$$

The optimal interest rate rule above states that the central bank must increase the short-term rate, r^s_t, upon inflationary pressures. Plugging the equilibrium inflation, equation (16), one can get the equilibrium interest rate under optimal discretionary policy. Finally, using the resource constraint of the economy, the optimal criteria (13) and (14) and the expression for *child*'s consumption (15), we arrive at the usual inflation-output gap trade-off, which for simplicity we can express in the absence of credit shocks (i.e. $\theta_t = 0$) as follows:

$$x_t = -\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\frac{\tau QE}{2\lambda_{qe} Y \Psi} - \lambda_\pi \left[\frac{(1-z)^2\gamma\zeta}{\lambda_C} + \frac{\gamma\zeta - \frac{\gamma\sigma}{1-z}}{\lambda_{qe}\Psi}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)^2\right]\pi_t \tag{23}$$

[23] In the online Appendix A.6 we show that the log-linearized resource constraint reads as: $y_t = (1-z)c_t + zc_{b,t} + \left(\frac{\tau QE}{Y}\right) qe_t$.

The above trade-off criterion reflects a well-established result from the standard three-equation New Keynesian model: after a cost-push shock that raises inflation, the central bank must trigger a decline in the output gap to stabilize prices. The size of the trade-off in equation (23), however, contains richer information than its counterpart in the three equation New Keynesian model. Non-zero central bank bond holdings at the steady state ($\bar{b}^{cb}, QE \neq 0$) together with the presence of borrowers ($z \neq 0$) make the inflation-output gap trade-off heavier. This is captured by the second term in the bracket of the coefficient on inflation in (23). At the same time, the induced decline in the output gap is alleviated by the supply channel of the central bank balance sheet policy. As the supply channel strengthens (i.e. higher $\frac{\gamma\sigma}{1-z}$) the inflation-output gap trade-off is alleviated, partly mitigating the induced output gap losses.

3.2 The case of de-anchored expectations

Before discussing the results of optimal policy under de-anchored expectations, it is useful to note that when expectations are de-anchored, as described in (9), iterating backwards allows us to write the expectations for a generic variable z as:

$$\mathbb{E}_t^d z_{t+1} \equiv \omega_t^z = \bar{g} \sum_{s=0}^{\infty} (1-\bar{g})^s z_{t-s-1} \tag{24}$$

We apply thus this expression in the forward looking variables of the model. We assume that the central bank has model-consistent expectations knowing the way the households form expectations and the actual law of motion of the economy. Minimizing the welfare loss under de-anchored expectations leads to the following targeting criteria:[24]

$$\pi_t = -\frac{\lambda_C}{\lambda_\pi \gamma\zeta (1-z)} c_t + \frac{\beta\lambda_C \bar{g}}{\lambda_\pi \gamma\zeta (1-z)} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{s+1} (1-\bar{g})^s c_{t+s+1} \tag{25}$$

$$\begin{aligned} qe_t = &-\frac{\tau QE}{2\lambda_{qe} Y\Psi} - \frac{\lambda_\pi}{\lambda_{qe}\Psi}\left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\pi_t - \frac{\lambda_{C_b}\bar{b}^{cb}\bar{b}^{FI}}{\lambda_{qe}\Psi}\theta_t \\ &+ \frac{\lambda_C}{\lambda_{qe}\Psi\gamma\zeta(1-z)}\left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\beta\bar{g}\mathbb{E}_t \sum_{s=0}^{\infty}\beta^{s+1}(1-\bar{g})^s c_{t+s+1} \end{aligned} \tag{26}$$

[24]Complete derivations are provided in online Appendix B.3.

where the expectations operator refers to the expectations of the central bank. The criterion in (25) shows the trade-off between inflation and the consumption of the *parent*. As in the literature, this trade-off contains an intratemporal element, which is current consumption, and an intertemporal one captured by the sum of the expected discounted consumption of the *parent*.[25] The intertemporal trade-off arises because of the expectations formation mechanism. Clearly, when expectations are de-anchored, the central bank must consider their backward looking nature when deciding upon its policy. That is, policy decisions have a delayed impact on the economy due to slow adjustment in agents' expectations. In the absence of learning (i.e., $\bar{g} = 0$), the trade-off reverts to the optimal trade-off under optimal discretion with rational expectations given in (13).

The second criterion in (26) describes the optimal setting of the central bank balance sheet when expectations are de-anchored. Comparing this with its counterpart under rational expectations in (14), the optimality condition (26) states that the central bank balance sheet should react to contemporaneous inflation and to a credit shock in exactly the same way as it does under rational expectations. The coefficient on contemporaneous inflation is not affected by the expectation formation mechanism: it includes the interaction between the supply and demand channel which limits the effectiveness of the balance sheet in responding to inflationary pressures. However, under de-anchored expectations, the central bank balance sheet also depends on the expected discounted sum of the future consumption of the *parent*. Hence expectations matter when deciding upon the balance sheet policy in this case. But how important are they? A closer look at the coefficient on the expected discounted sum of future *parent*'s consumption reveals that, first, it is a function of the term capturing the interaction between the demand and the supply channel of balance sheet policy (i.e. $\gamma\zeta - \frac{\gamma\sigma}{1-z}$), second it is subject to heavy discounting and lastly depends on the gain parameter, $\bar{g}$. All three factors tend to mitigate the impact of expectations on the optimal balance sheet setting. Given that the demand and the supply channels of balance sheet policy work in opposite directions, a strong supply channel captured by a higher $\frac{\gamma\sigma}{1-z}$ tends to limit the impact of expectations on optimal balance sheet decisions. Similarly, heavy discounting reinforces this attenuation. Finally, the low values of the gain parameter, $\bar{g}$, advocated in the literature (see Carvalho et al., 2023; Eusepi et al., 2020) further

[25] In the literature (see Gaspar et al., 2010; Gáti, 2023; Molnár and Santoro, 2014, among others) the intra- and intertermporal trade-off includes the output gap. These models however assume one representative household.

contribute to dampening the importance of expectations on the optimal balance sheet decisions. Our quantitative analysis below corroborates this showing that the expectations formation mechanism plays a limited role in the optimal balance sheet path. In this case as well, when there is no learning (i.e. $\bar{g} = 0$) we are back to the optimality condition for the central bank balance sheet with rational expectations, as summarized in (14).

Combining the intertemporal trade-off in (25) with the Phillips curve and exploiting the backward looking behavior in expectations as described in (24) we can express equilibrium inflation under optimal policy with de-anchored expectations as follows:[26,27]

$$\pi_t = \frac{\omega^\pi(1-\beta)}{\tilde{\varrho}} + \omega_\theta^\pi \sum_{s=0}^{\infty} \left(\frac{\beta\bar{g}\iota}{\varsigma\tilde{\varrho}}\right)^s \theta_{t-s} + \omega_{cp}^\pi \sum_{s=0}^{\infty} \left(\frac{\beta\bar{g}\iota}{\varsigma\tilde{\varrho}}\right)^s cp_{t-s} \tag{27}$$

where

$$\tilde{\varrho} = 1 + \tilde{\delta}\frac{(\beta\bar{g})^2}{\varsigma}\frac{\lambda_\pi\gamma\zeta(1-z)}{\lambda_C}\tilde{P}$$

$$\tilde{\delta} = \left[\gamma\zeta(1-z) + \left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)^2 \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)^2 \frac{\lambda_C}{\lambda_{qe}\Psi\gamma\zeta(1-z)}\right]$$

and

$$\iota = \sum_{s=0}^{\infty} \left[(1-\bar{g})^s - \tilde{\delta}\beta\bar{g}\frac{\lambda_\pi\gamma\zeta(1-z)}{\lambda_C}\left((1-\bar{g})^s + \prod_{j=0}^{s}\left(1+\beta^{j+2}\bar{g}\right)\beta^{j+1}(1-\bar{g})^{j+1}\right)\right]$$

Expression (27) above shows that equilibrium inflation under de-anchored expectations depends not only on current, but also on past credit and cost-push shocks. This contrasts starkly with the equilibrium inflation under RE in (16) which depends only on contemporaneous shocks. De-anchored expectations thus introduce persistence in the model that the central bank needs to take into account. When there is no learning (i.e., $\bar{g} = 0$) we have $\tilde{\delta} = 0$ and $\tilde{\rho} = 1$, the equilibrium inflation coincides with that under optimal discretion with RE in (16) and past shock realizations no longer play a role. Plugging the equilibrium

[26] A complete derivation is provided in the online Appendix B.3. As with rational expectations, we restrict our focus to the case of zero shock persistence ($\rho_\theta, \rho_{cp} = 0$) for tractability.

[27] In our derivations we have also used the fact that:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{s+1}(1-\bar{g})^s c_{t+s+1} = -\beta\bar{g}\frac{\lambda_\pi\gamma\zeta(1-z)}{\lambda_C}\left[\tilde{P}_t\pi_t + \sum_{s=0}^{\infty}\left((1-g)^s + \prod_{j=0}^{s}\left(1+\beta^{j+2}\bar{g}\right)\beta^{j+1}(1-\bar{g})^{j+1}\right)\pi_{t-s-1}\right]$$

where $\tilde{P} = \prod_{s=0}^{\infty}\left(1+\beta^{s+2}\bar{g}\right)\beta^{s+1}(1-\bar{g})^{s+1}$ as we also show in Appendix B.3.

inflation (27) in (26) ,we obtain the equilibrium of central bank asset holdings. Proceeding analogously to the case with RE, we then derive the corresponding equilibrium output gap.[28]

We now examine how de-anchored expectations influence the optimal interest rate setting. By substituting the solutions for the equilibrium inflation and central bank asset purchases in the IS equation (1) and solving for the short-term rate, we obtain the optimal interest rate rule under de-anchored expectations:

$$\begin{aligned} r_t^s &= r_t^f + \gamma\sigma\zeta\frac{\lambda_\pi}{\lambda_C}(1-z)\left(1+(\beta\bar{g})^2\tilde{P}\right)\pi_t \\ &+ \sigma\bar{g}\sum_{s=0}^{\infty}\left[(1-\bar{g})^s - \beta^2\bar{g}\frac{\lambda_\pi\gamma\zeta(1-z)}{\lambda_C}\left((1-g)^s + \prod_{j=0}^{s}\left(1+\beta^{j+2}\bar{g}\right)\beta^{j+1}(1-\bar{g})^{j+1}\right)\right]\pi_{t-s-1} \end{aligned} \tag{28}$$

Comparing the optimal interest rate rule above with its counterpart under RE in (22), two observations stand out. First, the reaction to contemporaneous inflation is stronger when expectations are de-anchored. Specifically, the reaction to contemporaneous inflation obtained under rational expectations (i.e. $\gamma\sigma\zeta\frac{\lambda_\pi}{\lambda_C}(1-z)$) is now multiplied by $1+(\beta\bar{g})^2\tilde{P}$ which is positive given $\tilde{P}>0$. As a result, for a given adverse cost-push shock that raises inflation, the central bank must raise the short-term rate more than under RE. Second, de-anchored expectations make the interest rate rule history dependent, as the sum of past inflation realizations now enters the rule. The historical component amplifies the policy response, as the central bank must now take into account the slow adjustment of expectations and their persistent impact on inflation dynamics. Note that when there is no learning (i.e. $\bar{g}=0$) the optimal interest rate rule above collapses to that under RE in (22). The more aggressive interest rate response to inflation is determined by the way de-anchored expectations affect the inflation-output gap trade-off. In fact, the sluggish adjustment of expectations deteriorates the transmission of monetary policy. Consequently, the central bank has to tolerate larger output losses to achieve a given path for inflation. This mechanism is formalized in Proposition 2.

[28] To save space we do not report the resulting expressions here.

Proposition 2. *De-anchored expectations result in a heavier inflation-output gap trade-off.*
Proof. In online Appendix B.4. ∎

The analysis of optimal policy thus shows that the central bank has to adjust its policy rate more aggressively following cost-push shocks when expectations are de-anchored, compared to rational expectations, leading to a heavier inflation-output gap trade-off. At the same time, its optimal decision on the balance sheet is only marginally affected by the way expectations are formed. In the following section, we provide a quantitative evaluation of the analytical findings above.

4 Optimal policy simulations

In this section, we turn to quantitative simulations. This section extends the analytical findings presented in Section 3 in several relevant ways. In our simulations, we consider persistent cost-push shocks, whereas in our analytical results above we considered $i.i.d.$ shocks for tractability. Second, we consider the scenario where changes in the policy rate are costly - an element not accounted for in the analytical framework. Finally, we also study a setting in which the economy is hit by both cost-push and credit shocks simultaneously, to explore whether this raises the importance of balance sheet policy. In Section 4.1, we start by studying the optimal policy path for the two policy instruments - the short-term policy rate and the balance sheet - under scenarios of RE and de-anchored expectations. To be complete, in the case of RE, we present the results under commitment and discretion, with only the latter described in detail in Section 3. Subsequently, in Section 4.2 we analyze optimal policy when fluctuations in the short-term rate are costly, while in Section 4.3 we focus on optimal policy when the economy is subject to both cost-push and negative credit shocks.

Details on simulations To replicate an environment with a high balance sheet that gradually returns to the steady state (defined as 10% of annualized output), we introduce a persistent positive QE shock.[29] Specifically, we assume that central bank asset holdings fol-

[29] The argument of a positive balance sheet in the steady state has been recently emphasized in policy discussions. See, e.g. 9 November 2023, Philip R. Lane, Member of the Executive Board of the ECB, at the ECB Conference on Money Markets: *"In particular, the appropriate level of central bank reserves can be expected to remain much higher and be more volatile in this new steady state compared to the relatively-low levels that prevailed before the global financial crisis (GFC)."*

low an AR(1) process initially, as specified in equation (7), and hence we shock this process. This allows us to generate an environment of central bank asset holdings that lie above the steady state. We also assume that the short-term rate is above the ZLB. In period 5, which is four quarters after the initial QE shock, we introduce a one standard deviation persistent price cost-push shock.[30] We compute optimal policy in the different cases that we consider from that period (i.e., period 5) onward.[31] Before period 5, the short-term rate is assumed to follow the interest rate rule specified in equation (6) and the central bank asset holdings follow the AR(1) process that is shocked in period 1, as discussed above. We mainly focus on the case in which both instruments, the short-term rate and the central bank asset holdings, are set optimally from period 5 onward.[32] In our results below, we show the impulse responses to a cost-push shock under optimal policy, starting from the period in which this shock hits, i.e. period 5, omitting the first 4 periods for clarity. Across all cases, the analysis takes place in a context of balance sheet normalization—that is, a transition from elevated central bank holdings back to their steady-state level.

Computational details We use a linear-quadratic approach to optimal policy, drawing on de Groot et al. (2021), Hebden and Winkler (2021) and McKay and Wolf (2023), which allows us to derive model-consistent forecast targeting criteria, both under de-anchored and rational expectations. We provide more details in our online Appendix C.1. We leverage on the first-order conditions of our optimal policy formulation, in which the constraints for each variable are defined using the impulse response representation of the model. We subsequently solve for the set of monetary policy shocks that satisfies the optimal policy problem.[33] This means that we find the short-term interest rate and balance sheet sequences

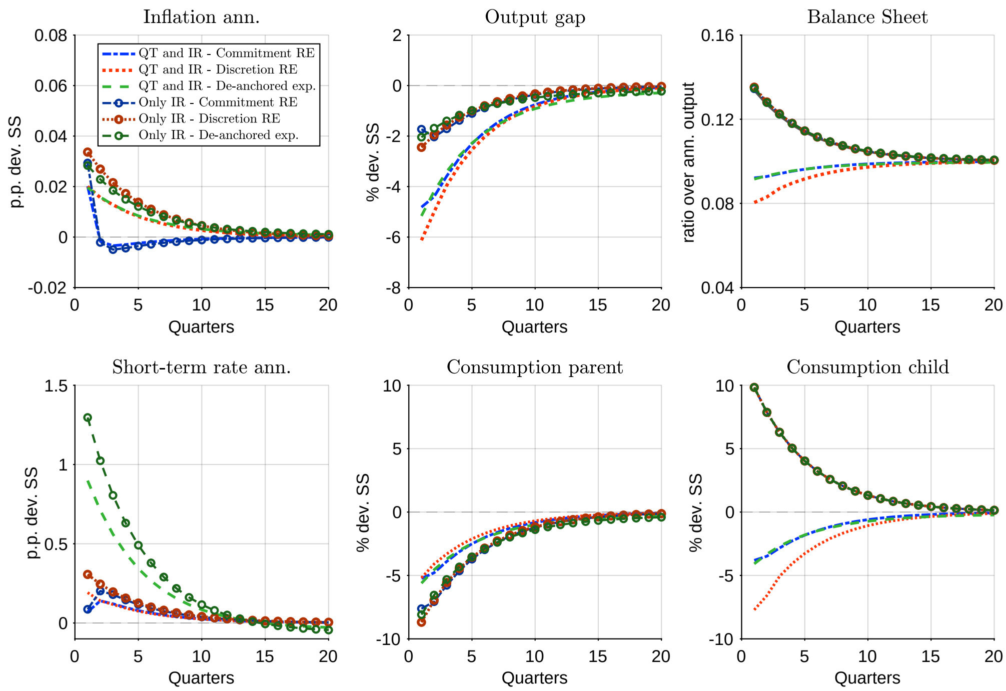
[30]We deviate from the assumption of i.i.d. shocks taken in Section 3, instead considering a persistent cost-push shock scenario, which aligns more closely with recent empirical observations during the inflation surge. This results in further differences between rational and de-anchored expectations.

[31]Note that the choice of period 5 is arbitrary and that our policy prescriptions are not sensitive to it qualitatively. The implications are only quantitative.

[32]In our analysis, we also compare the welfare implications of using both instruments versus one of the two only. In the case where only the short-term rate is set optimally, the central bank asset holdings are assumed to continue following the AR(1) rule throughout the entire horizon. When only the central bank asset holdings are set optimally (optimal QT), the short-term rate is assumed to continue following the interest rate rule assumed in the initialization of our simulations, as specified above. In this case, the path of the central bank asset holdings is decided optimally from period 5 onward. Before period 5, they simply follow the AR(1) process that is shocked in period 1.

[33]Our computations benefit from the extensive work outlined in the COPPs toolkit developed by de Groot et al. (2021), which provided valuable insights for solving our optimal policy problems. The toolkit can be accessed here: https://github.com/COPPsToolkit/COPPs.

Figure 1: Optimal monetary policy, in response to a cost-push shock



Notes: Optimizing the balance sheet and the short-term interest rate, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The dash-dotted blue lines depict optimal policy under commitment with fully RE, while the red dotted lines show the case of discretion with fully RE. The green dashed lines are optimal policy under fully de-anchored expectations. The circle-marked lines are the cases where only the short-term rate is set optimally (Only IR). The lines without markers are the cases in which both instruments are set optimally (QT and IR). To implement a transition from a high balance sheet level to a lower (i.e. steady state) level, we first apply a positive QE shock. Then, four periods after this initial shock, we introduce a cost-push shock. We show the impulse responses under optimal policy from this period onwards, when the cost-push shock hits.

that implement our forecast targeting criteria.

4.1 Optimal policy in response to a persistent cost-push shock

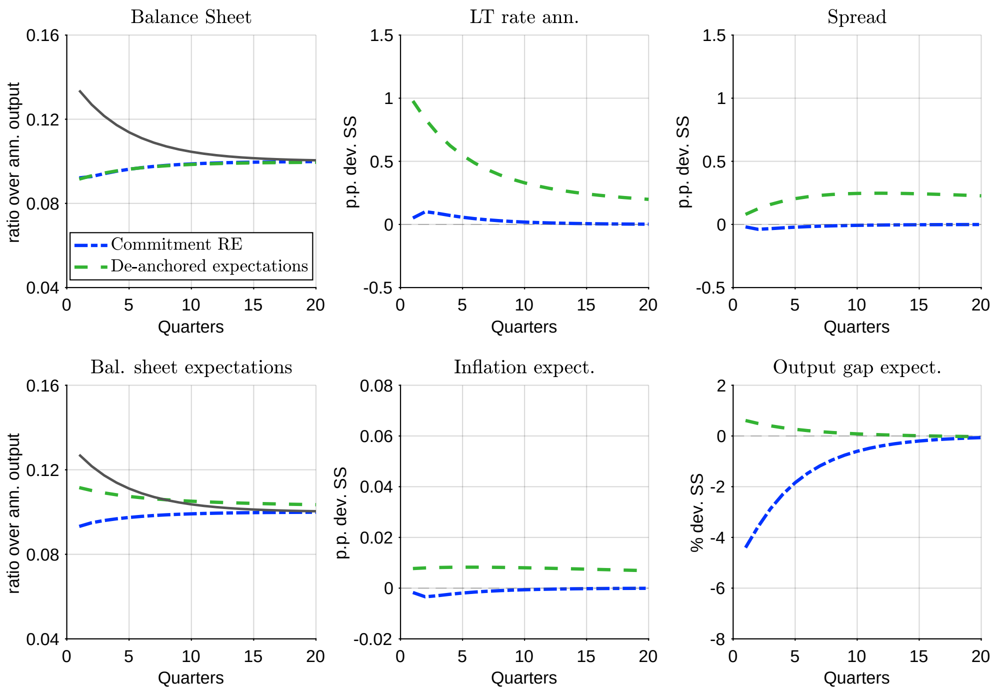
In this subsection, we analyze how the central bank can set both of its monetary policy instruments so as to minimize our model-consistent loss function, shown in Proposition 1, in response to a persistent price cost-push shock that induces upward pressure on inflation. Figure 1 plots the response of inflation, the output gap, the consumption of the *parent* and the *child* along with the two monetary policy instruments: the short-term policy rate and the balance sheet. This is presented in a context in which both instruments are used to minimize the loss function, in comparison to a scenario where only the short-term interest rate is set optimally, and the balance sheet follows the AR(1) process described in (7). Colored lines with circle markers are the cases in which only the short-term interest rate is set in an optimal way, while lines without markers represent scenarios where both instruments are set optimally.

The dash-dotted blue lines depict the case of commitment under RE, while the dotted red lines depict the case of discretion under RE. In these two cases, it appears that the short-term interest rate is a more powerful tool to attain inflation stabilization, causing a milder recession. Moreover, the path of the policy rate when both instruments are set optimally remains slightly below the level when only the short-term interest rate is set optimally (lines with circle markers). Consistent with this, the implementation of an optimal QT — reflected in a decrease in the balance sheet — results in a moderate additional reduction in inflation, although this comes at the cost of a significant decline in the output gap. Under de-anchored expectations, depicted in dashed green lines, the effectiveness of the short-term interest rate diminishes, where now the optimal rate hike is more pronounced and persistent. Yet, the paths of inflation and the output gap remain broadly similar to those under RE. As shown in (28), the optimal response of the short-term rate to inflation rises when expectations are de-anchored. The central bank accounts in this case for the slow adjustment of inflation expectations that keep inflation higher than otherwise, all else equal. Thus, to counteract the pressure from expectations, it needs to hike more forcefully.

Specifically, the more aggressive hike incorporates not only the stronger reaction to inflation contemporaneously, but also the reaction to elevated lagged inflation inherited due to de-anchored expectations. In this context as well, the balance sheet remains a less effective tool to fight inflationary pressures as it brings only limited gains on inflation stabilization at the expense of substantially higher output losses. In fact, as (26) shows, the demand and supply channels of balance sheet policy work in opposite directions, thereby reducing its overall efficacy. As a result, stronger cuts in the balance sheet are necessary to achieve a given decline in inflation. However, this comes at the expense of a deeper contraction in economic activity. Specifically, from (15), the consumption of the *child* declines considerably adding to the induced contraction.[34] These conclusions hold both under RE and de-anchored expectations, given that the expectations formation mechanism has a negligi-

[34]Recall that the *child* finances her consumption by issuing long-term bonds partly held by the central bank. Hence, unwinding the central bank balance sheet reduces the consumption of the *child*.

Figure 2: Optimal monetary policy, the role of expectations



Notes: The dynamics of expectations, the long-term rate and the spread when both instruments are set optimally, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The dash-dotted blue lines depict optimal policy under commitment with fully RE. The green dashed lines are optimal policy under fully de-anchored expectations. The solid black line shows the baseline QT trajectory where the central bank balance sheet follows an AR(1) process. To implement a transition from a high balance sheet level to a lower (i.e. steady state) level, we first apply a positive QE shock. Then, four periods after this initial shock, we introduce a cost-push shock. We show the impulse responses under optimal policy from this period onwards, when the cost-push shock hits.

ble effect on the optimal balance sheet path, as discussed in the previous section.

Overall, the description of Figure 1 allows us to emphasize the following main takeaways about the optimal normalization policy following cost-push shocks in the case of rational or de-anchored expectations. These findings quantify our analytical results in Section 3. First, the interest rate stands out as the primary instrument for macroeconomic stabilization, due to its higher efficacy in tempering inflationary pressures at a lower output cost. Consistent with the literature, it proves highly powerful under RE and less so under de-anchored expectations. Second, in both cases of expectation formation, rational and de-anchored, QT proves broadly ineffective, offering few additional gains in terms of inflation reduction while imposing significant costs on real activity. Furthermore, while the interest rate trajectory differs between the two expectation scenarios, the optimal balance sheet strategy remains largely unaffected by the type of expectations.

The role of expectations Figure 2 provides further details on the mechanism through which optimal interest rate and balance sheet policy operates, emphasizing the role of expectations in shaping the effectiveness of the policy. It shows the dynamics of the balance sheet, the long-term rate and the interest rate spread, as well as inflation, the output gap and balance sheet expectations.[35] When expectations are well-anchored, agents recognize that the policy response would prevent inflation from staying persistently elevated but that output will fall. In addition, agents' beliefs track precisely the announced path by the central bank, enabling the central bank to effectively manage long-term rates.[36] This allows the central bank to calibrate QT to the necessary degree, tightening long-term rates just enough to complement the increase in the short-term rate and thus counteract the elevated inflation.

In the case of de-anchored expectations, agents' beliefs are more sluggish. They perceive that inflation will remain elevated for an extended period, prompting the central bank to raise the short-term policy rate to a higher level and maintain it for longer, as depicted in Figure 1. The effect of de-anchored expectations on the path of expected inflation is captured analytically by the sum of lagged inflation terms in the optimal interest rate rule in (28) which forms a part of the stronger rate hike in this case, as explained above. On the side of the balance sheet, agents do not factor in the central bank's future QT path. This results in a significant rise in long-term rates due to the misalignment between the actual QT level and expectations about it. However, given the similar divergence in the short-term rate, in both cases (RE and de-anchored expectations), the spread between the long-term and the short-term interest rate does not vary markedly.

Importantly, in this case, the expectations formation of the private sector about the QT trajectory is of relatively low importance. The intuition for this result is that, here, smaller adjustments in the long-term rate are required when both instruments are set optimally, since the short-term interest rate remains the primary tool for controlling inflation, irrespective of the prevailing expectations formation scenario. Consequently, any variations in the long-term rate are a direct consequence of changes in QT expectations and the more forceful short-term rate response rather than the central bank's balance sheet actions per

[35]These variables are all explicitly part of the structural model outlined in Section 2.

[36]As detailed in online Appendix A, real long-term rates are a linear function of the expected changes in the size of the balance sheet $\mathbb{E}_t r^b_{t+1} - \mathbb{E}_t \pi_{t+1} = \sigma \left[\mathbb{E}_t c_{b,t+1} - c_{b,t}\right] = \sigma \left[\bar{b}^{FI} \left(\mathbb{E}_t \theta_{t+1} - \theta_t\right) + \bar{b}^{cb} \left(\mathbb{E}_t qe_{t+1} - qe_t\right)\right]$.

Table 2: Welfare costs

	Commitment RE	Discretion RE	De-anchored exp.
Interest rate only, parent	0.198	0.315	0.236
Balance sheet only, parent	0.485	0.616	0.815
Interest rate only, child	-0.670	-0.845	-0.687
Balance sheet only, child	0.281	0.674	0.623
Interest rate only, planner	0.017	0.050	0.034
Balance sheet only, planner	0.275	0.395	0.379

Notes: This table presents the welfare costs associated with using one instrument only, either the short-term interest rate or the balance sheet, relative to using both instruments. The values of the welfare cost represent the percentage increase in the steady-state consumption of the *parent* or the *child*, or the social planner, that would make her indifferent between using both instruments optimally and setting only the interest rate or the only balance sheet optimally.

se. The derivation of the optimal criterion in (26) revealed that the conflicting demand and supply channels of balance sheet policy mitigate the importance of de-anchored expectations in the decision of the central bank about its balance sheet. Their importance is further dampened due to discounting and a low constant gain parameter. Hence, it is the interaction between the two channels of transmission of balance sheet policy and the slow adjustment of expectations that makes the latter less relevant in the optimal decision of the central bank.

The welfare costs of using one relative to both instruments We now turn to quantifying the welfare costs of using one instrument (the short-term rate or the balance sheet) relative to using both to minimize our model consistent loss function. We obtain the welfare costs following Ravenna and Walsh (2011) and Mavromatis (2018), as the percentage increase in steady-state consumption that would make the *parent* and/or *child* indifferent between the optimal policy using one instrument only and the fully optimal policy where both instruments are set optimally.

We define α_p^{IR} as the welfare cost for the *parent* of not implementing the fully optimal policy, measured as the percentage increase in steady-state consumption that would make her indifferent between interest rate only optimal policy and the fully optimal policy. Similarly, α_p^{QT} is the welfare cost for the *parent* of not implementing the fully optimal policy, measured as the percentage increase in steady-state consumption that would make her indifferent between balance sheet only optimal policy and the fully optimal policy. The terms α_c^{IR} and α_c^{QT} denote the respective welfare costs for the child. Finally, α_w^{IR} and α_w^{QT}

represent the respective welfare costs for the economy as a whole, derived from the social welfare. Formally, α_p^k, α_c^k and α_w^k, with $k = \{IR, QT\}$ can be obtained from:

$$\frac{1}{1-\beta}\bar{U}\left(\left(1+\alpha_p^k\right)\bar{C},\bar{L}\right)+V_t^k=\frac{1}{1-\beta}\bar{U}\left(\bar{C},\bar{L}\right)+V_t^{fo} \tag{29}$$

$$\frac{1}{1-\beta^b}\bar{U}\left(\left(1+\alpha_c^k\right)\bar{C}^b\right)+V_{t,b}^k=\frac{1}{1-\beta^b}\bar{U}\left(\bar{C}^b\right)+V_{t,b}^{fo} \tag{30}$$

$$\begin{aligned}&\frac{1}{1-\beta}\bar{U}\left(\left(1+\alpha_w^k\right)\bar{C},\bar{L}\right)+\frac{1}{1-\beta}\bar{U}\left(\left(1+\alpha_w^k\right)\bar{C}^b\right)+W_t^k\\&=\frac{1}{1-\beta}\bar{U}\left(\bar{C},\bar{L}\right)+\frac{1}{1-\beta}\bar{U}\left(\bar{C}^b\right)+W_t^{fo}\end{aligned} \tag{31}$$

with V_t^k and $V_{t,b}^k$ the welfare of the *parent* and the *child*, respectively, in the case with only one instrument being set optimally, while V_t^{fo} and $V_{t,b}^{fo}$ are the welfare of the parent and the child in the case that we consider as fully optimal, where both instruments are set optimally.[37] Similarly, W_t^k and W_t^{fo} is the aggregate welfare, derived in Proposition 1, evaluated when only one instrument is set optimally and when both instruments are set optimally, respectively.

Table 2 summarizes the welfare costs. The first row shows the welfare costs for the *parent* of setting only the interest rate optimally, relative to the fully optimal policy involving both instruments. A positive value implies that when the central bank follows the fully optimal policy this is preferable for the *parent*, compared to implementing a single instrument optimally. The second row focuses on the scenario in which only the balance sheet is set optimally, relative to the fully optimal policy. The subsequent rows summarize the respective results for the *child*, as well as for aggregate welfare (denoted as *planner*). The first column is the case with RE and commitment, the second corresponds to the case with RE and discretion, and the third to the case with de-anchored expectations.

When comparing the scenarios where both instruments are optimized to those where only the interest rate is set optimally, Table 2 supports the findings illustrated in Figure 1. From the perspective of the social planner, the benefits of using both instruments optimally are small for the economy on aggregate. Specifically, setting only the interest rate optimally would require a rise in steady state consumption of 0.05% on average under discretion with

[37]The welfare of the *parent* and the *child*, respectively, are derived from a second order approximation to the utility function of each group, as summarized in the online Appendix B in equations (B.11) and (B.13).

RE and of 0.034% under de-anchored expectations, respectively, in order for the economy as a whole to be as well off as under the fully optimal policy. Focusing on the *parent* and the *child* individually, we find that relying solely on the interest rate is suboptimal for the *parent*, as her consumption — which depends on the real short-term interest rate — declines more significantly. Conversely, the *child* would benefit more from using only the interest rate optimally, as its consumption is directly linked to the path of the balance sheet, which in this scenario follows the stable AR(1) process.

Finally, when comparing the use of both instruments to using only the balance sheet, our results consistently indicate that the interest rate is the most effective instrument. Setting only the balance sheet optimally results in significantly higher welfare costs, both on aggregate and for each type of agent individually, compared to the fully optimal policy. This finding holds true across all scenarios.[38]

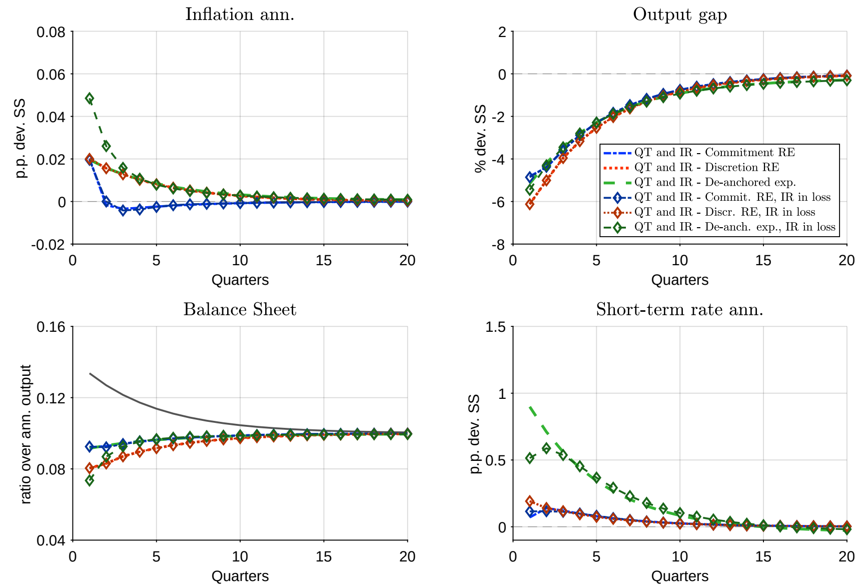
4.2 Penalizing changes in the short-term rate

In the previous subsection, we established that the interest rate is the best-suited tool to curb inflationary pressures caused by cost-push shocks and that reducing the size of the balance sheet brings negligible improvements in aggregate welfare. Importantly, we showed that this conclusion also holds under de-anchored expectations, albeit more aggressive hikes are necessary in response to cost-push shocks. In practice, this might prove too costly as central banks may need to adjust interest rates gradually or may want to avoid excess interest rate volatility. To assess how this affects our policy prescriptions, we append to the loss function derived in Proposition 1 the change in the short-term rate squared. The loss function thus receives the following form:

$$\begin{aligned} W_t \approx W - U_C Y \Bigg\{ & (1-z)\left(\frac{C}{2}\right) c_t^2 + z\left(\frac{C_b}{2}\right) c_{b,t}^2 + \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)}\pi_t^2 \\ & + \frac{1}{2}(\tilde{v}_t^p)^2 + \tau\frac{Qb^{cb}}{Y} qe_t + \tau Qb^{cb}\frac{Qb^{cb}}{2Y} qe_t^2 + \lambda_r \left(r_t^s - r_{t-1}^s\right)^2 \Bigg\} \end{aligned} \tag{32}$$

[38]In online Appendix C.3, we show that in scenarios in which the central bank balance sheet is the sole policy tool set optimally, inflation is less well contained at the expense of a strong decrease in output.

Figure 3: Optimal monetary policy, penalizing the change in interest rate



Notes: Optimizing the balance sheet and the short-term interest rate, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock, adding the change in the short-term interest rate in the loss function. The weight on the change in the short-term rate is set to $\lambda_r = 1$. The solid blue lines depict optimal policy under commitment with full RE, and the red dashed-dotted lines show the case of discretion with full RE. The green dashed lines are optimal policy under fully de-anchored expectations. The solid black line shows the baseline QT trajectory where QT follows an AR(1) process. The diamond-marked lines indicate the cases with the change in the interest rate in the loss function. To implement a transition from a high balance sheet level to a lower (i.e. steady state) level, we first apply a positive QE shock. Then, four periods after this initial shock, we introduce a cost-push shock. We show the impulse responses under the optimal policy from this period onwards when the cost-push shock hits.

with λ_r the weight associated with the change in the short-term rate.[39] The results are displayed in Figure 3, which plots the response of inflation, the output gap and the two monetary policy instruments when both are set optimally. The colored lines marked with diamonds plot optimal policy under RE (commitment and discretion) and de-anchored expectations when changes in the short-term rate are costly. For comparison, in the colored lines without markers we show the responses from Figure 1, where an interest rate stabilization objective is absent in the loss function.

Penalizing changes in the short-term rate has an impact on the conduct of optimal policy under de-anchored expectations, depicted with the green diamond-marked lines. In this case, the central bank is more constrained in the hikes that it can implement, and that is why the rise in the short-term rate is now dampened and hump-shaped compared to the

[39]We calibrate $\lambda_r = 1$ as in, e.g., Caravello et al. (2024).

case where variations in the short-term rate are not penalized (green dashed lines without markers). To compensate for this limitation, the central bank must initially shrink its balance sheet further. Given the costly short-term rate fluctuations, without the balance sheet as an additional instrument at its disposal, inflation would have jumped even higher on impact. In contrast, under rational expectations, penalizing interest rate variations leads to negligible changes in the prescribed optimal policy paths of both instruments. The key conclusion is that, under de-anchored expectations when interest rate variations are penalized, balance sheet management can contribute to macroeconomic stabilization and help avoid large changes in the short-term rate. It does so through further balance sheet reduction that tightens long-term rates and offsets some of the inflationary pressure.

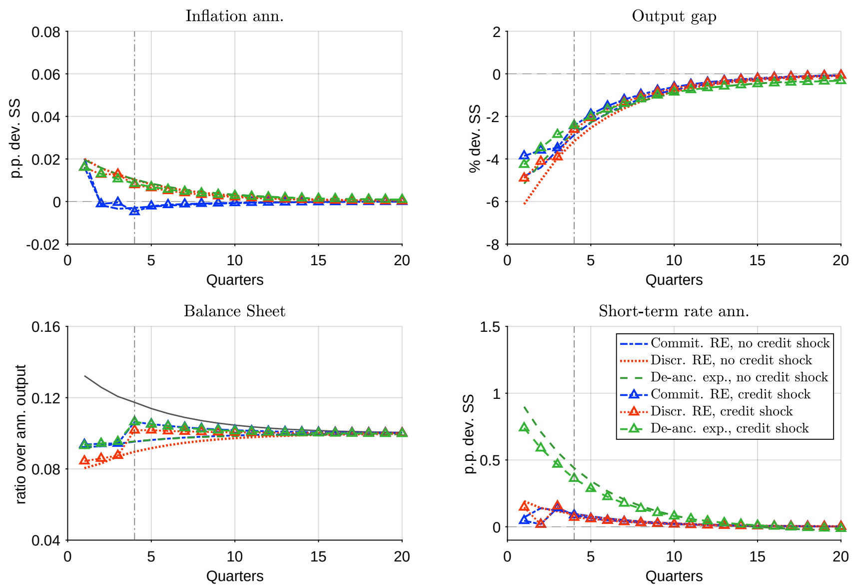
4.3 Facing multiple shocks, the effects of credit disturbances

Balance sheet policies are also frequently used as a way to stabilize stressed financial markets. When addressing inflationary shocks during monetary policy normalization, policymakers may concurrently face shocks affecting the financial system.[40] How should they respond optimally to such a combination of disturbances? Does the nature of the shock and the expectations formation mechanism matter for the performance of each policy tool? This section answers these questions by computing optimal monetary policy using both instruments, under RE and de-anchored expectations. We assume that a substantial credit shock occurs one year after the initial cost-push shock. The credit shock resembles a severe tightening of financial intermediaries' balance sheets, akin to a credit crunch. The responses to this combination of shocks are represented by the colored lines marked by triangles in Figure 4. For convenience, we also display the responses obtained in the previous sections under a cost-push shock only, in colored lines without triangle markers.

Sims et al. (2023) show that the balance sheet is the best-suited tool to respond to credit shocks. Our optimal policy results draw the same conclusion. Moreover, we show that this conclusion is robust to the expectations formation mechanism. Compared to the case with cost-push shocks only, we find that large credit disturbances can push the central bank to slow down and even partially reverse its normalization effort, both under RE and de-anchored expectations. As depicted in Figure 4, the balance sheet becomes the primary

[40] A recent example is the Federal Reserve's intervention in the wake of the failure of Silicon Valley Bank and Signature Bank.

Figure 4: Optimal monetary policy, in response to a cosh-push and a credit shock



Notes: Optimizing the balance sheet and the short-term interest rate, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock, and a five standard deviation persistent ($\rho_{\theta} = 0.8$) credit shock. The solid blue lines depict optimal policy under commitment with fully RE, while the red dashed-dotted lines show the case of discretion with fully RE. The vertical dashed line represents the period when the credit shock hits. The colored lines without markers are optimal policy when facing a cost-push only, while the colored lines with markers are when facing a cost-push and a credit shock. The solid black line shows the baseline QT trajectory where QT follows an AR(1) process. To implement a transition from a high balance sheet level to a lower (i.e. steady state) level, we first apply a positive QE shock. Then, four periods after this initial shock, we introduce a cost-push shock. We add a credit shock eight periods after the initial QE shock. We show the impulse responses under optimal policy from period 5 onwards, when the cost-push shock hits.

tool for responding to credit shocks. In contrast, when it comes to the optimal interest rate path, optimal policy does not prescribe significant movement to it at the time the credit shock hits.

The discussion above leads to the conclusion that the preference for the short-term rate over the balance sheet is in fact shock dependent. Under de-anchored expectations the optimal short-term rate path is slightly dampened relative to the case with a cost-push shock only. Conversely, the central bank should halt balance sheet unwinding early and temporarily overshoot its steady-state level to mitigate the credit shock. Similar patterns hold under RE. Our results indicate that the optimal balance sheet path in response to credit shocks achieves inflation and output gap outcomes nearly identical to those seen without credit shocks, under both types of expectations.

In summary, we draw two noteworthy results. First, the primacy of the policy rate, in terms of its ability to respond to shocks, depends on the nature of the shock. When confronted with cost-push and credit shocks while setting both policy instruments optimally, the central bank's approach is to tailor each tool to respond to a specific shock: the short-term policy rate is best suited to control inflationary pressures arising from a cost-push shock, while the balance sheet is the appropriate tool to address a credit shock. Second, this conclusion holds regardless of the expectations formation mechanism.

5 Conclusion

This paper is motivated by recent rate hikes implemented by major central banks, partly in response to supply shocks, and by the ongoing unwinding of large central bank balance sheets. We study optimal normalization policy in a framework in which agents' expectations can deviate from rational expectations, and the central bank is faced with cost-push shocks. Using an extended version of the four equation New Keynesian model of Sims et al. (2023) that incorporates a resource cost for central bank bond holdings, we examine whether balance sheet adjustments should play a more prominent role when expectations are de-anchored.

Our first contribution is to show analytically that the interest rate remains the most effective tool for managing inflationary pressures arising from cost-push shocks, regardless of the expectations formation process. While de-anchored expectations necessitate a more forceful policy response, the interest rate continues to outperform balance sheet adjustments. Moreover, expectations have only a limited influence on the optimal path of balance sheet normalization in response to such supply shocks.

Optimal policy simulations further reinforce these findings. We show that the balance sheet is less effective than the interest rate in controlling inflationary pressures and leads to larger welfare losses. We also examine scenarios in which balance sheet adjustments become more important. Although the balance sheet plays only a secondary role in addressing cost-push shocks, it gains relevance when interest rate adjustments are constrained and de-anchored expectations would otherwise necessitate strong rate increases. Importantly, we find that the balance sheet is more effective in responding to credit disturbances, regardless of how expectations are formed. This suggests that the optimal choice between policy tools depends on the nature of the shock: the interest rate is better suited for managing inflationary supply shocks, while the balance sheet is more effective in addressing credit shocks.

We acknowledge, however, that our conclusions are drawn from a rather simplified framework and, as such, a richer model featuring additional transmission channels is deemed necessary. A notable limitation is that the model does not capture potential endogenous market functioning issues that could be triggered by abrupt policy shifts. At the same time, this paper represents a first attempt to incorporate state-of-the-art techniques to analyze optimal normalization policy, involving both the short-term interest rate and the balance sheet, in a model with bounded rationality.

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Online Appendix (Not for Publication)

A Extending the Sims et al. (2023) with a quadratic efficiency cost

In this section, we present an extension of the original model by Sims et al. (2023) with a quadratic efficiency cost.[41]

The model makes several simplifying assumptions to reduce to four equations in log-linearized form. The quantitative implications of the four equation model are similar to more complicated models.

The economy is populated by the following agents: two types of households (the *parent* and the *child*, with respective shares of z and $1-z$), a representative financial intermediary, production firms, and a central bank. As detailed in Sims et al. (2023), the dynamics of the child's consumption in the model are in-line with the behavior of investment in Sims and Wu (2021).

A.1 Households

A.1.1 Parent, or patient household

A representative parent maximizes its discounted lifetime utility from consumption, C_t and labor L_t:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} \right] \tag{A.1}$$

with $\sigma > 0$ is the inverse elasticity of intertemporal substitution, $\chi \geq 0$ is the inverse Frisch elasticity, and $\psi > 0$ is a scaling parameter.

The parent's budget constraint is:

$$P_t C_t + S_t \leq W_t L_t + R^s_{t-1} S_{t-1} + P_t D_t + P_t D_t^{FI} + P_t T_t - P_t X_t^b - P_t X_t^{FI} \tag{A.2}$$

in which P_t is the nominal price of consumption. The parent earns income from W_t the

[41] For simplicity, we present the model under rational expectations, as in the original paper by Sims et al. (2023). The inclusion of boundedly rational expectations are discussed in the main text, in Section 2.

nominal wage, D_t dividends from ownership in firms, D_t^{FI} dividends from ownership in financial intermediaries, and from lump-sum transfers, T_t, arising via central bank surplus on asset holdings to be specified below. It can save in nominal short-term bonds S_{t-1} which pay a gross interest rate R_{t-1}^s. Finally, it makes transfers to the child X_t^b, as well as a transfer, X_t^{FI} to financial intermediaries, each period. The first-order conditions for C_t, L_t and S_t are:

$$\psi L_t^\chi = C_t^{-\sigma} w_t \tag{A.3}$$

$$\Lambda_{t-1,t} = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \tag{A.4}$$

$$1 = R_t^s \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \tag{A.5}$$

with $\Lambda_{t-1,t}$ the stochastic discount factor of the parent, $w_t = \frac{W_t}{P_t}$ the real wage and $\Pi_t = \frac{P_t}{P_{t-1}}$ the gross inflation rate.

A.1.2 Child, or impatient household

The child does not supply labor, he thus only gets utility from consumption, $C_{b,t}$:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_b^t \left[\frac{C_{b,t}^{1-\sigma} - 1}{1-\sigma} \right] \tag{A.6}$$

It is less patient than the parent, i.e. $\beta_b < \beta$.

The child can borrow/save using long-term bonds B_t. As in Woodford (2001), long-term bonds are modeled as perpetuities with geometrically decaying coupon payments. Coupon payments decay at rate denoted by $\kappa \in [0, 1]$. Issuing a bond in time t leads to payments of 1, κ, κ^2, . . . in the following periods. Using decaying coupon bonds allows to only keep track of the total outstanding bonds, rather than individual issues. The new issuance of long-term bonds is as follows:

$$NB_t = B_t - \kappa B_{t-1} \tag{A.7}$$

Newly issued bonds trade at market price Q_t, such that the total value of the bond portfolio equals $Q_t B_t$. The gross return on the long bond is defined by:

$$R_t^b = \frac{1 + \kappa Q_t}{Q_{t-1}} \tag{A.8}$$

The child has the following budget constraint:

$$P_t C_{b,t} + B_{t-1} \leq Q_t \left(B_t - \kappa B_{t-1}\right) + P_t X_t^b \tag{A.9}$$

This means that the sum of the nominal value of consumption and coupon payments on outstanding debt cannot exceed the value of new bond issuances plus the nominal value of the transfer from the parent.

First-order conditions for the consumption of the child and the long-term bond give:

$$\Lambda_{b,t-1,t} = \beta_b \left(\frac{C_{b,t}}{C_{b,t-1}}\right)^{-\sigma} \tag{A.10}$$

$$1 = \mathbb{E}_t \Lambda_{b,t,t+1} R_{t+1}^b \Pi_{t+1}^{-1} \tag{A.11}$$

A.2 Financial Intermediaries

Unlike Gertler and Karadi (2011), and Sims and Wu (2021), Sims et al. (2023) assume for simplicity and tractability that the representative financial intermediary (FI) is born each period and exits the industry in the subsequent period. It receives an exogenous amount of net worth from the parent household, $P_t X_t^{FI}$:

$$P_t X_t^{FI} = P_t \bar{X}^{FI} + \kappa Q_t B_{t-1}^{FI} \tag{A.12}$$

It consists of two components: fixed amount of new equity $\bar{X}^{FI}$, and outstanding long-bonds inherited from past intermediaries $\kappa Q_t B_{t-1}^{FI}$.

The FI has the following balance sheet condition:

$$Q_t B_t^{FI} + RE_t^{FI} = S_t^{FI} + P_t X_t^{FI} \tag{A.13}$$

The left-hand side of this equation corresponds to the assets of the FI, i.e. long-term lendings to the child $Q_t B_t^{FI}$ and central bank's reserves RE_t^{FI}. The right-hand side, the liabilities, is comprised of short-term deposits from the parent S_t^{FI} and the transfer $P_t X_t^{FI}$.

The FI pays interest, R_t^s, on short-term debt. It earns interest, R_t^{re}, on reserves, as well as a return on long-term bonds R_{t+1}^b. Upon exiting after period t, the FI's dividend to the parent household are therefore equal to:

$$P_{t+1}D_{t+1}^{FI} = \left(R_{t+1}^{b} - R_t^s\right) Q_t B_t^{FI} + \left(R_t^{re} - R_t^s\right) RE_t^{FI} + R_t^s P_t X_t^{FI} \tag{A.14}$$

The FI maximizes expected dividends, which are discounted by the nominal stochastic discount factor of the parent $\Lambda_{t,t+1}$, subject to a risk-weighted leverage constraint in which long-term bonds receive a risk weight of unity, while reserves on account with the central bank have a risk weight of zero:

$$Q_t B_t^{FI} \leq \Theta_t P_t \bar{X}^{FI} \tag{A.15}$$

This states that the value of the long-term loan to the child cannot be larger than a multiple Θ_t of the value of its equity. Θ_t is viewed as a credit shock, and obeys an AR(1) process.

The first-order conditions of the FI with respect to its choice variables, the quantity of long bonds and reserves are:

$$\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^{b} - R_t^s\right) = \Omega_t \tag{A.16}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left(R_t^{re} - R_t^s\right) = 0 \tag{A.17}$$

with Ω_t the multiplier on the leverage constraint. The binding constraint, i.e. $\Omega_t > 0$, generates excess returns between the expected return on long bonds and the cost of funds.

A.3 Production

The production side of the economy consists of three sectors: final output, retail output, and wholesale output. There is a representative final good firm and representative wholesale producer. There is a continuum of retailers, indexed by $f \in [0, 1]$.

Final output:

Each final output firm faces a downward-sloping demand function given by:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon} Y_t \tag{A.18}$$

with $\epsilon > 1$ the elasticity of substitution.

This gives rise to an aggregate price index:

$$P_t = \left[\int_0^1 P_t(f)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}} \tag{A.19}$$

Retail output:

Retailers purchase wholesale output at price $P_{m,t}$, which can be viewed as nominal marginal costs and repackage it for sale at $P_t(f)$. Retailers are subject to a Calvo (1983) pricing friction. Each period, there is a probability $1 - \phi$ that a retailer adjusts its price, with $\phi \in [0, 1]$.

When a retailer does adjust its prices, it picks a price to maximize the present value of expected profits, discounted by the stochastic discount factor of the parent household. Optimization results in an optimal reset price, $P_{*,t}$, that is common across updating retailers. Denote real marginal costs as $p_{m,t} = \frac{P_{m,t}}{P_t}$, the optimal reset price then satisfies:

$$P_{*,t} = \frac{\epsilon}{\epsilon - 1} \frac{X_{1,t}}{X_{2,t}} \tag{A.20}$$

$$X_{1,t} = P_t^{\epsilon} p_{m,t} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{1,t+1} \tag{A.21}$$

$$X_{2,t} = P_t^{\epsilon-1} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{2,t+1} \tag{A.22}$$

Wholesale output:

The wholesale firm produces output $Y_{m,t}$ using a production function which takes labor as input:

$$Y_{m,t} = A_t L_t \tag{A.23}$$

with A_t an exogenous productivity disturbance following a known stochastic process.

Real wages $w_t = \frac{W_t}{P_t}$ are determined by:

$$w_t = p_{m,t} A_t \tag{A.24}$$

A.4 The central bank

The central bank has two instruments, namely the policy rate R_t^s, and its portfolio of long-term bonds issued by the child B_t^{cb}. The policy rate evolves according to a simple Taylor rule subject to the ZLB:

$$\ln R_t^s = \max\left[0\ ;\ \phi_\pi \left(\ln \Pi_t - \ln \Pi\right) + \phi_x \left(\ln Y_t - \ln Y_t^*\right) + s_r \varepsilon_{r,t}\right] \tag{A.25}$$

The central bank finances its portfolio of long-term bonds through the creation of reserves RE_t. Its balance sheet condition is:

$$Q_t B_t^{cb} = RE_t \tag{A.26}$$

QE is defined as the real value of the central bank's bond portfolio:[42]

$$QE_t = Q_t b_t^{cb} \tag{A.27}$$

where $b_t^{cb} = \frac{B_t^{cb}}{P_t}$.

In their original specification, Sims et al. (2023) assume that QE is costless. We follow Gertler and Karadi (2011), Karadi and Nakov (2021) and Kabaca et al. (2023) and assume that the central bank pays an efficiency cost, Γ_t, whenever it conducts QE or QT. In this case, the central bank earns a surplus on its asset holdings which is transferred lump-sum to the parent household and takes the following form, in real terms:

$$T_t = \frac{R_t^b Q_{t-1}}{\pi_t} b_{t-1}^{cb} - \frac{R_{t-1}}{\pi_t} re_{t-1} - \Gamma_t \tag{A.28}$$

where $re_t = RE_t / P_t$, with

$$\Gamma_t = \frac{\tau}{2}\left(Q_t b_t^{cb}\right)^2 \tag{A.29}$$

As in Kabaca et al. (2023), these costs reflect the notion that the central bank faces several distortions, such as political costs and other implementation constraints (e.g., costs of maintaining a large balance sheet or identifying preferred government sector markets)

[42] In our baseline calibration of the model, the QE policy follows a simple AR(1) process.

when purchasing or selling long-term government bonds. Given its quadratic structure, both QE and QT incur costs to the central bank. We think of QT as active and passive, in the sense that it coincides with active selling of central bank asset holdings and with no re-investments of the proceeds of maturing bonds.

A.5 Aggregate conditions

Market clearing requires the following conditions:

$$RE_t = RE_t^{FI} \tag{A.30}$$

$$S_t = S_t^{FI} \tag{A.31}$$

$$S_t = S_t^{FI} \tag{A.32}$$

$$B_t = B_t^{FI} + B_t^{cb} \tag{A.33}$$

$$Y_t v_t^p = A_t L_t \tag{A.34}$$

A key feature of Sims et al. (2023) is that they assume that the transfer from parent to child, X_t^b, is time-varying, but that neither the parent nor the child behaves as though it can influence the value:

$$P_t X_t^b = (1 + \kappa Q_t)\, B_{t-1} \tag{A.35}$$

This assumption, referred to as the "full-bailout"[43] implies that, even though the child solves a dynamic problem and has a forward-looking Euler equation, its consumption is, in fact, static:

$$P_t C_{b,t} = Q_t B_t \tag{A.36}$$

This assumption by Sims et al. (2023) on the parent-child transfer allows one to eliminate a state variable and simplifies the system to four equations, although it is not crucial for the qualitative or quantitative properties of the model.

In our model, due to the introduction of the QE cost, the aggregate resource constraint of the economy changes. Considering the budget constraint of the parent and plugging in

[43] At each period, the parent pays off the child's debt. Sims et al. (2023) provide evidence that relaxing this assumption does not fundamentally alter the behavior of the model in response to shocks.

dividends from firm financial intermediary ownership as well as the expression for nominal remittances (A.28), we obtain:

$$
\begin{aligned}
P_t C_t = {} & R^S_{t-1} S_{t-1} + P_t Y_t + \left(R^b_t - R^S_{t-1}\right) Q_{t-1} B^{FI}_{t-1} + \left(R^{RE}_{t-1} - R^S_{t-1}\right) RE^{FI}_{t-1} \\
& + R^S_{t-1} P_{t-1} X^{FI}_{t-1} + R^b_t Q_{t-1} B^{CB}_{t-1} - R^{RE}_{t-1} RE_{t-1} - P_t \Gamma_t - \left(1 + \kappa Q_t\right) B_{t-1} \\
& - Q_t B^{FI}_t - RE^{FI}_t
\end{aligned}
\tag{A.37}
$$

Using the definition of the balance sheet of the financial intermediary (A.13), we can substitute out for $P_{t-1} X^{FI}_{t-1}$ in (A.37) above and then simplify to get:

$$
P_t C_t = P_t Y_t + R^b_t Q_{t-1} B_{t-1} - \left(1 + \kappa Q_t\right) B_{t-1} - P_t \Gamma_t - Q_t B^{FI}_t - RE^{FI}_t \tag{A.38}
$$

Accounting for the fact that long-term rates are defined as $R^b_t = \frac{1+\kappa Q_t}{Q_{t-1}}$ and using the bonds market clearing condition (A.33), the balance sheet of the central bank (A.26) and the fact that the full bailout condition (A.36), we arrive at:

$$
C_t + C_{b,t} = Y_t - \Gamma_t \tag{A.39}
$$

Using the definition of for QE_t in (A.27), we may rewrite the efficiency cost expression for Γ_t as follows:

$$
\Gamma_t = \frac{\tau}{2} QE_t^2 \tag{A.40}
$$

This allows us to rewrite the resource constraint (A.39) as follows:

$$
Y_t = C_t + C_{b,t} + \frac{\tau}{2} QE_t^2 \tag{A.41}
$$

A_t and Θ_t obey conventional log AR(1) processes. Potential output $Y_{*,t}$ is defined as the equilibrium level of output consistent with price flexibility (i.e. $\phi = 0$) and where the credit shock is constant, i.e. $\Theta_t = 0$. The natural rate of interest, R^f_t, is the gross real short-term interest rate consistent with this level of output. $X_t = \frac{Y_t}{Y_{*,t}}$ is the output gap.

A.6 The full log-linearized model

We write below all log-linearized conditions of the model, under rational expectations. The majority of these equations are similar to Sims et al. (2023), with the exception of the introduction of the QE cost. Lowercase variables denote log, i.e. percentage change, deviations from steady state. We use a "hat" when the corresponding level variable is already lower-case. Variables without a time subscript denote non-stochastic steady state values. The model is linearized around a steady state with zero trend inflation (i.e. $\Pi = 1$) where the leverage constraint on intermediaries binds.

$$\chi l_t = -\sigma c_t + \widehat{w}_t \tag{A.42}$$

$$\lambda_{t-1,t} = -\sigma \left(c_t - c_{t-1}\right) \tag{A.43}$$

$$0 = \mathbb{E}_t \lambda_{t,t+1} + r_t^s - \mathbb{E}_t \pi_{t+1} \tag{A.44}$$

$$\lambda_{b,t-1,t} = -\sigma \left(c_{b,t} - c_{b,t-1}\right) \tag{A.45}$$

$$r_t^b = \frac{\kappa}{R^b} q_t - q_{t-1} \tag{A.46}$$

$$0 = \mathbb{E}_t \lambda_{b,t,t+1} + \mathbb{E}_t r_{t+1}^b - \mathbb{E}_t \pi_{t+1} \tag{A.47}$$

$$q_t + \widehat{b}_t^{FI} = \theta_t \tag{A.48}$$

$$\left[Qb^{FI}(1-\kappa)\right] q_t + Qb^{FI}\widehat{b}_t^{FI} - \kappa Qb^{FI}\widehat{b}_{t-1}^{FI} + \kappa Qb^{FI}\pi_t + re \cdot \widehat{re}_t = s \cdot \widehat{s}_t \tag{A.49}$$

$$\mathbb{E}_t \lambda_{t,t+1} - \mathbb{E}_t \pi_{t+1} + \frac{R^b}{sp}\mathbb{E}_t r_{t+1}^b - \frac{R^s}{sp} r_t^s = \omega_t \tag{A.50}$$

$$r_t^{re} = r_t^s \tag{A.51}$$

$$\widehat{p}_{*,t} = \widehat{x}_{1,t} - \widehat{x}_{2,t} \tag{A.52}$$

$$\widehat{x}_{1,t} = (1-\phi\beta)\widehat{p}_{m,t} + (1-\phi\beta)y_t + \phi\beta\mathbb{E}_t\lambda_{t,t+1} + \epsilon\phi\beta\mathbb{E}_t\pi_{t+1} + \phi\beta\mathbb{E}_t\widehat{x}_{1,t+1} \tag{A.53}$$

$$\widehat{x}_{2,t} = (1-\phi\beta)y_t + \phi\beta\mathbb{E}_t\lambda_{t,t+1} + (\epsilon-1)\phi\beta\mathbb{E}_t\pi_{t+1} + \phi\beta\mathbb{E}_t\widehat{x}_{2,t+1} \tag{A.54}$$

$$\widehat{w}_t = \widehat{p}_{m,t} + a_t \tag{A.55}$$

$$(1-z)c_t + zc_{b,t} + \left(\frac{\tau QE}{Y}\right) qe_t = y_t \tag{A.56}$$

$$\widehat{v}_t^p + y_t = a_t + l_t \tag{A.57}$$

$$\widehat{v}_t^p = 0 \tag{A.58}$$

$$\pi_t = \frac{1-\phi}{\phi}\widehat{p}_{*,t} \tag{A.59}$$

$$q_t + \widehat{b}_t^{cb} = \widehat{re}_t \tag{A.60}$$

$$\widehat{b}_t = \frac{b^{FI}}{b}\widehat{b}_t^{FI} + \frac{b^{cb}}{b}\widehat{b}_t^{cb} \tag{A.61}$$

$$c_{b,t} = q_t + \widehat{b}_t \tag{A.62}$$

$$a_t = \rho_a a_{t-1} + s_a \varepsilon_{a,t} \tag{A.63}$$

$$\theta_t = \rho_\theta \theta_{t-1} + s_\theta \varepsilon_{\theta,t} \tag{A.64}$$

$$r_t^f = \rho_f r_{t-1}^f + s_f \varepsilon_{f,t} \tag{A.65}$$

$$cp_t = \rho_{cp} cp_{t-1} + s_{cp} \varepsilon_{cp,t} \tag{A.66}$$

$$qe_t = \rho_q qe_{t-1} + s_q \varepsilon_{q,t} \tag{A.67}$$

$$r_t^s = \max\left[0\,;\phi_\pi \pi_t + \phi_x x_t + s_r \varepsilon_{r,t}\right] \tag{A.68}$$

$$qe_t = \widehat{re}_t \tag{A.69}$$

$$x_t = y_t - y_t^* \tag{A.70}$$

A.7 Updated IS and Phillips Curve Equations

In this subsection we reduce the system of log-linearized equations to get to the four equations presented in the main text.

To obtain the *IS curve*, we start by combining first-order conditions on consumption of each type of households and interest rates (A.43)-(A.45) and (A.47) with the aggregate resource constraint (A.56):

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - \mathbb{E}_t \pi_{t+1}\right) - \frac{z}{\sigma}\left(\mathbb{E}_t r_{t+1}^b - \mathbb{E}_t \pi_{t+1}\right) - \left(\frac{\tau QE}{Y}\right)\left[\mathbb{E}_t qe_{t+1} - qe_t\right] \tag{A.71}$$

Combining (A.60)-(A.62) with (A.69) and the binding leverage constraint (A.48) allows us to write the consumption of the child as a function of credit shocks and QE:

$$c_{b,t} = \frac{b^{FI}}{b}\left(q_t + \widehat{b}_t^{FI}\right) + \frac{b^{cb}}{b} qe_t = \bar{b}^{FI}\theta_t + \bar{b}^{cb} qe_t \tag{A.72}$$

with $\bar{b}^{FI} = b^{FI}/b$ and $\bar{b}^{cb} = b^{cb}/b$ the steady state fraction of total bonds held by financial intermediaries and the central bank, respectively.

Using the child's first-order condition we get:

$$\mathbb{E}_t r_{t+1}^b - \mathbb{E}_t \pi_{t+1} = \sigma\left[\mathbb{E}_t c_{b,t+1} - c_{b,t}\right] = \sigma\left[\bar{b}^{FI}\left(\mathbb{E}_t\theta_{t+1} - \theta_t\right) + \bar{b}^{cb}\left(\mathbb{E}_t qe_{t+1} - qe_t\right)\right] \tag{A.73}$$

Combining these two equations we get:

$$\begin{aligned} y_t = {} & \mathbb{E}_t y_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - \mathbb{E}_t\pi_{t+1}\right) - \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\left(\mathbb{E}_t qe_{t+1} - qe_t\right) \\ & - z\bar{b}^{FI}\left(\mathbb{E}_t\theta_{t+1} - \theta_t\right) \end{aligned} \tag{A.74}$$

This equation can also be written as a function of the interest rate spread:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t^s - \mathbb{E}_t\pi_{t+1}\right) - \frac{z}{\sigma}\left(\mathbb{E}_t r_{t+1}^b - r_t^s\right) - \left(\frac{\tau QE}{Y}\right)\left[qe_{t+1} - qe_t\right] \tag{A.75}$$

Define the hypothetical natural rate of output, y_t^*, as the level of output consistent with flexible prices and no credit market shocks. That is, y_t^* is the level of output consistent with $\widehat{p}_{m,t} = \theta_t = 0$, or:

$$y_t^* = \frac{(1+\chi)(1-z)}{\chi(1-z)+\sigma} a_t \tag{A.76}$$

The natural rate of interest, r_t^f, is the real rate consistent with the IS equation holding at the natural rate of output absent credit shocks. It can be expressed as:

$$r_t^f = \frac{\sigma}{1-z}\left(\mathbb{E}_t y_{t+1}^* - y_t^*\right) = \frac{\sigma\left(\rho_A - 1\right)}{1-z} y_t^* \tag{A.77}$$

Using (A.70), this allows to express the IS curve as a function of the output gap:

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - \mathbb{E}_t\pi_{t+1} - r_t^f\right) - \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\left(\mathbb{E}_t qe_{t+1} - qe_t\right) - z\bar{b}^{FI}\left(\mathbb{E}_t\theta_{t+1} - \theta_t\right) \tag{A.78}$$

The *Phillips curve* can be derived as follows. First, we can combine (A.52)-(A.54) such as to have an equation linking prices and marginal costs:

$$\pi_t = \gamma\widehat{p}_{m,t} + \beta\mathbb{E}_t\pi_{t+1} \tag{A.79}$$

with $\gamma = \frac{(1-\phi)(1-\phi\beta)}{\phi}$. Combining (A.42) with (A.55) and (A.57), taking note of the fact that $\widehat{v}_t^p = 0$ around a zero inflation steady state, gives an expression for the marginal costs:

$$\widehat{p}_{m,t} = \chi y_t - (1+\chi)a_t + \sigma c_t \tag{A.80}$$

Using the aggregate resource constraint (A.56) allows us to write this as:

$$\widehat{p}_{m,t} = \frac{\chi(1-z)+\sigma}{1-z}y_t - (1+\chi)a_t - \frac{\sigma z}{1-z}c_{b,t} - \frac{\sigma}{(1-z)}\frac{\tau QE}{Y}qe_t \tag{A.81}$$

Using the definition for c_t^b derived in (A.72):

$$\widehat{p}_{m,t} = \frac{\chi(1-z)+\sigma}{1-z}y_t - (1+\chi)a_t - \frac{\sigma z}{1-z}\left[\bar{b}^{FI}\theta_t + \bar{b}^{cb}qe_t\right] - \frac{\sigma}{(1-z)}\frac{\tau QE}{Y}qe_t \tag{A.82}$$

This can be expressed as a function of output gap, using (A.76):

$$\widehat{p}_{m,t} = \frac{\chi(1-z)+\sigma}{1-z}x_t - \frac{\sigma z}{1-z}\left[\bar{b}^{FI}\theta_t + \bar{b}^{cb}qe_t\right] - \frac{\sigma}{(1-z)}\frac{\tau QE}{Y}qe_t \tag{A.83}$$

Plugging this equation into (A.79), and defining $\zeta = \frac{\chi(1-z)+\sigma}{1-z}$ gives the *Phillips curve* in this four equation model.

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \gamma\zeta x_t - \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)qe_t - \frac{\gamma\sigma z}{1-z}\bar{b}^{FI}\theta_t + cp_t \tag{A.84}$$

To this equation we add an ad-hoc price cost-push shock cp_t.

B Supplementary analytical details

This appendix provides additional details to complement Section 3. We derive first the welfare criterion summarized in Proposition 1. Subsequently, we derive optimal discretionary normalization policy under rational expectations (RE) away from the zero lower bound. We conclude this section by deriving optimal normalization policy under de-anchored expectations.

B.1 Derivation of the welfare criterion

We define aggregate welfare as the sum of the utility functions of the parent and the child:

$$W_t = V_t + V_{b,t} \tag{B.1}$$

We follow the approach of Rotemberg and Woodford (1997) and take a second-order approximation of the utility function of the parent and the child, respectively.

A second-order approximation of the utility function of the child reads as follows:

$$V_{b,t} = V_b + U_{C_b} C_b \left(c_{b,t} + \frac{1}{2}(1 + \frac{U_{C_b C_b} C_b}{U_{C_b}}) c_{b,t}^2 \right) \tag{B.2}$$

A second-order approximation of the utility function of the parent reads as follows:

$$V_t = V + U_C C \left(c_t + \frac{1}{2}(1 + \frac{U_{CC} C}{U_C}) c_t^2 \right) - U_L L \left(l_t + \frac{1}{2}(1 + \frac{U_{LL} L}{U_L}) l_t^2 \right) \tag{B.3}$$

where $U_{C_b} = U_C = C^{-\sigma}$, $U_{C_b C_b} = U_{CC} = -\sigma C^{-\sigma-1}$, $U_L = L^{\chi}$ and $U_{LL} = \chi L^{\chi-1}$. Log-linearizing the aggregate production function, we get that $y_t = a_t - \hat{\upsilon}_t^p + l_t$.[44] Taking instead a second-order Taylor approximation of the production function around the zero steady-state inflation, we receive:

$$l_t = y_t + \frac{Y}{2} y_t^2 + \frac{1}{2}(\hat{\upsilon}_t^p)^2 - \frac{L}{2} l_t^2 + t.i.p. \tag{B.4}$$

where $t.i.p.$ stands for terms irrelevant of policy, such as the productivity shock in this case. Note that in this case $\hat{\upsilon}_t^p = log(\upsilon_t^p) - log(\upsilon^p)$. From ch. 6 of Woodford (2003) we can write,

[44] Note that we are log-linearizing around the zero steady-state inflation, which implies that price dispersion is irrelevant up to first order.

under a Calvo price setting mechanism:

$$y_t^2 = \epsilon var(p_{*,t}) \tag{B.5}$$

and, in turn, the variance of optimal relative prices reads as follows:

$$\sum_{t=0}^{\infty} \beta^t var(log(p_{*,t})) = \frac{1}{1-\phi\beta} \sum_{t=0}^{\infty} \beta^t \left[\frac{\phi}{1-\phi} \pi_t^2 \right] + t.i.p. + O\left(||\xi^3|| \right) \tag{B.6}$$

Substituting (B.4) in (B.3), we receive:

$$\begin{aligned} V_t = V + U_C C \left(c_t + \frac{1}{2}(1-\sigma)c_t^2 \right) - U_L L \left(y_t + \frac{Y}{2} y_t^2 + \frac{1}{2}(\tilde{v}_t^p)^2 - \frac{L}{2} l_t^2 \right) \\ - \frac{1}{2} U_L L (1+\chi) l_t^2 + t.i.p. \end{aligned} \tag{B.7}$$

Note that from equations (B.5) and (B.6), we have:

$$y_t^2 = \frac{\epsilon\phi}{(1-\phi\beta)(1-\phi)} \pi_t^2 + t.i.p. + O\left(||\xi^3|| \right) \tag{B.8}$$

where $O\left(||\xi^3|| \right)$ captures terms of order higher than two. Substituting the above expression in (B.7), we receive:

$$\begin{aligned} V_t = V + U_C C \left(c_t + \frac{1}{2}(1-\sigma)c_t^2 \right) - U_L L \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)} \pi_t^2 \\ - U_L L \left(y_t + \frac{1}{2}(\tilde{v}_t^p)^2 - \frac{L}{2} l_t^2 \right) - \frac{1}{2} U_L L (1+\chi) l_t^2 + t.i.p. + O\left(||\xi^3|| \right) \end{aligned} \tag{B.9}$$

Note that from the first order condition of the parent's problem with respect to labor supply (and after normalizing steady state real wage to one), we have that $\psi U_L = U_C$. We may thus rewrite (B.9) as follows:

$$\begin{aligned} V_t = V + U_C Y \Bigg\{ \left(\frac{C}{Y} c_t + \frac{C}{2Y}(1-\sigma)c_t^2 \right) - \frac{Y\epsilon\phi}{2\psi(1-\phi\beta)(1-\phi)} \pi_t^2 \\ - \frac{1}{\psi} \left(y_t + \frac{1}{2}(\tilde{v}_t^p)^2 - \frac{L}{2} y_t^2 \right) - \frac{1}{2\psi}(1+\chi) y_t^2 \Bigg\} + t.i.p. + O\left(||\xi^3|| \right) \end{aligned} \tag{B.10}$$

where we have used the square of the expression (B.4) to substitute out for l_t^2. We now normalize steady state output to one. Gathering terms and setting $\psi = 1$, we can write:

$$V_t = V + U_C Y \Bigg\{ \left(\frac{C}{Y} c_t + \frac{C}{2Y}(1-\sigma) c_t^2 \right) - \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)} \pi_t^2 - y_t - \frac{1}{2}(\tilde{v}_t^p)^2 - \frac{\chi}{2} y_t^2 \Bigg\} + t.i.p. + O\left(||\xi^3||\right) \tag{B.11}$$

Taking a second order approximation of the resource constraint (A.41), we can write:

$$y_t - \frac{C}{Y} c_t - \frac{C_b}{Y} c_{b,t} = -\frac{Y}{2} y_t^2 + C \frac{C}{2Y} c_t^2 + C_b \frac{C_b}{2Y} c_{b,t}^2 + \tau \frac{b^{cb}}{Y} qe_t + b^{cb} E \frac{\tau b^{cb}}{2Y} qe_t^2 \tag{B.12}$$

Returning now to the welfare of the child, we can rewrite it as follows, using the marginal utility of consumption and the marginal disutility from labor at the steady state:

$$V_{b,t} = V_b + U_{C_b} Y \left(\frac{C_b}{Y} c_{b,t} + \frac{C_b}{2Y}(1-\sigma) c_{b,t}^2 \right) \tag{B.13}$$

Assuming that at the steady state $U_C = U_{C_b}$, and plugging (B.11) and (B.13) in (B.1), we receive:

$$W_t = W - U_C Y \Bigg\{ \frac{C}{Y} \left(\frac{C - 1 + \sigma}{2} \right) c_t^2 + \frac{C_b}{Y} \left(\frac{C_b - 1 + \sigma}{2} \right) c_{b,t}^2 + \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)} \pi_t^2 + \frac{\chi - Y}{2} y_t^2 + \frac{1}{2}(\tilde{v}_t^p)^2 + \tau \frac{Q b^{cb}}{Y} qe_t + \tau Q b^{cb} \frac{Q b^{cb}}{2Y} qe_t^2 \Bigg\} + t.i.p. + O\left(||\xi^3||\right) \tag{B.14}$$

Clearly, by using the calibration of Sims et al. (2023), where $\sigma = \chi = 1$ and accounting for a normalization of output at the steady state, $Y = 1$, we can simplify further to get:

$$W_t = W - U_C Y \Bigg\{ \frac{C}{Y} \left(\frac{C}{2} \right) c_t^2 + \frac{C_b}{Y} \left(\frac{C_b}{2} \right) c_{b,t}^2 + \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)} \pi_t^2 + \frac{1}{2}(\tilde{v}_t^p)^2 + \tau \frac{Q b^{cb}}{Y} qe_t + \tau Q b^{cb} \frac{Q b^{cb}}{2Y} qe_t^2 \Bigg\} + t.i.p. + O\left(||\xi^3||\right) \tag{B.15}$$

Note that the linear term, qe_t, in the loss function above arises because in our extension of the model to allow for quadratic efficiency costs of QE/QT, the steady state is inefficient, since $QE \neq 0$ at the steady state. As shown in Benigno and Woodford (2005), linear terms show up in the welfare criterion. Note also that $C/Y = 1 - z$ and $C_b/Y = z$.

Following the calibration described in Section 2 and taking $\phi = 0.75$ and $\epsilon = 11$ as in Sims et al. (2023), we get the weights in the loss function:

$$\lambda_C = \frac{C}{Y}\left(\frac{C}{2}\right) = 0.67\left(\frac{0.67}{2}\right) = 0.2244$$

$$\lambda_{C_b} = \frac{C_b}{Y}\left(\frac{C_b}{2}\right) = 0.33\left(\frac{0.33}{2}\right) = 0.0545$$

$$\lambda_\pi = \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)} = 65.0246$$

$$\lambda_{qe} = \tau Q b^{cb}\frac{Qb^{cb}}{2Y} = 8e - 04$$

B.2 Optimal Discretionary Normalization Policy under Rational Expectations

Given that the model is purely forward-looking under rational expectations, the central bank minimizes the welfare loss (B.15) subject to the Phillips curve and equation (A.72) that describes the dynamics of the consumption of the *child* conditional of full bailout from the *parent* taking expectations as given:

$$\begin{aligned}
\min_{\pi_t, c_t, c_{b,t}, qe_t} \Bigg[& -U_C Y \left\{ \lambda_C c_t^2 + \lambda_{C_b} c_{b,t}^2 + \lambda_\pi \pi_t^2 + \frac{1}{2}(\tilde{v}_t^p)^2 + \tau \frac{Qb^{cb}}{Y} qe_t + \lambda_{qe} qe_t^2 \right\} \\
& - \xi_\pi \left(\pi_t - \gamma\zeta x_t + \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) qe_t + \frac{\gamma\sigma z}{1-z}\bar{b}^{FI}\theta_t - cp_t \right) \\
& - \xi_{c_b}\left(c_{b,t} - \bar{b}^{FI}\theta_t - \bar{b}^{cb} qe_t \right) \Bigg]
\end{aligned}$$

The first-order conditions read as follows:

$$c_t : \quad -2U_C Y \lambda_C c_t + \gamma\zeta(1-z)\xi_\pi = 0 \tag{B.16}$$

$$c_{b,t} : \quad -2U_C Y \lambda_{C_b} c_{b,t} + \gamma\zeta z \xi_\pi - \xi_{c_b} = 0 \tag{B.17}$$

$$\pi_t : \quad -2U_C Y \lambda_\pi \pi_t - \xi_\pi = 0 \tag{B.18}$$

$$qe_t : \quad -U_C Y \tau \frac{QE}{Y} - 2U_C Y \lambda_{qe} qe_t + \xi_\pi \left(\gamma\zeta\tau\frac{QE}{Y} - \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right) \right) + \xi_{c_b}\bar{b}^{cb} = 0 \tag{B.19}$$

where we have exploited the resource constraint (A.56) to substitute out for x_t in the Phillips curve. When taking thus the first-order derivatives with respect to the central bank balance sheet, qe_t, the effect of the latter on the output gap has been taken into account. This explains the presence of coefficient ζ in the first-order condition with respect to qe_t. This formulation allows us to reduce the number of Lagrange multipliers.

Combining the FOC (B.16) and (B.18) we arrive at the first trade-off that the central bank is facing:

$$c_t = -\gamma\zeta\left(1-z\right)\frac{\lambda_\pi}{\lambda_C}\,\pi_t \tag{B.20}$$

Solving for ξ_{c_b} in (B.17) and plugging the resulting expression together with (B.18) in (B.19) we arrive at the relationship describing the optimal setting of the central bank balance sheet as a function of inflation:

$$qe_t = -\frac{\tau QE}{2\lambda_{qe}Y} - \frac{\lambda_\pi}{\lambda_{qe}}\left[\left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\right]\pi_t - \bar{b}^{cb}\frac{\lambda_{C_b}}{\lambda_{qe}}c_{b,t} \tag{B.21}$$

Plugging expression (A.72) in (B.21) to substitute out for $c_{b,t}$ and gathering terms we arrive at the following expression for the optimal central balance sheet, as reported in the text:

$$qe_t = -\frac{\tau QE}{2\lambda_{qe}Y\Psi} - \frac{\lambda_\pi}{\lambda_{qe}\Psi}\left[\left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\right]\pi_t - \bar{b}^{cb}\frac{\lambda_{C_b}}{\lambda_{qe}\Psi}\bar{b}^{FI}\theta_t \tag{B.22}$$

where $\Psi = 1 + (\bar{b}^{cb})^2 \frac{\lambda_{C_b}}{\lambda_{qe}}$. Plugging (B.20), (A.72) in the resource constraint (A.56) yields:

$$y_t = -\left(1-z\right)^2 \gamma\zeta \frac{\lambda_\pi}{\lambda_C}\pi_t + z\bar{b}^{FI}\theta_t + \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) qe_t \tag{B.23}$$

Inserting the above expression in the Phillips curve to substitute out for the output gap, we obtain:[45]

$$\begin{aligned}\pi_t = \beta\mathbb{E}_t\pi_{t+1} - \left(\gamma\zeta\right)^2 \left(1-z\right)^2 \frac{\lambda_\pi}{\lambda_C}\pi_t + \left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) qe_t \\ + \left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right) z\bar{b}^{FI}\theta_t + cp_t\end{aligned} \tag{B.24}$$

Using (B.22) to substitute for qe_t above, iterating forward and assuming zero persistence in the credit and cost-push shocks (i.e. $\rho_\theta, \rho_{cp} = 0$) we receive the equilibrium inflation under optimal discretion with rational expectations:

$$\pi_t = -\omega^\pi + \omega^\pi_\theta\theta_t + \omega^\pi_{cp}cp_t \tag{B.25}$$

where:

$$\begin{aligned}\omega^\pi &= -\frac{\tau QE}{2\lambda_{qe}Y\Psi\varsigma\left(1-\beta\right)}\varpi \\ \omega^\pi_\theta &= \left[\left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right) z - \bar{b}^{cb}\frac{\lambda_{C_b}\varpi}{\lambda_{qe}\Psi}\right]\frac{\bar{b}^{FI}}{\varsigma} \\ \omega^\pi_{cp} &= \frac{1}{\varsigma} \\ \varpi &= \left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) \\ \varsigma &= 1 + \left(\gamma\zeta(1-z)\right)^2\frac{\lambda_\pi}{\lambda_C} + \frac{\lambda_\pi}{\lambda_{qe}\Psi}\varpi^2\end{aligned}$$

Plugging (B.25) in (B.22), we receive the equilibrium central bank balance sheet under

[45] Given the definition of the output gap in (A.70), the definition of the flexible price level of output in (A.76) and the fact that we abstract from productivity shocks, it follows that $x_t = y_t$.

optimal discretion:

$$qe_t = \omega^{qe} - \omega_\theta^{qe}\theta_t - \omega_{cp}^{qe}cp_t \tag{B.26}$$

where:

$$\omega^{qe} = \frac{\tau QE}{2\lambda_{qe}Y\Psi}\left[\frac{\lambda_\pi}{\lambda_{qe}\Psi}\frac{\varpi^2}{\varsigma\left(1-\beta\right)} - 1\right] \tag{B.27}$$

$$\omega_\theta^{qe} = \frac{\lambda_\pi}{\lambda_{qe}\Psi}\varpi\left[\left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)z - \bar{b}^{cb}\frac{\lambda_{C_b}\varpi}{\lambda_{qe}\Psi}\right]\frac{\bar{b}^{FI}}{\varsigma} + \bar{b}^{cb}\frac{\lambda_{C_b}}{\lambda_{qe}\Psi}\bar{b}^{FI} \tag{B.28}$$

$$\omega_{cp}^{qe} = \frac{\lambda_\pi}{\lambda_{qe}\Psi}\frac{\varpi}{\varsigma} \tag{B.29}$$

Finally, combining the FOC (B.20) with (A.72), (B.25), (B.26) and the resource constraint (A.56) we arrive at the equilibrium output gap under optimal discretionary policy:

$$x_t = \omega^x - \omega_\theta^x\theta_t - \omega_{cp}^x cp_t \tag{B.30}$$

where:

$$\omega^x = -\left(1-z\right)^2\gamma\zeta\frac{\lambda_\pi}{\lambda_C}\omega^\pi + \left(z\bar{b}^c b + \frac{\tau QE}{Y}\right)\omega^{qe} \tag{B.31}$$

$$\omega_\theta^x = \left[\left(1-z\right)^2\gamma\zeta\frac{\lambda_\pi}{\lambda_C}\omega_\theta^\pi + \left(z\bar{b}^c b + \frac{\tau QE}{Y}\right)\omega_\theta^{qe}\right]z\bar{b}^{FI} \tag{B.32}$$

$$\omega_{cp}^x = \left(1-z\right)^2\gamma\zeta\frac{\lambda_\pi}{\lambda_C}\omega_{cp}^\pi + \left(z\bar{b}^c b + \frac{\tau QE}{Y}\right)\omega_{cp}^{qe} \tag{B.33}$$

Plugging the resource constraint in the IS equation and using (B.20) to (A.72) to substitute for c_t and $c_{b,t}$, and considering the case of zero shock persistence ($\rho_\theta, \rho_{cp} = 0$), we arrive at the optimal interest rate rule that is reported in equation (22) in the main text:

$$r_t^s = r_t^f + \gamma\sigma\zeta\left(1-z\right)\frac{\lambda_\pi}{\lambda_C}\pi_t \tag{B.34}$$

To derive the equilibrium interest rate consistent with optimal discretionary policy, one has to only plug in equilibrium inflation (B.25) in the optimal interest rate rule above, so that the interest rate can be expressed as a function of the cost-push and the credit shocks.

B.3 Optimal Normalization Policy under De-Anchored Expectations

Before formulating the minimization problem of the central bank in the presence of de-anchored expectations it is useful to work on inflation expectations. Exploiting the law of motion of de-anchored expectations (9), and iterating backwards we may write the law of motion of inflation expectations as follows:

$$\omega_t^{\pi} \approx \bar{g} \sum_{s=0}^{\infty} (1-\bar{g})^s \, \pi_{t-s-1} \tag{B.35}$$

The Phillips curve under de-anchored expectations thus reads as follows:

$$\pi_t = \beta\omega_t^{\pi} + \gamma\zeta x_t - \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) qe_t - \frac{\gamma\sigma z}{1-z}\bar{b}^{FI}\theta_t + cp_t \tag{B.36}$$

where we can plug in expression (B.35) for inflation expectations. The minimization problem of the central bank under de-anchored expectations reads as follows:[46]

$$\begin{aligned}
\min_{\pi_t, c_t, c_{b,t}, qe_t} \mathbb{E}_t \sum_{t=1}^{\infty} \Bigg[& -U_C Y \left\{ \lambda_C c_t^2 + \lambda_{C_b} c_{b,t}^2 + \lambda_{\pi}\pi_t^2 + \frac{1}{2}(\tilde{v}_t^p)^2 + \tau\frac{Qb^{cb}}{Y} qe_t + \lambda_{qe} qe_t^2 \right\} \\
& - \xi_{\pi,t}\left(\pi_t - \beta\left(\bar{g}\sum_{s=0}^{\infty}(1-\bar{g})^s \pi_{t-s-1} \right) - \gamma\zeta x_t + \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right) qe_t + \frac{\gamma\sigma z}{1-z}\bar{b}^{FI}\theta_t - cp_t \right) \\
& - \xi_{c_b,t}\left(c_{b,t} - \bar{b}^{FI}\theta_t - \bar{b}^{cb} qe_t \right) \Bigg]
\end{aligned}$$

[46]Note that the expectation operator in the minimization problem of the central bank corresponds to the expectations of the central bank itself. As mentioned in the main body of the text, we have assumed that the central bank is rational and has full knowledge of the actual law of motion of the economy and the way agents form their expectations. As a result, the expectations of the central bank are model-consistent.

where $\xi_{\pi,t}$ and $\xi_{c_b,t}$ are Lagrange multipliers. The first-order conditions read as follows:

$$c_t: \quad -2U_C Y \lambda_C c_t + \gamma\zeta(1-z)\xi_{\pi,t} = 0 \tag{B.37}$$

$$c_{b,t}: \quad -2U_C Y \lambda_{C_b} c_{b,t} + \gamma\zeta z \xi_{\pi,t} - \xi_{c_b,t} = 0 \tag{B.38}$$

$$\pi_t: \quad -2U_C Y \lambda_\pi \pi_t - \xi_\pi + \bar{g}\beta \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{s+1} (1-\bar{g})^s \xi_{\pi,t+s+1} = 0 \tag{B.39}$$

$$qe_t: \quad -U_C Y \tau \frac{QE}{Y} - 2U_C Y \lambda_{qe} qe_t + \xi_{\pi,t} \left(\gamma\zeta\tau \frac{QE}{Y} - \frac{\gamma\sigma}{1-z} \left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right) \right) + \xi_{c_b,t} \bar{b}^{cb} = 0 \tag{B.40}$$

where solving for $\xi_{\pi,t}$ in (B.37), and plugging the resulting expression in (B.39), we obtain:

$$\pi_t = -\frac{\lambda_C}{\lambda_\pi \gamma\zeta (1-z)} c_t + \frac{\beta\lambda_C \bar{g}}{\lambda_\pi \gamma\zeta (1-z)} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{s+1} (1-\bar{g})^s c_{t+s+1} \tag{B.41}$$

which is the intertemporal trade-off reported in the text. Plugging the same expression for $\xi_{\pi,t}$ in (B.40) and substituting out $\xi_{C_b,t}$ using (B.39), we receive:

$$-\frac{\tau QE}{2Y} - \lambda_{qe} qe_t + \frac{\lambda_C}{\gamma\zeta(1-z)} \left(\gamma\zeta - \frac{\gamma\sigma}{1-z} \right) \left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right) c_t - \lambda_{C_b} \bar{b}^{cb} c_{b,t} = 0 \tag{B.42}$$

Solving in (B.41) for C_t and plugging the resulting expression in (B.42) we get:

$$\begin{aligned} qe_t = &-\frac{\tau QE}{\lambda_{qe} Y \Psi} - \frac{\lambda_\pi}{\lambda_{qe}\Psi} \left(\gamma\zeta - \frac{\gamma\sigma}{1-z} \right) \left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right) \pi_t - \frac{\lambda_{C_b} \bar{b}^{cb} \bar{b}^{FI}}{\lambda_{qe}\Psi} \theta_t \\ &+ \frac{\lambda_C}{\lambda_{qe}\Psi\gamma\zeta(1-z)} \left(\gamma\zeta - \frac{\gamma\sigma}{1-z} \right) \left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right) \beta\bar{g} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{s+1} (1-\bar{g})^s c_{t+s+1} \end{aligned} \tag{B.43}$$

Plugging the intertermporal trade-off in (B.41) and (B.43) in the Phillips curve under de-anchored expectations (B.36) we arrive at:

$$\pi_t = \frac{\beta\bar{g}}{\varsigma}\sum_{s=0}^{\infty}(1-\bar{g})^s\pi_{t-s-1} + \omega^{\pi}(1-\beta) + \omega^{\pi}_{\theta}\theta_t + \omega^{\pi}_{cp}cp_t + \tilde{\delta}\frac{\beta\bar{g}}{\varsigma}\mathbb{E}_t\sum_{s=0}^{\infty}\beta^{s+1}(1-\bar{g})^s c_{t+s+1} \tag{B.44}$$

where

$$\tilde{\delta} = \left[\gamma\zeta(1-z) + \left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)^2\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)^2\frac{\lambda_C}{\lambda_{qe}\Psi\gamma\zeta(1-z)}\right] \tag{B.45}$$

In the expressions above, parameters ω^{π}, ω^{π}_{θ} and ς have been defined below equation (B.25) above. Expanding the last sum in (B.44) and plugging the intertemporal trade-off (B.41) repeatedly for current and future consumptions we arrive at:

$$\begin{aligned}&\mathbb{E}_t\sum_{s=0}^{\infty}\beta^{s+1}(1-\bar{g})^s c_{t+s+1}\\ &\quad= -\beta\bar{g}\frac{\lambda_{\pi}\gamma\zeta(1-z)}{\lambda_C}\left[\tilde{P}\pi_t + \sum_{s=0}^{\infty}\left((1-g)^s + \prod_{j=0}^{s}\left(1+\beta^{j+2}\bar{g}\right)\beta^{j+1}\left(1-\bar{g}\right)^{j+1}\right)\pi_{t-s-1}\right]\end{aligned} \tag{B.46}$$

where

$$\tilde{P} = \prod_{s=0}^{\infty}\left(1+\beta^{s+2}\bar{g}\right)\beta^{s+1}\left(1-\bar{g}\right)^{s+1} \tag{B.47}$$

Notice that in the above equation setting $\bar{g}=0$ (i.e. no learning), leads the term on the left hand to equal zero. Plugging (B.46) in (B.44), gathering terms and iterating backwards we arrive at the equilibrium inflation under optimal policy with de-anchored expectations:

$$\pi_t = \frac{\omega^{\pi}(1-\beta)}{\tilde{\varrho}} + \omega^{\pi}_{\theta}\sum_{s=0}^{\infty}\left(\frac{\beta\bar{g}\iota}{\varsigma\tilde{\varrho}}\right)^s\theta_{t-s} + \omega^{\pi}_{cp}\sum_{s=0}^{\infty}\left(\frac{\beta\bar{g}\iota}{\varsigma\tilde{\varrho}}\right)^s cp_{t-s} \tag{B.48}$$

where

$$\tilde{\varrho} = 1 + \tilde{\delta}\frac{(\beta\bar{g})^2}{\varsigma}\frac{\lambda_{\pi}\gamma\zeta(1-z)}{\lambda_C}\tilde{P} \tag{B.49}$$

and

$$\iota = \sum_{s=0}^{\infty}\left[(1-\bar{g})^s - \tilde{\delta}\beta\bar{g}\frac{\lambda_\pi\gamma\zeta(1-z)}{\lambda_C}\left((1-g)^s + \prod_{j=0}^{s}\left(1+\beta^{j+2}\bar{g}\right)\beta^{j+1}\left(1-\bar{g}\right)^{j+1}\right)\right] \quad \text{(B.50)}$$

B.4 Proof of Proposition 2

The log-linearized resource constraint reads as follows:

$$y_t = (1-z)c_t + zc_{b,t} + \left(\frac{\tau QE}{Y}\right)qe_t$$

Plugging the intertemporal trade-off (B.41) in the resource constraint and using the fact that $c_{b,t} = \bar{b}^{FI}\theta_t + \bar{b}^{cb}qe_t$:

$$y_t = -\frac{(1-z)^2\lambda_\pi\gamma\zeta}{\lambda_C}\pi_t + z\bar{b}^{FI}\theta_t + \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)qe_t + (1-z)\beta\bar{g}\mathbb{E}_t\sum_{s=0}^{\infty}\beta^{s+1}\left(1-\bar{g}\right)^s c_{t+s+1} \tag{B.51}$$

Recall that in the absence of productivity shocks $y_t = x_t$ as explained in footnote 37. Plugging expression (B.43) for the optimal qe_t in the expression above and gathering terms, we obtain:

$$\begin{aligned} x_t = &-\frac{\tau QE}{2\lambda_{qe}Y\Psi}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) - \lambda_\pi\left[\frac{(1-z)^2\gamma\zeta}{\lambda_C} + \frac{\gamma\zeta - \frac{\gamma\sigma}{1-z}}{\lambda_{qe}\Psi}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)^2\right]\pi_t \\ &+ \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\frac{\lambda_{C_b}\bar{b}^{cb}\bar{b}^{FI}}{\lambda_{qe}\Psi}\theta_t \\ &+ \beta\bar{g}\left[(1-z) + \frac{\lambda_C}{\lambda_{qe}\Psi\gamma\zeta(1-z)}\left(\gamma\zeta - \frac{\gamma\sigma}{1-z}\right)\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)^2\right]\mathbb{E}_t\sum_{s=0}^{\infty}\beta^{s+1}\left(1-\bar{g}\right)^s c_{t+s+1} \end{aligned} \tag{B.52}$$

Using the expression in (B.46) to substitute out for for $\mathbb{E}_t\sum_{s=0}^{\infty}\beta^{s+1}\left(1-\bar{g}\right)^s c_{t+s+1}$ and gathering terms, we receive:

$$
\begin{aligned}
x_t = &- \left(z\bar{b}^{cb} + \frac{\tau QE}{y} \right) \frac{\tau QE}{2\lambda_{qe} Y \Psi} - \left(\Omega^{RE} + \Omega^{DE} \tilde{P} \right) \pi_t \\
&- \left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right) \frac{\lambda_{C_b} \bar{b}^{cb} \bar{b}^{FI}}{\lambda_{qe} \Psi} \theta_t \\
&- \Omega^{DE} \sum_{s=0}^{\infty} \left((1-g)^s + \prod_{j=0}^{s} \left(1 + \beta^{j+2} \bar{g}\right) \beta^{j+1} \left(1 - \bar{g}\right)^{j+1} \right) \pi_{t-s-1}
\end{aligned}
\tag{B.53}
$$

where:

$$
\begin{aligned}
\Omega^{RE} &= \lambda_\pi \left[\frac{(1-z)^2 \gamma\zeta}{\lambda_C} + \frac{\gamma\zeta - \frac{\gamma\sigma}{1-z}}{\lambda_{qe} \Psi} \left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right)^2 \right] \\
\Omega^{DE} &= \beta \bar{g} \frac{\lambda_\pi \gamma\zeta (1-z)}{\lambda_C} \left[(1-z) + \frac{\lambda_C}{\lambda_{qe} \Psi \gamma\zeta (1-z)} \left(\gamma\zeta - \frac{\gamma\sigma}{1-z} \right) \left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right)^2 \right]
\end{aligned}
$$

The coefficient Ω^{RE} coincides with the coefficient on inflation in the inflation-output gap trade-off under RE as displayed in equation (23) in the main text. The coefficient Ω^{DE} is the additional trade-off arising due to de-anchored expectations. As we have argued in the main text $\tilde{P} > 0$ and given the definition of $\zeta = \frac{\chi(1-z)+\sigma}{1-z}$, it follows that $\gamma\zeta - \frac{\gamma\sigma}{1-z} > 0$. Consequently, $\Omega^{DE} > 0$. This then implies that the aggregate coefficient on inflation in the inflation-output gap trade-off under de-anchored expectations is higher (in absolute terms) than its counterpart under RE. As a result, this means that in the face of a given rise in inflation from a cost-push shock, the central bank must respond by lowering the output gap more when expectations are de-anchored than when they are rational. In other words, de-anchored expectations lead to a heavier inflation-output gap trade-off. Note that under no learning (i.e. $\bar{g} = 0$) the trade-off under de-anchored expectations collapses to that under RE, as portrayed in equation (23) in the main text.

C Supplementary details on the optimal policy simulations

This appendix presents additional material for our quantitative optimal policy analysis described in Section 4. In Appendix C.1, we provide additional details on the solution method we used to construct the optimal short-term interest rate and balance sheet trajectories.

Appendix C.2 explores how the gain parameter in the forecasting process of de-anchored agents influences our findings. Appendix C.3 shows the outcome of implementing only the balance sheet in an optimal way, while the short-term interest rate follows the Taylor rule described in (6). Finally, C.4 presents the optimal path of inflation, the output gap and the two policy instruments in a scenario with agents having heterogeneous expectations.

C.1 Computational details

When solving for optimal policy, we use a method similar to de Groot et al. (2021), Hebden and Winkler (2021) and McKay and Wolf (2022) and construct sequence-space linear-quadratic policy problems. Doing so, we use impulse response functions to monetary policy and balance sheet shocks from the model specified in Section 2, and a baseline (which is here the response to a cost-push shock in the same model as the one used for the IRFs). Similar solution methods can be found in McKay and Wolf (2023) and Barnichon and Mesters (2023) using empirical models.

As regularly discussed in the literature (e.g. Fernández-Villaverde et al., 2016 and Auclert et al., 2021), by certainty equivalence, the first-order perturbation solution of models with aggregate risk is identical to the solution of the model in linearized perfect-foresight. Under perfect-foresight, each variable can be written in the sequence space as:

$$Z = Z^B + \mathcal{A}^{z,\varepsilon_p}\,\varepsilon_p = \mathcal{A}^{Z,\varepsilon_s}\varepsilon_s + \mathcal{A}^{Z,\varepsilon_p}\varepsilon_p \tag{C.1}$$

In this equation, $Z^B = \left\{Z_t^B\right\}_{t=0}^{H} = \left\{Z_0^B, ..., Z_H^B\right\}$ is the baseline path for each variable Z in the model, over all periods within a defined projection horizon H. This consists of the impulse responses $\mathcal{A}^{Z,\varepsilon_s}$ of a specific variable to a set of structural shocks ε_s, under the prevailing baseline policy rules. In our simulations, the baseline path for each variable will correspond to the transition path of this variable in response to a cost-push shock (and a credit shock, if applicable) either in the rational expectation or the de-anchored expectation model, under the Taylor rule and AR(1) QE/QT process defined in Section 2. The object $\mathcal{A}^{Z,\varepsilon_p}$ collects the impulse responses of each variable Z under the baseline policy rules, to a set of contemporaneous and expected (i.e. news) policy shocks ε_p.[47] We obtain these

[47] We expose the solution for one policy instrument, but it is straightforward to extend it to allow for multiple instruments.

matrices of impulse responses to policy shocks using the model under rational expectations, and under de-anchored expectations.[48]

In this Appendix, we focus on the case of commitment and rational expectations. However, the process for solving optimal policy under de-anchored expectations is analogous; it simply requires substituting the set of impulse responses with those obtained from the model featuring de-anchored agents.[49] For scenarios with optimal policy under discretion and rational expectations, we follow the recursive algorithm of de Groot et al. (2021).

Under commitment and rational expectations, identifying the optimal trajectory of the desired policy instrument requires solving for a sequence of policy deviations that satisfy the first-order conditions of the optimal policy problem, as defined in a sequence space representation:[50]

$$\begin{aligned}
&\min_{\{\Pi, C, C_b, QE, \varepsilon_p\}} \mathrm{E}_0 \Big[\frac{1}{2} \left(\Pi' \Lambda_\Pi \Pi + C' \Lambda_C C + C_b' \Lambda_{C_b} C_b + QE' \Lambda_{QE} QE \right) \\
&+ \Xi^{\Pi'} \left(-\Pi + \Pi^B + \mathcal{A}^{\Pi, \varepsilon_p} \varepsilon_p \right) + \Xi^{C'} \left(-C + C^B + \mathcal{A}^{C, \varepsilon_p} \varepsilon_p \right) \\
&+ \Xi^{C_b'} \left(-C_b + C_b^B + \mathcal{A}^{C_b, \varepsilon_p} \varepsilon_p \right) + \Xi^{QE'} \left(-QE + QE^B + \mathcal{A}^{QE} \varepsilon_p \right) \Big]
\end{aligned} \tag{C.2}$$

with $\Pi = \{\pi_t\}_{t=0}^H = \{\pi_0, ..., \pi_H\}$, $C = \{c_t\}_{t=0}^H = \{c_0, ..., c_H\}$, $C_b = \{c_{b,t}\}_{t=0}^H = \{c_{b,0}, ..., c_{b,H}\}$ and $QE = \{qe_t\}_{t=0}^H = \{qe_0, ..., qe_H\}$ the perfect-foresight sequence of inflation, the consumption of the *parent*, the consumption of the *child* and the path of the balance sheet. Additionally, $\Xi^i \equiv \mathrm{diag}\left(1, \beta, \ldots, \beta^T\right) \otimes \xi$, $i = \{\Pi, C, C_b, QE\}$, are the Lagrange multipliers for both constraints with discount factor $\beta \in (0,1)$, and $\Lambda_i \equiv \mathrm{diag}\left(1, \beta, \ldots, \beta^T\right) \otimes \lambda_j$, $j = \{\pi, x\}$, are the weights associated with each term in the loss function.

The first-order conditions are:

$$\Lambda_\Pi \, \Pi = \Xi^\Pi \tag{C.3}$$

$$\Lambda_C \, C = \Xi^C \tag{C.4}$$

[48]de Groot et al. (2021) show how to obtain these matrices from the state-space representation of the model.

[49]In the case with de-anchored expectations, agents are entirely backward-looking and are therefore influenced solely by shocks from the point at which they occur, without any anticipation. This contrasts with the case of rational expectations, in which agents react to anticipated future events. Consequently, in the case of de-anchored expectations, impulse response matrices are lower triangular.

[50]We omit the linear qe_t term from the loss function in our analysis, considering its impact to be marginal. The inclusion of this term would introduce only a minor adjustment (a "shifter") to the optimal policy rule, which would be insignificant for the scope of our results.

$$\Lambda_{C_b}\ C_b = \Xi^{C_b} \tag{C.5}$$

$$\Lambda_{QE}\ QE = \Xi^{QE} \tag{C.6}$$

$$\mathcal{A}^{\Pi,\varepsilon_p\prime}\ \Xi^{\Pi} + \mathcal{A}^{C,\varepsilon_p\prime}\ \Xi^{C} + \mathcal{A}^{C_b,\varepsilon_p\prime}\ \Xi^{C_b} + \mathcal{A}^{QE,\varepsilon_p\prime}\ \Xi^{QE} = 0 \tag{C.7}$$

Combining them gives:

$$\mathcal{A}^{\Pi,\varepsilon_p,\prime}\ (\Lambda_{\Pi}\ \Pi) + \mathcal{A}^{C,\varepsilon_p,\prime}\ (\Lambda_C\ C) + \mathcal{A}^{C_b,\varepsilon_p,\prime}\ (\Lambda_{C_b}\ C_b) + \mathcal{A}^{QE,\varepsilon_p,\prime}\ (\Lambda_{QE}\ QE) = 0 \tag{C.8}$$

Relationship with the targeting criteria In line with McKay and Wolf (2022), equation (C.8) is the forecast targeting criterion consistent with our welfare criterion as a function of impulse responses to monetary policy shocks, under RE. It summarizes the trade-off faced by a policymaker, independently of the non-policy shocks hitting the economy. It expresses how a policymaker can set available policy instruments to align the projected path of key macroeconomic variables — such as inflation and the output gap — with the central bank's objectives.

Additionally, mapping the matrices of impulse responses of two of the central bank's targets, e.g. those of inflation $\mathcal{A}^{\Pi,\varepsilon_p}$ and of the consumption of the *parent* $\mathcal{A}^{C,\varepsilon_p}$, provides the dynamic relationship over time (i.e. trade-off) between these two targets for an optimal policy under commitment.

Finding the optimal sequences of policy shocks To find the set of policy shocks that implement the optimal trajectory for a given policy instrument, we substitute for the law of motion of all endogenous variables in the loss function:

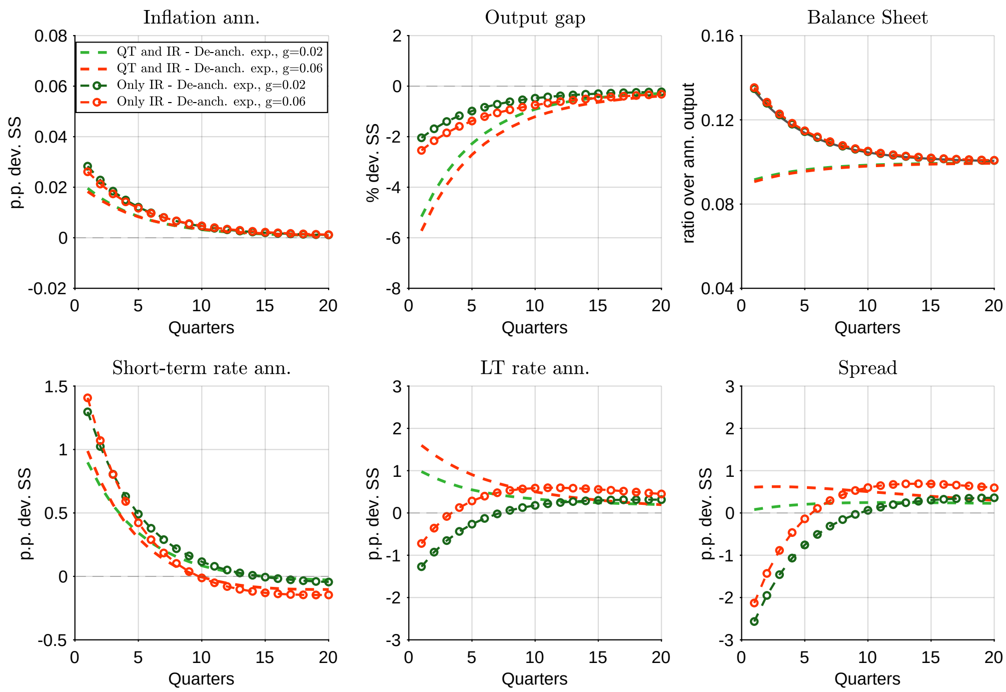
$$\begin{aligned}
&\mathcal{A}^{\Pi,\varepsilon_p\prime}\left(\Lambda_{\Pi}\ \left(\Pi^B + \mathcal{A}^{\Pi,\varepsilon_p}\varepsilon_p\right)\right) + \mathcal{A}^{C,\varepsilon_p\prime}\left(\Lambda_C\ \left(C^B + \mathcal{A}^{C,\varepsilon_p}\varepsilon_p\right)\right) + \\
&\mathcal{A}^{C_b,\varepsilon_p\prime}\left(\Lambda_{C_b}\left(C_b^B + \mathcal{A}^{C_b,\varepsilon_p}\varepsilon_p\right)\right) + \mathcal{A}^{QE,\varepsilon_p\prime}\left(\Lambda_{QE}\left(QE^B + \mathcal{A}^{QE,\varepsilon_p}\varepsilon_p\right)\right) = 0
\end{aligned}$$

We can then solve for the optimal sequence of policy shocks $\tilde{\varepsilon_p}$. For simplicity, stacking all loss function variables into Z_l and all weights into Λ gives:

$$\tilde{\varepsilon_p} = -\left(\mathcal{A}^{Z_l,\varepsilon_p\prime}\Lambda\mathcal{A}^{Z_l,\varepsilon_p}\right)^{-1}\left(\mathcal{A}^{Z_l,\varepsilon_p\prime}\Lambda\ Z_l^B\right) \tag{C.9}$$

Optimal deviations $\tilde{\varepsilon_p}$ to the baseline path of the policy instrument are set to offset as well as possible (in a weighted least-squares sense) the deviations of the policy targets incurred

Figure 5: Optimal monetary policy, in response to a cost-push shock, with different constant gains



Notes: Optimizing the balance sheet and the short-term interest rate, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The green dashed lines are optimal policy under fully de-anchored expectations for $\bar{g} = 0.02$, while red dashed lines are optimal policy under fully de-anchored expectations for $\bar{g} = 0.06$. The solid black line shows the baseline QT trajectory where QT follows an AR(1) process. The circle-marked lines are the cases in which only the interest rate is used to minimize the loss function. The lines without markers are the cases in which both instruments are used in an optimal way.

by the exogenous shocks.

C.2 Role of the gain

In our model, unlike in the case of the endogenous gain approach used in Gáti (2023) or Carvalho et al. (2023), we calibrate the constant gain parameter which enters the forecasting rule of de-anchored agents to a fixed value. But what would be the impacts of a different gain? Figure 5 presents the outcomes for different gain calibrations. We compare our baseline calibration with a gain of $\bar{g} = 0.02$ (green dashed lines), to a higher gain of $\bar{g} = 0.06$ (red dashed lines), which is the estimated value in the sample pre-1999 in Eusepi et al. (2020).

As the gain parameter increases, reflecting a greater influence of recent forecast errors on agents with de-anchored expectations, macroeconomic fluctuations become more pronounced. Inflation and the output gap deviate further from their targets, prompting a stronger response from the central bank.

More specifically, adjustments to the policy rate and balance sheet become more pronounced. However, the change in the optimal trajectory of either the interest rate alone or both instruments remains relatively small. The long-term interest rate and the spread also reflect this dynamic, adjusting in line with the degree of de-anchoring of expectations. Overall, the central bank calibrates its policy instruments slightly more aggressively to mitigate the destabilizing effects of higher gain values.

C.3 Setting optimally only the balance sheet

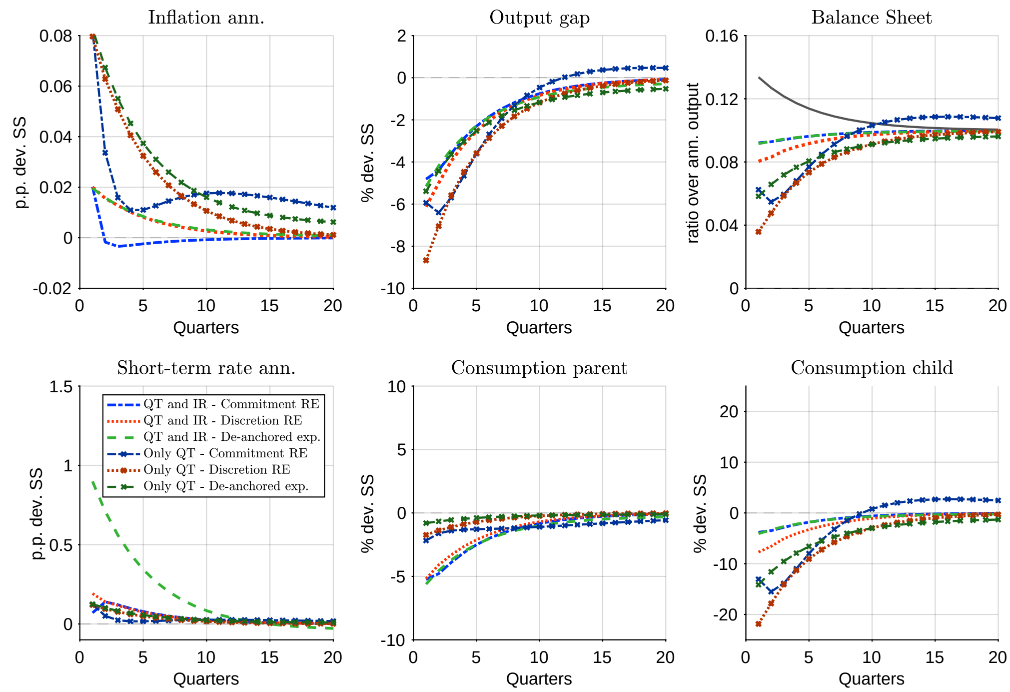
In this appendix, we present the outcome of a scenario in which the central bank optimally conducts its balance sheet policy in response to a persistent cost-push shock, for a given interest rate rule described in equation (6).

Figure 6 presents the optimal balance sheet policy for commitment and discretion under RE, and de-anchored expectations. The cross-marked colored lines are the cases in which only the balance sheet is set in an optimal way, while the lines without markers represent scenarios where both instruments are set optimally.

The dash-dotted blue lines depict the case of commitment and RE, the dotted red lines the case of discretion and RE. The analysis reveals that, in response to a cost-push shock, the central bank is required to significantly contract the size of its balance sheet when QT is the sole tool that is set optimally. As a consequence, this reduction in the balance sheet size dampens inflation; however, it comes at the expense of considerable output reductions. Moreover, inflation now jumps more on impact.

Furthermore, the trajectories under commitment with RE and de-anchored expectations exhibit important differences. In the RE framework, the central bank can partially offset the initial fall in the output gap by sustaining a balance sheet above the steady state (overshooting) in the medium-run. Therefore, it can commit to keeping a future positive output gap as inflation decreases, provided that past reductions have been sufficient. In the case of de-anchored expectations, such commitment is not feasible, thus necessitating a consistently lower balance sheet, with no subsequent elevation above the steady state. In fact,

Figure 6: Optimal monetary policy with both instruments and optimal QT, in response to a cost-push shock



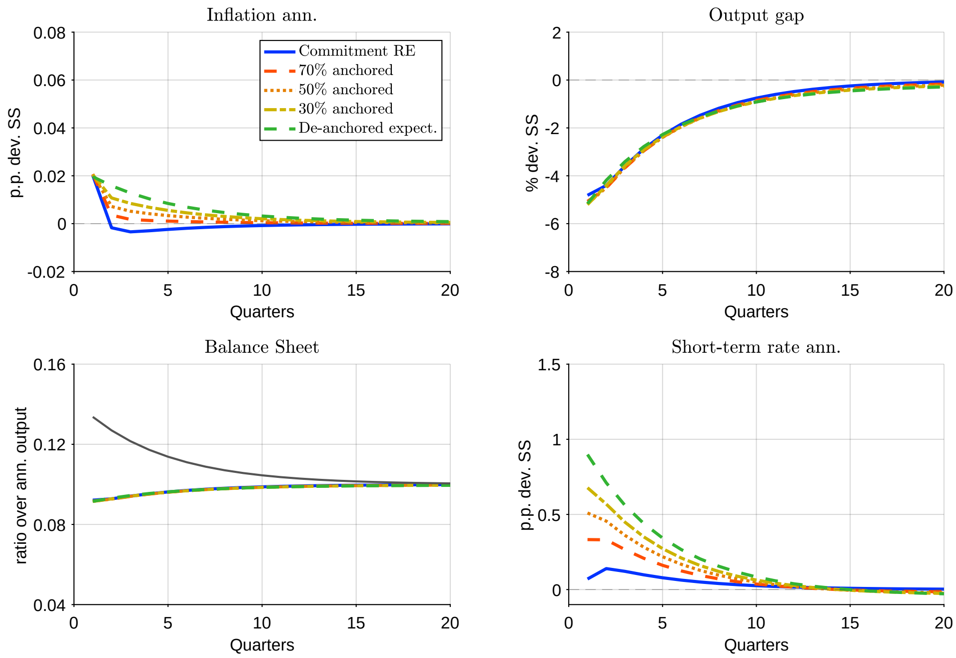
Notes: Optimizing QT for a given short-term interest rate rule, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The dash-dotted blue lines depict optimal policy under commitment with fully RE, while the red dotted lines show the case of discretion with fully RE. The green dashed lines are optimal policy under fully de-anchored expectations. The solid black line shows the baseline QT trajectory where QT follows an AR(1) process. The cross-marked lines are the cases in which only the balance sheet is used to minimize the loss function (Only QT). The lines without markers are the cases in which both instruments are used in an optimal way (QT and IR). To implement a transition from a high balance sheet level to a lower (i.e. steady state) level, we first apply a positive QE shock. Then, four periods after this initial shock, we introduce a cost-push shock. We show the impulse responses under optimal policy from this period onwards, when the cost-push shock hits.

under de-anchored expectations the central bank balance sheet remains below the steady state throughout, perpetuating a lower output gap.

C.4 The case of heterogeneous expectations

In Section 4, we detailed optimal monetary policy trajectories under our two extreme expectation scenarios (RE and de-anchored expectations). In this appendix, we examine how the central bank should implement its normalization strategy, for different shares of agents having anchored or de-anchored expectations. In this case, the shares of each type of ex-

Figure 7: Optimal monetary policy, with heterogeneous expectations



Notes: Optimizing QT and the interest rate, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The solid blue lines depict optimal policy under commitment with fully RE. The green dashed lines are optimal policy under fully de-anchored expectations. The remaining lines are different levels for the fraction of anchored (i.e., RE) agents. The solid black line shows the baseline QT trajectory where QT follows an AR(1) process. To implement a transition from a high balance sheet level to a lower (i.e. steady state) level, we first apply a positive QE shock. Then, four periods after this initial shock, we introduce a cost-push shock. We show the impulse responses under optimal policy from this period onwards, when the cost-push shock hits.

pectations formation are uniform across the two types of households, the *parent* and the *child*.

Figure 7 reports the results when the central bank uses both instruments, the short-term policy rate and the balance sheet, in response to a cost-push shock. The different trajectories depicted reflect varying proportions of agents having anchored expectations, each signifying a distinct level of confidence in the central bank's commitment. This figure confirms that while expectations anchoring significantly affects the interest rate path, the QT policy path remains consistent across different expectation profiles. This invariance indicates that the central bank's QT strategy is robust to variations in the degree of expectations anchoring. In essence, whether a larger fraction of agents are fully anchored or not, the balance sheet adjustments that the central bank must implement do not vary, highlighting a clear distinction in the sensitivity of monetary policy instruments to the anchoring of expectations under the optimal policy with both instruments in this model.