Assessing the Aggregate and Distributional Implications of Large-Scale Bond Purchases in the Euro Area*

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Abstract

This paper studies how central bank large-scale bond purchases (namely *Quantitative Easing*) affected euro area economic activity and income inequality, within a New Keynesian model with limited asset market participation. I estimate the model using Bayesian methods, considering the occasionally binding constraint on the policy rate and the *public sector purchase programme* (PSPP) implemented by the ECB in 2015. The results suggest that the PSPP has effectively affected both output and inflation. On income inequality, the impact was modest on average. However, it was non-linear, income inequality increased during the early stages of the program due to asset price effects. In contrast, the medium-term impact on income inequality was characterized by growth in labor income, which became the main driver. These results show that the distributional impact of QE is time-varying, due to differences in the responsiveness of income components.

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1 Introduction

The Great Recession prompted major central banks to lower short-term policy rates to the effective lower bound (ELB). As conventional monetary policy became less effective, central banks implemented several extraordinary measures, with one of the most prominent being large-scale asset purchase (LSAP) programs, often referred to as Quantitative Easing (QE). The recent COVID-19 outbreak further emphasized the reliance on these policies. Despite their frequent use, the implication of these measures for macroeconomic variables is still subject to debate. In addition, the rising concerns about income and wealth inequality have drawn attention to the distributional consequences of QE policies.

In this paper, therefore, I seek to answer the following questions: *How did the European Central Bank's (ECB) large-scale bond purchases affect inflation and economic activity in the euro area*? *What were the implications for income inequality*? *Which channel, the labor market, or the financial market, has played a dominant role in shaping the distributional effects of QE*?

The question of how monetary policy, and especially QE, affects income inequality is still unresolved. Specifically, the literature tends to point out that the distributional effects of this policy can vary across regions and are often ambiguous (e.g. Saiki et al., 2020 and Colciago et al., 2019). The distributional consequences of QE are ambivalent, as it affects inequality through various channels. One distributional channel that is commonly emphasized relates to households' income composition, which highlights the fact that households have different sources of income, reacting differently to a specific shock or policy. In the case of QE, on the one hand, it can boost asset prices, primarily benefiting a fraction of households and potentially exacerbating inequality. This is generally referred to as the financial channel. On the other hand, QE stimulates demand, which increases labor earnings by boosting employment and wages, thus favoring households at the bottom of the distribution. This is usually referred to as the labor market channel. To understand the effects of QE on inequality, it is therefore essential to understand the relative contributions of these two channels.

I develop a model that incorporates the two competing effects. It features two types of households: one is financially constrained (called *Rule-of-Thumb consumers*, ROT henceforth) and relies only on labor income and transfers, while the other type (called *Ricardian*) can invest in several assets: stocks, short-term bonds, and long-term bonds. Similar to Vayanos and Vila (2021), following a 'preferred habitat' motive, bonds are imperfect substitutes. Ricardian households face portfolio adjustment costs when allocating their assets between short-term and long-term government bonds. This gives rise to the portfolio rebalancing channel of QE. The central bank has two policy instruments. Its primary tool is the short-term policy rate, which follows a Taylor rule subject to the ELB. The second, QE, occurs when the central bank buys long-term public debt directly from financially unconstrained households. Portfolio adjustment costs create a wedge between returns on short-term and long-term bonds which provides a role for central bank asset purchases. The model also contains price and wage rigidity. Real wage rigidity plays a crucial role in this framework: it determines the effect on labor earnings, which in turn affects the income of constrained households and the scale of general equilibrium effects.

The model is estimated using Bayesian methods with euro data. Importantly, I explicitly take into account the binding ELB on the nominal interest rate, and the *public sector purchase programme* (PSPP) conducted by the ECB from 2015. Counterfactual simulations reveal that the PSPP had a measurable impact on euro area economic activity over this period. Normalizing the asset purchases to 1% of euro area GDP, I find that it had a peak impact of 0.07% and 0.03 percentage points on real GDP, and y-o-y inflation, respectively. These estimates are conservative and aligned with the most recent literature documenting the impacts of LSAPs in the euro area. Rostagno et al. (2021) report that studies using estimated DSGE models for the euro area find peak real GDP impacts of QE ranging between 0.02% and 0.2%. Similarly, these studies report that y-o-y inflation has been 0.01 to 0.1 percentage points higher due to the QE policy implemented by the ECB. The results in this paper are therefore in the mid-low range of these intervals. In addition, this paper underscores the importance of analyzing the effects of agents' attention to central bank announcements, in measuring the effects of QE. In particular, a higher degree of attentiveness leads to non-proportional increases in the magnitude of the PSPP's effects on aggregate variables.

Next, I analyze the consequences of the PSPP on income inequality in the euro area. In the model, the incomes of the two types differ mainly because one type has access to the financial market, while the other cannot. This framework effectively isolates the distinct roles played by the financial channel and the labor market channel. The financial channel benefits

Ricardian households, while the labor market channel favors financially constrained households, whose income depends mainly on it. I find that QE had a small, non-linear, impact on income inequality. Due to QE, income inequality first increased in the early periods of the program, as purchases of long-term bonds by the central bank led to a sizeable surge in asset prices. However, in the medium term, labor earnings, which reacted with a lag, rose markedly and became dominant, so that on average over the entire course of the policy, the effects on income inequality were modest. This mainly arose due to higher demand for labor, as real wages are estimated to be highly sticky, aligned with the findings of Lenza and Slacalek (2018). Compared to the existing literature, this demonstrates the importance of accounting for the differences in the sensitivity of income components, which can lead to time-varying effects of monetary policy.

Furthermore, I examine the role of labor market structures, particularly real wage rigidity, in shaping the consequences of asset purchases for income inequality. The analysis suggests that the degree of wage stickiness can significantly influence the distributional effects of QE policies. More flexible wages lead to a larger role for the labor market channel, while more rigid wages limit the extent to which labor earnings compensate for the surge in financial returns thus increasing income inequality. These findings may have important implications for understanding the varying effects of QE policies across different regions and economies.

Finally, due to rising inflation rates, central banks have started reducing the size of their balance sheets. I examine the impact of a central bank's reduction in bond holdings on income inequality. I find that the trajectory of the reduction plays an important role in determining the distributional impacts.

Related Literature:

(*Empirical Literature*):

This paper contributes to the literature on monetary policy and inequality, see Colciago et al. (2019) for a recent survey. First, this paper relates to several empirical studies. Most of this literature focuses on conventional monetary policy, which is a seminal paper is Coibion et al. (2017). Using the Consumer Expenditures Survey, they show that contractionary monetary policy increases both consumption and income inequality. Another example for the US is Davtyan (2017), which contradicts Coibion et al. (2017) and finds that contractionary monetary policy decreases income inequality. The literature on the distributional unconventional monetary policies is more limited and highlights cross-region differences. Saiki and Frost (2014) and Saiki et al. (2020) show that the QE policy in Japan increased income inequality. Similarly, Taghizadeh-Hesary et al. (2020) find that the Bank of Japan's asset purchase policies raised income inequality through an increase in the price of assets, which benefited only to richer households. In contrast, studies focusing on the euro area are less clear-cut and demonstrate predominantly equalizing (or negligible) effects of asset purchase policies on income and consumption inequality. Lenza and Slacalek (2018) find that the ECB's programs reduced the income Gini index in European countries mainly by increasing the employment of low-income groups. Casiraghi et al. (2018) point out comparable results for Italy. Using a VAR model, Guerello (2018) shows that there is high heterogeneity in the results.

(*Theoretical Literature*):

This paper is also related to two strands of the theoretical literature: the literature studying the effects of QE within a New Keynesian model, and the literature using a heterogeneous agent framework to examine macroeconomic fluctuations.

In the New Keynesian literature, there are several ways of rendering QE effective for real variables. One way is to introduce frictional financial intermediation, as in Gertler and Karadi (2011), or more recently, Sims and Wu (2021). Another possibility, which is what this paper uses, is to model the portfolio rebalancing channel of QE, using imperfect substitutability between assets, as in Chen et al. (2012) or Harrison (2017). Within the New Keynesian literature, several papers quantify the macroeconomic effect of QE in the euro area. A common approach is to use a set of shadow rates as a proxy for unconventional measures, as in Mouabbi and Sahuc (2019) and Hohberger et al. (2023). Hohberger et al. (2019*b*) set up an estimated two-region model, to estimate the effects of all QE policies in the euro area. Andrade et al. (2016) analyze the effects of the ECB's Asset Purchase Programme on yields and economic activity using a calibrated model close to Gertler and Karadi (2018). This paper adds to this literature by quantifying the effects of the PSPP while accounting for the binding lower bound up until the start of the COVID crisis.

Furthermore, this paper adds to the literature on heterogeneous agent models and monetary policy. Seminal HANK models focused on conventional monetary policy are Kaplan et al. (2018), which stress the importance of the indirect effects of monetary policy shocks, compared to the standard RANK model, and Gornemann et al. (2016) which add matching frictions to obtain countercyclical labor market risks, endogenous to monetary policy. Two recent papers use a HANK framework to study the consequences of QE: Lee (2021) builds an estimated model to study how it affects household welfare across the wealth distribution in the US, and Cui and Sterk (2021) examine whether QE is an effective substitute for conventional monetary policy. Cui and Sterk (2021) show that it effectively mitigated the Great Recession but also comes with side effects on inequality and welfare.

Mostly, this paper follows the TANK literature inspired by Bilbiie (2008) and Debortoli et al. (2017). It features sticky prices and wages as in Colciago (2011), and Ascari et al. (2017). Similar two agent frameworks focused on QE policies in the euro area are Tsiaras (2021) and Hohberger et al. (2019*a*) which find that consumption and income inequality decrease after a QE shock. This paper supplements this literature by showing that QE is likely to have a time-varying impact, due to differences in the responsiveness of income components. This paper also aims to connect theoretical and empirical literature by investigating how real wage rigidity, which shapes the response of labor income, influences the impact of QE on income inequality.

The remainder of the paper proceeds as follows. Section 2 describes the model. Section 3 presents the estimation methodology for the euro area and the data used in this exercise. Section 4 describes the estimated impacts of QE on euro area economic activity and income inequality. Section 5 discusses the role of expectations in quantifying the implications of this policy. Section 6 studies the role of real wage rigidities. Section 7 examines the consequences of a reduction in the central bank's bond holdings. Section 8 concludes.

2 Model

The economy is composed of several types of agents. There are two types of households differing in their ability to access financial markets: financially unconstrained households (called *Ricardian*) and financially constrained households (called *ROT*). Ricardian households can invest in three types of assets: long-term bonds, short-term bonds and stocks. Long-term and short-term bonds are imperfect substitutes, this is meant to break the neutrality condition that renders QE irrelevant in standard macroeconomic models (see Wallace, 1981). There are both wage and price stickiness in the model. The production side is divided between a representative competitive final good firm and a continuum of monopolistically competitive intermediate good firms. In the case of the labor market, households provide differentiated labor input, there is a labor packer that combines it into a composite labor good. Finally, the treasury runs a balanced budget and the central bank implements conventional monetary policy subject to the ELB and QE.

2.1 Households

Households are divided into two types based on their access to financial markets. *Ricardian* households have access to asset markets and can smooth their consumption over time. They have the ability to invest in long-term bonds, short-term bonds, and stocks. On the other hand, *ROT* (also referred to as *hand-to-mouth*) households have limited access to financial markets and cannot smooth their consumption. They consume their net income flow each period. Both types have identical standard preferences:

$$U_{j,0} = \mathbb{E}_{j,0} \sum_{t=0}^{\infty} \beta^t \mu_t \left\{ \frac{\left(C_{j,t} - hC_{j,t-1}\right)^{1-\gamma}}{1-\gamma} - \chi \frac{N_{j,t}^{1+\sigma_N}}{1+\sigma_N} \right\} \tag{1}$$

where $j = r$ for the Ricardian household, or $j = k$ for the ROT.

$0 < \beta < 1$ is the discount factor, $0 < h < 1$ is the degree of habits formation, γ is the risk aversion parameter, σ_N is the inverse Frisch elasticity of labor supply and χ is the disutility of labor. μ_t is an exogenous variable meant to represent a preference shock.

2.1.1 Limited asset market participation

Ricardian Households:

The real budget constraint of the Ricardian household is:

$$\begin{aligned} &C_t^r + b_t^{ST} + q_t^B \frac{b_t^{LT,h}}{\varepsilon_t^b} + q_t^s S_t = \\ &w_t^r N_t^r + q_t^s S_{t-1} + Div_t S_{t-1} + \left(1 + \kappa_b q_t^B\right) b_{t-1}^{LT,h} \Pi_t^{-1} + R_{t-1} b_{t-1}^{ST} \Pi_t^{-1} - T_t - Adj_t \end{aligned} \tag{2}$$

Ricardian households can invest in three different types of assets: long-term government bonds $b_t^{LT,h}$, short-term government bonds b_t^{ST} and stocks distributed by intermediate good firms S_t. Long-term and short-term bonds are imperfect substitutes (*detailed below*). They also receive income from labor $w_t^r N_t^r$ and dividends from intermediate good producers Div_t. T_t are lump-sum taxes.

Long-term bonds are modelled as in Woodford (2001), as a perpetuity with geometrically decaying coupon payments with coupon rate κ_b. That is, a coupon due at time $t+s$ on debt issued in period t is κ_b^{s-1}. Using this notation, it is possible to write the budget constraint using a single stock of long-term bonds. The quantity of long-term debt purchased at all previous dates is summarised in terms of a quantity of bonds issued in the previous period, $b_{t-1}^{LT,h}$. This pays a coupon of 1 per bond and has a value of $\kappa_b q_t^B$. The short-term bond pays a gross rate of return R_t. Adj_t is the portfolio adjustment cost that households have to pay due to imperfect substitutability. ε_t^b is a risk premium shock that affects the return on long-term bonds.

ROT Households:

ROT consumers do not have access to financial markets and can only consume their net income every period. It is composed of labor income net of taxes. The real budget constraint for this type is:

$$C_t^k = w_t^k N_t^k - T_t \tag{3}$$

2.1.2 Bond market friction and asset pricing

In the model, investors have preferred habitat. This assumption allows to break the no-arbitrage condition between short-term and long-term rates that prevails in the standard New-Keynesian model and makes QE ineffective in affecting economic activity. Therefore, short-term and long-term bonds are imperfect substitutes, such that investors face portfolio adjustment costs, modelled similar to Harrison (2017) or Hohberger et al. (2019*a*):

$$Adj_t = \frac{\nu}{2}\left[\delta \frac{b_t^{ST}}{b_t^{LT,h}} - 1\right]^2 \tag{4}$$

where b^{ST} and $b^{LT,h}$, are the steady-state levels of short-term debt and long-term debt held by the Ricardian household, respectively. The Ricardian household targets a mix of long-term and short-term bonds, determined by δ.[1]. Deviations from the target incur a quadratic adjustment cost whose strength is shaped by the parameter ν.

First-order conditions for each type of assets give the following real pricing conditions:

(*Short-term bonds*):

$$\lambda_t + \lambda_t \frac{\nu\delta}{b_t^{LT,h}} \left[\delta \frac{b_t^{ST}}{b_t^{LT,h}} - 1 \right] \; = \beta R_t \mathbb{E}_t \lambda_{t+1} \Pi_{t+1}^{-1} \tag{5}$$

(*Long-term bonds*):

$$\lambda_t \frac{q_t^B}{\varepsilon_b^t} - \lambda_t \frac{\nu\delta b_t^{ST}}{\left(b_t^{LT,h}\right)^2} \left[\delta \frac{b_t^{ST}}{b_t^{LT,h}} - 1 \right] = \beta \mathbb{E}_t \lambda_{t+1} \left(1 + \kappa_B q_{t+1}^B\right) \Pi_{t+1}^{-1} \tag{6}$$

(*Stocks*):

$$\lambda_t q_t^S = \beta \lambda_{t+1} \left(q_{t+1}^S + div_{t+1} \right) \tag{7}$$

where λ_t, is the marginal utility of consumption of Ricardians.

2.1.3 Labor market

Given the focus on the income composition channel and the contrast between the effects on labor and financial income, I keep the labor market side of the model simple (see Appendix B for the full derivations). Therefore, I assume that labor earnings are the same for all households. The two types do not differ in terms of skills and therefore earn the same real wage $w_t^k = w_t^r = w_t$. Moreover, firms are indifferent to the type of household they hire, such that aggregate employment is distributed uniformly among households: $N_t^k = N_t^r = N_t$.

Households supply differentiated labor input which gives them some pricing power. Total labor input is given by:

$$N_t = \left(\int_0^1 N_t(l)^{\frac{\varepsilon_t^w - 1}{\varepsilon_t^w}} dl \right)^{\frac{\varepsilon_t^w}{\varepsilon_t^w - 1}} \tag{8}$$

where l indexes the differentiated labor input and ε_t^w represents the degree of substitutability among different types of labor and governs the desired markup of wages over the household's

[1]Which is the inverse of the steady-state level of the mix, such that the cost disappears in steady-state

marginal rate of substitution. It is time-varying and follows an AR(1) process:

$$\log \varepsilon_t^w = (1-\rho_w)\log \varepsilon^w + \rho_w \log \varepsilon_{t-1}^w + \eta_{w,t} - \iota_w \eta_{w,t-1} \tag{9}$$

with $\eta_{w,t}$ the wage mark-up shock.

There exists a labor packer firm which bundles differentiated labor input into an aggregate labor input. The maximisation of this labor packer yields the following demand curve for each labor input and the aggregate real wage index:

$$N_t(l) = \left(\frac{w_t(l)}{w_t}\right)^{-\varepsilon_t^w} N_t \tag{10}$$

$$w_t = \left(\int_0^1 w_t(l)^{1-\varepsilon_t^w} dl\right)^{\frac{1}{1-\varepsilon_t^w}} \tag{11}$$

Households are then not able to adjust their real wage every period.[2] Each period, there is a probability of $1-\phi_w$ that the real wage is set optimally. I assume that Ricardian households have all the bargaining power. Thus, the unconstrained type sets the wage to maximize its objective function, while the constrained type takes real wages as given. The wage dynamic is given by the following equation:

$$w_t(l) = \begin{cases} w_t^{\#}(l) & \text{if } w_t(l) \text{ chosen optimally} \\ (\Pi_t)^{-1}(\Pi_{t-1})^{\zeta_w} w_{t-1}(l) & \text{otherwise} \end{cases} \tag{12}$$

With probability $1-\phi_w$ an household receives a new optimal real wage $w_t^{\#}(l)$. With probability ϕ_w, its wage remains unchanged, except that it is adjusted by the inflation rate and it is partially indexed to lagged inflation. Indexation is governed by the parameter $\zeta_w \in [0,1]$.

2.2 Production

There are two layers of production: a representative competitive final good firm, and a continuum of monopolistically competitive intermediate goods firms that are subject to price stickiness à la

[2] I follow Galí et al. (2007), such that the fraction of constrained households is uniformly distributed across types of workers, such that aggregate demand for labor type l is the same across households.

Calvo (1983).

Final good producers:

Final good output Y_t is a CES aggregate of the intermediate outputs $Y_t(j)$:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon_t^p - 1}{\varepsilon_t^p}} \right)^{\frac{\varepsilon_t^p}{\epsilon p - 1}} \tag{13}$$

where ε_t^p is the time-varying elasticity of substitution which evolves according to:

$$\log \varepsilon_t^p = (1 - \rho_p) \log \varepsilon^p + \rho_p \log \varepsilon_{t-1}^p + \eta_{p,t} - \iota_p \eta_{p,t-1} \tag{14}$$

with $\eta_{p,t}$ the price mark-up shock.

Profit maximisation by the final good producer yields a standard demand function for each intermediate good and an aggregate price index:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t^p} Y_t \tag{15}$$

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon_t^p} dj \right)^{\frac{1}{1-\varepsilon_t^p}} \tag{16}$$

where P_t is the final output price and differentiated output prices are denoted as $P_t(j)$.

Intermediate producers:

Intermediate producers produce output according to the following one factor function:

$$Y_t(j) = A_t N_t(j) \tag{17}$$

where A_t is an exogenous productivity shock which is common to all firms and follows a stationary AR(1) process.

Intermediate producers cannot freely adjust their price: there is a probability $1 - \phi_p$ that a producer can adjust its price to the optimal price $P_t^{\#}(j)$, and a probability ϕ_p that the price is not updated but is indexed to lagged inflation depending on the parameter $\zeta_p \in [0, 1]$.[3] Prices

[3]See Appendix B for the full derivations

evolve according to the following equation:

$$P_t(j) = \begin{cases} P_t^{\#}(j) & \text{if } P_t(j) \text{ chosen optimally} \\ (\Pi_{t-1})^{\zeta_p} P_{t-1}(j) & \text{otherwise} \end{cases} \tag{18}$$

2.3 Treasury and Monetary Authority

Treasury:

The government simply taxes households and issues debt. It also collects a lump-sum transfer from the central bank (i.e. its profits).

The flow budget constraint in real terms is:

$$T_t + T_t^{cb} + q_t^B b_t^{LT} + b_t^{ST} = R_{t-1} b_{t-1}^{ST} \Pi_t^{-1} + \left(1 + \kappa_b q_t^B\right) b_{t-1}^{LT} \Pi_t^{-1} \tag{19}$$

where T_t^{cb} is the profit of the central bank, and b_t^{LT} is the total amount of long-term debt, defined as the sum of private holdings and central bank holdings:

$$b_t^{LT} = b_t^{LT,h} + b_t^{CB} \tag{20}$$

Monetary authority:

(*Conventional monetary policy*):

The central bank sets the desired interest rate according to a Taylor rule:

$$\begin{aligned} \ln R_t^{tr} &= (1-\rho_{mp}) \ln R^{tr} + \rho_{mp} \ln R_{t-1}^{tr} \\ &+ (1-\rho_{mp}) \left[\theta_\pi \left(\ln \Pi_t - \ln \Pi\right) + \theta_y \left(\ln Y_t - \ln Y\right) + \theta_{dy} \left(\ln Y_t - \ln Y_{t-1}\right)\right] + \varepsilon_{r,t} \end{aligned} \tag{21}$$

It responds to deviation of inflation from its target, to deviation of output to its steady-state, and to output growth. ρ_{mp} is the interest rate smoothing rate and θ_π, θ_y and θ_{dy} are feedback coefficients to inflation, output and output growth. $\varepsilon_{r,t}$ is the monetary policy shock which follows an AR(1). The actual interest rate is subject to the ELB:

$$R_t = \max\left\{1, R_t^{tr}\right\} \tag{22}$$

(*Quantitative Easing*):

As in Gertler and Karadi (2011), the central bank can buy a fraction of long-term government issued bonds as part of its QE policy. As in Sims and Wu (2021), real bond holdings follow a simple exogenous auto-regressive process:

$$b_t^{CB} = (1 - \rho_{cb})\, b^{CB} + \rho_{cb} b_{t-1}^{CB} + s_{qe}\varepsilon_{qe,t} \tag{23}$$

b^{CB} is the steady-state central bank holdings of long-term government bonds, ρ_{cb} is the persistence of the program, and s_{cb} the standard deviation of the QE shock.

In this model, QE policies generate real effects through the portfolio rebalancing channel. Unconstrained households target a specific mix of bonds of different maturities, by taking away a fraction of their long-term bond holdings, QE pushes unconstrained households to rebalance their portfolio, which incurs some transaction costs scaled by the parameter ν. This transaction cost creates a wedge between the return between short-term and long-term bonds, breaking the no-arbitrage condition, which can be influenced by the QE policy.

2.4 Market clearing and aggregate conditions

The full nonlinear equations is described in Appendix C. Below, I present market clearing and aggregate conditions of the model.

Price and real wage indexes can be written as:

$$P_t^{1-\varepsilon_t^p} = (1 - \phi_p)\, P_t^{\#,1-\varepsilon_t^p} + \phi_p P_{t-1}^{1-\varepsilon_t^p} \left(\Pi_{t-1}\right)^{\zeta_p(1-\varepsilon_t^p)} \tag{24}$$

$$w_t^{1-\varepsilon_t^w} = (1 - \phi_w)\, w_t^{\#,1-\varepsilon_t^w} + \phi_w \left(\Pi_t\right)^{\varepsilon_t^w - 1} \left(\Pi_{t-1}\right)^{\zeta_w(1-\varepsilon_t^w)} w_{t-1}^{1-\varepsilon_t^w} \tag{25}$$

Integrating over the demand curves, aggregate production function is given by:

$$Y_t = \frac{A_t N_t}{v_t^p} \tag{26}$$

where v_t^p is a measure of price dispersion:

$$v_t^p = \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t^p} dj \tag{27}$$

The good market clearing is:

$$Y_t = C_t + \frac{\nu}{2} \left[\delta \frac{b_t^{ST}}{b_t^{LT,h}} - 1 \right]^2 \tag{28}$$

where aggregate consumption C_t is the weighted sum of the consumption of each type:

$$C_t = \omega C_t^r + (1 - \omega) C_t^k \tag{29}$$

ω is the fraction of Ricardian households, $(1 - \omega)$ is the fraction of ROT households.

The total stock of long-term bonds is in positive net supply, such that market clearing for long-term bonds yields:

$$b_t^{LT,h} + b_t^{CB} = b^{LT} \tag{30}$$

Market clearing for the overall bond market requires that:

$$b_t^{ST} + b^{LT} = \bar{b} \tag{31}$$

Such that the total stock of bond in the model is in positive net supply, at a constant level $\bar{b}$ as in Harrison (2017).

3 Estimation methodology

Since October 2014, the ECB has conducted several asset purchase programs to provide the necessary level of policy accommodation to ensure price stability. In Section 3 and Section 4, I carry out a Bayesian estimation of the model for the euro area, to understand the effects of the ECB's PSPP, taking into account the ELB on the policy rate that was binding during this period. Using this estimated model, I seek to quantify the macroeconomic consequences of QE in the euro area, and to assess how it impacted income inequality, by examining their effects on the different components of income. In Section 3, I sketch the method used to estimate the model in the presence of a binding ELB, describe the set of parameters that are calibrated, and discuss the choice of observables, as well as the prior and posterior estimates. Estimated results are then described in Section 4.

3.1 Method

To accurately account for the ELB on the short-term policy rate, I use the solution method from Guerrieri and Iacoviello (2015) which applies a first-order perturbation to a piecewise linear model to deal with the presence of the occasionally binding constraint. When this constraint is binding (i.e. as soon as the gross notional interest rate is below 1), the model switches to a second regime in which the gross policy rate R_t is pegged at the lower bound. As the model becomes non-linear, I use an inversion filter as in Guerrieri and Iacoviello (2017), Atkinson et al. (2020) and Jondeau et al. (2022) to back out structural shocks. This filter extracts the sequence of innovations recursively by inverting the observation equations (see Appendix A for details).[4]

3.2 Calibrated parameters

Table 1 shows the calibrated parameters. The discount factor is set to $\beta = 0.995$ to imply an annualized steady-state risk-free interest rate of 2%. The coupon decay of long-term bonds, κ_B, is set to $1 - 40^{-1}$ to match the duration of a 10-year government bond. χ, the labor disutility parameter, is set so that steady-state labor hours equal $1/3$. $\varepsilon^w = 12$ and $\varepsilon_p = 12$ are set to give steady-state markups of around 10%. For debt-related parameters, the steady-state level of government debt held by the central bank b^{cb} is chosen to match a ratio of ECB holdings over annualized GDP at the end of 2014. The steady-state level of government debt $\bar{b}$ matches the ratio of government debt to GDP of 0.92 at the end of 2014. The calibration for δ follows Hohberger et al. (2019*a*) and is set to 0.916. As in Lee (2021), I fix the auto-correlation coefficient of the central bank's asset holdings to $\rho_{qe} = 0.99$. Finally, I follow Slacalek et al. (2020) and set the share of hand-to-mouth in the EA to 0.216%. The remaining parameters are estimated using Bayesian techniques.

[4] For the comparative performance of the inversion filter compared to other nonlinear filters, I refer to Cuba-Borda et al. (2019) and Atkinson et al. (2020).

Table 1: Calibrated Parameters

Parameter	Value or Target	Description
κ_B	$1-40^{-1}$	Coupon decay
β	0.995	Discount Factor
χ	$N=1/3$	Labor disutility
ε^w	12	Steady-state elasticity of substitution labor
ε^p	12	Steady-state elasticity of substitution goods
b^{cb}	$\frac{b^{cb}q^b}{4Y}=0.021$	SS CB Treasury holdings
$\bar{b}$	$\frac{\bar{b}\,q^b}{4Y}=0.92$	Total debt stock relative to output
δ	0.916	Ratio of SS LT to ST debt
ω	0.216	Share of ROT households
ρ_{cb}	0.99	Auto-correlation of QE shocks

3.3 Data

I estimate the model parameters with Bayesian methods using quarterly data from 1999:I to 2019:IV.

I use the following set of 6 observables in the estimation:

$$\left[\Delta\log C_t, \log\Pi_t, \Delta\log w_t, \log R_t, re_t^{gdp}, \log\left(R_{t+1}^{LT}-R_t\right)\right] \tag{32}$$

where C_t, Π_t, w_t, R_t, re_t^{gdp} and R_{t+1}^{LT} are i) consumption, ii) the inflation rate, iii) real wages, iv) the gross nominal interest rate, v) the central bank's real holdings of government bonds over annual real GDP and vi) the long-term rate.

I measure consumption as real private consumption expenditures per capita. The inflation rate is defined as the quarterly percentage change of the GDP deflator. The real wage is computed by dividing the compensation of employees by employment and the GDP deflator. All these data are from the OECD. The nominal short-term rate is the Euribor 3-month provided by the ECB, while the long-term rate is the OIS 10-year provided by Refinitiv. For the asset purchase policy, I take ECB data of net purchases of public sector securities under the PSPP between March 9, 2015, and December 19, 2018. Besides, I use the following shock processes: i) a preference shock, ii) a monetary policy shock, iii) a QE shock, iv) a price mark-up shock, v) a wage mark-up shock, vi) a risk-premium shock on long-term bonds. [5]

[5] One drawback of the inversion filter is that the number of shocks has to be the same as the number of

3.4 Prior and posterior

Table 2: Distribution of estimated parameters

	Details	Prior			Posterior		
		density	mean	std	mean	5%	95%
γ	CRRA	gamma	1.50	0.2	1.388	1.374	1.403
σ_N	Inverse Frisch	gamma	1.50	0.2	1.529	1.484	1.621
h	Habit	beta	0.7	0.1	0.834	0.823	0.849
θ_π	TR inflation	normal	1.50	0.1	1.525	1.517	1.541
θ_y	TR output	normal	0.5	0.1	0.428	0.416	0.438
θ_{dy}	TR output growth	normal	0.05	0.05	0.079	0.068	0.087
ρ_{mp}	TR smoothing	beta	0.8	0.1	0.945	0.931	0.949
ϕ_p	Price Calvo	beta	0.6	0.2	0.788	0.730	0.857
ϕ_w	Wage Calvo	beta	0.6	0.2	0.870	0.849	0.884
ζ_p	Price indexation	beta	0.6	0.1	0.097	0.060	0.113
ζ_w	Wage indexation	beta	0.6	0.1	0.416	0.386	0.430
ι_p	Coeff. MA term price	beta	0.5	0.2	0.587	0.563	0.614
ι_w	Coeff. MA term price	beta	0.5	0.2	0.760	0.728	0.779
ν	Port. adj cost	gamma	0.0015	0.0006	$1.44e-04$	0.0001	0.0002

Table 3: Prior and posterior distribution of shock processes

	Details	Prior			Posterior		
		density	mean	std	mean	5%	95%
ρ_c	AR(1) preference	beta	0.6	0.2	0.523	0.507	0.588
ρ_p	AR(1) price mark-up	beta	0.6	0.2	0.937	0.923	0.955
ρ_w	AR(1) wage mark-up	beta	0.6	0.2	0.922	0.896	0.953
ρ_r	AR(1) IR	beta	0.6	0.2	0.446	0.416	0.438
ρ_b	AR(1) risk prem.	beta	0.5	0.2	0.820	0.792	0.869
σ_c	Std. dev preference	inv. gamma	0.01	0.1	0.048	0.043	0.056
σ_p	Std. dev price	inv. gamma	0.01	0.1	0.077	0.061	0.117
σ_w	Std. dev wage	inv. gamma	0.01	0.1	0.240	0.225	0.250
σ_r	Std. dev IR	inv. gamma	0.001	0.1	$8.1e-04$	0.0007	0.0009
σ_{cb}	Std. dev QE	inv. gamma	0.001	0.1	$2.5e-04$	0.0002	0.0003
σ_b	Std. dev risk prem.	inv. gamma	0.01	0.1	$9e-04$	0.001	0.001

Table 2-3 show the priors estimates of structural parameters of the model and the shock processes. The priors closely follow the New-Keynesian literature. For the degree of habit h, I follow Smets and Wouters (2007) and use beta distribution with a mean equal to 0.7 and a

innovations to allow the inversion of the observation equations. Therefore, I restrict the number of structural shocks used in the estimation to six.

standard deviation equal to 0.1. The prior of the coefficient of relative risk aversion is a normal distribution with a mean to 1.5 and a standard deviation equal to 0.2. Standard priors are used for the Taylor rule parameters. I use normal distributions, with a mean of 1.5 and a standard deviation of 0.1 for the response to inflation, a mean of 0.5 and a standard deviation of 0.1 for the response to output, and a mean of 0.05 and a standard deviation of 0.05 for the response to output growth. Taylor rule smoothing uses a beta distribution with mean of 0.8 and a standard deviation of 0.1.

For parameters related to price and wage settings, I assume that wage Calvo ϕ_w and price Calvo ϕ_p parameters follow a beta distribution with a mean of 0.6 and a standard deviation of 0.2. In line with Carlstrom et al. (2017), I assume a beta distribution for indexation parameters ζ_w and ζ_p with a mean of 0.6 and a standard deviation of 0.1. The priors for the shock autocorrelations are set to beta distributions with a mean of 0.6 and a standard deviation of 0.2.

For the standard deviations of the shocks, I follow closely Garín et al. (2016) and use inverse gamma distributions with a mean of 0.01 for preference, price mark-up and wage mark-up shocks, and a slightly lower mean of 0.001 for conventional and unconventional monetary policy shocks. Standard deviations are all equal to 0.1.

One crucial parameter is the portfolio adjustment cost, which governs the wedge between the returns on short-term and long-term bonds, and ultimately the scale of the impacts of QE. To estimate it, I use the prior set by Hohberger et al. (2019*b*) which is a gamma distribution with a mean of 0.0015 and a standard deviation of 0.0006.

The results of the posterior distribution, which is drawn from two parallel chains of $200,000$ iterations of the Metropolis-Hasting algorithm, are also reported in Table 2. The estimated parameters suggest a high degree of wage and price rigidities. I find that real wages are more rigid than prices in the euro area over this period. The parameter for habit formation implies a significant degree of inertia in the model. The results for the monetary policy rule indicate a high level of gradualism with a smoothing parameter $\rho_{mp} = 0.95$, a moderate response to inflation, around 1.53, a positive response to output gap close to around 0.43 and a negative but small response to output growth of -0.01. Importantly, the estimated portfolio adjustment cost gives a result of $1.44e - 04$.[6]

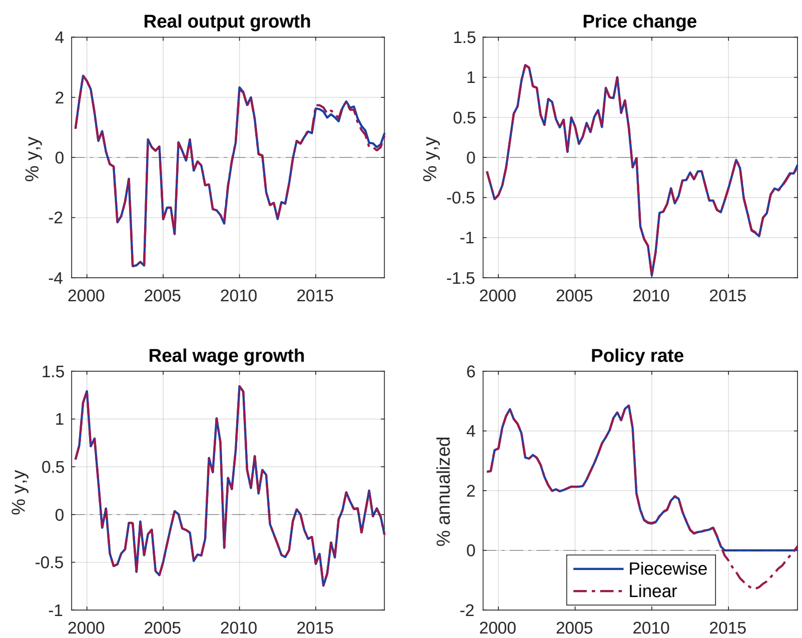
[6]See Appendix A for estimated IRFs to policy shocks, and comparison to a RANK framework.

4 Estimated results

In this section, I present the quantitative results concerning the impacts of the PSPP conducted by the ECB between 2015 and the end of 2018 in the euro area. Mostly, I answer the following questions: How has the ECB's QE policy affected economic activity? What have been the consequences in terms of income inequality?

I conduct a counterfactual analysis to compare the estimated behavior of the economy to an alternative without a QE policy. I use the same series of shocks, but this time I assume that the ECB did not conduct any QE policy from 2015, such that reserves stay at their pre-2015 level. Besides, to explicitly account for the role of the binding ELB, I perform this exercise in the context of a piecewise linear model in which the interest rate is constrained by the ELB.

Figure 1: Effect of the lower bound in the filtered series



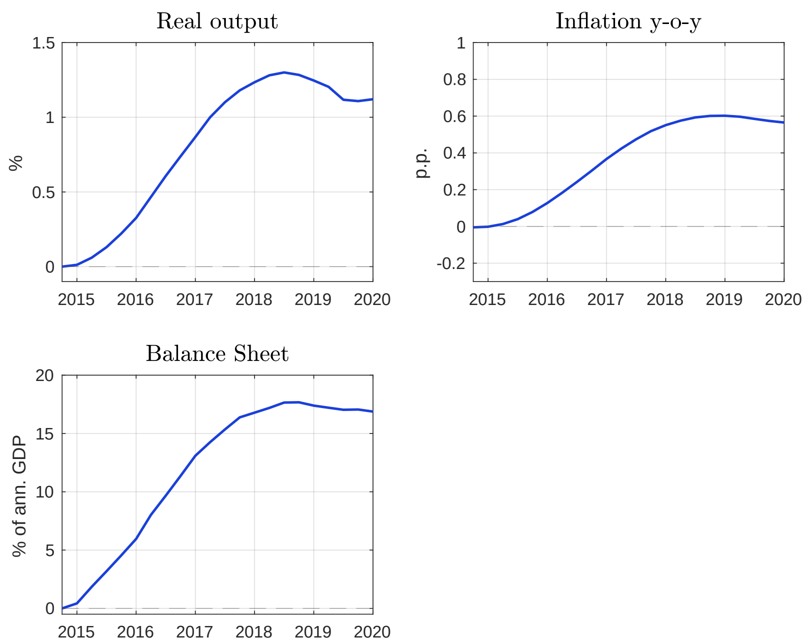
Notes: The solid blue line shows the paths of filtered series in the piecewise-linear model, while the dotted purple line is the linear model. Real consumption, real wage and price change are demeaned year-on-year growth rates, the policy rate is in annualized percentage points, and the CB balance sheet is the ECB's holding of government bonds shown as a ratio to annual real GDP.

Filtered series in both models can be found in Figure 1. The blue solid line is the piecewise linear model which fully considers the binding lower bound, the dashed purple line is the linear model, in which the policy rate follows an unconstrained Taylor rule. It shows that without the ELB, the Taylor rule would have suggested a further decrease of the nominal rate, to an annualized level of about -1.3% at the start of 2017.

4.1 Aggregate effects

Figure 2 shows the estimated effects of the PSPP on real output and y-o-y inflation, calculated as the difference between the model with estimated series accounting for the binding lower bound on the policy rate and parameters fixed at the posterior means, and the same model without the asset purchase policy.

Figure 2: Estimated effects of the PSPP



Notes: All variables are shown in deviation from the counterfactual without QE. Year-on-year inflation is shown in percentage points deviation, while real output is in percentage deviation. QE is expressed as the ratio of real reserves over annualized real GDP.

The analysis reveals that the ECB's asset purchase policy had a sizeable impact on euro area economic activity. It had a positive effect on output, with an average increase of 0.84% between 2015Q1 and 2019Q4, peaking at 1.3% in 2018, compared to a scenario without a balance sheet expansion. Besides, I find that inflation has also been higher due to the asset purchase program, with an average increase of 0.38 percentage points and a peak of 0.60 percentage points.

The magnitude of these results is conservative and aligns with existing literature. Scaling these results to make it comparable across studies, I find that a purchase of bonds increasing the balance sheet by 1% of euro-area GDP, leads to a rise of output and y-o-y inflation of 0.07% and 0.03 percentage points, respectively. These estimates are in the mid-low range of those reported in the literature. Rostagno et al. (2021) describe that, across studies using quarterly DSGE models and focusing on the euro area, asset purchases amounting to 1% of euro area GDP lead to an effect on real output ranging between 0.02% and 0.2%.[7] The baseline impact of this paper is also close to results reported by empirical studies. Using a large-scale Bayesian VAR, Rostagno et al. (2021) find that it led to a rise of GDP of 0.12%, while Gambetti and Musso (2017) report an increase of 0.02% using an SVAR with quarterly GDP. In the case of inflation, this paper's estimated impact of 0.03 percentage points is also aligned with existing euro area studies. The same range of studies that use a structural model finds that a purchase program equal to 1% of euro area GDP leads to an increase in y-o-y inflation between 0.01 and 0.1 percentage points.

4.2 Distributional effects

In this subsection, I turn to the distributional consequences of the PSPP in the euro area. Income inequality, for the purposes of this paper, is defined as the ratio of income held by financially unconstrained households to the total income of both financially constrained and unconstrained households. Accordingly, a rise in the proportion of total household income earned by unconstrained households is considered an increase in income inequality.

The focus of this paper is to examine the impact of asset purchase policies on income inequality by analyzing their effects on the different components of income. Households in the economy differ in their ownership of assets, with only one group receiving financial income. Therefore, understanding the dynamics of income inequality involves studying the interplay between the relative effects on labor and wages (i.e. the labor market channel) and the effects on asset prices and asset rate of returns (i.e. the financial market channel). When the central bank purchases assets, this tends to have a direct effect on asset prices through various channels. In terms of

[7] The studies used to construct this range are Andrade et al. (2016), Burlon et al. (2019), Cova et al. (2019), Kühl (2018), Mouabbi and Sahuc (2019) and Sahuc (2016), to which I add Hohberger et al. (2019*b*).

income, this primarily benefits asset holders through capital gains. However, the impact on asset markets is also likely to stimulate demand, which ultimately benefits households that primarily depend on employment and wages.

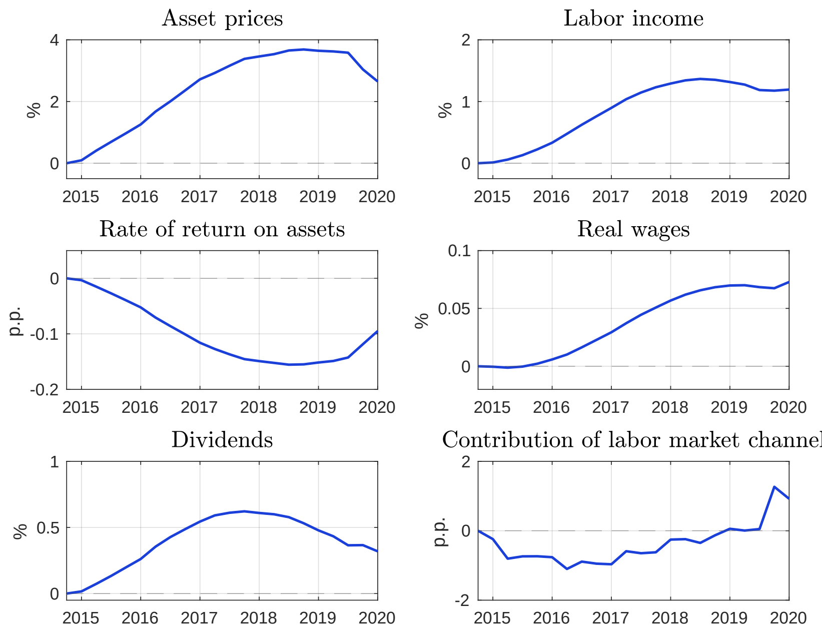
Figure 3: Impact of QE on income inequality

Notes: Income inequality is in percentage points deviation from the counterfactual without QE. The variable is defined as the ratio of income held by financially unconstrained households to the total income of both types of households.

Figure 3 shows the effects of the ECB's PSPP on income inequality. It indicates that these effects have been non-linear. The program initially widened the income gap for the first two and a half years, peaking at 0.21 percentage points. However, in 2018, the effects reversed, and the asset purchase policies began to narrow the income ratio, reaching a maximum decline of 0.32 percentage points by the end of the sample period. Nevertheless, the overall consequences for income inequality remained modest, with an average of 0.03 percentage points since the start of the PSPP.

To gain a deeper insight into the factors influencing the dynamics of income inequality, Figure 4 provides an overview of the effects of the PSPP on income components. This shows that QE had a positive impact on financial and labor income, but that the response of each channel varied over time. It first led to a rise in financial market variables. Asset prices surged sharply, up to

Figure 4: Effects of QE on income components



Notes: The contribution of the labor market channel is the share of the sum of household income originating from labor income. All variables are shown in deviation from the counterfactual without QE. Asset prices, real wages, dividends and labor income are in percentage deviation, while rate of return on assets and contribution of labor market channel are in percentage points.

a maximum close to 4%[8], in contrast to the rate of returns on assets which steadily declined, due to short-term rates staying at the ELB level and declining long-term yields. As depicted in Figure 3, this raised income inequality as continuously rising asset prices benefited unconstrained households through rapid positive capital gains. Dividends also contributed to the initial widening of the income gap. In this model, due to the estimated high degree of real wage rigidity in the euro area, dividends respond procyclically to the expansionary balance sheet program. This is consistent with empirical studies that have shown that an expansionary policy leads to an upward shift in both real wages and profits, which is not the case in the traditional New Keynesian model.[9]

Labor income also increased, up to 2%, but this affected income inequality with a lag. This was mainly due to increased demand for labor, as wage growth remained limited. As shown in

[8]This effect is close to previous estimates in the literature, Rostagno et al. (2021) show that QE pushed stock market returns up to almost 10%. Using a similar counterfactual exercise in the US, Lee (2021) finds that equity prices peaked close to 2% higher than in the counterfactual case without an asset purchase policy.

[9]See Christiano et al. (1999), Coibion et al. (2017) and Cantore et al. (2021) for further discussion.

the right bottom panel, the impact of QE on the contribution of the labor market channel to total household income was initially negative due to the larger role of financial income through capital gains. However, over time, the impact became positive. The dynamic interplay between these two factors explains the total response of income inequality in this framework.

The empirical literature has significantly studied the impact of both the labor and financial market channels, as they play a crucial role in shaping the overall effects of asset purchase policies. Saiki et al. (2020) find that higher returns for financial assets have outpaced the gains in labor income, showing that asset purchases by the Bank of Japan led to an increase in income inequality in the country. In contrast, Casiraghi et al. (2018) demonstrate that the effects on labor earnings outweighed those on financial variables in the euro area. In this case of the distributional consequences of the PSPP in the euro area, this paper suggests that it fluctuated over time, due to different degrees of responsiveness of income components. Financial variables reacted more quickly, widening the income gap. Asset prices increased steadily, as well as dividends, which reacted procyclically, aligned with empirical evidence. Nonetheless, throughout the course of the program, the role of the labor market channel gradually increased and eventually became the dominant factor, narrowing income inequality. This shift primarily resulted from higher aggregate demand for labor, while real wage growth remained subdued.

5 The role of expectations in quantifying the aggregate effects of QE

This section delves into the central role of expectations regarding the future policy path when assessing the impact of the PSPP in the euro area. Policymakers frequently preannounce the details of future bond purchases within a specific program, and agents' interpretation of these announcements significantly influences the outcomes of these policies on the economy by shaping their expectations. However, as described in Section 4, to assess the impact of a specific policy, it is standard in the literature, both in VARs and DSGE models, to build counterfactual without the policy intervention by setting structural or policy shocks to 0, depending on whether the policy is assumed to adhere to an explicit rule or is considered entirely exogenous.[10]

In this model, QE is treated as a purely exogenous policy. Therefore, constructing a counterfactual of this form requires setting QE shocks to 0 from the start of the program, thereby maintaining the central bank's balance sheet at its pre-program level. However, this implies that agents' expectations regarding the future conduct of the policy are irrelevant.

In this section, I design counterfactuals without the QE policy to capture the influence of expectations about the future path of central bank balance sheet expansion.

To do so, I follow de Groot et al. (2021) and Hebden and Winkler (2021) and write the counterfactual path for any variable in the model as follows:

$$Z^c = \{Z_t^c\}_{t=0}^H = Z^b + \mathcal{A}^{Z,\varepsilon_{qe}}\tilde{\varepsilon}_{qe} = \mathcal{A}^{Z,\varepsilon_s}\tilde{\varepsilon}_s + \mathcal{A}^{Z,\varepsilon_{qe}}\tilde{\varepsilon}_{qe}$$

This expresses that the counterfactual for any variable $Z^c = \{Z_t^c\}_{t=0}^H$ can be written in deviation from a baseline Z^b. In the case of this paper, the baseline is the estimated model written as the Impulse Responses of the variable of interest $\mathcal{A}^{Z,\varepsilon_s}$ to all shocks used in the estimation over the sample $\tilde{\varepsilon}_s$. To get this counterfactual, I solve for the vector of, at least partially, *anticipated* QE shocks $\tilde{\varepsilon}_{qe}$ that implements the desired trajectory of the balance sheet during the years in which the PSPP was implemented. To get the impact on the macroeconomic variables, I can then use the impulse responses of these variables (stored in $\mathcal{A}^{Z,\varepsilon_{qe}}$) to each individual current and future QE shocks. This ensures that, in the case in which agents are fully attentive they are influenced in 2015 by the policy decisions made throughout the program.[11]

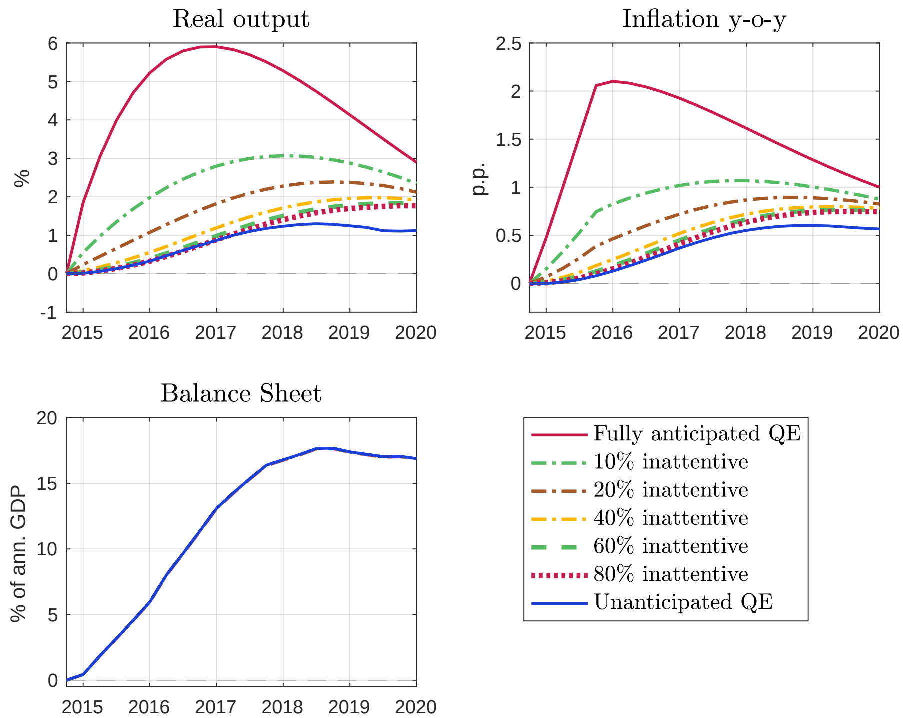
To analyze in depth the role of expectations, I adopt the approach detailed in de Groot and Mazelis (2020) and vary the fraction of agents that are attentive to the communication of the central bank.

Figure 5 presents the impacts of introducing a role for agents' expectations in quantifying the impact of the PSPP on real output and inflation. This reveals that allowing some agents to observe the future path of the balance sheet has measurable and non-proportional implications on the effects of the bond purchase program on the economy. Setting agents to be fully inattentive

[10] See Mouabbi and Sahuc (2019) and Lee (2021) for recent examples in the case of QE policies.

[11] As described in McKay and Wolf (2023), in the case of a fully specified policy rue, this would also allow agents to understand that they has been a systematic change in the policy.

Figure 5: Role of expectations on the effects of the PSPP



Notes: All variables are shown in deviation from the counterfactual without QE. Year-on-year inflation is shown in percentage points deviation, while real output is in percentage deviation. QE is represented by the ratio of real reserves over annualized real GDP. The different colored lines depict different level of attentiveness.

(i.e. the QE policy is fully unanticipated), acts as a lower bound. With a high degree of inattentiveness, the increase in real economic activity and year-on-year inflation remains relatively consistent. However, significantly increasing the degree of attentiveness amplifies the policy's substantive effects on real output and inflation. When all agents are attentive, this gives rise to a form of Forward Guidance Puzzle, whereby PSPP announcements generate unrealistically positive effects on economic activity and inflation.

This section therefore underscores the significance of accounting for agents' attentiveness to the central bank's communication about the future path of policy. As the degree of attentiveness to the entire course of policy increases, it can result in markedly different aggregate consequences.

6 The role of labor market rigidities

Section 4 highlights the importance of the sensitivity of financial market and labor market variables to QE policies in determining their impact on income inequality. Recent empirical studies point to the role of labor market structures in shaping the consequences of asset purchases for inequality (e.g. see Saiki et al., 2020). Therefore, in this section, I investigate the effect of real wage rigidities on the findings. Would the PSPP have led to different conclusions if real wages had been more flexible? More rigid?

Real wage rigidity affects the degree of responsiveness of labor market variables to QE policies. This has both direct and indirect consequences. First, it directly affects labor earnings through higher real wages. Besides, through its effect on households' disposable income, it has an impact on consumption. This is mainly the case for constrained households as it is a substantial share of their revenues, and they tend to have higher marginal propensities to consume (MPCs).[12] This in turn influences variables as well as other income components, such as labor demand, but also asset prices, through higher aggregate demand.[13] Finally, the cyclicality of dividends is also affected by the degree of real wage rigidity.[14]

To study the role of the degree of real wage rigidity, I perform a similar counterfactual analysis as in Section 4. I quantify the effects of the PSPP on aggregate variables and on income inequality for three different values of real wage rigidity ϕ_w. One is the estimated value $\phi_w = 0.87$, the second is meant to represent an economy with more rigid real wages $\phi_w = 0.95$ and the third depicts an economy with more flexible real wages $\phi_w = 0.75$.[15]

Figure 6 shows the results for real output and inflation. An economy with more flexible real wages leads to a higher intial impact of the PSPP on real output, but less persistent. Besides, it leads to an amplification of the implications for inflation. In contrast, if real wages are more rigid, both output and inflation are lower than in the estimated scenario. In the case of inflation, a higher ϕ_w would lead to smaller wage adjustments, which translates into smaller and more

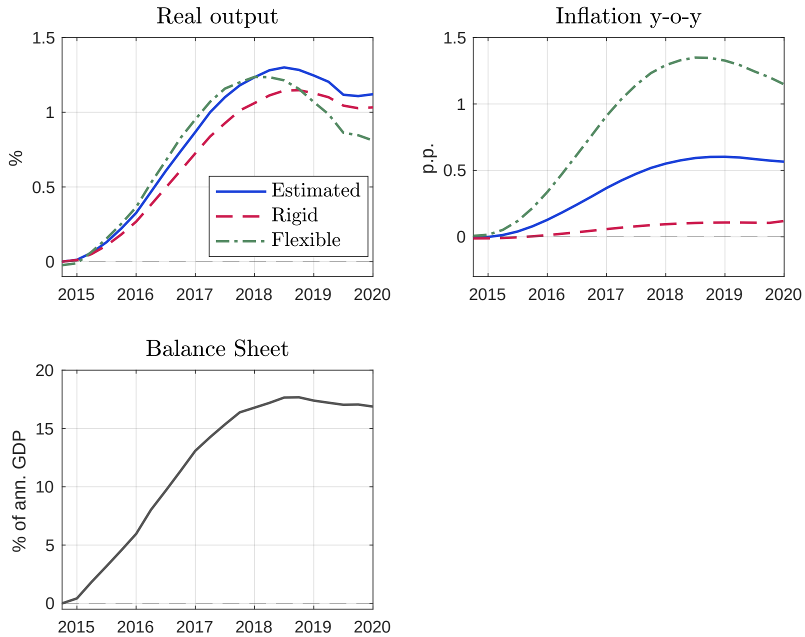
[12] Households that have little assets, also called hand-to-mouth are often estimated to exhibit larger MPCs. This is documented in Aguiar et al. (2020), and Kaplan et al. (2014).

[13] In the model, financial market variables are affected to the stochastic discount factor.

[14] See Ascari et al. (2017), Bilbiie et al. (2022) for related discussion of the role of sticky wages in heterogeneous agent models.

[15] As ϕ_w rises, the fraction of firms that can reset wages diminish. This increases real wage rigidity.

Figure 6: Role of real wage rigidities for the effects of QE on economic activity



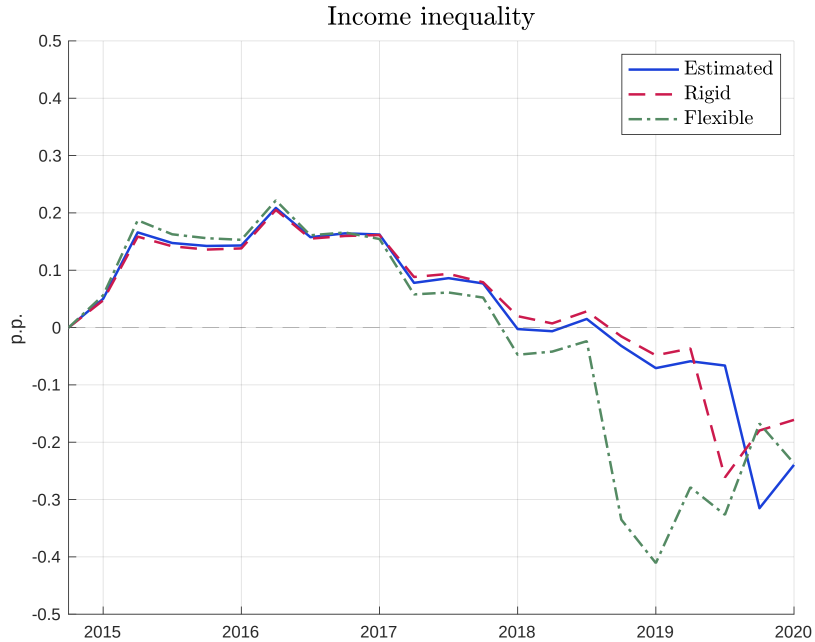
Notes: All variables are in deviation from a counterfactual without QE. The three colored lines depict different levels of wage rigidities. $\phi_w = 0.95$ is the case with more rigid real wages, $\phi_w = 0.75$ is the case with more flexible real wages, and the estimated case is with $\phi_w = 0.87$.

persistent movements of the inflation rate.[16]

Next, Figures 7-8 show the responses of income inequality and its components to the three distinct scenarios. The level of real wage stickiness plays a crucial role in how asset purchases affect income inequality. As shown in Figure 8, assuming that real wages are more flexible enhances the general equilibrium effects of QE. In this case, this starts by affecting financial income, through higher asset prices and a negative effect on dividends and rate of return on assets, which in turn slightly increases income inequality in the first half of the program, as presented in Figure 7. In addition, labor income is also markedly affected. Real wages and overall labor income increase more when real wages are more flexible, which pushes the contribution of the labor market channel at the end of the program. Ultimately, the rise in income inequality is short-lived, while the subsequent decline is more pronounced. Such that, on average income inequality decreases by 0.01 percentage points due to QE.

[16]Blanchard and Galí (2007) and Christoffel and Linzert (2010) provide further details on the role of real wage rigidities for inflation and employment in New-Keynesian models.

Figure 7: Role of real wage rigidities for the effects of QE on income inequality

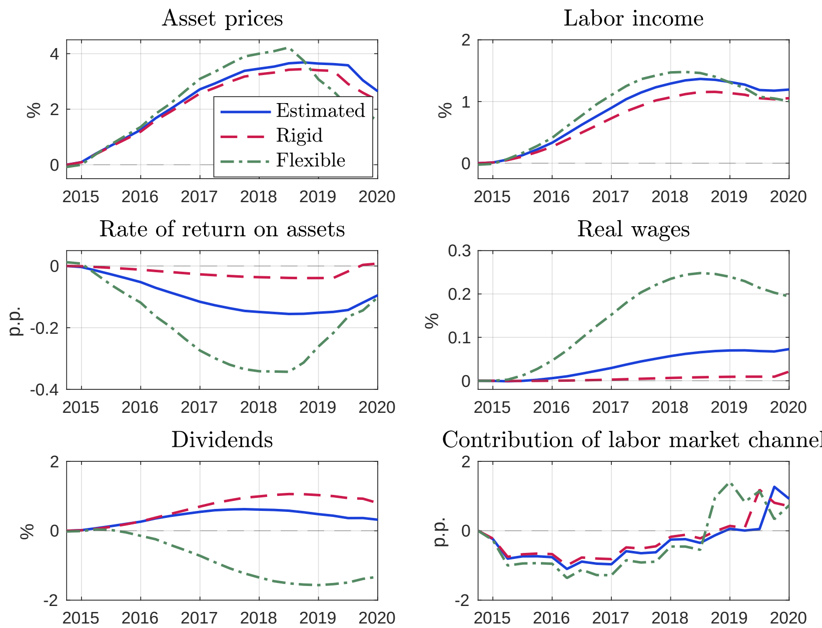


Notes: Income inequality is in percentage points deviation from the counterfactual without QE. The variable is defined as the ratio of income held by financially unconstrained households to the total income of both types of households. The three colored lines depict different levels of real wage rigidities. $\phi_w = 0.95$ is the case with more rigid real wages, $\phi_w = 0.75$ is the case with more flexible real wages, and the estimated case is with $\phi_w = 0.87$.

In contrast, more rigid real wages would cause income inequality to be higher due to the balance sheet expansion during the majority of the sample, such that income inequality would have surged by 0.04 percentage points. This divergence arises because, with insufficient increases in real wages and employment, constrained households benefit less from rising labor earning compared to unconstrained households, which gain from the positive asset price effects and dividends. Therefore, real wage rigidities tend to strengthen the role of the financial market channel in determining the impact of QE policies on income inequality, while a more flexible labor market leads to a greater role for the labor market channel.

In the empirical literature, several studies also highlighted the role of real wage rigidities in shaping the consequences of asset purchases for income inequality. Saiki et al. (2020), which examine the distributional consequences of central bank asset purchases in Japan, state the impacts for Japan may differ from those in the euro area due to differences in labor market structures, especially wage stickiness. Therefore, this paper tends to corroborate the findings of this literature. Different

Figure 8: Role of real wage rigidities for the effects of QE on income components



Notes: All variables are in percentage deviation from a counterfactual without QE. The three colored lines depict different levels of real wage rigidities. $\phi_w = 0.95$ is the case with more rigid real wages, $\phi_w = 0.75$ is the case with more flexible real wages, and the estimated case is with $\phi_w = 0.87$.

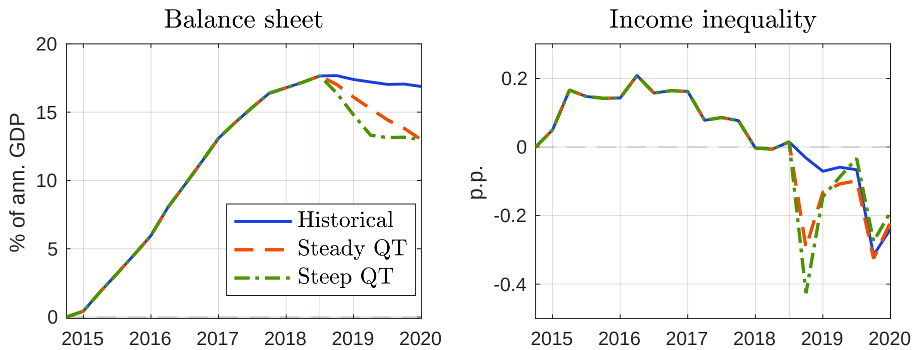
levels of wage rigidity could explain cross-region differences in the consequences of asset purchases for income inequality. Furthermore, this section shows that real wage stickiness role is twofold. It exerts a direct influence by contributing to the rise in wages and the decline in profits, and generates a boost in labor earnings through the general equilibrium effects on aggregate demand.

7 Quantitative Tightening and inequality

The results presented so far study the case of a central bank expanding its balance sheet. This has been predominantly the case over the past decade. However, from 2021 inflation rates in most of the advanced economies have increased. This pushed central banks to normalize their monetary policy. In such a context, several central banks, including the ECB, have decided to reduce the size of their balance sheet. In this section, I study the implications of a reduction in the central bank's bond holding for income inequality.

I conduct the following exercise. Starting from the end of the second quarter of 2018, I

Figure 9: Impact of QT on income inequality



Notes: All variables are in deviation from the counterfactual without QE. The three colored lines depict different path for the ECB's balance sheet. *Historical* is the estimated model with the realized path. *Steady QT* depicts a scenario in which the reduction occurs at a constant rate each quarter. *Steep QT* depicts a scenario in which the case for which most of the decline takes place in the first few quarters. The vertical red line is the start of the counterfactuals.

assume that the ECB reduces its bond holding by a fixed percentage, resulting in a balance sheet approximately 5 percentage points lower than the baseline scenario in terms of annual GDP. Two different scenarios are considered. In the first scenario, the reduction occurs at a constant rate each quarter (*Steady QT*), while in the second scenario, most of the decline takes place in the first few quarters (*Steep QT*). To solely focus on the effects of balance sheet policies, I keep the policy rate unchanged at the ELB, which allows me to study the impact of balance sheet policies independently from interest rate policies. As presented in Section 5, I use de Groot et al. (2021) and assume that the central bank preannounced the path for QT, which agents partly internalized.[17]

Figure 9 presents the effect on income inequality. In both scenarios, income inequality initially decreases more significantly than in the baseline, specifically in the case of a *steep QT*. However, by 2019, the effects on income inequality in both scenarios become similar to the baseline in 2019. Notably, compared to the initial period, the *steep QT* scenario leads to a smaller fall in income inequality. Table 3 confirms that while the *steep QT* scenario is associated with a larger initial decrease in inequality, it also leads to a smaller subsequent increase in income inequality towards the end of the sample. The *steady QT* scenario has the opposite pattern.

Figure 10 provides additional insight into these outcomes. It demonstrates that in the *steep*

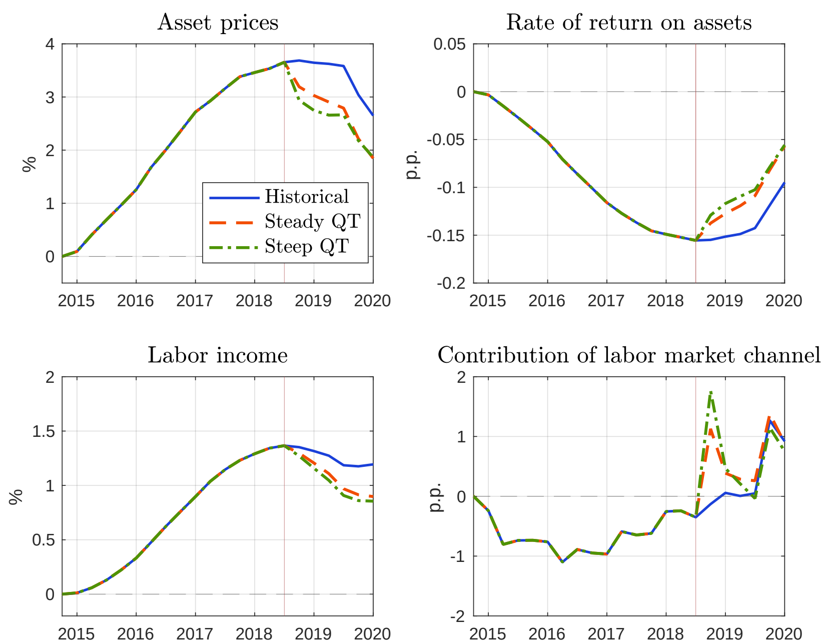
[17] As in de Groot et al. (2021), the exercise is designed such that 70% of agents are attentive to the communication of central bank.

Table 4: Impact on income inequality

	2018 Q2 - 2019 Q1	2019 Q1 - 2019 Q4	2018 Q2 - 2019 Q4
Baseline	-0.05	-0.21	-0.13
Steady QT	-0.18	-0.21	-0.20
Steep QT	-0.22	-0.17	-0.19

Notes: These results are in percentage points deviation from a counterfactual without QE.

Figure 10: Impacts of QT on income components



Notes: All variables are in deviation from the counterfactual without QE. The three colored lines depict different path for the ECB's balance sheet. *Historical* is the estimated model with the realized path. *Steady QT* depicts a scenario in which the reduction occurs at a constant rate each quarter. *Steep QT* depicts a scenario in which the case for which most of the decline takes place in the first few quarters. The vertical red line is the start of the counterfactuals.

QT scenario, both labor income and asset prices undergo more substantial declines in the initial periods compared to the baseline and the *steady QT* scenario, particularly with regard to asset prices. This suggests a more prominent role for the labor market channel in the *steep QT* case. But, as time progresses, while the decline in labor income remains similar in both scenarios, asset prices fall more strongly in the *steady QT* scenario. This accounts for the smaller decrease in income inequality during the later periods in the case of *steep QT*.

Overall, the distributional consequences of a reduction in bond holdings from the central bank are non-trivial. The amount of the reduction can have important consequences for income inequality, but so does the way in which a central bank reduces its bond holdings. Specifically, these implications are tightly linked with developments in financial markets following the implementation of the QT policy, which are likely to be more volatile than other income components, adding to concerns about financial stability. This suggests that central banks need to consider the potential distributional consequences of their balance sheet policies, including the trajectory of their reduction in asset holdings, if they aim to avoid any adverse effects on income inequality.

8 Conclusion

In this paper, I develop a New-Keynesian model with limited asset market participation, wage and price rigidities, and an occasionally binding constraint on the nominal interest rate to study the effects of asset purchase policies in the euro area on inflation and economic activity, as well as on income inequality. I conduct an estimation of the model using Bayesian methods, taking into account the occasionally binding zero lower bound for the euro area.

The counterfactual analyses underline that the PSPP implemented by the ECB effectively affected inflation and economic activity, while leading to modest effects on income inequality on average. Nonetheless, the distributional impacts varied over time. The income gap widened in the early phases of the program due to strong asset price effects, before narrowing later as labor income rose. This highlights the importance of examining the relative strength of the labor market and financial market channels and how they interact over time.

The results of this paper also suggest that labor market structures and real wage rigidities are key to the distributional consequences of QE policies. More flexible real wages lead to a larger role for the labor market channel, while more rigid wages limit the extent to which labor earnings compensate for the surge in financial returns thus increasing income inequality. These findings may have important implications for understanding the varying effects of QE policies across different regions and economies.

While much attention has been paid to the effects of central bank bond purchases on income inequality, less is known about the consequences of balance sheet reductions. This paper assesses the effects of a reduction in the central bank's bond holdings. I find that the shape of the path of Quantitative Tightening is crucial for the effects on income inequality.

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A Details on the estimation

In this appendix, I provide supplementary details on the estimation data in Appendix A.1, and method in Appendix A.2. Appendix A.3 presents estimated IRFs to policy shocks, in the benchmark TANK model, and compares them to the RANK counterpart.

A.1 Observables and related model equations

For the estimation I use data collected from the OECD, the ECB and Refinitiv. I consider the period from 1999Q1 to 2019Q4. Data and observation equations are as follows:

i) Consumption:

- Model: $\tilde{C}_t^{obs} = \Delta \log C_t$
- Observable: Nominal private consumption expenditures (OECD) divided by GDP deflator (OECD) and active population (OECD), which is log-transformed, first differenced and demeaned.

ii) Real wage:

- Model: $\tilde{w}_t^{obs} = \Delta \log w_t$
- Observable: Compensation of employees (OECD) divided by employment (OECD) and GDP deflator (OECD), which is log-transformed, first differenced and demeaned.

iii) Inflation:

- Model: $\tilde{\Pi}_t^{obs} = \log \Pi_t$
- Observable: GDP deflator (OECD) log-transformed, first-differenced and demeaned.

iv) Interest rate:

- Model: $\tilde{R}_t^{obs} = \log R_t$
- Observable: Euribor 3-month (ECB), divided by 400 and added to 1 in order to make it quarterly gross rate, log-transformed.

v) Purchase government bonds under the PSPP as a ratio of annualized GDP:

- Model: $\tilde{re}_t^{obs} = \frac{re_t}{4 * y_t}$

- Observable: ECB holding of government bonds, divided by GDP deflator. Divided by annualized real GDP.

vi) Spread between long-term and short-term bonds:

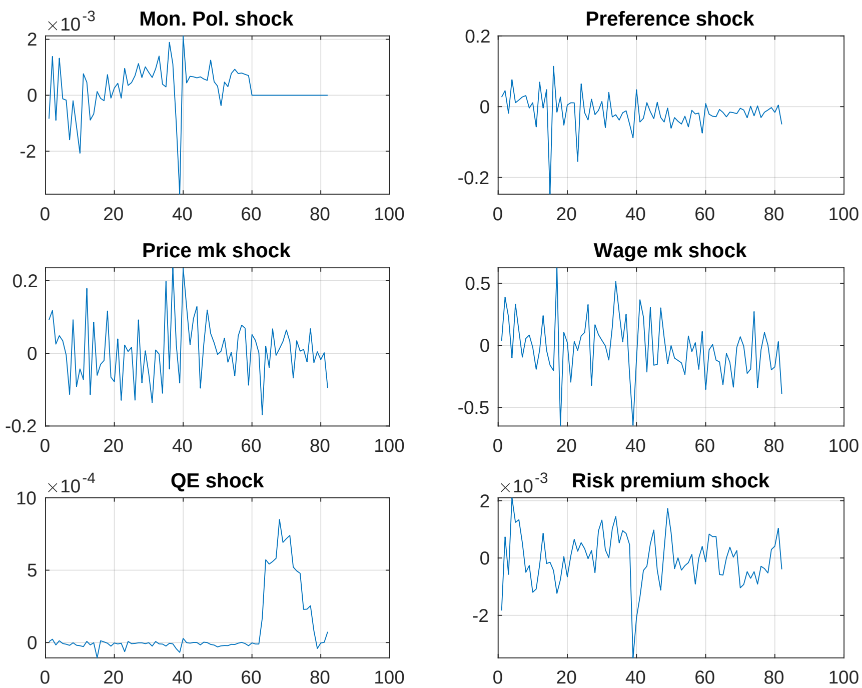
- Model: $\tilde{Spr}_t^{obs} = \log\left(R_{t+1}^{LT} - R_t\right)$

- Observable: Difference between the OIS 10-year (Refinitiv) and the Euribor 3-month (ECB).[18] The spread is then divided by 400 and added to 1 in order to make them quarterly gross rates, and log-transformed.

The inversion filter requires the use of the same number of shocks as observables in the estimation. The following 6 shock processes are thus employed: price mark-up $\eta_{p,t}$, wage mark-up $\eta_{w,t}$, preference η_t, risk-premium $\eta_{b,t}$, conventional monetary policy $\eta_{r,t}$ and QE $\eta_{cb,t}$. Filtered shock processes are shown in Figure 11.

[18] Due to missing values at the start of the serie for the OIS 10-year, I backcast it using the growth rate of the 10-year German Bund over the period in which the data are missing.

Figure 11: Filtered shocks



Notes: Time series of filtered shocks over the the sample period.

A.2 Inversion filter

Following Guerrieri and Iacoviello (2017), Cuba-Borda et al. (2019), and Jondeau et al. (2022) this paper applies the inversion filter to back out time series of structural shocks. In this subsection, we provide further details on this method.

The model solution is:

$$Z_t = \mathbf{P}\left(Z_{t-1}, \varepsilon_t\right) Z_{t-1} + \mathbf{D}\left(Z_{t-1}, \varepsilon_t\right) + \mathbf{Q}\left(Z_{t-1}, \varepsilon_t\right) \varepsilon_t \tag{33}$$

with $\mathbf{P}$, $\mathbf{D}$ and $\mathbf{Q}$ the structural matrices of coefficients the model. Z_t is the vector of endogenous variables and ε_t the vector of shocks. The vector of observables Y_t is a subset of the model endogenous variables such that $Y_t = HZ_t$. This can therefore be expressed as:

$$Y_t = \mathbf{HP}\left(Z_{t-1}, \varepsilon_t\right) Z_{t-1} + \mathbf{HD}\left(Z_{t-1}, \varepsilon_t\right) + \mathbf{HQ}\left(Z_{t-1}, \varepsilon_t\right) \varepsilon_t \tag{34}$$

Conditional on the matrix $\mathbf{HQ}$ being convertible, we can retrieve the shocks as:

$$\varepsilon_t = [\mathbf{HQ}(Z_{t-1}, \varepsilon_t)]^{-1} [Y_t - \mathbf{HD}(Z_{t-1}, \varepsilon_t) - \mathbf{HP}(Z_{t-1}, \varepsilon_t) Z_{t-1}] \tag{35}$$

This filter therefore requires the same number of observed variables in the vector Y_t as shocks used for the estimation ε_t.

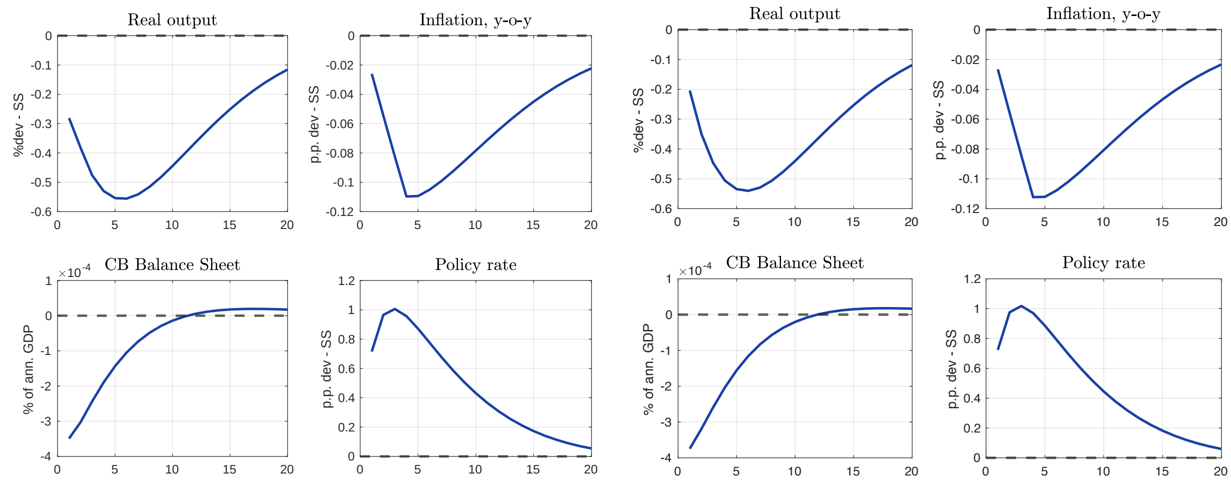
A.3 Estimated IRFs

This appendix shows Impulse Response Functions (IRFs) to a monetary policy and a QE shock in the benchmark estimated model, setting parameters to their posterior mean, and compares them to the representative agent (RANK) version of the estimated model.

IRFs to a policy rate shock are shown in Figure 12. The shock has less effect on real output in the RANK framework. This is because there are no indirect effects of monetary policy shocks from the impact on consumption of ROT households. The implications on inflation are broadly similar between the two frameworks.

Figure 13 presents the impact of a positive QE shock in the TANK and RANK models, at and away from the ELB. I emphasize the following two points. First, the presence of ROT households push the initial impact of the shock on real output, while having very small effects on the response of inflation. Secondly, the binding nature of the ELB amplifies the impact of the shock on inflation and, to an even bigger extent, on real output.

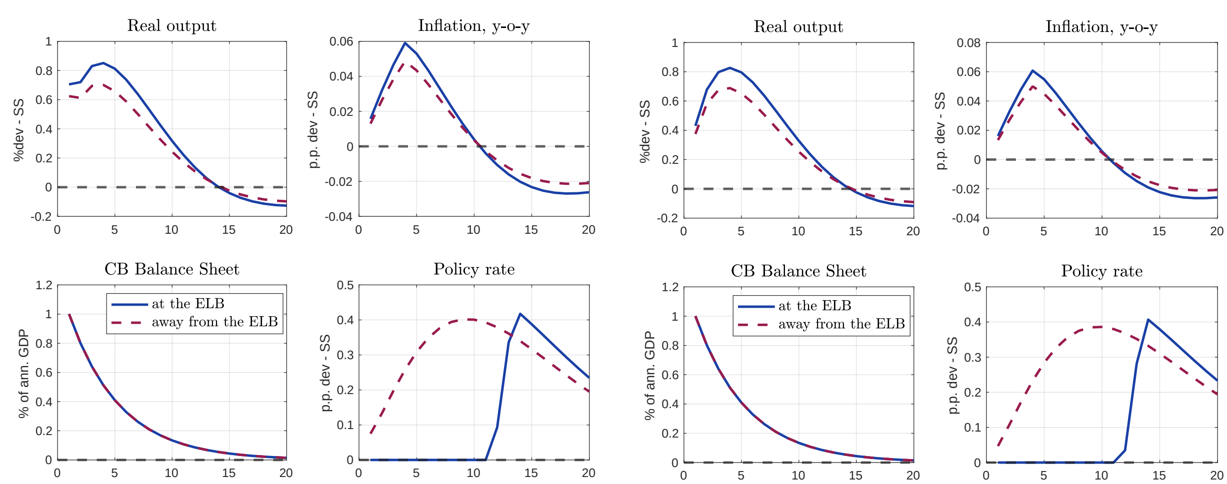
Figure 12: IRF to a Monetary Policy shock in the TANK and RANK models



(a) Estimated TANK model (b) RANK model

Notes: Impulse responses to a standard short-term interest rate shock (normalized to 100 basis points). Parameters are fixed at their posterior mean. The left panel is the benchmark model, the right panel uses the same filtered shocks and parameters, but in a representative agent setting.

Figure 13: IRF to a QE shock in the TANK and RANK models



(a) Estimated TANK model (b) RANK model

Notes: Impulse responses to a standard QE shock (normalized to an increase of 1% in the ratio of real reserves to ann. GDP). Parameters are fixed at their posterior mean, with the exception of the persistence of the QE shock, set to $\rho_{cb} = 0.8$. The left panel is the benchmark model, the right panel uses the same filtered shocks and parameters, but in a representative agent setting.

B Wage and price settings

This section describes the derivations of real wages and prices in this economy. It follows closely Carlstrom et al. (2017).

B.1 Wages

In this model, by construction, labor earnings are the same for the two types of agents, such that real wages are equal to $w_t^r = w_t^k = w_t$ and labor is such that $N_t^r = N_t^k = N_t$.

The labor packer:

Total labor input is given by equation (8):

$$N_t = \left(\int_0^1 N_t(l)^{\frac{\varepsilon_t^w - 1}{\varepsilon_t^w}} dl \right)^{\frac{\varepsilon_t^w}{\varepsilon_t^w - 1}}$$

The real profit maximization problem of the competitive labor packer is then:

$$\max_{N_t(l)} w_t \left(\int_0^1 N_t(l)^{\frac{\varepsilon_t^w - 1}{\varepsilon_t^w}} dl \right)^{\frac{\varepsilon_t^w}{\varepsilon_t^w - 1}} - \int_0^1 w_t(l) N_t(l) dl$$

The first order condition is:

$$w_t \frac{\varepsilon_t^w}{\varepsilon_t^w - 1} \left(\int_0^1 N_t(l)^{\frac{\varepsilon_t^w - 1}{\varepsilon_t^w}} dl \right)^{\frac{\varepsilon_t^w}{\varepsilon_t^w - 1} - 1} \frac{\varepsilon_t^w - 1}{\varepsilon_t^w} N_t(l)^{\frac{\varepsilon_t^w - 1}{\varepsilon_t^w} - 1} = w_t(l)$$

which can be simplified into equation (10):

$$N_t(l) = \left(\frac{w_t(l)}{w_t} \right)^{-\varepsilon_t^w} N_t$$

To derive the aggregate real wage index, we can define:

$$w_t N_t = \int_0^1 w_t(l) N_t(l) dl = \int_0^1 w_t(l)^{1-\varepsilon_t^w} w_t^{\varepsilon_t^w} N_t dl$$

which can be simplified into (11):

$$w_t = \left(\int_0^1 w_t(l)^{1-\varepsilon_t^w} dl \right)^{\frac{1}{1-\varepsilon_t^w}}$$

Households:

Wages are fully determined by financially unconstrained households. Constrained households take the wage as given. The households problem is given by equation (1) subject to the budget constraint of unconstrained households (2) and labor demand (10). Using (10) to replace $N_t(l)$ in the Lagrangian, such that the problem is now to choose $w_t(l)$. As with prices, households are assumed to be unable to choose their wages every period. With probability $1 - \phi_w$ they can adjust, and with probability ϕ_w they keep the same wage. Non-updated wages may be indexed to lagged inflation with a degree of indexation of $\zeta_w \in [0, 1]$. Ultimately, wages for both types of households changes with the same probabilities.

We can define the Lagrangian in real terms (related to the choice of labor only for simplicity) as:

$$\begin{aligned}
&\widetilde{\mathcal{L}} = E_t \sum_{s=0}^{\infty} (\beta\phi_w)^s \\
&\left(-\chi\mu_{t+s} \frac{\left(\frac{w_t(l)\Pi_{t,t+s}^{-1}}{w_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\zeta_w} \right)^{-\varepsilon_t^w(1+\sigma_N)} N_{t+s}^{1+\sigma_N}}{1+\sigma_N} \right. \\
&\left. + \lambda_{t+s} P_{t+s} \left(w_t(l) \Pi_{t,t+s}^{-1} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\zeta_w} \left(\frac{w_t(l)\Pi_{t,t+s}^{-1}}{w_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\zeta_w} \right)^{-\varepsilon_t^w} N_{t+s} \right) \right.
\end{aligned}$$

The first-order condition for $w_t(l)$ yields:

$$\begin{aligned}
\frac{\partial \widetilde{\mathcal{L}}}{\partial w_t(l)} =& \varepsilon_t^w w_t(l)^{-\varepsilon_t^w(1+\sigma_N)-1} E_t \sum_{s=0}^{\infty} (\beta\phi_w)^s \chi\mu_{t+s} w_{t+s}^{\varepsilon_t^w(1+\sigma_N)} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{-\zeta_w \varepsilon_t^w (1+\sigma_N)} \Pi_{t,t+s}^{\varepsilon_t^w(1+\sigma_N)} N_{t+s}^{1+\sigma_N} \\
&+ (1-\varepsilon_t^w) w_t(l)^{-\varepsilon_t^w} E_t \sum_{s=0}^{\infty} (\beta\phi_w)^s \lambda_{t+s} P_{t+s} w_{t+s}^{\varepsilon_t^w} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\zeta_w(1-\varepsilon_t^w)} \Pi_{t,t+s}^{\varepsilon_t^w - 1} N_{t+s} = 0
\end{aligned}$$

Simplifying we can get to the reset wage:

$$w_t^{\#,1+\varepsilon_t^w\sigma_N} = \frac{\varepsilon_t^w}{\varepsilon_t^w - 1}\frac{E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s\chi\mu_{t+s}w_{t+s}^{\varepsilon_t^w(1+\sigma_N)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\zeta_w\varepsilon_t^w(1+\sigma_N)}\Pi_{t,t+s}^{\varepsilon_t^w(1+\sigma_N)}N_{t+s}^{1+\sigma_N}}{E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s\lambda_{t+s}P_{t+s}w_{t+s}^{\varepsilon_t^w}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w(1-\varepsilon_t^w)}\Pi_{t,t+s}^{\varepsilon_t^w-1}N_{t+s}}$$

which can be rewritten as:

$$w_t^{\#,1+\varepsilon_t^w\sigma_N} = \frac{\varepsilon_t^w}{\varepsilon_t^w - 1}\frac{H_{1,t}}{H_{2,t}} \tag{36}$$

with:

$$H_{1,t} = \chi\mu_t w_t^{\varepsilon_t^w(1+\sigma_N)}N_t^{1+\sigma_N} + \beta\phi_w E_t\Pi_t^{-\zeta_w\varepsilon_t^w(1+\sigma_N)}\Pi_{t+1}^{\varepsilon_t^w(1+\sigma_N)}H_{1,t+1} \tag{37}$$

$$H_{2,t} = C_t^{-\gamma}\mu_t w_t^{\varepsilon_t^w}N_t + \beta\phi_w E_t\Pi_{t+1}^{\varepsilon_t^w-1}\Pi_t^{\zeta_w(1-\varepsilon_t^w)}H_{2,t+1} \tag{38}$$

Rewriting the aggregate real wage index:

The real wage index is given by (11):

$$w_t = \left(\int_0^1 w_t(l)^{1-\varepsilon_t^w}dl\right)^{\frac{1}{1-\varepsilon_t^w}}$$

Using the fact that $1-\phi_w$ reset their wages, while ϕ_w do not, and plugging in indexation we have:

$$w_t^{1-\varepsilon_t^w} = (1-\phi_w)\,w_t^{\#,1-\varepsilon_t^w} + (\Pi_t)^{\varepsilon_t^w-1}\int_{1-\phi_w}^1\left((\Pi_{t-1})^{\zeta_w}w_{t-1}(l)\right)^{1-\varepsilon_t^w}dl$$

Using the Calvo assumption, we can integrate the heterogeneity, such that we arrive to:

$$w_t^{1-\varepsilon_t^w} = (1-\phi_w)\,w_t^{\#,1-\varepsilon_t^w} + \phi_w\,(\Pi_{t-1})^{\zeta_w(1-\varepsilon_t^w)}\,(\Pi_t)^{\varepsilon_t^w-1}\,w_{t-1}^{1-\varepsilon_t^w} \tag{39}$$

Such that the aggregate is the sum of the reset wage and the wage in the previous period adjusted by the inflation rate, weighted by the Calvo probabilities.

B.2 Prices

In this appendix, I focus on intermediate producers. The final good producers problem, that leads to equations (15) and (16) is standard.

Intermediate firms produce output according using only labor, and face a common productivity shock:

$$Y_t(j) = A_t N_t(j)$$

Their cost minimization problem is:

$$\min_{N_t(j)} W_t N_t(j)$$

s.t.

$$A_t N_t(j) \geq \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_t^p} Y_t$$

The Lagrangian is then:

$$\mathcal{L} = -W_t N_t(j) + \varphi_t(j)\left(A_t N_t(j) - \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_t^p} Y_t\right)$$

this gives nominal marginal cost:

$$\varphi_t = \frac{W_t}{A_t}$$

Real profit for the each intermediate firm is:

$$Div_{int,t}(j) = \frac{P_t(j)}{P_t} Y_t(j) - mc_t Y_t(j)$$

where mc_t is the real marginal cost. The intermediate firms' dynamic problem consists in picking the optimal price so as to optimize profits, subject to the constraint that only a fraction $1-\phi_p$ of firm can reset the price. There is indexation to lagged inflation, with a price indexation parameter

ζ_p. The price setting problem is therefore:

$$\max_{P_t(j)} E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s$$
$$\frac{\lambda_{t+s}}{\lambda_t}\left(\frac{P_t(j)}{P_{t+s}}\left(\frac{P_t(j)}{P_{t+s}}\right)^{-\varepsilon_t^p}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\left(1-\varepsilon_t^p\right)\zeta_p} Y_{t+s} - mc_{t+s}\left(\frac{P_t(j)}{P_{t+s}}\right)^{-\varepsilon_t^p}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\varepsilon_t^p\zeta_p} Y_{t+s}\right)$$

where $\beta\frac{\lambda_{t+s}}{\lambda_t}$ is the stochastic discount factor of the ricardian household. As we did for the reset wage, we can write the first-order condition as a ratio:

$$P_t(j) = \frac{\varepsilon_t^p}{\varepsilon_t^p - 1}\frac{E_t\sum_{s=0}^{\infty}(\beta\phi_p)^s \lambda_{t+s} mc_{t+s}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\varepsilon_t^p\zeta_p} P_{t+s}^{\varepsilon_t^p} Y_{t+s}}{E_t\sum_{s=0}^{\infty}(\beta\phi_p)^s \lambda_{t+s}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\left(1-\varepsilon_t^p\right)\zeta_p} P_{t+s}^{\varepsilon_t^p-1} Y_{t+s}}$$

This yields the following reset price, which is the same for each firm:

$$P_t^{\#} = \frac{\varepsilon_t^p}{\varepsilon_t^p - 1}\frac{X_{1,t}}{X_{2,t}} \tag{40}$$

where:

$$X_{1,t} = \lambda_t mc_t P_t^{\varepsilon_t^p} Y_t + \phi_p \beta E_t \Pi_t^{-\varepsilon_t^p\zeta_p} X_{1,t+1} \tag{41}$$

$$X_{2,t} = \lambda_t P_t^{\varepsilon_t^p - 1} Y_t + \phi_p \beta E_t \Pi_t^{\left(1-\varepsilon_t^p\right)\zeta_p} X_{2,t+1} \tag{42}$$

where mc_t is real marginal cost.

Aggregation:

Production of each intermediate firm is:

$$Y_t(j) = A_t N_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_t^p} Y_t$$

We define v_p as the measure of price dispersion:

$$v_t^p = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_t^p} dj$$

This can be derived using Calvo properties and indexation to lagged inflation:

$$v_t^p = \int_0^{1-\phi_p} \left(\frac{P_t^\#}{P_t}\right)^{-\varepsilon_t^p} dj + \int_{1-\phi_p}^1 \left(\frac{\Pi_{t-1}^{\zeta_p} P_{t-1}(j)}{P_t}\right)^{-\varepsilon_t^p} dj \tag{43}$$

With $\Pi_t^\# = \frac{P_t^\#}{P_t}$, this can be simplify into:

$$v_t^p = \int_0^{1-\phi_p} \left(\frac{P_t^\#}{P_t}\right)^{-\varepsilon_t^p} dj + \Pi_{t-1}^{-\gamma_p \varepsilon_t^p} P_t^{\varepsilon_t^p} P_{t-1}^{-\varepsilon_t^p} \int_{1-\phi_p}^1 \left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\varepsilon_t^p} dj \tag{44}$$

Such that we have:

$$v_t^p = (1-\phi_p)\, \Pi_t^{\#-\varepsilon_t^p} + \phi_p\, \Pi_t^{\varepsilon_t^p} \Pi_{t-1}^{-\varepsilon_t^p \zeta_p} v_{t-1}^p \tag{45}$$

Using v_t^p, we can write the production function as:

$$Y_t = \frac{A_t N_t}{v_t^p} \tag{46}$$

Now, let's rewrite the aggregate price index in a way close to what we did for the aggregate wage level. Aggregate price index is defined in (16):

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon_t^p} dj\right)^{\frac{1}{1-\varepsilon_t^p}}$$

We assumed that a fraction $1-\phi_p$ of intermediate firms can reset their prices and the remaining fraction ϕ_p cannot. We can then write the integral as:

$$P_t^{1-\varepsilon_t^p} = \int_0^{1-\phi_p} P_t^{\#,1-\varepsilon_t^p} dj + \int_{1-\phi_p}^1 \Pi_{t-1}^{\left(1-\varepsilon_t^p\right)\zeta_p} P_{t-1}(j)^{1-\varepsilon_t^p} dj$$

Now using the Calvo assumption and price indexation, this can be simplified into:

$$P_t^{1-\varepsilon_t^p} = (1-\phi_p)\, P_t^{\#,1-\varepsilon_t^p} + \phi_p \Pi_{t-1}^{\left(1-\varepsilon_t^p\right)\zeta_p} P_{t-1}^{1-\varepsilon_t^p}$$

Dividing by $P_t^{1-\varepsilon_t^p}$ we arrive to the equation for price evolution:

$$1 = (1 - \phi_p) \left(\Pi_t^{\#} \right)^{1-\varepsilon_t^p} + \phi_p \Pi_{t-1}^{\left(1-\varepsilon_t^p\right)\zeta_p} \Pi_t^{\varepsilon_t^p - 1} \tag{47}$$

where $\Pi_t^{\#} = \frac{P_t^{\#}}{P_t}$.

Finally, we define $x_{1,t} = X_{1,t}/P_t^{\varepsilon_t^p}$ and $x_{2,t} = X_{2,t}/P_t^{\varepsilon_t^p - 1}$, in order to rewrite the reset price as:

$$\Pi_t^{\#} = \frac{\varepsilon_t^p}{\varepsilon_t^p - 1} \frac{X_{1,t}}{X_{2,t}} \tag{48}$$

where:

$$x_{1,t} = \lambda_t mc_t Y_t + \phi_p \beta E_t \Pi_{t+1}^{\varepsilon_t^p} \Pi_t^{-\varepsilon_t^p \zeta_p} x_{1,t+1} \tag{49}$$

$$x_{2,t} = \lambda_t Y_t + \phi_p \beta E_t \Pi_{t+1}^{\varepsilon_t^p - 1} \Pi_t^{\left(1-\varepsilon_t^p\right)\zeta_p} x_{2,t+1} \tag{50}$$

C Full Set of Equilibrium Conditions

This appendix summarizes the full set of equations of the non-linear model.

Households

(Marginal utility of Ricardian):

$$\lambda_t = \mu_t \left(C_t^r - hC_{t-1}^r \right)^{-\gamma} - h\beta\, E_t \mu_{t+1} \left(C_{t+1}^r - hC_t^r \right)^{-\gamma} \tag{51}$$

(Consumption of ROT):

$$C_t^k = w_t N_t - T_t \tag{52}$$

Asset pricing

(*Short-term bonds*):

$$\lambda_t + \lambda_t \frac{\nu\delta}{b_t^{LT,h}} \left[\delta \frac{b_t^{ST}}{b_t^{LT,h}} - 1 \right] = \beta \mathbb{E}_t \lambda_{t+1} R_t \Pi_{t+1}^{-1} \tag{53}$$

(*Long-term bonds*):

$$\lambda_t \frac{q_t^B}{\varepsilon_b^t} - \lambda_t \frac{\nu\delta b_t^{ST}}{\left(b_t^{LT,h}\right)^2}\left[\delta \frac{b_t^{ST}}{b_t^{LT,h}} - 1\right] = \beta \mathbb{E}_t \lambda_{t+1}\left(1+\kappa_B q_t^B\right)\Pi_{t+1}^{-1} \tag{54}$$

(*Stocks*):

$$\lambda_t q_t^S = \beta\lambda_{t+1}\left(q_{t+1}^S + div_{t+1}\right) \tag{55}$$

Wages

(Reset wage):

$$w_t^{\#,1+\varepsilon_t^w \sigma_N} = \frac{\varepsilon_t^w}{\varepsilon_t^w - 1}\frac{H_{1,t}}{H_{2,t}} \tag{56}$$

$$H_{1,t} = \chi\mu_t w_t^{\varepsilon_t^w(1+\sigma_N)} N_t^{1+\sigma_N} + \beta\phi_w E_t \Pi_t^{-\zeta_w \varepsilon_t^w(1+\sigma_N)} \Pi_{t+1}^{\varepsilon_{t+1}^w(1+\sigma_N)} H_{1,t+1} \tag{57}$$

$$H_{2,t} = \lambda_t \mu_t w_t^{\varepsilon_t^w} N_t + \beta\phi_w E_t \Pi_{t+1}^{\varepsilon_{t+1}^w - 1}\Pi_t^{\zeta_w(1-\varepsilon_t^w)} H_{2,t+1} \tag{58}$$

Production

(Marginal cost):

$$mc_t = \frac{w_t}{A_t} \tag{59}$$

(*Inflation evolution*):

$$\Pi_t^{\#} = \frac{\varepsilon_t^p}{\varepsilon_{t,p} - 1}\frac{x_{1,t}}{x_{2,t}} \tag{60}$$

$$x_{1,t} = \lambda_t mc_t Y_t + \phi_p \beta E_t \Pi_{t+1}^{\varepsilon_t^p} \Pi_t^{-\varepsilon_t^p \zeta_p} x_{1,t+1} \tag{61}$$

$$x_{2,t} = \lambda_t Y_t + \phi_p \beta E_t \Pi_{t+1}^{\varepsilon_t^p - 1} \Pi_t^{\left(1-\varepsilon_t^p\right)\zeta_p} x_{2,t+1} \tag{62}$$

Treasury and Monetary Authority

(Treasury):

$$T_t + T_t^{cb} + q_t^B b_t^{LT} + b_t^{ST} = R_{t-1} b_{t-1}^{ST}\Pi_t^{-1} + \left(1+\kappa_b q_t^B\right) b_{t-1}^{LT}\Pi_t^{-1} \tag{63}$$

(Conventional monetary Policy):

$$\ln R_t^{tr} = (1-\rho_{mp}) \ln R^{tr} + \rho_{mp} \ln R_{t-1}^{tr} \\ + (1-\rho_{mp}) \left[\theta_\pi \left(\ln \Pi_t - \ln \Pi\right) + \theta_y \left(\ln Y_t - \ln Y\right) + \theta_{dy} \left(\ln Y_t - \ln Y_{t-1}\right)\right] + s_r \varepsilon_{r,t} \tag{64}$$

$$R_t = \max\left\{1, R_t^{tr}\right\} \tag{65}$$

(CB's reserve):

$$q_t^B b_t^{CB} = re_t \tag{66}$$

(Quantitative Easing):

$$b_t^{CB} = (1-\rho_{cb})\, b^{CB} + \rho_{cb} b_{t-1}^{CB} + s_{cb} \eta_{cb,t} \tag{67}$$

(CB's profit):

$$T_{cb,t} = \left(R_t^{LT} - R_{t-1}\right) \Pi_t^{-1} q_{t-1}^B B_{t-1}^{CB} \tag{68}$$

Aggregate conditions

(*Aggregate resource constraint*):

$$Y_t = C_t + \frac{\nu}{2}\left[\delta \frac{b_t^{ST}}{b_t^{LT,h}} - 1\right]^2 \tag{69}$$

(*Aggregate consumption*):

$$C_t = \omega C_t^r + (1-\omega) C_t^k \tag{70}$$

(*Full stock of long-term bond*):

$$b_t^{LT,h} + b_t^{CB} = b^{LT} \tag{71}$$

(*Full stock of Government debt*):

$$b_t^{ST} + b_t^{LT,h} + b_t^{CB} = \bar{b} \tag{72}$$

(*Aggregate production function*):

$$Y_t = \frac{A_t N_t}{v_t^p} \tag{73}$$

(*Aggregate dividends*):

$$Div_t = Y_t - w_t N_t \tag{74}$$

(*Price dispersion*):

$$v_t^p = (1-\phi_p)\,\Pi_t^{\#-\varepsilon_t^p} + \phi_p\,\Pi_t^{\varepsilon_t^p}\Pi_{t-1}^{-\varepsilon_t^p\zeta_p}v_{t-1}^p \tag{75}$$

(*Wage evolution*):

$$w_t^{1-\varepsilon_t^w} = (1-\phi_w)\,w_t^{\#,1-\varepsilon_t^w} + \phi_w\,(\Pi_{t-1})^{\zeta_w(1-\varepsilon_t^w)}\,(\Pi_t)^{\varepsilon_t^w-1}\,w_{t-1}^{1-\varepsilon_t^w} \tag{76}$$

(*Price evolution*):

$$1 = (1-\phi_p)\left(\Pi_t^{\#}\right)^{1-\varepsilon_t^p} + \phi_p\Pi_{t-1}^{\left(1-\varepsilon_t^p\right)\zeta_p}\Pi_t^{\varepsilon_t^p-1} \tag{77}$$

Exogenous processes:

$$\log A_t = \rho_a \ln A_{t-1} + s_a\eta_{a,t} \tag{78}$$

$$\log\varepsilon_t^p = (1-\rho_p)\log\varepsilon^p + \rho_p\log\varepsilon_{t-1}^p + \eta_{p,t} - \iota_p\eta_{p,t-1} \tag{79}$$

$$\log\varepsilon_t^w = (1-\rho_w)\log\varepsilon^w + \rho_w\log\varepsilon_{t-1}^w + \eta_{w,t} - \iota_w\eta_{w,t-1} \tag{80}$$

$$\log\mu_t = \rho_\mu \ln\mu_{t-1} + s_\mu\eta_{\mu,t} \tag{81}$$

$$\log\varepsilon_{r,t} = \rho_r \ln\varepsilon_{r,t-1} + s_r\eta_{r,t} \tag{82}$$

$$\log\varepsilon_{b,t} = \rho_b \ln\varepsilon_{b,t-1} + s_b\eta_{b,t} \tag{83}$$

D Solving for counterfactuals

As described in Section 4 and Section 5, assessing the impact of policy requires the construction of a counterfactual, in which the specific policy had either never being implement, or would have followed an alternative trajectory.

Similar to Section 4, a standard approach is to find the shocks, in each period t, that implement the desired counterfactual path for the policy instrument.[19] In this model, QE is modeled as a

[19]As such, in the case of a specific policy rule, it possible to write the rule as a function of structural shocks

purely exogenous policy. Therefore, constructing a counterfactual of this form requires setting QE shocks to 0 from the start of the program. Nonetheless, encoded in this method is that idea that agents solve a static problem, with no role for the future path of policy through their expectations.

As discussed in McKay and Wolf (2023), it is possible to study the effect of a systematic change in policy, using the implications of multiple distinct policy shocks on aggregate variables of the economy. Using the Structural Vector Moving Average (SVMA) representation of the model, one can construct policy counterfactuals in which private agents are affected by a change in policy (in this case, the absence of the PSPP) not only as a surprise each period, but in expectations.

Using the approach outlined in de Groot et al. (2021) and Hebden and Winkler (2021), I recast the model as a linear combination of impulse responses to contemporaneous and anticipated (i.e. news) QE shocks.[20] Any variable Z can be written as in sequence space as:

$$Z^c = \{Z_t^c\}_{t=0}^{H} = Z^b + \mathcal{A}^{Z,\varepsilon_{qe}}\tilde{\varepsilon}_{qe} = \mathcal{A}^{Z,\varepsilon_s}\tilde{\varepsilon}_s + \mathcal{A}^{Z,\varepsilon_{qe}}\tilde{\varepsilon}_{qe}$$

with $\mathcal{A}^{Z,\varepsilon_s}$ the impulse responses of a variable Z in periods $t = 0$ to H, to a set structural shocks stacked in $\tilde{\varepsilon}_s$. Similarly, $\mathcal{A}^{Z,\varepsilon_{qe}}$ is the matrix of impulse responses of a variable Z to a vector of current and QE contemporaneous and news shocks $\tilde{\varepsilon}_{qe}$. In this representation, news shocks have a direct impact on agents' expectations as soon as the program is implemented, through the Impulse Response of each variable to these shocks, throughout the entire projection from period $t = 0$ to H. I use this representation of the model to solve for the vector of QE shocks that imposes the balance sheet is fixed at 0, instead of expanding following the onset of the program.[21]

affecting the policy instrument via the systematic component of the rule or via deviations of this rule, or policy shocks.

[20] This is done under the assumption of perfect foresight, and that the model is linearized except for occasionally binding constraints with respect to the policy instruments

[21] I also use interest rate shock to ensures that the policy rate stays at the ELB.